# Safety of Automatic Emergency Braking in Platooning 

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#### Abstract

A platoon comprises a string of consecutive highly automated vehicles traveling together. Platooning allows for increased road utilization and reduced fuel consumption due to short intervehicular distances. Safety in terms of guaranteeing no rear-end collisions is of utmost importance for platooning systems to be deployed in practice. We compare how safely emergency braking can be handled by emerging vehicle-to-vehicle (V2V) communications on the one hand and by radar-based measurements of existing automatic emergency braking systems (AEBS) on the other. We show that even under conservative assumptions on the V2V communications, such an approach significantly outperforms AEBS with an ideal radar sensor in terms of allowed inter-vehicle distances and response times. Furthermore, we design two emergency braking strategies for platooning based on V2V communications. The first braking strategy assumes centralized coordination by the leading vehicle and exploits necessary optimal conditions of a constrained optimization problem, whereas the second - the more conservative solution - assumes only local information and is distributed in nature. Both strategies are also compared with the AEBS.


Index Terms-Platooning, safety, vehicle-to-vehicle communication (V2V), radar, automatic emergency braking systems.

## I. Introduction

APLATOON comprises a number of vehicles traveling autonomously together with short inter-vehicle distances [2] and facilitated by both traditional radar sensors and emergent $V 2 V$ radio communication. Radar sensors are used to obtain the information about the preceding platoon member, while wireless sensors allow for the increased awareness horizon beyond line-of-sight with information about all platoon members exchanged almost instantaneously. In addition to improved safety

[^0]with respect to manual driving with no automation in place, platooning technology furthermore also contributes to better fuel economy [3], [4] due to the slipstream effect, thus, also reducing the impact on the climate. [4] reports figures from $2 \%$ to $21 \%$ for fuel savings, which means that the vehicle can save up to $21 \%$ of the fuel if it follows another one in a platooning formation.

Currently, automotive radars are deployed in many vehicles for safety enhancements via features such as adaptive cruise control (ACC) [5] and AEBS [6]. With the latter, a vehicle can sharply decelerate without driver involvement to avoid a potential collision or to mitigate the consequences thereof. Usual AEBS has one or several "warning modes" and an "emergency braking phase". In the warning phase/mode, the AEBS provides the driver with an audio or video warning signal. A collision warning braking, i.e., short harsh braking, can also be applied to attract the driver's attention to the critical situation. In the emergency braking phase, automatic braking is applied to avoid the upcoming collision or to mitigate the severity of it if the collision is unavoidable.

In AEBS, tVime to collision (TTC) is often used as a trigger for entering "warning" or "emergency braking" phases. TTC is the time remaining before the rear-end collision happens if the course and relative speed of vehicles are maintained constant. A high TTC-threshold implies a safe system. However, lower numbers are dictated by a desire to avoid a high number of false positive braking and an unnecessary nuisance when driving.

Introduction of V2V communications enables cooperative adaptive cruise control (CACC) [7] and emergency electronic brake lights (EEBL) [8]. With the latter, a vehicle broadcasts Decentralized Environmental Notification Message (DENM) when its deceleration value reaches a predefined emergency braking threshold value. Reception of DENMs by another vehicle triggers automatic emergency braking. The functioning of the EEBL depends on the reliability of the V2V communication channel - the higher the packet error rate is, the more DENM repetitions are needed to inform other vehicles about the critical situation. In the context of platooning, so-called Platooning Control Message (PCM) can be used instead of DENM [9]. Two main V2V radio technologies are local area network based IEEE 802.11p/802.11bd and broadband cellular based C-V2X/5G NR [10], [11].

Emergency braking, when the leading vehicle is braking sharply and unexpectedly due to some obstacle on the road, is a situation where safety needs to be guaranteed to avoid rear-end
collisions throughout the string of vehicles. Standard ISO/PAS 21448, known as Safety of the Intended Functionality or SOTIF, provides guidance on minimizing safety hazards that occur due to fundamental deficiencies of sensor technologies [12]. In this context, inherent unreliability of V2V communications is one kind of such a sensing limitation. The SOTIF, similarly to functional safety frameworks [13], operates with the notion of risk, which incorporates two components - a likelihood of a crash and an estimate of its severity.

## A. Related Work on Platooning

Platooning is a generalization of the two-vehicle case mostly considered in the contexts above, as it implies a string of multiple vehicles. Safety and stability need to be ensured for the platooning system as a whole. Safety usually refers to the avoidance of rear-end collisions between consecutive vehicles, whereas stability usually refers to a string stability [14], i.e., no amplification of propagating disturbances throughout the platoon. A recent unification of various definitions on string stability is provided in [15]. Tracking performance for a range of string stable controllers can be found in the references therein. Various recent MPC-based approaches have been suggested for improved safety in [16], [17]. In [18], a string stable controller is designed that is safe under communication delays.

A key aspect when designing a platooning control strategy is the spacing policy [19], [20], i.e., the choice of desired, possibly time-varying, inter-vehicle distances. This aspect has been addressed in many of the early works on string stability [21]-[23]. In [24], some requirements for an ideal spacing policy are listed. Amongst those are guaranteed stability and string stability, but also further requirements such as smooth traffic flow and reasonable control effort. The constant time gap spacing policy, defined via relative position and velocity, is first described, followed by a proposed non-linear "ideal" spacing policy. In [25], the effect of a vehicle look ahead for the constant spacing policy is studied in the presence of "parasitic lags". Examples of nonlinear spacing policies are presented in [20], [26]. In [27], [28], delay-based spacing policies are introduced for guaranteed string stability subject to external noise. In [29], a decentralized spacing policy is presented, which shows robustness to communication drop-outs.

The platooning controller defines the topology and scheduling of the V2V communication. Commonly used communication topologies are predecessor-following, bidirectional, bidirectional-leader, predecessor-following-leader [30], where the two latter imply direct communication with the leader. Platooning control requires ultra-reliable and low latency communication, which becomes vital, especially at high speeds scenarios [31].

Thus, the most common objectives in platoon control are safety and string stability, addressed by choosing a proper control law, relative spacing policy, communication topology and scheduling. Most of the research in platooning is focused on ensuring stability of a controller and improved safety by achieving string stability. However, in terms of safety, there
is little work on ensuring safety and lower bounds on spacing policy in braking scenarios when joint V2V communication is used, which is the topic of this paper.

## B. Our Proposed Safety Analysis

We focus on emergency braking scenarios and collision avoidance in platooning and derive minimum safe inter-vehicle distances, or lower bounds, that ensure no rear-end collisions. We calculate such a relative spacing policy lower bounds for two different setups and then compare their performance. In the first setup, only on-board sensor measurements are available, and in the second one V2V communication is assumed to be used. For both setups, metrics characterizing the likelihood of crash are also derived.

Key contributions of the paper are:

1) Safety metrics that link platooning operation to the safety requirements discussed in the SOTIF standard are introduced. We demonstrate that based on those metrics, minimum inter-vehicle distance (IVD)s that allow achieving the predefined level of safety, can be calculated.
2) Safety analysis is linked to fuel efficiency considerations and two V2V-enabled emergency braking strategies are proposed. In the decentralized strategy, only local information about the preceding vehicle is used, whereas in the centralized strategy, information from the whole platoon is gathered by the leader to improve the decentralized solution.
3) An explicit comparison between a radar-based AEBS and V2V-based EEBL for emergency braking scenarios in platooning is presented. We show that having equivalent safety guarantees, the V2V-strategy outperforms the radarbased solution in terms of minimum allowed IVDs.
In more details on the second bullet, two presented braking strategies for V2V-based EEBL are:
4) distributed approach where each vehicle uses only information about the vehicle in front and calculates the minimum safe distance to it. This allows each vehicle to minimize its own fuel consumption independently of others, by choosing the smallest possible safe distance to the previous one. The distributed approach can be adopted in multi-brand platoons [9], i.e., platoons of vehicles from different manufacturers.
5) centralized approach where the best braking strategy is calculated centrally by a platoon leader using information from all the vehicles in the platoon. It provides globally optimal fuel savings (given the modeling scheme) for the whole platoon, and it can be deployed by mono-brand platoons, i.e., platoons of vehicles from the same manufacturer. Obeying restrictions on deceleration capacity dictated by others - which is implicated by this centralized approach - is seen as a principal implementation obstacle for multi-brand platoons, where the distributed approach is most likely preferred.
For a comparative study between radar sensor and wireless sensor based emergency braking systems, we use a model of the platoon where solutions can be derived analytically. This


Fig. 1. The scheme of the considered platooning model in the case of $N=3$.
means that we can compute bounds on probabilities on rearend collisions that are correct under the modeling assumptions. Thus, there is no need for, e.g., time-consuming Monte Carlo methods.

The probability of rear-end collisions is estimated in [4] by calculating the overall stopping distance of a platoon leader and followers using Monte-Carlo simulations. In [32], safe distance sets for heavy-duty vehicle platooning are numerically computed through a game theoretical framework. Communication delays are represented as changes in relative velocities between vehicles at the moment when braking is initiated by the leader. However, this approach is computationally expensive for realtime applications. An emergency braking strategy is presented in [33], where the braking capability of the platoon is limited by the vehicle with the least deceleration capability. This idea is extended to a coordinated emergency brake protocol in [34] where vehicles form groups that brake together using the lowest common brake capability among the vehicles. A minimum safe time headway corresponded to this braking strategy is calculated using learning-based testing. According to the space-buffer scheme proposed in [35], platooning vehicles are required to be sorted in the order of increasing stopping distances. In [36], all vehicles brake synchronously, some milliseconds after the leader has sent an emergency brake command. It is assumed that all followers receive the braking message successfully during the introduced delay. Our work differs from those above in a way that a predefined probability of no-collision is used to analytically calculate minimum safe distances between vehicles given V2V communication channel characteristics.

The rest part of this paper is structured as follows: platooning model and considered sensors are discussed in Section II; safety metrics are introduced in Section III; braking strategies and corresponding safe IVDs are presented in Section IV. Section VI discusses the simulation results, and the paper is concluded by Section VII.

The main notations that are used in this paper are summarized in Table I.

## II. System Model

We consider a platoon of $N$ vehicles, each moving at a constant speed $v_{0}$. The vehicles are enumerated from the front of the platoon from 0 to $(N-1)$, see Fig. 1. The 0 -th vehicle is often referred to as the leader, others - followers. For each vehicle $i \geq 1$, the IVD between the $(i-1)$-th and the $i$-th vehicle is $d_{i}$.

Each vehicle $i$ has a maximum braking capacity with an absolute value $\bar{a}_{i}$. In other words, the $i$-th vehicle is able to brake with a deceleration $a_{i}$ where $0<a_{i} \leq \bar{a}_{i}$. The $\bar{a}_{i}$ 's may differ

TABLE I
Main Notations

| Notation | Definition |
| :---: | :---: |
| $N$ | number of vehicles in the platoon |
| $i$ | ordinal number of the vehicle in the platoon; 0 is referred to the leader, whereas $1, \ldots, N-1$ are followers |
| $d_{i}$ | the IVD between vehicles $(i-1)$-th and $i$-th at the moment when emergency braking starts |
| $r_{i}(t)$ | the IVD between vehicles $(i-1)$-th and $i$-th during emergency braking |
| $\Delta_{w}, f_{w}$ | update period and frequency of the wireless sensor, $\Delta_{w}=$ $1 / f_{w}$ |
| $\Delta_{r}, f_{r}$ | update period and frequency of the radar sensor, $\Delta_{r}=1 / f_{r}$ |
| $p_{i}$ | packet loss probability between the leader and the $i$-th vehicle |
| $\bar{a}_{i}$ | maximum deceleration capability of the $i$-th vehicle |
| $a_{i}$ | chosen deceleration of the $i$-th vehicle during emergency braking, $0<a_{i} \leq \bar{a}_{i}$ |
| $v_{0}$ | velocity of the platoon in the normal mode |
| $Q_{W}$ | probability of no collision for the whole platoon. "W" refers to the wireless case. |
| $Q_{W}^{i}$ | probability of no collision between neighboring vehicles ( $i-$ 1 )-th and $i$-th, $i \geq 1$. "W" refers to the wireless case. |
| $Q_{R}$ | probability of no collision for the whole platoon. " R " refers to the radar case. |
| $Q_{R}^{i}$ | probability of no collision between neighboring vehicles ( $i-$ 1) and $i, i \geq 1$. 'R" refers to the radar case. |
| $C_{*}^{i}$ | desirable safety requirement for probabilities $Q_{R}^{i}$ or $Q_{W}^{i}$ |
| $Q_{*}$ | desirable safety requirement for probabilities $Q_{R}$ or $Q_{W}$ |
| $T_{T T C}^{i}(t)$ | time to collision of the $i$-th vehicle with the preceding one, $(i-1)$-th, on the assumption that the current and the preceding vehicle continue driving with constant speeds till the collision |
| $T_{A E B S}$ | threshold for $T_{T T C}^{i}(t)$ when AEBS is allowed to start harsh braking |
| $t^{*}$ | time when $T_{T T C}^{i}\left(t^{*}\right)=T_{A E B S}$ |
| $\bar{\tau}_{i}$ | maximum time delay allowed for the vehicle $i$ to start braking in order to avoid collision with the vehicle $(i-1)$ |
| $S\left(d_{i}\right)$ | safety threshold defined as $T_{T T C}^{i}\left(\bar{\tau}_{i}\left(d_{i}\right)-C_{*}^{i} \Delta_{r}\right)$ |

among the vehicles to reflect heterogeneous platoons where different vehicle models, trailer loads, tire conditions, etc. affect the braking capacity. It is assumed that exact $a_{i}$ 's are known; however, we do not specify in the paper how those values are obtained. The lateral dynamics of vehicles is not considered here.

Two modes of platooning operation are considered: a normal mode and an emergency braking mode (these should not be mixed up with the various phases of AEBS presented in Section I):

- In the normal mode, the platoon moves at a constant speed $v_{0}$ in an unchanging environment, that is, the road friction coefficient, the road slope, etc., are all considered as time invariants or as being constant.
- During the emergency braking mode, the leader applies maximum deceleration $\bar{a}_{0}$ until stopping. We assume constant deceleration model for all the vehicles. How followers decide on the braking, i.e., when and with what deceleration value to brake, depends on which sensors are in use. Respective approaches are explained in two subsections below.
Below in the paper, we refer to the "wireless case" if the platoon is equipped with V2V sensors, and to the "radar case" if only radars are used to manage a platoon. It should be emphasized that for the wireless case, it is assumed that vehicles can
also be equipped with radar sensors and receive information about relative velocities and distances to their neighbors by means of it. Moreover, we do not distinguish in the wireless case how the local information is obtained. However, only in the wireless case platooning vehicles can obtain extra global information such as maximum deceleration capabilities of platoon members, their decisions, current decelerations, etc. In the paper, we assume that IEEE 802.11p protocol is used for V2V communication.


## A. Wireless Sensor

At the moment of applying maximum deceleration, the leading vehicle repeatedly with a time-period $\Delta_{w}=1 / f_{w}$ sends an emergency message (EM) to the other vehicles using V2V communication, where $f_{w}$ is the EM update frequency. The EM informs other vehicles about the hard braking situation, i.e., implicitly asking vehicles to enter the emergency braking mode. In practical implementations, either PCM or DENM are natural candidates for such an EM.

Only the leading vehicle is transmitting information about the hard-braking situation, and no re-transmission by the other vehicles in the platoon is performed, i.e., we consider one-hop broadcast communication. This assumption is adopted since we assume the length of the platoon will not exceed the intended communication range of the leader in early practical phases of platooning technology deployment, which makes multi-hop retransmissions redundant.

The $i$-th vehicle either receives the packet with the probability $\left(1-p_{i}\right)$ or does not receive it with the probability $p_{i}$. All packet receptions are independent. Furthermore, $p_{i}<1$ and wireless connectivity can not be lost entirely. It is assumed that for each vehicle, the packet loss probability $p_{i}$ remains constant during the entire braking process. This is justified by the fact that relative distances and velocities exhibit only minor changes during a short period of time after the leading vehicle starts braking. Thus, these variations do not pose any major changes on the communication channel (the large scale fading), and therefore, do not significantly change the distribution of the $p_{i}$ 's.

When the $i$-th vehicle (where $i>0$ ) receives the EM, it requests itself to start braking by applying constant deceleration with an absolute value $a_{i} \leq \bar{a}_{i}$ (reasoning behind applying non-maximal deceleration is provided in Section IV).

## B. Radar Sensor

We consider AEBS with a "warning phase" that does not include a speed reduction due to a collision warning braking. It is also assumed that AEBS is functioning without any outside intervention, e.g., from the driver side. This can happen, for example, when the driver is asleep at the wheel or unconscious. Instead, the deceleration only occurs during the emergency braking mode. Such an assumption is valid for highly automated platooning vehicles.

Automotive radar measures the range, angle, and relative radial velocity of targets. We assume that all platooning vehicles are equipped with "perfect" radars, i.e., measurements of radars do not contain errors and are obtained every $\Delta_{r}$ s. For the
platoon, TTC of the $i$-th vehicle with the preceding $(i-1)$-th can be calculated as:

$$
\begin{equation*}
T_{T T C}^{i}(t)=-\frac{r_{i}(t)}{v_{i}(t)} \tag{1}
\end{equation*}
$$

where $r_{i}(t)$ - the relative distance between two considered vehicles, and $v_{i}(t)$ - the relative velocity, both obtained by the front radar installed on the $i$-th vehicle. Note that $T_{T T C}^{i}$ is updated with a radar update rate $f_{r}$ since new measurements are available only with a radar sampling period $\Delta_{r}=1 / f_{r}$. We assume that time for $T_{T T C}^{i}$ calculation is negligibly small.

Once the $T_{T T C}^{i}$ reaches threshold value $T_{A E B S}$, the $i$-th vehicle requests itself to start braking by applying maximum deceleration with an absolute value $\bar{a}_{i}$.

## III. Safety Analysis

Different performance metrics can be introduced to assess safety in platooning. In this paper, we use a generalization of the quantitative metric $Q$ for safe braking, which was introduced in [37], where it was defined as the probability that no collisions between the $N$ vehicles occur during an emergency braking.

We introduce two symbols $Q_{W}$ and $Q_{R}$ corresponding to the probability of no collision during emergency braking when wireless sensors or radar sensors are used, respectively. When we write $Q$, we refer to any of these. Furthermore, we use indexes to directly refer to the probability of no collision between two consecutive vehicles. For example, $Q_{R}^{i}$ is the probability of no collision between vehicles $(i-1)$-th and $i$-th when only radar sensors are used.

The performance metric $Q$ is a function that maps system parameters (variables) to a value in [0,1]. The system parameters include those for the dynamical model of the vehicles, i.e., $\bar{a}_{i}$ 's, the $d_{i}$ 's, and $v_{0}$. Furthermore, there are additional parameters for the different $Q_{W}$ and $Q_{R}$. For example, for $Q_{W}$, the $p_{i}$ 's comprise additional system parameters.

## A. Wireless Sensor

We denote by $\bar{\tau}_{i}$ the maximum time delay allowed for the $i$-th vehicle to start braking to avoid collision with the vehicle $(i-1)$-th. For the wireless case, $\bar{\tau}_{i}$ can be seen as a maximum communication delay during which the EM has to be received by the $i$-th vehicle to initiate braking and avoid a crash with the ( $i-1$ )-th vehicle [37].

In the wireless setting $Q$, or $Q_{W}$, represents the probability that each vehicle receives the EM no later $\bar{\tau}_{i}$ seconds have passed since the time point $\tau_{i-1}$ when the $(i-1)$-th vehicle received EM. It is assumed that $\Delta_{w} \leq \bar{\tau}_{i}$ for every $i$, which reflects the fact that at least one packet transmission attempt can be completed within the feasible delay region. In this section, we follow the model from [37], where no braking lags are considered. Thus, for every vehicle $i$, the time point when the vehicle receives the EM and the time point when it starts braking coincide in $\bar{\tau}_{i}$. We provide the following proposition:

Proposition 1: Safe braking probability $Q_{W}$ justifies

$$
\underline{Q}_{W} \leq Q_{W} \leq \bar{Q}_{W}
$$



Fig. 2. The tree formed by all possible time slots when the EM can be delivered to all vehicles in a platoon. Numbers on the top of the scheme are vehicles' order numbers; a node's number is a number of a corresponding time slot.
given that $\bar{\tau}_{i} \geq \Delta_{w}$ holds for each $i$, where

$$
\underline{Q}_{W}=\prod_{i=1}^{N-1}\left(1-p_{i}^{\left\lfloor\frac{\bar{\tau}_{i}}{\Delta \Delta_{w}}\right\rfloor}\right)
$$

and

$$
\bar{Q}_{W}=\prod_{i=1}^{N-1}\left(1-p_{i}^{\sum_{j=1}^{i}\left\lfloor\frac{\bar{\tau}_{j}}{\Delta w}\right\rfloor}\right)
$$

serve as lower and upper bounds to $Q_{W}$, respectively. It may be noted that $\bar{Q}_{W}$ represents the case where each vehicle receives the EM at the most postponed attempt but still in time to brake safely without a crash, i.e., the vehicles start braking one by one in chronological order according to their enumeration.

Proof: The possible time slots when EMs have to be delivered to platooning vehicles in order to avoid a crash, can be represented by a tree (Fig. 2). The nodes in the $i$-th "column" of the tree corresponds to the $i$-th vehicle. Each vehicle has a maximum allowed communication delay $\bar{\tau}_{i}$ given respectively to the previous vehicle's braking which gives maximum $K_{i}=\left\lfloor\frac{\bar{\tau}_{i}}{\Delta_{w}}\right\rfloor$ attempts for the successful delivery. For each vehicle $i$, the probability that the EM was delivered in the $j$-th time slot is $\left(1-p_{i}\right) p_{i}^{j-1}$, where $1 \leq j \leq K_{i}$. Thus, the probability that the EM was delivered to all vehicles in the first time slot is corresponding to the topmost branch, and equals to $\prod_{i=1}^{N-1}\left(1-p_{i}\right)$. The probability that the EM was delivered to every vehicle in the last time slot $K_{i}$ corresponds to the bottom-most branches and is given by $\prod_{i=1}^{N-1}\left(1-p_{i}\right) p_{i}^{\sum_{j=1}^{i} K_{j}}$. Summing over all branches gives:

$$
\begin{aligned}
Q_{W}= & \prod_{i=1}^{N-1}\left(1-p_{i}\right) \cdot\left[\sum_{j_{1}=0}^{\left\lfloor\frac{\bar{\tau}_{1}}{T}\right\rfloor-1} p_{1}^{j_{1}} \cdot \sum_{j_{2}=0}^{j_{1}+\left\lfloor\frac{\bar{\tau}_{2}}{T}\right\rfloor-1} p_{2}^{j_{2}} \cdots\right. \\
& \left.\cdot \sum_{j_{N-1}=0}^{j_{N-2}+\left\lfloor\frac{\bar{\tau}_{N-1}}{T}\right\rfloor-1} p_{N-1}^{j_{N-1}}\right],
\end{aligned}
$$

from which it can easily be deduced that $\underline{Q}_{W}$ and $\bar{Q}_{W}$ bound $Q_{W}$ from below and above, respectively.

If $\underline{Q}_{W} \geq Q_{*}$, then the platoon can be considered as safe in an emergency braking scenario with the probability no less than $Q_{*}$, where $Q_{*}$ should be induced by desirable safety requirements.

The metric $Q_{W}$ requires a readjustment of IVDs whenever a new vehicle joins the platoon. In this context, $Q_{W}$ is well suited for centralized schemes for controlling IVDs. Distributed safety metrics, stable in terms of dynamic additions and removals of vehicles in the platoon, may be considered by introducing a set of $(N-1)$ probabilities $Q_{W}^{i}$, each capturing only collisions between consecutive vehicles. Explicitly, for the $(i-1)$-th and the $i$-th vehicles, the following is introduced:

$$
\begin{equation*}
Q_{W}^{i}=\left(1-p_{i}^{\left\lfloor\frac{\bar{\tau}_{i}}{\Delta w}\right\rfloor}\right) \tag{2}
\end{equation*}
$$

which gives the lower bound on the probability of avoiding collision between $(i-1)$-th and $i$-th vehicles during emergency braking. The metric (2) is calculated under the assumption that the $(i-1)$-th vehicle receives the EM at the first attempt and starts braking right thereafter (at time $t=\Delta_{w}$ ). So that the $i$-th vehicle has $\bar{\tau}_{i}+\Delta_{w}$ s to receive the EM and safely enter the emergency braking mode.

By analogy with the centralized metric $Q_{W}$, if

$$
\begin{equation*}
Q_{W}^{i} \geq C_{*}^{i}, \tag{3}
\end{equation*}
$$

then the crash between $(i-1)$-th and $i$-th vehicles can be avoided in the considered scenario with the probability no less than $C_{*}^{i}$.

It is worth noticing in this context that if

$$
\begin{equation*}
C_{*}^{1}=C_{*}^{2}=\cdots=C_{*}^{N-1}=\left(Q_{*}\right)^{\frac{1}{N-1}} \tag{4}
\end{equation*}
$$

then the chosen IVDs meet the criteria $Q \geq Q_{*}$. However, the constraints in (4) can be challenging to satisfy.

## B. Radar Sensor

In this section, we define $Q_{R}$, i.e., the probability of safe braking using a radar-based AEBS system.

By inspecting (1), we see that, as long as the $i$-th vehicle is in a normal mode and the $(i-1)$-th vehicle is emergency braking, $T_{T T C}^{i}(t)$ is monotonically decreasing function of time $t$. Due to this property, it holds that the collision can be avoided if and only if there are any radar measurements received at $t^{*} \leq t \leq \bar{\tau}_{i}$ where $t^{*}$ is a moment when the function $T_{T T C}^{i}$ reaches $T_{A E B S}$ value, i.e., $T_{T T C}^{i}\left(t^{*}\right)=T_{A E B S}$. From this follows, if

$$
\begin{equation*}
T_{T T C}^{i}\left(\bar{\tau}_{i}\right)>T_{A E B S}, \tag{5}
\end{equation*}
$$

then a collision is unavoidable, and $Q_{R}^{i}=0$. Since TTC updates happen with a period $\Delta_{r}$, the in-time decision of emergency braking is guaranteed to be done for updates received at $t \leq$ $\bar{\tau}_{i}-\Delta_{r}$.

So, it holds that if

$$
\begin{equation*}
T_{T T C}^{i}\left(\bar{\tau}_{i}-\Delta_{r}\right)<=T_{A E B S} \tag{6}
\end{equation*}
$$

then there is no collision, and $Q_{R}^{i}=1$. The remaining case, when $T_{T T C}^{i}$ reaches $T_{A E B S}$ value at the moment $t^{*}$ where


Fig. 3. A schematic graph of the probability of no collision $Q_{R}^{i}$ between two vehicles for the radar case. On the x-axis is $t^{*}$ - time when $T_{T T C}^{i}\left(t^{*}\right)=$ $T_{A E B S}$. Clearly, having fixed parameters $\left(a_{i-1}, a_{i}, v_{0}, \Delta_{r}\right), t^{*}$ is dependent on the initial distance $d_{i} ; \bar{\tau}_{i}$ is also dependent on $d_{i}$.
$\bar{\tau}_{i}-\Delta_{r}<t^{*}<\bar{\tau}_{i}$, is not so obvious since the start of the braking of the $(i-1)$-th vehicle and radar measurements are not synchronized. It can be assumed that possible radar measurement in $\left[\bar{\tau}_{i}-\Delta_{r} ; \bar{\tau}_{i}\right]$ is a uniformly distributed random variable. In other words, it is equally probable that one radar's measurement can be received at any time moment between $\bar{\tau}_{i}-\Delta_{r}$ and $\bar{\tau}_{i}$. We are now ready to define $Q_{R}^{i}$ as a function of time $t^{*}$ :

$$
Q_{R}^{i}\left(t^{*}\right)=\left\{\begin{array}{l}
0 \text { if } \quad \bar{\tau}_{i} \leq t^{*}  \tag{7}\\
1-\frac{t^{*}-\left(\bar{\tau}_{i}-\Delta_{r}\right)}{\Delta_{r}} \text { if } \quad \bar{\tau}_{i}-\Delta_{r} \leq t^{*} \leq \bar{\tau}_{i} \\
1 \text { if } \quad \bar{\tau}_{i}-\Delta_{r} \geq t^{*}
\end{array}\right.
$$

where the middle line defines the probability that one radar measurement falls into $\left[t^{*}, \bar{\tau}_{i}\right]$. The corresponding schematic graph is shown on the Fig. 3.

By analogy with the wireless case, one can introduce $Q_{R}$ for the case of $N$ platooning vehicles which will define the probability of no collision in an emergency braking scenario for the whole platoon under some assumptions:

$$
Q_{R}=Q_{R}^{1} \cdot Q_{R}^{2} \cdots Q_{R}^{N-1}
$$

If IVDs are chosen such that $Q_{R} \geq Q_{*}$ then the platoon can be considered as safe in an emergency braking situation with a probability $Q_{*}$.

Furthermore, other types of safety metrics may be introduced, such as ones capturing computation of probability of collision and quantification of the severity of the crash. This could be performed by using relative velocities or impacts at the time of the crash. In this paper, the analysis is restricted, in the context of distributed-suited safety metrics, to the above-introduced $Q_{W}^{i}$ and $Q_{R}^{i}$.

## IV. Braking Strategies

This section addresses the following overall question: how to select IVDs $d_{i}$ 's to ensure safe braking in the emergency braking mode and, at the same time, ensure fuel-efficient operation in the normal mode?

It is well known that reducing IVDs decreases air-drag, which, in turn, reduces fuel consumption [4]. On the contrary, close distances between vehicles increase safety concerns and the risk of collisions. Hence, the aim here is to find optimal IVDs that minimize the fuel consumption of the platoon moving at a certain predefined speed $v_{0}$ without compromising safety.

To succeed in addressing the set-out problem, the following is needed: a notion of safety metrics, see Section III; an approach for fuel consumption modeling, see Section IV-A1; and formulation of an optimization problem with the use of those notions together. In the wireless case, the structure of optimization problems allows for different types of implementation, presented in Sections IV-B2 and IV-B3, whereas in the radar case, only a distributed approach is possible (see Section IV-C).

1) Fuel Consumption: For two consecutive vehicles, $(i-1)$ and $i$, the aerodynamic drag acting on the $i$-th vehicle can be modeled as a function of the distance $d_{i}$ [38]:

$$
\begin{align*}
F_{a . d, i} & =\frac{1}{2} \rho A_{i} c_{d}\left(d_{i}\right) v_{0}^{2} \\
c_{d}\left(d_{i}\right) & =c_{d, 0}\left(1-\frac{c_{d, 1}}{c_{d, 2}+d_{i}}\right) \tag{8}
\end{align*}
$$

where $A_{i}$ is the cross-sectional area of the $i$-th vehicle, $\rho$ is the air density, $c_{d, 0}$ is the air drag coefficient and $c_{d, 1}, c_{d, 2}$ are constants, obtained by fitting experimental data [38].

## A. Wireless Case

1) Optimization Problem: To minimize air-drag force (8) applied to the $i$-th vehicle, the minimum possible $d_{i}$ must be retrieved. In other words, a minimum of the objective function $J_{i}=A_{i} d_{i}$ needs to be found where $d_{i}$ is considered to be safe in terms of selected safety metric, and $A_{i}>0$ is the penalty coefficient on the cross-size of the $i$-th vehicle. Note that the safe distance $d_{i}$ can be changed by varying deceleration capacities $a_{i-1}$ and $a_{i}$, i.e., $J_{i}$ implicitly depends on the $a_{i-1}, a_{i}$.

In the case of $N$-vehicle platoon, all IVDs should be minimized simultaneously. However, it is not conceivable since such minimizing points are outside the feasible region, e.g., when $d_{1}$ decreases by changing the deceleration capacity $a_{1}$, $d_{2}$ is increasing (having fixed $a_{0}$ and $a_{2}$ ). Here a multi-objective optimization problem arises for which one can resort to Pareto optimal solutions as a way forward. To decide upon which solution on the Pareto boundary to be preferred, a new objective function is introduced as a weighted sum of all IVDs in the platoon:

$$
\begin{equation*}
J=\sum_{i=1}^{N-1} A_{i} d_{i} \tag{9}
\end{equation*}
$$

The interpretation of this objective function $J$ is as follows. It captures the fuel consumption of the whole platoon by relating the $d_{i}$ 's to air drag forces. It is worth noticing that other coefficients than the $A_{i}$ 's can be used, such as ones reflecting
engine efficiency. The particular case when $J$ takes the form $J=\sum_{i=1}^{N-1} d_{i}$ can be chosen with an intention to minimize the road space occupied by the whole platoon. Without loss of generality, coefficients $A_{i}$ 's are considered as dimensionless.

The objective function (9) has to be minimized subject to safety constraints (3) and constraints on the deceleration capabilities $a_{i}: 0<a_{i} \leq \bar{a}_{i}$ 's, $i \in[1, N-1]$. The deceleration capacity $a_{0}$ is regarded as a fixed parameter: $a_{0}=\bar{a}_{0}$, since the leading vehicle has to use its maximum possible deceleration in the case of an emergency situation. Therefore, $a_{0}$ is not part of the variable vector in the optimization problem above.

This problem can be seen as centralized, since the objective function $J$ (9) binds together all braking capabilities $a_{1}, \ldots, a_{N-1}$ through the $d_{i}$ 's. The solution to this optimization problem gives IVDs that provide the best fuel saving for the whole platoon during the normal mode and do not compromise safety in the emergency braking scenario.

The problem of optimizing fuel consumption in a platoon in a distributed way is also investigated. Here, every vehicle minimizes its own fuel consumption under the assumption that all preceding vehicles' deceleration capacities and distances are fixed or have already been chosen. Thus, instead of one centralized problem, a set of optimization problems is considered:

$$
\begin{aligned}
& \min _{d_{i}, a_{i}} A_{i} d_{i} \\
& \text { s.t. } C_{i-1, i} \geq C_{i-1, i}^{*}, d_{i} \geq 0,0<a_{i} \leq \bar{a}_{i}
\end{aligned}
$$

where $1 \leq i \leq N-1$.
In the following subsections, the set of distributed optimization problems is solved, and then a solution for the centralized approach is presented.
2) Distributed Approach: Consider the following set of ( N 1) optimization problems presented in order:

$$
\begin{aligned}
& \min _{d_{1}, a_{1}} A_{1} d_{1} \\
& \text { s.t. } \quad C_{0,1} \geq C_{0,1}^{*}, \quad d_{1} \geq 0, \quad 0<a_{1} \leq \bar{a}_{1} \\
& \vdots \\
& \min _{d_{i}, a_{i}} A_{i} d_{i} \\
& \text { s.t. } \quad C_{i-1, i} \geq C_{i-1, i}^{*}, \quad d_{i} \geq 0, \quad 0<a_{i} \leq \bar{a}_{i} ; \\
& \vdots \\
& \min _{d_{N-1}, a_{N-1}} A_{N-1} d_{N-1} \\
& \text { s.t. } \quad C_{N-2, N-1} \geq C_{N-2, N-1}^{*}, \quad d_{N-1} \geq 0 \\
& 0<a_{N-1} \leq \bar{a}_{N-1} .
\end{aligned}
$$

Solving each such problem independently can be regarded as a distributed approach, in that computations and decision-making are made by every vehicle. In other words, the $i$-th vehicle solves the $i$-th optimization problem locally.

The algorithm for solving the above set of problems is as follows: $(i)$ the optimal distance $d_{1}$ and the corresponding deceleration capacity $a_{1}$ are obtained by solving the first constrained optimization problem; (ii) the optimal distance $d_{2}$ and corresponding $a_{2}$ are computed by solving the second problem, having chosen $d_{1}$ and $a_{1}$; (iii) the procedure is repeated in a consecutive order for all vehicles such that distance $d_{N-1}$ and deceleration $a_{N-1}$ are computed from the last constrained optimization problem by assuming $d_{1}, \ldots d_{N-2}, a_{1}, \ldots a_{N-2}$ have already been selected.

The solution of the $i$-th optimization problem is given below. By imposing the assumption introduced in Section II that $p_{i}$ can be considered as constant on the IVD interval, and keeping in mind that from a practical point of view, probability $0<p_{i}<1$, $0<C_{i-1, i}^{*}<1$, the constraint (3) on the safety metric can be rewritten as:

$$
\begin{equation*}
\left\lfloor\frac{\bar{\tau}_{i}}{\Delta_{w}}\right\rfloor \geq \frac{\ln \left(1-C_{i-1, i}^{*}\right)}{\ln p_{i}} \tag{10}
\end{equation*}
$$

which is equivalent to:

$$
\begin{equation*}
\bar{\tau}_{i} \geq \frac{1}{f_{w}}\left\lceil\frac{\ln \left(1-C_{i-1, i}^{*}\right)}{\ln p_{i}}\right\rceil . \tag{11}
\end{equation*}
$$

We will use the following formula from [37], which shows how much time, $\tau_{\text {max }}^{i}$, the $i$-th vehicle has before it has to start braking in order to avoid a collision with the $(i-1)$-th vehicle. This time is calculated from the moment when the $(i-1)$-th started braking by using a double-integrator model for vehicles:

$$
\tau_{\max }^{i}=\left\{\begin{array}{cl}
\sqrt{\frac{2 d_{i}\left(a_{i}-a_{i-1}\right)}{a_{i-1} a_{i}}} & \text { if }\left\{\begin{array}{c}
\sqrt{\frac{2 d_{i} a_{i-1}}{a_{i}\left(a_{i}-a_{i-1}\right)}} \leq \frac{v_{0}}{a_{i}} \\
a_{i}>a_{i-1}
\end{array}\right.  \tag{12}\\
\frac{d_{i}}{v_{0}}+\frac{v_{0}}{2} \frac{a_{i}-a_{i-1}}{a_{i} a_{i-1}} & \text { otherwise },
\end{array}\right.
$$

Here, $d_{i}$ presents the IVD between the $i$-th and $(i-1)$-th vehicles in the normal mode of platooning, i.e., at the beginning of the emergency mode.

In [37], no braking lags were considered, thus the maximum communication delay $\bar{\tau}_{i}$ used for safety metric calculation, and $\tau_{\max }^{i}$ (12) were equal. Here, to make the model more realistic, physical delays, or braking lags $\tau_{d, i}$ are introduced. It implies that after the $i$-th vehicle receives the EM at the moment $\tau_{i}$, its deceleration reaches commanded maximum at $t=\tau_{i}+\tau_{d, i}$ whereas $a_{i}=0$ for $t<\tau_{i}+\tau_{d, i}$. Hence, it follows that

$$
\begin{equation*}
\bar{\tau}_{i}+\left(\tau_{d, i}-\tau_{d, i-1}\right)=\tau_{\max }^{i} \tag{13}
\end{equation*}
$$

Now, combining (12) with (11), the lower bound $\underline{d}_{i}$ can be obtained for a safe distance $d_{i}$. In other words, $d_{i} \geq \underline{d}_{i}$, where $\underline{d}_{i}$ is expressed as:

$$
\underline{d}_{i}=\left\{\begin{array}{l}
\frac{a_{i-1} \cdot a_{i}}{2 f_{i}^{2} \cdot\left(a_{i}-a_{i-1}\right)} \quad \text { if } a_{i-1} \leq \frac{v_{0} f_{i} \cdot a_{i}}{v_{0} f_{i}+a_{i}},  \tag{14}\\
\frac{v_{0}}{f_{i}}-\frac{v_{0}^{2}}{2} \cdot \frac{a_{i}-a_{i-1}}{a_{i} \cdot a_{i-1}} \quad \text { otherwise }
\end{array}\right.
$$

where $f_{i}=\left(\left(\tau_{d, i}-\tau_{d, i-1}\right)+\frac{1}{f_{w}}\left\lceil\frac{\ln \left(1-C_{i-1, i}^{*}\right)}{\ln p_{i}}\right\rceil\right)^{-1}$. Note that $\underline{d}_{i}$ is a function of $\left(a_{i-1}, a_{i}\right)$. However, for ease of notation and to save space, this explicit dependency is omitted.

These equations can be rewritten as:

$$
\underline{d}_{i}=\left\{\begin{array}{l}
d_{0, i}+\frac{a_{i-1} \cdot a_{i}}{2 f_{i}^{2} \cdot\left(a_{i}-a_{i-1}\right)} \quad \text { if } a_{i-1} \leq \frac{v_{0} f_{i} \cdot a_{i}}{v_{0} f_{i}+a_{i}}  \tag{15}\\
d_{0, i}+\frac{v_{0}}{f_{i}}-\frac{v_{0}^{2}}{2} \cdot \frac{a_{i}-a_{i-1}}{a_{i} \cdot a_{i-1}} \quad \text { otherwise }
\end{array}\right.
$$

where $d_{0, i}$ is a buffer added to cope with measurement uncertainties and can account for stand-still distances as well.

After safety constraint transformation from (3) to (15), the optimization problem for the $i$-th vehicle, $i \in[1, N-1]$, can be rewritten as:

$$
\begin{align*}
& \min _{d_{i}, a_{i}} A_{i} d_{i} \\
& \text { s.t. } d_{i} \geq \underline{d}_{i}, \quad 0<a_{i} \leq \bar{a}_{i} \tag{16}
\end{align*}
$$

where $\underline{d}_{i}$ is defined by equations (15). Note that a feasible set on $a_{i}$ is not closed. However, $a_{i}=0$ can not give a minimum to the posed optimal problem because the minimum safe distance $\underline{d}_{i}$ tends to infinity when $a_{i}$ tends to 0 .

Since the derivative $\left(\underline{d}_{i}\right)_{a_{i}}^{\prime}<0$ for all valid $a_{i}$ (see (15)), the solution to the constrained optimization problem (16) is straightforward:

$$
\begin{align*}
& a_{i}=\bar{a}_{i},  \tag{17}\\
& d_{i}= \begin{cases}d_{0, i}+\frac{\bar{a}_{i-1} \cdot \bar{a}_{i}}{2 f_{i}^{2} \cdot\left(\bar{a}_{i}-\bar{a}_{i-1}\right)}, & \text { if } \bar{a}_{i-1} \leq \frac{v_{0} f_{i} \cdot \bar{a}_{i}}{v_{0} f_{i}+\bar{a}_{i}} \\
d_{0, i}+\frac{v_{0}}{f_{i}}-\frac{v_{0}^{2}}{2} \cdot \frac{\bar{a}_{i}-\bar{a}_{i-1}}{\bar{a}_{i} \cdot \bar{a}_{i-1}}, & \text { otherwise. }\end{cases} \tag{18}
\end{align*}
$$

Thus, the solution for the distributed approach is obtained where (17) represents a braking strategy, and minimum safe distances between vehicles are defined by (18). This solution states that in order to minimize its own fuel consumption by means of reducing air drag, every vehicle applies its maximum possible deceleration in the emergency braking mode.
3) Centralized Approach: Having (3) as a safety requirement for each vehicle $i$ and transforming it in the same way as for the distributed approach above, the centralized optimization problem is formulated as:

$$
\begin{align*}
& \underset{\mathbf{d}, \mathbf{a}}{ } \sum_{i=1}^{N-1} A_{i} d_{i}  \tag{19}\\
& \text { s.t. } \quad \underline{d}_{i} \leq d_{i}, \quad i \in[1, N-1]  \tag{20}\\
& 0<a_{i} \leq \bar{a}_{i}, \quad i \in[1, N-1] \tag{21}
\end{align*}
$$

where function $\underline{d}_{i}=\underline{d}_{i}\left(a_{i-1}, a_{i}\right)$ is defined by (15), and bold letters are used to denote vectors: $\mathbf{d}=\left(d_{1}, \ldots, d_{N-1}\right), \mathbf{a}=$ $\left(a_{1}, \ldots, a_{N-1}\right)$.

To solve the problem above, a generalized Lagrange multiplier method is used in the form of Karush-Kuhn-Tucker (KKT) conditions [39], [40]. For this purpose, restrictions on the deceleration capabilities are expanded to non-strict inequalities $0 \leq a_{i} \leq \bar{a}_{i}$, and then shown that KKT points do not lie on the zero boundary. Strictly speaking, the minimum safe distance $\underline{d}_{i}(15)$ is not defined if $a_{i}=0$. But as argued above, no such $a_{i}$ can be a solution for the problem (19)-(21) since the value of the objective function tends to plus infinity in this case. The Lagrangian function for the posed problem (19)-(21) is
expressed as:

$$
\begin{align*}
& L(\mathbf{d}, \mathbf{a}, \boldsymbol{\gamma}, \boldsymbol{\mu}, \boldsymbol{\eta})=\sum_{i=1}^{N-1} A_{i} d_{i}+ \\
& +\sum_{i=1}^{N-1}\left[\gamma_{i} \cdot\left(\underline{d}_{i}-d_{i}\right)+\mu_{i} \cdot\left(a_{i}-\bar{a}_{i}\right)-\eta_{i} \cdot a_{i}\right] \tag{22}
\end{align*}
$$

where $\gamma_{i}, \mu_{i}, \eta_{i}$ are Lagrange multipliers [39], [40].
Gradients of active inequalities can be shown to be linearly independent (linear independence constraint qualification [41]), which means that the problem is well-posed. Necessary KKT conditions can be used to prove that $\eta_{i}=0$ for all $i \in[1, N-1]$. Details can be found in [1]. Also, the next two theorems follow by exploiting the KKT conditions:

Theorem 1: The optimal IVDs for the centralized approach are defined by the equations $d_{i}=\underline{d}_{i}(15)$, where $a_{0}=\bar{a}_{0}$.

Theorem 2: To minimize $J$, the last vehicle in a platoon has to apply maximum possible deceleration $\bar{a}_{N-1}$ in the emergency braking mode.

We skip the proof of those theorems and refer the interested reader to [1].

In the two-vehicle platoon case, Theorems 1 and 2 give the same solution as the distributed approach. It is logical since the centralized and distributed optimization problems coincide for $N=2$. In the case with $N \geq 3$, the optimal braking capabilities of middle vehicles in the platoon have to be defined. The explicit solution for the case $N=3$ is given in subsection IV-B4. For the case $N \geq 4$, the solution can be obtained numerically. In Section VI, an example with $N=4$ vehicles is presented.

It should be noted that introduced buffers $d_{i, 0}$ do not affect the braking strategy defined by the solution of the centralized problem. They contribute only with a constant $\sum_{i=1}^{N-1} A_{i} d_{0, i}$ to Lagrangian, which does not change the optimal solution.
4) Centralized Approach for $N=3$ : Taking into account Theorems 1 and 2, the Lagrangian function for the centralized optimization problem in the case $N=3$ can be expressed as:

$$
L\left(a_{1}, \mu_{1}\right)=A_{1} \underline{d}_{1}+A_{2} \underline{d}_{2}+\mu_{1} \cdot\left(a_{1}-\bar{a}_{1}\right)
$$

where functions $\underline{d}_{1}, \underline{d}_{2}$ are defined by (15) having $a_{0}=\bar{a}_{0}, a_{2}=$ $\bar{a}_{2}$, and $\mu_{1}$ is the Lagrange multiplier.

When the constraint on $a_{1}$ is not active, i.e., $\mu_{1}=0$, the stationarity condition of KKT expresses as:

$$
\begin{equation*}
A_{1}\left(\underline{d}_{1}\right)_{a_{1}}^{\prime}+A_{2}\left(\underline{d}_{2}\right)_{a_{1}}^{\prime}=0 \tag{23}
\end{equation*}
$$

where functions $\underline{d}_{1}$ and $\underline{d}_{2}$ take one of the two possible forms of (15), depending on the ratio of coefficients. Combinations of those forms in (23) yield the next candidate points for the minimum:

$$
\begin{aligned}
& a_{1,1}=\frac{v_{0} f_{1} \bar{a}_{0}}{v_{0} f_{1}-\bar{a}_{0}}, \\
& a_{1,2}=\frac{v_{0} f_{2} \bar{a}_{2}}{v_{0} f_{2}+\bar{a}_{2}}, \\
& a_{1,3}=\frac{\bar{a}_{0}}{1-\frac{\bar{a}_{0}}{f_{1} v_{0}} \sqrt{\left(\frac{A_{1}}{A_{2}}\right)}},
\end{aligned}
$$

$$
\begin{aligned}
a_{1,4} & =\frac{\bar{a}_{2}}{1+\frac{\bar{a}_{2}}{f_{2} v_{0}} \sqrt{\left(\frac{A_{2}}{A_{1}}\right)}} \\
a_{1,5} & =\frac{Z_{1} \bar{a}_{2}+Z_{2} \bar{a}_{0}}{Z_{1}+Z_{2}}
\end{aligned}
$$

where $Z_{1}=\frac{\bar{a}_{0} \sqrt{A_{1}}}{f_{1}}, Z_{2}=\frac{\bar{a}_{2} \sqrt{A_{2}}}{f_{2}}$.
Instead of checking the conditions on the coefficients to determine which point from the above is the minimum, the primal feasibility condition is applied. Then the final solution of the centralized optimal problem for the $N=3$ case is expressed as:

$$
a_{1}^{*}=\arg \min J(x),
$$

where $x \in X=\left(\bar{a}_{1} \cup a_{1,1} \cup a_{1,2} \cup a_{1,3} \cup a_{1,4} \cup a_{1,5}\right) \cap\left[0, \bar{a}_{1}\right]$.

## B. Radar Case

Without wireless communication, a decentralized approach where every vehicle minimizes its own distance to the preceding vehicle in order to minimize fuel consumption or platoon length is the most logical and straightforward. Since vehicles can not obtain information further their neighbors, a centralized approach can not be realized.

From the introduced definition $Q_{R}^{i}$ (7), one can conclude that a collision between vehicles can be avoided with a certain probability $C_{*}^{i}$ if $t^{*} \geq \bar{\tau}_{i}-C_{*}^{i} \Delta_{r}$. Since $T_{T T C}^{i}$ is a monotonically decreasing function of time $t$ in the considered scenario, the condition above is equivalent to:

$$
\begin{equation*}
T_{A E B S} \leq T_{T T C}^{i}\left(\bar{\tau}_{i}-C_{*}^{i} \Delta_{r}\right) \tag{24}
\end{equation*}
$$

In the considered here scenario, the $(i-1)$-th vehicle starts emergency braking at the time $T_{b}^{i-1}$ with maximum possible deceleration $\bar{a}_{i-1}$, whereas $i$-th maintains constant velocity $v_{0}$. Without losing generality, we assume $T_{b}^{i-1}=0$. Then $T_{T T C}^{i}$ can be expressed as:

$$
T_{T T C}^{i}(t)= \begin{cases}\frac{d_{i}-\frac{\bar{a}_{i-1} t^{2}}{2}}{\bar{a}_{i-1} \cdot t}, & \text { if }\left\{\begin{array}{l}
t \leq \frac{v_{0}}{\bar{a}_{i-1}} \\
t \leq \sqrt{\frac{2 d_{i}}{\bar{a}_{i-1}}} \\
\left(\frac{d_{i}}{v_{0}}+\frac{v_{0}}{2 \bar{a}_{i-1}}\right)-t,
\end{array}\right.  \tag{25}\\
\text { if }\left\{\begin{array}{l}
\frac{v_{0}}{\bar{a}_{i-1}} \leq t \leq \frac{v_{0}}{2 \bar{a}_{i-1}}+\frac{d_{i}}{v_{0}} \\
d_{i} \geq \frac{v_{0}^{2}}{2 \bar{a}_{i-1}}
\end{array}\right.\end{cases}
$$

where $d_{i}$ in this explicit context is the initial distance between vehicles $(i-1)$-th and $i$-th at the moment $T_{b}^{i-1}$, and $t>0$. It is worth noting that given (25), one can explicitly verify that $T_{T T C}^{i}$ is a monotonically decreasing function of time.

We assume that the deceleration of the $(i-1)$-th vehicle, as well as $\bar{\tau}_{i}$, can be estimated by the $i$-th vehicle. How this is done is out of the scope of this paper. Here, we simply want to maintain a principal comparison of minimum safe IVDs in the platoon with AEBS, based only on radar measurements, and the solution based on V2V communication. As mentioned before, we do not assume any radar measurement errors.

Now, by combining (25) and (12), we define a what we call the safety threshold $S\left(d_{i}\right)=T_{T T C}^{i}\left(\bar{\tau}_{i}-C_{*}^{i} \Delta_{r}\right)$. For the case
$\bar{a}_{i} \leq \bar{a}_{i-1}$, we have:
$S\left(d_{i}\right)=\left\{\begin{array}{l}\frac{d_{i}-\frac{\bar{a}_{i-1}}{2}\left(\tau_{\max , 2}^{i}-C_{*}^{i} \Delta_{r}\right)^{2}}{\bar{a}_{i-1}\left(\tau_{\max , 2}^{i}-C_{i}^{i} \Delta_{r}\right)}, \\ \text { if } v_{0} C_{*}^{i} \Delta_{r}-\frac{v_{0}^{2}}{2} Y_{i} \leq d_{i} \leq \frac{v_{0}^{2}}{2} \frac{\bar{a}_{i-1}+\bar{a}_{i}}{\bar{a}_{i-1} \bar{a}_{i}}+v_{0} C_{*}^{i} \Delta_{r}, \\ \frac{v_{0}}{2 a_{i}}+C_{*}^{i} \Delta_{r}, \\ \text { if } \quad d_{i} \geq \frac{v_{0}^{2}}{2}\left(\frac{1}{\overline{a_{i-1}}}+\frac{1}{\bar{a}_{i}}\right)+v_{0} C_{*}^{i} \Delta_{r},\end{array}\right.$
where $Y_{i}=\frac{1}{\overline{a_{i-1}}}-\frac{1}{\overline{a_{i}}}, \tau_{\text {max }, 2}^{i}=\frac{d_{i}}{v_{0}}+\frac{v_{0}}{2} Y_{i}$.
For the case $\bar{a}_{i}>\bar{a}_{i-1}$, the safety threshold $S$ is defined as:

$$
S\left(d_{i}\right)=\left\{\begin{array}{l}
\frac{d_{i}-\frac{\bar{a}_{i-1}}{2}\left(\sqrt{2 d_{i} Y_{i}}-C_{*}^{i} \Delta_{r}\right)^{2}}{\bar{a}_{i-1}\left(\sqrt{2 d_{i} Y_{i}}-C_{i}^{i} \Delta_{r}\right)}, \text { if } \frac{\left(C_{*}^{i} \Delta_{r}\right)^{2}}{2 Y_{i}} \leq d_{i} \leq \frac{v_{0}^{2}}{2} Y_{i},  \tag{27}\\
\frac{d_{i}-\frac{\bar{a}_{i-1}}{2}\left(\tau_{\max , 2}^{i}-C_{*}^{i} \Delta_{r}\right)^{2}}{\bar{a}_{i-1}\left(\tau_{\text {max }, 2}^{i}-C_{*}^{i} \Delta_{r}\right)}, \\
\text { if } \quad \frac{v_{0}^{2}}{2} Y_{i} \leq d_{i} \leq \frac{v_{0}^{2}}{2}\left(\frac{1}{\bar{a}_{i-1}}+\frac{1}{\bar{a}_{i}}\right)+v_{0} C_{*}^{i} \Delta_{r}, \\
\frac{v_{0}}{2 a_{i}}+C_{*}^{i} \Delta_{r}, \\
\text { if } \quad d_{i} \geq \frac{v_{0}^{2}}{2}\left(\frac{1}{\bar{a}_{i-1}}+\frac{1}{\bar{a}_{i}}\right)+v_{0} C_{*}^{i} \Delta_{r},
\end{array}\right.
$$

Note that we have chosen to denote $S$ as a function of $d_{i}$, however it is also parameterized by $\bar{a}_{i-1}, \bar{a}_{i}, v_{0}, C_{*}^{i}$, and $\Delta_{r}$. Furthermore, $Y_{i}, \tau_{\text {max, } 2}^{i}$ are, in turn, parameterized by most of those parameters as well. With a possible lack of stringency, we allow ourselves to let $S$ be a function of any of these parameters, depending on the context. It is worth noting, $S$ as a function of initial distance $d_{i}$ is not monotonic in general. The behavior of this function depends on the deceleration capabilities of vehicles. However, for the case $\bar{a}_{i} \leq \bar{a}_{i-1}$, equations (26) define monotonically non-increasing function from $d_{i}$.

From (24) follows that if $d_{i}$ is chosen so that $T_{A E B S} \leq S\left(d_{i}\right)$ then a collision between vehicles $(i-1)$ and $i$ can be avoided with a $C_{*}^{i}$ guarantee taking into account all the introduced assumptions.

## V. Comparison of Minimum Distances

Since above we have defined minimum safe distances both for radar and wireless cases, now we can compare them. Let us define the quality of communication $p_{i}$ corresponding to certain predefined $d_{i}$ so that the collision is avoidable with the probability $C_{*}^{i}$. From (10) follows that:

$$
\begin{equation*}
p_{i} \leq\left(1-C_{*}^{i}\right)^{\left\lfloor\frac{\bar{\tau}_{i}}{\Delta w}\right\rfloor^{-1}} \tag{28}
\end{equation*}
$$

which defines the value of the packet loss probability that allows to have IVD $d_{i}$. Here, $\bar{\tau}_{i}$ is defined by (13) and depends on $d_{i}$.

## VI. Numerical Results

With wireless communication, all necessary information about the emergent situation can be shared directly with all involved vehicles. So, the following vehicle does not only notice a change of relative velocity and distance by means of its onboard sensors, but also understands the plan and decisions of the previous vehicles. That means that vehicles that received the EM, can emergency brake even if TTC is exceeding those prescribed numbers $T_{A E B S}$ for the non-wireless case. According to European regulations on advanced emergency braking systems [42], no emergency braking should start "before TTC equal to or less


Fig. 4. Safety threshold $S\left(d_{i}\right)$ versus the initial distance $d_{i}$. Here, $\bar{a}_{i}=$ $\bar{a}_{i-1}=7 \mathrm{~m} / \mathrm{s}^{2}$.
than 3 s." For some systems [43], as well as for non-assisted drivers [44], this threshold on hard braking initializing can be even lower than 2 s .

For numerical results, it is assumed that all platooning vehicles are equipped with onboard radars with an update period $\Delta_{r}=50 \mathrm{~ms}$. Deceleration performances of the vehicles were taken in a range between 2 and $9 \mathrm{~m} / \mathrm{s}^{2}$ [4]. The velocity of the platoon was varied in $[10,30] \mathrm{m} / \mathrm{s}$. Braking lag times are not considered here and were set to 0 for the wireless case as well as for the radar.

For the wireless case, all platooning vehicles broadcast PCMs with an update period $\Delta_{w}=50 \mathrm{~ms}$, i.e., frequency of 20 Hz [9]. This contains information about the current state of the vehicle, e.g., acceleration, velocity, as well as global parameters such as maximum braking capability. To be consistent with the model introduced earlier, it is assumed that the EM is included only in the leading vehicle's PCM which is generated synchronously with the beginning of its braking. All safety requirements in simulations are set to $C_{*}^{i}=0.99999$.

Below, a scenario with two identical platooning vehicles $\left(\bar{a}_{i}=\bar{a}_{i-1}\right)$ was considered. It is assumed that at the moment $t=0$, the $(i-1)$-th vehicle enters emergency braking mode by applying $\bar{a}_{i-1}$. First, the radar-based solution is presented, then it is compared with the wireless case.

In Fig. 4, a safety threshold $S$, calculated by using (26), is plotted versus initial distance $d_{i}$ for different initial velocities $v_{0}$. Those lines can be seen as a border between an unavoidable collision area and an area where collisions can be avoided by harsh braking. If we have a threshold for emergency braking mode initialization as $T_{A E B S}=3 s$ [42] then almost any initial distance between cars driving at velocity $v_{0}=10 \mathrm{~m} / \mathrm{s}$ can be seen as safe, whereas for the case of $v_{0}=30 \mathrm{~m} / \mathrm{s}$ only initial distances grater than $83.4 m$ can be considered as safe. For any distance shorter than 83.4 m , rear-end collision is unavoidable if harsh braking can not be applied earlier than $T_{T T C}^{i}(t)=3 \mathrm{~s}$. If we choose $T_{A E B S}=2 s$ as a threshold when AEBS can enter emergency


Fig. 5. TTC versus time for initial distances $d_{i}=40: 40: 160 \mathrm{~m}$. Here, $\bar{a}_{i}=\bar{a}_{i-1}=7 \mathrm{~m} / \mathrm{s}^{2}, v_{0}=30 \mathrm{~m} / \mathrm{s}$. Markers correspond to the time point $\bar{\tau}_{i}-C^{*} \Delta_{r}$ and $T_{A E B S}=3 \mathrm{~s}$.
braking mode, then having $v_{0}=30 \mathrm{~m} / \mathrm{s}$ as an initial velocity, a rear-end collision is unavoidable for any initial distance between cars. This is illustrated in Fig. 5, where TTC is plotted versus time for different initial distances between vehicles. The marker of the corresponding color represents the maximum delay $\bar{\tau}_{i}-C_{*}^{i} \Delta_{r}$ allowed for the $(i-1)$-th vehicle to start braking in order to avoid rear-end collision with probability $C_{*}^{i}$, and $T_{A E B S}=3 \mathrm{~s}$. For example, the blue line that corresponds to initial distance 40 m lies higher than the marker, which implies that $T_{T T C}^{i}(t)$ at the moment $t=\bar{\tau}_{i}-C_{*}^{i} \Delta_{r}$ exceeds $T_{A E B S}=3 \mathrm{~s}$. This means that with this initial distance, a collision between vehicles is unavoidable. As opposite, for initial distances more than 83.4 m , markers are shifted to the right of corresponding lines. That implies that $T_{T T C}^{i}(t)$ at the moment $\bar{\tau}_{i}-C_{*}^{i} \Delta_{r}$ is less then $T_{A E B S}=3 \mathrm{~s}$, and a collision can be avoided with probability no less than $C_{*}^{i}$. The yellow marker lying just at the corresponding line implies that 83.4 m is the minimum safe distance. However, it can be seen that for all presented lines $T_{T T C}^{i}(t)$ at the moment $t=\bar{\tau}_{i}-C_{*}^{i} \Delta_{r}$ exceeds 2 s , and no collision can be avoided by AEBS with such a threshold.

Since (26) represents a monotonic non-increasing function from $d_{i}$, we can calculate a minimum safe distance for the considered scenario with two identical vehicles. In Fig. 6, the minimum safe distance is plotted versus the deceleration capability $a_{i}$. It can be seen that for some deceleration values, no safe distance exists, having chosen initial velocity. For example, no safe initial distance can be found for vehicles with the maximum deceleration capability $5 \mathrm{~m} / \mathrm{s}^{2}$ and initial velocity $30 \mathrm{~m} / \mathrm{s}$. We should emphasize that this fact is due to the restriction of no hard braking before $T_{T T C}^{i}(t)$ is equal to or less than $T_{A E B S}=3 \mathrm{~s}$.

In order to compare the two approaches, we calculated packet loss probabilities for the wireless solution, which correspond to minimum safe distances related to the radar case (presented in Fig. 6). In Fig. 7, values of $p_{i}$ 's that allow to have such distances are presented. It is assumed that those $p_{i}$ 's account for both: the collision probability of the IEEE 802.11p protocol and the path loss effects. Packet loss probabilities lower than


Fig. 6. The radar case. Dependence of the minimum safe distance $d_{i}$ from the deceleration capability for different initial velocities $v_{0}$. Here, $\bar{a}_{i-1}=\bar{a}_{i}$.


Fig. 7. Dependence of the packet loss probability corresponding to the minimum safe distance in radar case (presented in Fig. 6), from the deceleration capability for different initial velocities. Here, $a_{i-1}=a_{i}$.
those presented in Fig. 7, let to decrease IVD and still to avoid rear-end collisions in the considered emergency scenario with probability $C^{*}$. Thus, for the already mentioned example of two identical vehicles with $a_{i-1}=a_{i}=7 \mathrm{~m} / \mathrm{s}^{2}$ and $v_{0}=30 \mathrm{~m} / \mathrm{s}$, the packet loss probability $p_{i}=0.81$ allow to have the same IVD as in the radar case. However, with improved quality of wireless communication, i.e., having $p_{i}<0.81$, the IVD between considered vehicles can be shortened. One can expect that in practice packet loss probabilities in V2V platooning communications are higher than this value.

In Figs. 8 and 9, introduced metrics of safe braking are presented. The probability of safe braking $Q_{W}$ is given for different values of packet loss probability $p_{i}$. It can be seen that especially for high velocities where it is critical to brake early, the wireless solution with a reliable channel quality outperforms the radar based solution in terms of allowed IVDs. In opposite, mainly

Q( $\left.\mathbf{d}_{\mathbf{i}}\right)$


Fig. 8. Probabilities of safe braking, $Q_{R}$ and $Q_{W}$, for the two vehicles case versus the initial distance $d_{i} . Q_{W}$ is shown for $p_{1}=0.1: 0.2: 0.9$. Here, $\bar{a}_{i}=$ $\bar{a}_{i-1}=7 \mathrm{~m} / \mathrm{s}^{2}, v_{0}=30 \mathrm{~m} / \mathrm{s}^{2}$.
$Q\left(d_{i}\right)$


Fig. 9. Probabilities of safe braking, $Q_{R}$ and $Q_{W}$, for the two vehicles case versus the initial distance $d_{i} . Q_{W}$ is calculated for $p_{1}=0.1: 0.2: 0.9$. Here, $\bar{a}_{i}=\bar{a}_{i-1}=7 \mathrm{~m} / \mathrm{s}^{2}, v_{0}=10 \mathrm{~m} / \mathrm{s}^{2}$.
for low velocities (Fig. 9), the radar based approach allows for shorter IVDs in conditions with a poor V2V communication quality. In real implementations, platoons will use a fusion of the radar based and V2V approaches. In other words, having degraded performance of wireless communication links, i.e., high values of $p_{i}$ 's, the platooning operation will fall back on radar sensors.

As was mentioned in Section IV, for the wireless solution having $N=2$, braking strategies for the decentralized and centralized approaches coincide. However, for $N>2$ centralized approach can give even better performance in terms of considered function $J$, i.e., the whole length of the platoon. Below we demonstrate how the performance of metric $J$ can be improved for platoons with $N=3$ and $N=4$ vehicles.

TABLE II
Comparison of the Distributed and the Centralized Approaches for Six Different Numerical Settings, $N=3$. In the "Setting" Column, "C" Refers to the Centralized Approach and "D" to the Distributed

| Setting | $A_{1}$ | $A_{2}$ | $a_{1}^{*}, m / s^{2}$ | $d_{1}, m$ | $d_{2}, m$ | $J, m$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1C | 1 | 1 | 4.9 | 8.1 | 8.03 | 16.1 |
| 1D | 1 | 1 | 7.5 | 1.7 | 30.15 | 31.85 |
| 2C | 1 | 2 | 4.63 | 11.73 | 5.3 | 22.34 |
| 2D | 1 | 2 | 7.5 | 1.7 | 30.15 | 62 |
| 3C | 2 | 1 | 5.23 | 4.86 | 12.1 | 21.82 |
| 3D | 2 | 1 | 7.5 | 1.7 | 30.15 | 33.55 |
| 4C | 1 | 1 | 4.9 | 20.7 | 20.7 | 41.38 |
| 4D | 1 | 1 | 7.5 | 3.12 | 42.78 | 45.9 |
| 5C | 1 | 2 | 3.9 | 36.98 | 5.3 | 47.6 |
| 5D | 1 | 2 | 7.5 | 3.12 | 42.78 | 88.68 |
| 6C | 2 | 1 | 6.64 | 4.86 | 37.35 | 47.07 |
| 6D | 2 | 1 | 7.5 | 3.12 | 42.78 | 49.02 |



Fig. 10. Graphs of $J$ corresponding to 6 different settings from the Table II. Number (.) near a line denotes the setting's number. Used values for the computation are $v_{0}=25 \mathrm{~m} / \mathrm{s}, p_{1}=0.1, p_{2}=0.2$.

## A. Distributed and Centralized Approaches for the $N=3$ Case

Results of the distributed and centralized approaches for a three-truck platoon are compared in Table II. Here, optimal values of the objective function $J\left(a_{2}\right)=A_{1} d_{1}+A_{2} d_{2}$ and corresponding IVDs are presented for 6 different numerical experiments. For the settings $1-3, \bar{a}_{0}=4.5, \bar{a}_{2}=5.5 \mathrm{~m} / \mathrm{s}^{2}$, i.e., $\bar{a}_{0}<\bar{a}_{2}$. For the settings $4-6$, values $\bar{a}_{0}$ and $\bar{a}_{2}$ swapped, such that $\bar{a}_{0}>\bar{a}_{2}$. The deceleration of the middle vehicle is not fixed, with a maximum possible $\bar{a}_{1}=7.5 \mathrm{~m} / \mathrm{s}^{2}$ in all settings.

In Fig. 10, $J$ is plotted versus deceleration $a_{1}$ where red markers correspond to the centralized approach solution. The distributed approach gives $a_{1}=\bar{a}_{1}$ as a solution, which means that corresponding to this approach points on the graphs are located in the most right position. For $a_{1}<3 \mathrm{~m} / \mathrm{s}^{2}$, all curves are monotonically decreasing and not included in Fig. 10 for a better scale.

It is worth mentioning that if penalty coefficients are equal, i.e., $A_{1}=A_{2}$, the objective function $J$ has a minimum interval opposite to one point in the other cases. It implies that every choice of braking capability $a_{1}$ from this interval gives the same minimum value of $J$. Basically, it means that the distance


Fig. 11. Surface $J$ for the case $N=4$. Used values for the computation are $v_{0}=25 \mathrm{~m} / \mathrm{s}, p_{1}=0.1, p_{2}=0.2, p_{3}=0.3$.
between the leading vehicle and the last one is fixed, whereas the middle vehicle can change its position in some interval by choosing an appropriate braking capacity. In Table II, one random point from every such interval is presented.

In Fig. 10, red lines are above the blue ones. It imposes that IVDs are shorter when the leader and the last vehicle are ordered according to their increasing braking capabilities (settings (1-3)) than in the case when their braking capabilities take a decreasing order (settings (4-6)). In the latter case, longer distances are a necessity between the trucks to reach safe-braking metrics close to 1 .

From Fig. 10 and Table II, it can be seen that the centralized approach achieves a smaller value of $J$, i.e., better fuel economy or shorter length of the whole platoon (depending on the type of chosen penalty coefficients). However, if the parameter $\bar{a}_{1}$ is chosen to the left of red markers, then the centralized approach gives $a_{1}=\bar{a}_{1}$, i.e., the same solution as the distributed approach.

## B. Distributed and Centralized Approaches for the $N=4$ Case

For the case $N=4$, the objective function $J$ depends on two variables: $J=J\left(a_{1}, a_{2}\right)$. Fig. 11 demonstrates the surface of this objective function for the following selected parameters setting $\bar{a}_{0}=4.5, \bar{a}_{1}=7, \bar{a}_{2}=7, \bar{a}_{3}=6.5 \mathrm{~m} / \mathrm{s}^{2}$. According to the results presented in Section IV-B, both distributed and centralized approaches give $a_{0}=\bar{a}_{0}, a_{3}=\bar{a}_{3}$ as a solution. Deceleration capabilities of the middle vehicles are restricted to $a_{1}=5.03$ and $a_{2}=5.64 \mathrm{~m} / \mathrm{s}^{2}$ in the centralized solution, whereas $a_{1}=\bar{a}_{1}$ and $a_{2}=\bar{a}_{2}$ for the distributed case. Restrictions posed on deceleration capacities of the 1 -st and the 2-nd vehicles by the centralized solution, achieve smaller IVDs: $d_{1}=6.46, d_{2}=5.81, d_{3}=6.45 \mathrm{~m}$ (corresponding value of $J$ is 18.72 m ), in comparison to the centralized approach where $d_{1}=1.91, d_{2}=12.5, d_{3}=17.18 \mathrm{~m}$ (corresponding value of $J$ is 31.59 m ).

## VII. CONCLUSION

Platooning holds a great promise of increasing safety of the traffic as well as increasing road throughput, but the safety analysis for platooning is still incomplete. This paper provides protocols that, using given V2V communication quality, select inter-vehicle distances between consecutive vehicles, ensuring the safety of platooning in emergency braking scenarios. The framework can be used both in a distributed and in a centralized manner, where in the former, each vehicle determines its own safe distance to the vehicle in front. In the latter, the platoon leader gathers relevant data from all platoon members to decide upon appropriate IVDs for all trucks. A key feature of the centralized protocol is the ability to decrease IVDs by temporally reducing braking capabilities of the vehicles in the platoon. Thus, better fuel economy, as well as less road space, are achieved. If the quality of V 2 V communication has degraded, the platoon can adjust to new circumstances by changing distances. Both approaches can outperform purely radar-based platoons where each vehicle gathers only information about neighboring vehicles. In such systems, longer IVDs are a disadvantage of the classical AEBS where no harsh braking is allowed to start until a certain threshold of such metrics as TTC is not reached. A lower threshold of TTC leads to the increased number of false positive braking, whereas V2V that allows to know explicitly plans and decisions of all platooning vehicles, helps to avoid such undesirable braking.

The following directions are considered for the future work:

- Incorporation of more realistic dynamic models of vehicles, e.g., time-varying decelerations;
- Incorporation of more realistic environment models, e.g., time-varying road slopes;
- Consideration of other types of AEBS, e.g., where slight braking is allowed during a "warning phase"; where a threshold of harsh braking is not a constant; where other than TTC metrics are used to trigger an "emergency phase".
- Fusion of wireless and radar based control systems to ensure safety in emergency braking scenarios.
- Applying the same approach of finding minimum safe distances to the closed-loop dynamics for specific platoon controllers. In such a setting, explicit or computationally efficient numerical solutions should be found and investigated to find tighter bounds on the inter-vehicle spacing policy.
- Further linking of the developed framework to the SOTIF standard [12] through quantification of a collision severity.


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