Abstract—

# Double-RIS Aided Multi-user MIMO Communications: Common

Reflection Pattern and Joint Beamforming Design

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tered the public consciousness as a promising technology for enhancing the performance of future wireless communication systems by dynamically constructing the wireless channels. In this letter, we study a double-RIS aided downlink multi-user multiple-input multiple-output (MIMO) communication system. We investigate the mean-square-error (MSE) minimization problem by jointly optimizing the active transmit beamforming, the receive equalizer and the passive beamforming at each RIS. Different from prior works, for the sake of reducing both communication overhead and signal processing complexity, we assume that the two RISs utilize the common reflection pattern. Under this assumption, the coupling of the variables becomes tighter, thereby making the optimization problem more challenging to solve. To effectively address this issue, we propose a majorization-minimization (MM)-based alternating optimization (AO) algorithm. Numerical results show that in high signal-tonoise ratio (SNR) region, the double-RIS with common reflection pattern can achieve nearly the same performance as that with separate reflection pattern whereas the complexity is only half of the latter. Thus, our proposed design enables an effective tradeoff between the performance and the implementation complexity of the considered system.

- Reconfigurable intelligent surface (RIS) has en-

Index Terms—Double-RIS, multi-user MIMO system, MSE, common reflection pattern.

#### I. INTRODUCTION

S a key enabling technology for the future generation wireless systems, reconfigurable intelligent surface (RIS) can adaptively reconfigure the wireless communication environment to improve both energy and spectrum efficiency [1], [2]. In general, RIS is an intelligent metasurface made up of a large number of passive reflecting elements. Each element can independently adjust the input electromagnetic (EM) signal's amplitude and phase [3]. In contrast to traditional multipleinput multiple-output (MIMO) relay communications [4], RIS can provide higher beamforming gains while consuming less power. Moreover, thanks to additional advantages of easy deployment, environment friendly, high compatibility and low cost [5], RIS has been widely used in wireless communication systems.

Most existing works only considered the passive beamforming design for the single-RIS scenario [6]–[8]. For example, [6] jointly designed the transmit covariance matrix and RIS reflection pattern matrix by investigating the channel capacity maximization problem considering a single-user MIMO communication system aided by one RIS. The work in [7] provided a general framework for the transceiver designs in the single-RIS aided single-user and multi-user MIMO communication systems. To further explore the RIS's potential in enhancing wireless communication performance, some works extend the single-RIS scenario to the multi-RIS scenario. For example, [9] studied the mean-square-error (MSE) minimization problem by jointly optimizing the RIS phase shifts, the transmit beamforming and the receive equalizers considering a multiple but non-cooperative RISs aided MIMO system. The work in [10] aimed to maximize the channel capacity considering a cooperative double-RIS empowered single-user MIMO system with line-of-sight (LoS) channel, while [11] further extended the above study to the cooperative double-RIS assisted multiuser MIMO system. The authors jointly optimized the receive beamforming and the cooperative reflection coefficients at two RISs aiming to maximize the worst signal-to-interference-plusnoise ratio (SINR) of all users.

In this letter, from the perspective of reducing the high communication overhead and signal processing complexity induced by the large-scale RIS, we consider the cooperative double-RIS aided downlink multi-user MIMO communication system with the common reflection pattern. We resort to minimize the average sum MSE of all data symbols by jointly optimizing the active transmit beamforming matrices, the receive equalizer and the common RIS reflection pattern. The formulated optimization problem is generally more challenging to solve than its counterpart with separate reflection pattern due to the strongly coupled optimization variables. To tackle this difficult problem, a majorization-minimization (MM)based alternating optimization (AO) algorithm is proposed. Numerical simulation results illustrate the better performance of the proposed algorithm when compared with the single-RIS system, and also show that our proposed algorithm can perform as best as the double-RIS system with the separate reflection pattern in the high-SNR region.

#### II. PROBLEM FORMULATION AND SYSTEM MODEL

As shown in Fig. 1, we consider a double-RIS aided downlink MIMO communication system with K users, where each user is equipped with  $N_{r,k}$  antennas and the BS is equipped with  $N_t$  antennas. The two distributed RISs, namely, RIS 1 and RIS 2, are assumed to be near the BS and the users, respectively, so that the productive path loss of the double-RIS cascaded channel is minimized [10]. Since the two RISs utilize the common reflection pattern, we assume they have the equal number of reflection elements, denoted as M. The direct channel between the BS and the users is ignored due to the blockages.

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Let  $\mathbf{T}_l \in \mathbb{C}^{M \times N_t}$ ,  $\mathbf{S} \in \mathbb{C}^{M \times M}$  and  $\mathbf{R}_{l,k} \in \mathbb{C}^{N_{r,k} \times M}$ denote the channel matrices for the BS  $\rightarrow$  RIS l, RIS  $1 \rightarrow$ RIS 2 and RIS  $l \rightarrow$  user k links, respectively, with l = 1, 2and k = 1, 2, ..., K. The common reflection coefficients of two RISs are introduced as  $\boldsymbol{\Theta} = \text{diag}\{\theta_1, \cdots, \theta_M\}$ , where the amplitude of each element is assumed to be 1. Under the aforementioned settings, an effective cascaded channel between the BS and the kth user is expressed as

$$\mathbf{H}_{k} = \mathbf{R}_{1,k} \Theta \mathbf{T}_{1} + \mathbf{R}_{2,k} \Theta \mathbf{T}_{2} + \mathbf{R}_{2,k} \Theta \mathbf{S} \Theta \mathbf{T}_{1}.$$
 (1)

We assume total N data streams are transmitted from the BS to all K users and  $\mathbf{x} \triangleq \left[\mathbf{x}_{1}^{\mathrm{H}}, \cdots, \mathbf{x}_{K}^{\mathrm{H}}\right]^{\mathrm{H}} \in \mathbb{C}^{N \times 1}$  is defined as the transmitted data symbol vector, where  $\mathbf{x}_{k} \in \mathbb{C}^{N_{k} \times 1}$  is the data symbols for the kth user, obeying Gaussian distribution, i.e.,  $\mathbf{x}_{k} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_{k}})$  and  $\sum_{k=1}^{K} N_{k} = N$ .  $\mathbf{F} \triangleq [\mathbf{F}_{1}, \cdots, \mathbf{F}_{K}] \in \mathbb{C}^{N_{t} \times N_{k}}$  is defined as the transmit beamformer, where  $\mathbf{F}_{k} \in \mathbb{C}^{N_{t} \times N_{k}}$  is the kth user's beamformer. For simplicity, we assume that the BS has perfect knowledge of system global channel state information (CSI). The received signal  $\mathbf{y}_{k} \in \mathbb{C}^{N_{r,k} \times 1}$  at the kth user side is then given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{F} \mathbf{x} + \mathbf{z}_k, \tag{2}$$

where  $\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{N_{r,k}})$  is the additive white Gaussian noise (AWGN). For the *k*th user, upon receiving  $\mathbf{y}_k$ , the equalizer  $\mathbf{G}_k \in \mathbb{C}^{N_k \times N_{r,k}}$  is used to decode the transmitted signal, i.e.,  $\hat{\mathbf{x}}_k = \mathbf{G}_k \mathbf{y}_k$ . Accordingly, the MSE matrix of the estimated signal  $\hat{\mathbf{x}}_k$  is expressed as

$$\begin{split} \Psi_{\text{MSE},k}(\mathbf{F}_k, \mathbf{G}_k, \mathbf{\Theta}) \\ &= \mathbb{E}\left\{ (\mathbf{x}_k - \hat{\mathbf{x}_k})^{\text{H}} (\mathbf{x}_k - \hat{\mathbf{x}_k}) \right\} \\ &= \mathbf{G}_k \mathbf{H}_k \mathbf{F} (\mathbf{G}_k \mathbf{H}_k \mathbf{F})^{\text{H}} - 2\Re(\mathbf{G}_k \mathbf{H}_k \mathbf{F}_k) + \sigma_n^2 \mathbf{G}_k \mathbf{G}_k^{\text{H}} + \mathbf{I}_{N_k}. \end{split}$$
(3)

In this letter, we aim to minimize the average sum MSE of all data symbols by jointly designing the BS transmit beamforming matrices  $\{\mathbf{F}_k\}_{k=1}^K$ , the linear receive equalizers  $\{\mathbf{G}_k\}_{k=1}^K$  and the common RIS reflection pattern  $\boldsymbol{\Theta}$ , subject to the total BS transmit power constraint and the unit-modulus constraints for all RIS reflecting elements. The corresponding optimization problem is formulated as

(P1) 
$$\min_{\{\mathbf{F}_{k},\mathbf{G}_{k}\}_{k=1}^{K},\mathbf{\Theta}} \sum_{k=1}^{K} \operatorname{tr}(\boldsymbol{\Psi}_{\mathrm{MSE},k}(\mathbf{F}_{k},\mathbf{G}_{k},\mathbf{\Theta})) \quad (4a)$$

s.t. 
$$\operatorname{tr}(\mathbf{F}\mathbf{F}^{\mathrm{H}}) \le P$$
, (4b)

$$\boldsymbol{\Theta} = \operatorname{diag}\{\theta_1, \cdots, \theta_M\},\tag{4c}$$

$$|\theta_i| = 1, \quad i \in \{1, \cdots, M\},\tag{4d}$$

where P is the BS maximum transmit power. It can be observed that problem (P1) is challenging to solve due to the strongly coupled variables in the objective function (4a) and the non-convex constraints in (4d). Moreover, the adopted common RIS reflection pattern leads to an intractable fourthorder term in the objective function (4a), which also greatly increases the difficulty of solving problem (P1) when compared to the separate reflection pattern case. Next, we present an MM-based AO algorithm to decouple the variables and tackle the intractable high-order term caused by the common reflection pattern.



Fig. 1. A double-RIS aided multi-user MIMO system.

#### III. PROPOSED MM-BASED AO ALGORITHM

In this section, an MM-based AO algorithm is proposed to solve the challenging problem (P1), where each optimization variable is updated while keeping the others fixed. By leveraging the MM technique, each subproblem admits a closed-form optimal solution.

#### A. Equalizer and Transmit Beamforming Optimization

Firstly, we derive the optimal equalizer  $\mathbf{G}_k$  with the fixed  $\{\mathbf{F}_k\}_{k=1}^K$  and  $\boldsymbol{\Theta}$ . In this case, It can be easily found that the problem (P1) w.r.t  $\mathbf{G}_k$  is an unconstrained convex problem. Based on the LMMSE criterion, the optimal  $\mathbf{G}_k^{\text{opt}}$  can be derived as

$$\mathbf{G}_{k}^{\text{opt}} = \mathbf{F}_{k}^{H} \mathbf{H}_{k}^{H} (\mathbf{H}_{k} \mathbf{F} \mathbf{F}^{H} \mathbf{H}_{k}^{H} + \mathbf{R}_{\mathbf{z}_{k}})^{-1}.$$
 (5)

Secondly, with the fixed  $\{\mathbf{G}_k\}_{k=1}^K$  and  $\boldsymbol{\Theta}$ , we optimize the active transmit beamforming  $\mathbf{F}_k$ . We reformulate problem (P1) as

(P2) 
$$\min_{\mathbf{F}_{k}} \sum_{k=1}^{K} \operatorname{tr}(\boldsymbol{\Psi}_{\mathrm{MSE},k}(\mathbf{F}_{k})),$$
(6a)

s.t. 
$$\operatorname{tr}(\mathbf{F}\mathbf{F}^{\mathrm{H}}) \leq P.$$
 (6b)

Problem (P2) can be easily found to be a quadratic convex problem w.r.t.  $\mathbf{F}_k$ . So the Karush-Kuhn-Tucker (KKT) conditions can be utilized to obtain the optimal  $\mathbf{F}_k$ . Specifically, defining  $\mu$  as the Lagrange multiplier, problem (P2)'s Lagrangian function is expressed as

$$L(\mathbf{F}_k, \mu) = \sum_{k=1}^{K} \operatorname{tr}(\boldsymbol{\Psi}_{\mathrm{MSE},k}(\mathbf{F}_k)) + \mu(\operatorname{tr}\left\{\mathbf{F}\mathbf{F}^{\mathrm{H}}\right\} - P).$$
(7)

Then the KKT conditions can be derived as

$$\mathbf{F}_{k} = \left(\sum_{i=1}^{N} \mathbf{H}_{i}^{H} \mathbf{G}_{i}^{H} \mathbf{G}_{i} \mathbf{H}_{i} + \mu \mathbf{I}_{Nt}\right)^{-1} \mathbf{H}_{k}^{H} \mathbf{G}_{k}^{H}, \quad (8a)$$

$$\mu(tr{FF^{H}} - P) = 0, \ \mu \ge 0,$$
 (8b)

$$\operatorname{tr}\{\mathbf{F}\mathbf{F}^{\mathrm{H}}\} - P \le 0. \tag{8c}$$

It follows from (8a) that the optimal  $\mathbf{F}_k$  depends on the Lagrange multiplier  $\mu$ . Specifically, if  $\operatorname{tr}(\mathbf{FF}^{\mathrm{H}}) \leq P$  and  $\mu = 0$ , we have  $\mathbf{F}_k = (\sum_{i=1}^{K} \mathbf{H}_i^H \mathbf{G}_i^H \mathbf{G}_i \mathbf{H}_i)^{-1} \mathbf{H}_k^H \mathbf{G}_k^{\mathrm{H}}$ . If  $\mu > 0$ , we have  $\operatorname{tr}\{\mathbf{FF}^{\mathrm{H}}\} = P$ . Let  $\sum_{i=1}^{K} \mathbf{H}_i^H \mathbf{G}_i^H \mathbf{G}_i \mathbf{H}_i = \mathbf{U} \Lambda \mathbf{U}^H$ , where  $\Lambda = \operatorname{diag}\{\lambda_1, \ldots, \lambda_M\}$  is the diagonal matrix consisting of eigenvalues of  $\sum_{i=1}^{K} \mathbf{H}_i^H \mathbf{G}_i^H \mathbf{G}_i \mathbf{H}_i$ . Substituting

this decomposition into (8a), the BS transmit power is monotonically decreasing over  $\mu$  as follows:

$$\operatorname{tr}(\mathbf{F}\mathbf{F}^{H}) = \operatorname{tr}\left[\sum_{k=1}^{K} (\sum_{i=1}^{K} \mathbf{H}_{i}^{H} \mathbf{G}_{i}^{H} \mathbf{G}_{i} \mathbf{H}_{i} + \mu \mathbf{I}_{Nt})^{-2} \mathbf{H}_{k}^{H} \mathbf{G}_{k}^{H} \mathbf{G}_{k} \mathbf{H}_{k}\right]$$
$$= \operatorname{tr}\left[(\mathbf{\Lambda} + \mu \mathbf{I}_{N})^{-2} \mathbf{R}\right] = \sum_{i=1}^{N} \frac{[\mathbf{R}]_{i,i}}{(\lambda_{i} + \mu)^{-2}},$$
(9)

where  $\mathbf{R} \triangleq \mathbf{U}^{H}(\sum_{k=1}^{K} \mathbf{H}_{k}^{H} \mathbf{G}_{k}^{H} \mathbf{G}_{k} \mathbf{H}_{k})\mathbf{U}$ . Thus, the optimal  $\mu$  can be efficiently found through one-dimensional search methods, e.g., bisection search [12]. Then the optimal  $\mathbf{F}_{k}$  can be obtained from (8a).

### B. Common Reflection Pattern Optimization

After obtaining  $\{\mathbf{F}_k\}_{k=1}^K$  and  $\{\mathbf{G}_k\}_{k=1}^K$ , we aim to optimize the common RIS reflection pattern  $\Theta$ . Different from the traditional double-RIS system with the separate reflection pattern, the common reflection pattern makes the objective function more complicated and higher-order. Instead of directly solving it, we explore its inherent structure to simplify the problem. To be specific, by omitting the irrelevant constants, the objective function can be written as

$$f_{\text{Obj}}(\Theta)$$

$$= \sum_{k=1}^{K} [\text{tr}(\sum_{i=1}^{2} \sum_{j=1}^{2} \mathbf{G}_{k} \mathbf{R}_{i,k} \Theta \mathbf{T}_{i} \mathbf{F} \mathbf{F}^{\text{H}} \mathbf{T}_{j}^{\text{H}} \Theta^{\text{H}} \mathbf{R}_{j,k}^{\text{H}} \mathbf{G}_{k}^{\text{H}}$$

$$+ \sum_{i=1}^{2} (\mathbf{G}_{k} \mathbf{R}_{i,k} \Theta \mathbf{T}_{i} \mathbf{F} \mathbf{F}^{\text{H}} \mathbf{T}_{1}^{\text{H}} \Theta^{\text{H}} \mathbf{S}^{\text{H}} \Theta^{\text{H}} \mathbf{R}_{2,k}^{\text{H}} \mathbf{G}_{k}^{\text{H}}$$

$$+ \mathbf{G}_{k} \mathbf{R}_{2,k} \Theta \mathbf{S} \Theta \mathbf{T}_{1} \mathbf{F} \mathbf{F}^{\text{H}} \mathbf{T}_{1}^{\text{H}} \Theta^{\text{H}} \mathbf{R}_{i,k}^{\text{H}} \mathbf{G}_{k}^{\text{H}})$$

$$+ \mathbf{G}_{k} \mathbf{R}_{2,k} \Theta \mathbf{S} \Theta \mathbf{T}_{1} \mathbf{F} \mathbf{F}^{\text{H}} \mathbf{T}_{1}^{\text{H}} \Theta^{\text{H}} \mathbf{R}_{2,k}^{\text{H}} \mathbf{G}_{k}^{\text{H}})$$

$$- 2 \Re (\text{tr}(\sum_{i=1}^{2} \mathbf{G}_{k} \mathbf{R}_{i,k} \Theta \mathbf{T}_{i} \mathbf{F} + \mathbf{G}_{k} \mathbf{R}_{2,k} \Theta \mathbf{S} \Theta \mathbf{T}_{1} \mathbf{F}))].$$
(10)

By stacking the diagonal elements of RIS reflection coefficients  $\Theta$  and a general matrix  $\mathbf{P}$  into the vectors  $\boldsymbol{\theta} \triangleq \operatorname{diag}(\boldsymbol{\Theta})$ and  $\mathbf{p} \triangleq \operatorname{diag}(\mathbf{P})$ , respectively, and using the identities  $\operatorname{tr}(\mathbf{X}\mathbf{Y}\mathbf{Z}\mathbf{W}) = \operatorname{vec}(\mathbf{X}^{\mathrm{H}})^{\mathrm{H}}(\mathbf{W}^{\mathrm{T}} \otimes \mathbf{Y})\operatorname{vec}(\mathbf{Z})$  and  $\operatorname{tr}(\boldsymbol{\Theta}\mathbf{P}) = \boldsymbol{\theta}^{\mathrm{H}}\mathbf{p}^{*}$ , we have

$$f_{\rm Obj}(\boldsymbol{\Theta})$$

 $= \operatorname{vec}(\Theta)^{\mathrm{H}} \mathbf{A} \operatorname{vec}(\Theta) + \operatorname{vec}(\Theta)^{\mathrm{H}} \mathbf{B} \operatorname{vec}(\Theta \mathbf{S} \Theta) + \operatorname{vec}(\Theta \mathbf{S} \Theta)^{\mathrm{H}} \mathbf{B}^{\mathrm{H}} \operatorname{vec}(\Theta) + \operatorname{vec}(\Theta \mathbf{S} \Theta)^{\mathrm{H}} \mathbf{C} \operatorname{vec}(\Theta \mathbf{S} \Theta) - 2\Re(\theta^{\mathrm{H}} \mathbf{p}^{*} + \operatorname{vec}(\Theta^{\mathrm{H}})^{\mathrm{H}} \mathbf{D} \operatorname{vec}(\Theta)),$  (11)

where

$$\mathbf{A} = \sum_{i=1}^{2} \sum_{j=1}^{2} ((\mathbf{T}_{i} \mathbf{F} \mathbf{F}^{\mathrm{H}} \mathbf{T}_{j}^{\mathrm{H}})^{\mathrm{T}} \otimes \sum_{k=1}^{K} \mathbf{R}_{j,k}^{\mathrm{H}} \mathbf{G}_{k}^{\mathrm{H}} \mathbf{G}_{k} \mathbf{R}_{i,k}),$$

$$\mathbf{B} = \sum_{i=1}^{2} (\mathbf{T}_{1} \mathbf{F} \mathbf{F}^{\mathrm{H}} \mathbf{T}_{i}^{\mathrm{H}})^{\mathrm{T}} \otimes \sum_{k=1}^{K} \mathbf{R}_{i,k}^{\mathrm{H}} \mathbf{G}_{k}^{\mathrm{H}} \mathbf{G}_{k} \mathbf{R}_{2,k},$$

$$\mathbf{C} = (\mathbf{T}_{1} \mathbf{F} \mathbf{F}^{\mathrm{H}} \mathbf{T}_{1}^{\mathrm{H}})^{\mathrm{T}} \otimes \sum_{k=1}^{K} \mathbf{R}_{2,k}^{\mathrm{H}} \mathbf{G}_{k}^{\mathrm{H}} \mathbf{G}_{k} \mathbf{R}_{2,k},$$

$$\mathbf{D} = \sum_{k=1}^{K} (\mathbf{T}_{1} \mathbf{F} \mathbf{G}_{k} \mathbf{R}_{2,k})^{\mathrm{T}} \otimes \mathbf{S}, \mathbf{P} = \sum_{k=1}^{K} (\sum_{i=1}^{2} \mathbf{T}_{i} \mathbf{F}_{k} \mathbf{G}_{k} \mathbf{R}_{i,k}).$$
(12)

Thanks to the sparse structure of  $\operatorname{vec}(\Theta)$ , the objective function (11) can be further simplified. Specifically, let  $\mathbf{A}_0 \in \mathbb{C}^{M \times M}$  and  $\mathbf{D}_0 \in \mathbb{C}^{M \times M}$  be made up of all elements at ] the intersections of the [(m-1)M + m]th column and the [(n-1)M + n]th row of  $\mathbf{A}$  and  $\mathbf{D}$  for  $m, n = 1, \ldots, M$ , respectively.  $\mathbf{B}_0$  is made up of all elements of the columns  $[(q-1)M + q], q = 1, \ldots, M$  of  $\mathbf{B}$  and  $\mathbf{C}_0 = \mathbf{C}$ . We define  $\mathbf{v} \triangleq \operatorname{vec}(\Theta \mathbf{S} \Theta) = (\Theta^{\mathrm{T}} \otimes \Theta) \operatorname{vec}(\mathbf{S})$ . Then the subproblem in terms of the common RIS reflection pattern is expressed as

(P3) 
$$\min_{\boldsymbol{\theta}} \boldsymbol{\theta}^{\mathrm{H}} \mathbf{A}_{0} \boldsymbol{\theta} + \boldsymbol{\theta}^{\mathrm{H}} \mathbf{B}_{0} \mathbf{v} + \mathbf{v}^{\mathrm{H}} \mathbf{B}_{0}^{\mathrm{H}} \boldsymbol{\theta} + \mathbf{v}^{\mathrm{H}} \mathbf{C}_{0} \mathbf{v} - 2\Re(\boldsymbol{\theta}^{\mathrm{H}} \mathbf{p}^{*} + \boldsymbol{\theta}^{\mathrm{H}} \mathbf{D}_{0}^{*} \boldsymbol{\theta}^{*})$$
  
s.t.  $\mathbf{v} = (\boldsymbol{\Theta}^{\mathrm{T}} \otimes \boldsymbol{\Theta}) \operatorname{vec}(\mathbf{S}),$   
 $\boldsymbol{\Theta} = \operatorname{diag} \{ \theta_{1}, \cdots, \theta_{M} \},$   
 $|\theta_{i}| = 1, \quad i \in \{1, \cdots, M\}.$  (13)

Problem (P3) is still hard to get the solutions due to the coupling of  $\Theta$  and  $\mathbf{S}$  in the vector  $\mathbf{v}$ . Next, we resort to separate them. Define  $\mathbf{B}_0 \triangleq [\mathbf{B}_{0,1}, \dots, \mathbf{B}_{0,M}] \in \mathbb{C}^{M \times M^2}$ , where  $\mathbf{B}_{0,p} \in \mathbb{C}^{M \times M}$  is the block matrix consisting of the [(p-1)M+1]th to the [pM]th column of  $\mathbf{B}_0$  for  $p=1, \dots, M$ .  $\mathbf{C}_0$  is similarly defined as  $\mathbf{C}_0 \triangleq \begin{bmatrix} \mathbf{C}_{0,1,1} & \cdots & \mathbf{C}_{0,1,M} \\ \vdots & \ddots & \vdots \\ \mathbf{C}_{0,M,1} & \cdots & \mathbf{C}_{0,M,M} \end{bmatrix}$ , where  $\mathbf{C}_{0,j,l} \in \mathbb{C}^{M \times M}$  is the block matrix consisting of the [(p-1)M+1]th to the lock matrix consisting of the  $[\mathbf{C}_{0,M,1} & \cdots & \mathbf{C}_{0,M,M}]$ .

where  $\mathbf{C}_{0,j,l} \in \mathbb{C}^{M \times M}$  is the block matrix consisting of the [(j-1)M+1]th row to the [jM]th row and the [(l-1)M+1]th column to the [lM]th column of  $\mathbf{C}_0$  for  $j, l=1, \ldots, M$ . Then we have

$$\boldsymbol{\theta}^{\mathrm{H}} \mathbf{B}_{0} \mathbf{v} = \sum_{i=1}^{M} \boldsymbol{\theta}^{\mathrm{H}} \mathbf{B}_{0,i} \mathrm{diag}(\mathbf{s}_{i}) \boldsymbol{\theta}_{i} \boldsymbol{\theta} = \boldsymbol{\theta}^{\mathrm{H}} \tilde{\mathbf{B}}(\boldsymbol{\theta} \otimes \boldsymbol{\theta}), \quad (14)$$
$$\mathbf{v}^{\mathrm{H}} \mathbf{C}_{0} \mathbf{v} = \sum_{l=1}^{M} \boldsymbol{\theta}^{\mathrm{H}} \sum_{j=1}^{M} \boldsymbol{\theta}_{j}^{\mathrm{H}} \mathrm{diag}(\mathbf{s}_{j}^{\mathrm{H}}) \mathbf{C}_{0,j,l} \mathrm{diag}(\mathbf{s}_{l}) \boldsymbol{\theta}_{l} \boldsymbol{\theta} \quad (15)$$
$$= (\boldsymbol{\theta} \otimes \boldsymbol{\theta})^{\mathrm{H}} \tilde{\mathbf{C}}(\boldsymbol{\theta} \otimes \boldsymbol{\theta}),$$

where  $\mathbf{s}_i$  is the *i*th column of  $\mathbf{S}$ ,  $\mathbf{\tilde{B}} = [\mathbf{B}_{0,1} \operatorname{diag}(\mathbf{s}_1), \ldots, \mathbf{B}_{0,M} \operatorname{diag}(\mathbf{s}_M)]$  and  $\mathbf{\tilde{C}} = [\mathbf{\tilde{C}}_1 \operatorname{diag}(\mathbf{s}_1), \ldots, \mathbf{\tilde{C}}_M \operatorname{diag}(\mathbf{s}_M)]$ with  $\mathbf{\tilde{C}}_l = [\operatorname{diag}(\mathbf{s}_1^{\mathrm{H}})\mathbf{C}_{0,1,l}, \ldots, \operatorname{diag}(\mathbf{s}_M^{\mathrm{H}})\mathbf{C}_{0,M,l}]^{\mathrm{T}}$ . Based on the above definitions, (P3) becomes

(P4) 
$$\min_{\boldsymbol{\theta}} \tilde{\boldsymbol{\theta}}^{\mathrm{H}} \tilde{\mathbf{W}} \tilde{\boldsymbol{\theta}} - 2\Re(\boldsymbol{\theta}^{\mathrm{H}} \mathbf{p}^{*} + \boldsymbol{\theta}^{\mathrm{H}} \mathbf{D}_{0}^{*} \boldsymbol{\theta}^{*})$$
  
s.t.  $|\theta_{i}| = 1, \quad i \in \{1, \cdots, M\},$  (16)

where  $\tilde{\theta} = [\theta, \theta \otimes \theta]^{\mathrm{T}}$ ,  $\tilde{\mathbf{W}} = \begin{bmatrix} \mathbf{A}_0 & \tilde{\mathbf{B}} \\ \tilde{\mathbf{B}}^{\mathrm{H}} & \tilde{\mathbf{C}} \end{bmatrix}$ . It can be readily found that  $\tilde{\mathbf{W}}$  is the Hermitian matrix. Then, in order to obtain the closed-form solution, a more tractable surrogate function can

be constructed by the MM technique. Assuming the optimal  $\theta_t$  at the *t*th iteration, a surrogate function of  $\tilde{\theta}^{\rm H} \tilde{\mathbf{W}} \tilde{\theta}$  w.r.t.  $\tilde{\theta}$  can be derived as [13]

$$\tilde{\boldsymbol{\theta}}^{\mathrm{H}} \tilde{\mathbf{W}} \tilde{\boldsymbol{\theta}} \leq \tilde{\boldsymbol{\theta}}^{\mathrm{H}} \boldsymbol{\Lambda} \tilde{\boldsymbol{\theta}} + 2\Re \left( \tilde{\boldsymbol{\theta}}^{\mathrm{H}} \left( \tilde{\mathbf{W}} - \boldsymbol{\Lambda} \right) \tilde{\boldsymbol{\theta}}_{t} \right) + \tilde{\boldsymbol{\theta}}_{t}^{\mathrm{H}} \left( \tilde{\mathbf{W}} - \boldsymbol{\Lambda} \right) \tilde{\boldsymbol{\theta}}_{t}, \quad (17)$$

where  $\mathbf{\Lambda} = \lambda_{max}(\mathbf{\tilde{W}})\mathbf{I}_{M^2+M}$ . In order to avoid the high complexity induced by the eigenvalue decomposition of  $\mathbf{\tilde{W}}$ , i.e.,  $\mathcal{O}(M^2+M)^3$ , we choose  $\mathbf{\Lambda} = \operatorname{tr}(\mathbf{\tilde{W}})\mathbf{I}_{M^2+M} = \operatorname{tr}(\mathbf{A}_0)\mathbf{I}_M + \operatorname{tr}(\mathbf{\tilde{C}})\mathbf{I}_{M^2}$  as another efficient auxiliary parameter. In addition, by employing the unit-modulus property of the RIS reflection

coefficients,  $\tilde{\theta}^H \Lambda \tilde{\theta}$  can be transformed as  $\operatorname{tr}(\tilde{\mathbf{W}})(M + M^2)$ , which is a constant term irrelevant to the optimization variable.  $\tilde{\theta}_t^H \left(\tilde{\mathbf{W}} - \Lambda\right) \tilde{\theta}_t$  is also a constant because  $\tilde{\theta}_t$  is already known at the *t*th iteration. Therefore, the objective function in problem (P4) is simplified as

$$2\Re \left( \tilde{\boldsymbol{\theta}}^{\mathrm{H}} \left( \tilde{\mathbf{W}} - \boldsymbol{\Lambda} \right) \tilde{\boldsymbol{\theta}}_{t} \right) - 2\Re (\boldsymbol{\theta}^{\mathrm{H}} \mathbf{p}^{*} - \boldsymbol{\theta}^{\mathrm{H}} \mathbf{D}_{0}^{*} \boldsymbol{\theta}^{*})$$
  
=2\R(\beta^{\mathrm{H}} \tilde{\mathbf{u}}\_{t} + (\boldsymbol{\theta}^{\mathrm{H}} \otimes \boldsymbol{\theta}^{\mathrm{H}}) \mathbf{V}\_{t} - \boldsymbol{\theta}^{\mathrm{H}} \mathbf{D}\_{0}^{\*} \boldsymbol{\theta}^{\*})  
=2\R(\beta^{\mathrm{H}} \tilde{\mathbf{u}}\_{t} + \boldsymbol{\theta}^{\mathrm{H}} \tilde{\mathbf{V}}\_{t} \boldsymbol{\theta}^{\*}), \qquad (18)

where  $\tilde{\mathbf{u}}_t = (\mathbf{A}_0 - \lambda_{max}(\tilde{\mathbf{W}})\mathbf{I}_M)\tilde{\boldsymbol{\theta}}_t + \tilde{\mathbf{B}}(\tilde{\boldsymbol{\theta}}_t \otimes \tilde{\boldsymbol{\theta}}_t) - \mathbf{p}^*, \mathbf{V}_t = \tilde{\mathbf{B}}^{\mathrm{H}}\tilde{\boldsymbol{\theta}}_t + (\tilde{\mathbf{C}} - \lambda_{max}(\tilde{\mathbf{W}})\mathbf{I}_{M^2})(\tilde{\boldsymbol{\theta}}_t \otimes \tilde{\boldsymbol{\theta}}_t)$ , and  $\tilde{\mathbf{V}}_t = \hat{\mathbf{V}}_t - \mathbf{D}_0^*$ .  $\hat{\mathbf{V}}_t$  is the reshaped version of  $\mathbf{V}_t$ , i.e.,  $\mathbf{V}_t = \operatorname{vec}(\hat{\mathbf{V}}_t)$ . Then problem (P4) is equivalently transformed into

(P5) 
$$\min_{\boldsymbol{\theta}} 2\Re(\boldsymbol{\theta}^{\mathrm{H}}\tilde{\mathbf{u}}_{t} + \boldsymbol{\theta}^{\mathrm{H}}\tilde{\mathbf{V}}_{t}\boldsymbol{\theta}^{*})$$
  
s.t.  $|\boldsymbol{\theta}_{i}| = 1, \quad i \in \{1, \cdots, M\}.$  (19)

Using MM algorithm, we have transformed the original fourthorder problem (P4) into a second-order problem (P5). Such procedure greatly decreases the difficulty and complexity for solving the problem. However, problem (P5) is still nonconvex. To get the solution, we consider using MM algorithm again to find a more tractable surrogate function. To be specific, define  $\overline{\theta} \triangleq [\Re\{\theta^T\} \Im\{\theta^T\}]^T$  and  $\overline{\mathbf{V}}_t \triangleq \begin{bmatrix} \Re\{\tilde{\mathbf{V}}_t\} & \Im\{\tilde{\mathbf{V}}_t\} \\ \Im\{\tilde{\mathbf{V}}_t\} & \Im\{\tilde{\mathbf{V}}_t\} \end{bmatrix}$ . The real function  $\Re\{\theta^H\tilde{\mathbf{V}}_t\theta^*\}$  can be expressed as  $\overline{\theta}^T\overline{\mathbf{V}\theta}$ . Then, based on the second-order

be expressed as  $\theta \ \nabla \theta$ . Then, based on the second-order Taylor expansion, a convex surrogate function of  $\overline{\theta}^{\mathrm{T}} \overline{\nabla \theta}$  (i.e.,  $\Re\{\theta^{\mathrm{H}} \widetilde{\nabla}_t \theta^*\}$ ) is derived as [14]

$$\begin{split} \overline{\boldsymbol{\theta}}^{\mathrm{T}} \overline{\mathbf{V}} \overline{\boldsymbol{\theta}} &\leq \overline{\boldsymbol{\theta}}_{t}^{\mathrm{T}} \overline{\mathbf{V}}_{t} \overline{\boldsymbol{\theta}}_{t} + \overline{\boldsymbol{\theta}}_{t}^{\mathrm{T}} (\overline{\mathbf{V}}_{t} + \overline{\mathbf{V}}_{t}^{\mathrm{T}}) (\overline{\boldsymbol{\theta}} - \overline{\boldsymbol{\theta}}_{t}) + \frac{\lambda}{2} (\overline{\boldsymbol{\theta}} - \overline{\boldsymbol{\theta}}_{t})^{\mathrm{T}} (\overline{\boldsymbol{\theta}} - \overline{\boldsymbol{\theta}}_{t}) \\ &= \Re \{ \boldsymbol{\theta}^{\mathrm{H}} \mathbf{U} \overline{\mathbf{v}}_{t} \} + c, \end{split}$$

(20) where  $\overline{\boldsymbol{\theta}}_t \triangleq [\Re\{\boldsymbol{\theta}_t^{\mathrm{T}}\} \Im\{\boldsymbol{\theta}_t^{\mathrm{T}}\}]^{\mathrm{T}}, \lambda \triangleq \lambda_{max}(\overline{\mathbf{V}}_t + \overline{\mathbf{V}}_t^{\mathrm{T}})$  and  $\overline{\mathbf{v}}_t \triangleq (\overline{\mathbf{V}}_t + \overline{\mathbf{V}}_t^{\mathrm{T}} - \lambda \mathbf{I}_{2M})\overline{\boldsymbol{\theta}}_t. c$  is a constant irrelevant to  $\boldsymbol{\theta}.$  $\mathbf{U} \triangleq [\mathbf{I}_M \, j \mathbf{I}_M]$  is defined to convert the real-valued function back to the original complex-valued function. So problem (P5) is equivalent to

(P6) 
$$\min_{\boldsymbol{\theta}} 2\Re(\boldsymbol{\theta}^{\mathrm{H}}\mathbf{f}_{t}), \quad \text{s.t. } |\theta_{i}| = 1, \forall i,$$
 (21)

where  $\mathbf{f}_t = \tilde{\mathbf{u}}_t + \mathbf{U}\overline{\mathbf{v}}_t$ . The optimal common RIS reflection coefficients are obtained as

$$\boldsymbol{\theta}^{\text{opt}} = -e^{j \arg(\mathbf{f}_t)}.$$
 (22)

In short, with the derivation of closed-form solutions for the transmit beamformers  $\{\mathbf{F}_k\}_{k=1}^K$ , the receive equalizers  $\{\mathbf{G}_k\}_{k=1}^K$  and the common RIS reflection pattern  $\boldsymbol{\Theta}$ , the overall optimization process is summarized in Algorithm 1. *C. Analysis of Convergence and Computational Complexity* 

To analyze the convergence of the proposed MM-based AO algorithm, in the *t*th iteration, we first define the optimal solutions of its involved three subproblems as  $\{\boldsymbol{G}_k^t\}_{k=1}^K$ ,  $\{\boldsymbol{F}_k^t\}_{k=1}^K$  and  $\boldsymbol{\Theta}^t$  shown in (5), (8a) and (22), respectively. The resultant objective value of problem (P1) is given by  $f(\{\boldsymbol{G}_k^t\}_{k=1}^K, \{\boldsymbol{F}_k^t\}_{k=1}^K, \boldsymbol{\Theta}^t)$ . Then we have

$$f(\{\boldsymbol{G}_{k}^{t+1}\}_{k=1}^{K}, \{\boldsymbol{F}_{k}^{t+1}\}_{k=1}^{K}, \boldsymbol{\Theta}^{t+1}) \leq f(\{\boldsymbol{G}_{k}^{t}\}_{k=1}^{K}, \{\boldsymbol{F}_{k}^{t}\}_{k=1}^{K}, \boldsymbol{\Theta}^{t}),$$
(23)

Algorithm 1 Proposed MM-based AO algorithm
<b>Input:</b> $T_1, T_2, R_{1,k}, R_{2,k}, S, P, \sigma_n^2$ .
1: Initialize: $\Theta^0 = \mathbf{I}_M, \mathbf{F}_k^{(0)} = [\mathbf{I}_{N_k}; 0_{(N_t - N_k) \times N_k}].$
2: repeat
3: Update $\{G_k^{t+1}\}_{k=1}^K$ based on (5).
4: Update $\{F_k^{t+1}\}_{k=1}^K$ based on (8a).
5: Update $\theta^{t+1}$ based on (22).
6: <b>until</b> the convergence is satisfied.
<b>Output :</b> $\{\boldsymbol{G}_{k}^{\text{opt}}\}_{k=1}^{K}, \{\boldsymbol{F}_{k}^{\text{opt}}\}_{k=1}^{K}, \boldsymbol{\Theta}^{\text{opt}}\}$

which holds since the optimal closed-form solution of each subproblem in the *t*th iteration is available. It follows from (23) that the objective value of problem (P1) is monotonically non-increasing throughout the iterations. In addition, it is readily inferred that the achievable value of problem (P1) is lower bounded by zero. As such, we can conclude that the proposed MM-based AO algorithm finally converges to a locally optimal solution of problem (P1) [15]. Assuming the required number of iterations to be *I*, the total complexity of the proposed algorithm is then calculated as  $O(I(K(N_k^3 + M^3 + N_{r,k}^3) + 8M^3))$ .

#### IV. NUMERICAL RESULT

In this section, we demonstrate the simulations to show the performance of the MM-based AO algorithm. In a 3D Cartesian coordination, we assume the BS, RIS 1 and RIS 2 are deployed at (1,0,5)m, (0,0,2)m and (0,50,2)m, respectively. Two users locate in a circle with the center at (1,50,0)m randomly. The numbers of BS transmit antennas and each user receive antennas are 16 and 4, respectively. There are 64 reflecting elements in each RIS. Moreover, we assume 2 data streams are transmitted to each user. Unless otherwise specified, the noise power is set to be -120dBm and the BS maximum transmit power is P = 0dBm. We assume Rician fading for each involved channel, i.e.,  $\mathbf{H} = \sqrt{\beta}(\sqrt{\kappa}\mathbf{H}_{\text{LoS}} + \sqrt{1-\kappa}\mathbf{H}_{\text{NLoS}}),$ where  $\kappa$  is the Rician factor assumed to be 0.75 and  $\beta$  is the distance-based path loss given by  $\beta = \beta_0 d^{-\gamma}$ .  $\beta_0$  denotes the reference path loss with d = 1m and set as -30dB. For the BS  $\rightarrow$  RIS 1 and RIS 2  $\rightarrow$  user k links, the path loss exponents are given by  $\gamma_{T_1} = \gamma_{R_{2,k}} = 2.2$ . For other channels, the path loss exponents are given by  $\gamma_{T_2} = \gamma_{R_{1,k}} = \gamma_S = 3.6$ .  $\mathbf{H}_{\mathrm{NLoS}}$ is the small-scale fading component, obeying the Rayleigh distribution, i.e.,  $\mathbf{H}_{NLoS} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ . Moreover, our proposed algorithm is compared with the following benchmarks: (1) Single-RIS (BS side): There is only RIS 1 in the multi-user MIMO system. (2) Single-RIS (UE side): There is only RIS 2 in the multi-user MIMO system. (3) Double-RIS-Separate: The system model is the same with Fig. 1, but the two RISs utilize two separate reflection coefficients.

First, the convergence of the MM-based AO algorithm is demonstrated in Fig. 2. It's observed that the proposed algorithm converges within 15 iterations under different initializations, showing the superior convergence behavior. In addition, our proposed algorithm can achieve the same performance under different initialization schemes.

Fig. 3 demonstrates the MSE performance of considered schemes versus the noise power. Compared with the Single-



Fig. 2. Convergence behavior of the proposed MM-based AO algorithm.



Fig. 3. MSE performance versus the noise power for different schemes comparison.

RIS schemes, i.e., Single-RIS (BS side) and Single-RIS (UE side), our proposed algorithm achieves lower MSE at the expense of the same signal processing overhead. Furthermore, it is observed that our proposed algorithm can also achieve nearly the same performance as the Double-RIS-Separate scheme in the high-SNR region, even using a half of signal processing overhead.



Fig. 4. MSE performance versus the number of RIS elements for different schemes comparison.

In Fig. 4, the MSE performance of different schemes versus the number of RIS elements is demonstrated. All schemes exhibit better MSE performance as the number of RIS elements increases, which is attributed to the increased degrees of freedom. Observing from Fig. 4, our system still outperforms the single-RIS systems and only has a slight performance loss as compared to the double-RIS system with separate reflection pattern. This indicates that the proposed algorithm can strike a better tradeoff between the MSE performance and the implementation complexity of the considered system.

## V. CONCLUSION

This letter studied a cooperative double-RIS aided multiuser MIMO system, where the common RIS reflection pattern leading to the low communication overhead and signal processing complexity is considered. We jointly optimized the active transmit beamforming at the BS, the equalizer at the receiver and the common reflection pattern at RISs by investigating the average sum MSE minimization problem. To solve this non-convex problem, we proposed an MMbased AO algorithm where the intractable original problem was decomposed into three subproblems. By exploiting the convex optimization theory and MM technology, each subproblem admitted a closed-form solution. Simulation results characterized the good convergence behavior of the proposed algorithm. Moreover, it was shown that the double-RIS with common reflection pattern achieved superior performance over the single-RIS systems and achieved nearly the same performance as the double-RIS with separate reflection pattern in the high-SNR region while the communication overhead is only half. This implies that a superior tradeoff between the performance and complexity of the implementation and signal processing can be achieved for practical applications.

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