### On the Capacity of Mobile Ad Hoc Networks with Delay Constraints

Cristina Comaniciu, Stevens Institute of Technology, e-mail: ccomanic@stevens.edu, H. Vincent Poor Princeton University poor@princeton.edu

Abstract-Previous work on ad hoc network capacity has focused primarily on source-destination throughput requirements for different models and transmission scenarios, with an emphasis on delay tolerant applications. In such problems, network capacity enhancement is achieved as a tradeoff with transmission delay. In this paper, the capacity of ad hoc networks supporting delay sensitive traffic is studied. First, a general framework is proposed for characterizing the interactions between the physical and the network layer in an ad hoc network. Then, CDMA ad hoc networks, in which advanced signal processing techniques such as multiuser detection are relied upon to enhance the user capacity, are analyzed. The network capacity is characterized using a combination of geometric arguments and large scale analysis, for several network scenarios employing matched filters, decorrelators and minimum-mean-square-error receivers. Insight into the network performance for finite systems is also provided by means of simulations. Both analysis and simulations show a significant network capacity gain for ad hoc networks employing multiuser detectors, compared with those using matched filter receivers, as well as very good performance even under tight delay and transmission power requirements.

Index Terms—ad hoc network, delay, capacity, CDMA, multiuser detection.

#### I. INTRODUCTION

A mobile ad hoc network consists of a group of mobile nodes that spontaneously form temporary networks without the aid of a fixed infrastructure or centralized management. The communication between any two nodes can be either direct or relayed through other nodes (if the direct transmission causes too much interference in the network or consumes too much power). Research on ad hoc networks has traditionally been focused on routing and medium access control, and only recently has there been an increased interest in characterizing the capacity of such networks. We mention here a few landmark papers that analyze network capacity in terms of achievable throughput under different system models and assumptions [3]. [5], [13]. In [13], the authors focus on fixed, finite networks and derive capacity regions under various predefined transmission protocols, under the assumption of omniscient nodes. Alternatively, [5] and [3] discuss the asymptotic throughput performance for fixed and mobile networks, respectively. In [5], the authors study the capacity of a fixed ad hoc network in which the nodes' locations are fixed but randomly distributed. They prove that, as the number of nodes (N) per unit area increases, the achievable throughput between any randomly selected source-destination pair is on the order of  $O(1/\sqrt{N})$ . In contrast to this somewhat pessimistic result, [3] shows that

exploiting mobility can result in a form of multiuser diversity and can improve the system capacity. The authors of [3] propose a two-hop transmission strategy in which the traffic is first randomly spread (first hop) across as many relay nodes as possible, and then it is delivered (second hop) as soon as any of the relaying nodes is close to the destination. The disadvantage of this scheme is that it involves large delays and therefore it is not suitable for delay sensitive traffic. A capacity increase with mobility has also been noticed in [4], in which the capacity is empirically determined for a different network model that exploits spatial diversity.

In this paper, we study the capacity of large mobile ad hoc networks carrying delay sensitive traffic. Because of tight delay requirements, we cannot take advantage of mobility as in [3]. To improve the capacity we rely on advanced signal processing techniques such as multiuser detection, which can be implemented adaptively and blindly (e.g. [16]).

We analyze the network for a given stationary distribution of the mobile nodes' locations with constraints on the maximum number of hops between any arbitrary source-destination pair. Using similar arguments as in [5] we show that limiting the maximum number of hops for any given transmission also improves the source-destination throughput by limiting the additional transmissions for the relayed traffic. On the other hand, reducing the number of hops has a negative impact on the capacity by increasing the interference level. Thus, for delay sensitive traffic, the network capacity is interference limited and multiuser receivers can significantly improve the performance.

We first propose a general framework for characterizing the interactions between the physical and the network layer in an ad hoc network. This is based on defining a link probability, which relates to the performance of both physical and network layers. We then focus on CDMA ad hoc networks, and determine link probability expressions at the network layer and physical layer.

To derive the network capacity, we characterize the asymptotic network performance based on geometric considerations, and obtain a constraint on the probability of maintaining a link such that the network's diameter is D, as the number of nodes increases without bound. As these results are asymptotic in nature, we also validate them through simulations for finite values of N. The network diameter represents the longest shortest path between any two nodes, and consequently, is the maximum number of hops required for transmission between any given pair of nodes. The link probability p also characterizes the physical layer and is defined to be the probability that the signal-to-interference ratio can be maintained above the desired target. We compute p for different scenarios

This paper was presented in part to the IEEE CAS Workshop on Wireless Communications and Networking, Pasadena, California, September 2002. This research was supported by the National Science Foundation under Grant CCR-02-05214 and by the New Jersey Center for Pervasive Information Technology.

(Code-division multiple-access (CDMA) with random spreading codes and matched filter, minimum-mean-square-error (MMSE) and decorrelating receivers) using an asymptotic analysis (both the number of nodes and the spreading gain are driven to infinity while their ratio is kept fixed) [14].

The paper is organized as follows. In Section 2 we introduce the system model, Section 3 presents the general cross-layer framework for analyzing ad hoc networks, while Section 4 presents the asymptotic capacity derivation and Section 5 presents simulations for finite systems. Finally, some concluding remarks are presented in Section 6.

#### II. SYSTEM MODEL

We consider an ad hoc network consisting of N mobile nodes, having a uniform stationary distribution over a square area, of dimension  $b \times b$ . The multiaccess scheme is directsequence CDMA (DS-CDMA) and three types of receivers are considered: the matched filter (MF), the decorrelator, and the linear minimum mean squared error receiver (MMSE). All nodes use independent, randomly generated and normalized spreading sequences of length L. For simplicity, we assume that all nodes transmit with the same power,  $P_t$ , and we define the signal-to-noise ratio (SNR), as the ratio between the transmitted power and the noise power:  $SNR = P_t/\sigma^2$ . As in [9], we consider a transmitter oriented protocol, in which each transmitting node has its own signature sequence. Although this implementation yields more complex receivers and longer acquisition times, it has very good capturing probabilities, allowing multiple packet reception at the same receiver node. To avoid collisions, multiple concurrent transmissions from the same node are not allowed; instead transmissions from one node to multiple destination nodes are time multiplexed. We assume that all nodes are active at a given time (to transmit their own packets or relayed traffic), although the analysis can be easily extended to the case in which only a fraction  $\beta$  of nodes are active (in which case interference is reduced by a factor of  $1/\beta$ ).

The path loss model is usually characterized by three zones: the near field zone, the free space path loss zone and the excess path loss zone. The near field zone extends to a distance of

$$d_1 = \frac{2D_{max}^2}{\lambda},\tag{1}$$

where  $D_{max}$  is the largest dimension of the antenna, and  $\lambda$  is the wavelength of the carrier. The signal attenuation in this zone is the highest and it is usually not modeled for typical applications. For an antenna dimension of  $D_{max} = \lambda/2$ , the near-field zone extends to  $d_1 = \lambda/2$ . In this paper we approximate the path loss model, and we assume that no reception is possible within distance  $d < d_m$ , where  $d_m = \lambda > d_1$ .

For distances  $d \ge d_m$ , and  $d \le d_2 = \frac{4h_th_r}{\lambda}$  ( $h_t$  and  $h_r$  are the heights of the transmitter and receiver antennas, respectively), the free space propagation model applies. For antenna heights greater or equal to 1 meter (a resonable value for ad hoc networks), and 3 GHz frequency ( $\lambda = 0.1$  meters), we have  $d_2 \ge 40$  meters. Thus, since we are considering low range transmissions in the ad hoc networks, in our analysis we

consider a free space propagation path loss model for which the received power is given as:

$$P_r = P_t^* G_t G_r \frac{\lambda^2}{(4\pi d)^2} = P_t \frac{\lambda^2}{d^2} = P_t h,$$
 (2)

where  $P_t$  represents the above-defined transmitted power, which incorporates also the transmitting and receiving antenna gains and the constant  $1/(4\pi)^2$ , and  $h = \frac{\lambda^2}{d^2}$  is the link gain.

Although not specifically addressed in this paper, the analysis can be extended to consider a general path loss exponent s > 2, which may be useful in characterizing the performance for long range ad hoc networks.

The traffic can be directly transmitted between any two nodes, or it can be relayed through intermediate nodes. It is assumed that the end-to-end delay can be measured in the number of hops required for a route to be completed. The quality of service (QoS) requirements for the ad hoc network are the bit error rate (mapped into a signal-to-interference ratio requirement: SIR), the average source-destination throughput  $(T_{S-D})$ , and the transmission delay. Both the throughput and the delay are influenced by the maximum number of hops allowed for a connection and consequently, by the network diameter D. Using arguments similar to those in [5], a simplified computation shows that, if the number of hops for a transmission is D, then each node generates Dl(N) traffic for other nodes, where l(N) represents the traffic generation rate for a given node. Thus, the total traffic in the network must meet the stability condition Dl(N)N < NW/L, where W is the system bandwidth. This implies that the average sourcedestination throughput that can be supported by the network must meet the condition

$$T_{S-D} \le \frac{W}{LD}.$$
(3)

We note that the throughput  $T_{S-D}(N) = l(N) \leq \frac{W}{LD(N)}$  is actually dependent on the number of nodes in the network N, which influences the achievable network diameter. For notation simplicity, for the remainder of the paper we denote D(N) =D and  $T_{S-D}(N) = T_{S-D}$ , while keeping in mind that both quantities are in fact dependent on N.

In [5], it was argued that although (3) shows that the throughput decreases with an increase in the number of hops required, this does not account for the fact that if the range of a node increases, more collisions occur and the throughput decreases. In our case, increasing the transmission range for the CDMA network is achieved as a result of improved physical layer reception (increased multi-packet reception capability), and thus directly yields increased network throughput for a reduced achievable network diameter.

In terms of SIR requirements, a connection can be established between two nodes if the SIR is greater than or equal to the target SIR  $\gamma$ . The obtained SIR for a particular link is random due to the randomness of the nodes' positions. To compute the probability of a connection between any two nodes we rely on results developed in [8] concerning the distribution of distances between any two nodes, when the nodes' locations are uniformly distributed in a rectangular area. In [8], an exact distribution for the distances is obtained, with the cumulative distribution function (CDF) given as

$$P(d \le bx) = \begin{cases} 0; & x < 0\\ x^2(1/2x^2 - 8/3x + \pi); & 0 \le x \le 1\\ 4/3\sqrt{x^2 - 1}(2x^2 + 1) - & \\ -(1/2x^4 + 2x^2 - 1/3) + & \\ +2x^2 \left[\sin^{-1}(1/x) - \cos^{-1}(1/x)\right]; & 1 \le x \le \sqrt{2}, \\ 1; & x \ge \sqrt{2}. \end{cases}$$
(4)

It is also shown in [8] that this model is very close to a model in which the nodes are distributed according to a Gaussian distribution having standard deviation  $\sigma_1 = b/k$ , with k = 3.5. The CDF of d under this new model is given by

$$P(d \le k\sigma_1 x) = 1 - \exp\left(-\frac{k^2}{4}x^2\right), \ x \ge 0.$$
 (5)

Equivalently, (5) can be expressed as:

$$F_d(y) = 1 - \exp\left(-\frac{k^2}{4b^2}y^2\right), \ y \ge 0.$$
 (6)

The similarity between these two models is illustrated in Fig. 1 for an example with b = 20. For simplicity, we use the expression in (5) throughout the analysis, while the simulations rely on the actual uniform distribution over the square area.



Fig. 1. Gaussian approximation model: CDF

We denote by  $d_m = \lambda$  the minimum distance for reception and by  $d_M = \sqrt{2}b$  the maximum distance between two nodes (nodes uniformly distributed in a square area). We also define the normalized distances:  $\delta_m = d_m/\lambda = 1$ , and  $\delta_M = d_M/\lambda$ . Hence, the link gain h takes values in the interval  $\left[1/\delta_M^2, 1/\delta_m^2\right]$  with high probability (e.g., according to the Gaussian model  $P(d \le \delta_m) \approx 8.5033 \times 10^{-4}$ , and  $P(d \ge \delta_M) \approx 0.0022$  for  $\lambda = 0.1$  m and b = 6).

As a consequence, the CDF for the link gain can be expressed as follows:

$$F_H(h) = 1 - F_d\left(\lambda/\sqrt{h}\right) = \exp\left(-\frac{C}{h}\right), \ h > 0,$$
 (7)

Taking the derivative of (7) we obtain the probability density function for the link gain:

$$f_H(h) = \frac{C}{h^2} \exp\left(-\frac{C}{h}\right), \ h \ge 0.$$
(8)

Using (8) the mean link gain can be easily computed to be:

$$E_H \approx C \left[ E_1(\delta_m^2 C) - E_1(\delta_M^2 C) \right], \tag{9}$$

where  $E_1(x) = \int_x^\infty \frac{1}{t} \exp(-t) dt$  is the exponential integral. We define the network capacity to be the maximum number

of nodes that can be supported such that both the SIR constraints and the delay constraints can be met for any arbitrary source-destination pair of nodes. We map the delay constraints into a maximum network diameter constraint D. In the following sections, we will characterize the ad hoc network asymptotic capacity for the case in which the number of nodes and the spreading gain go to infinity, while their ratio is fixed.

## III. GENERAL CROSS-LAYER FRAMEWORK FOR AD HOC NETWORKS

We start our discussion by characterizing the interactions between the physical and network layers for a wireless ad hoc network. The interactions between layers can be characterized based on a cross-coupling element, which represents the information shared between layers. This cross-coupling element is essentially related to the quality of the links. Since wireless systems are primarily interference limited, the physical layer transmission and reception parameters (e.g. transmitted powers, receiver design) influence the link quality, and consequently the cross-coupling information. On the other hand, the links constitute the basic element to construct routing graphs that are used to optimize routing at the network layer. Minimum cost routing may rely on cost definitions that include the shared links' quality information.

A question that arises is: what is an appropriate link quality metric to serve as a cross-coupling element? We note that there is no unique definition for the shared information measure across layers. We suggest that two appropriate selections for the cross-coupling element are the reliable transmission range  $d_r$ , and the link availability probability p (see Figure 2).



Fig. 2. General cross-layer model for ad hoc networks

The reliable transmission range is defined as the maximum source-destination transmission distance for which the BER target can be met. We can easily see that improvements at the physical layer will yield an extended transmission range, and consequently higher  $d_r$ . This translates into longer links being available for routing at the network layer. If a network delay is imposed, which requires a minimum hop solution, a certain constraint on the desired value of  $d_r$  can be enforced.

where  $C = \frac{k^2}{4b^2}\lambda^2$ .

The other link quality metric proposed, the link probability constraint p, is defined as the probability that a link is available for transmission, i.e., meets the SIR target constraints. We can see that this measure is more generally applicable than the previous one, as it relates to link quality variations that are not necessarily distance based. As a possible example, a power controlled network (equal SIR for all links) may result in different achievable target SIRs for the links as a function of the level of interference in the network. The network delay will include the effect of retransmissions for errored packets. For this case, at the network level, an average delay constraint as well as a variance delay constraint may be imposed (to model delay and jitter), which coupled with the physical layer characteristics will determine the network capacity (see Figure 3). In a similar context, this measure may be more suitable for power controlled users in fading environments, where the link quality varies with the fading process, irrespective of distance. Furthermore, the link availability model fits naturally with any random access based system model with or without multipacket reception, where the link availability can be translated into the probability of success for the current transmission on a particular link. We also note that the two information sharing measures are related for simple system models, and a relationship between them can be determined.



Fig. 3. Link probability and network capacity

In this work, we consider only distance based fading, and we ignore the effect of retransmissions on the packet transmission delay. Consequently, the reliable transmission range metric  $d_r$  is suitable to describe the cross-coupling between layers. However, since the link availability metric p is suitable to model a larger array of scenarios, we will mostly use p in our derivation, with the understanding that, for our system model, both metrics are equivalent. In Figure 4 we show the equivalence between these two metrics, which can be expressed analytically as:

$$p = P\left(d \le d_r\right) = F_d(d_r) = 1 - \exp\left(-\frac{k^2}{4b^2}{d_r}^2\right).$$
 (10)

## IV. ASYMPTOTIC CAPACITY FOR CDMA AD HOC NETWORKS

#### A. Physical Layer Performance

Based on the framework presented in Section III, we determine the number of active nodes that can be supported by the network, given a link probability constraint p. The link probability p is affected by the level of interference in the network and thus it will be very sensitive to the choice of the receiver.

We start our capacity analysis by considering the ideal synchronous transmission case, then we discuss performance



Fig. 4. Cross-layer coupling metrics equivalence

results for the asynchronous case. The asymptotic capacity is derived for three types of receivers: the matched filter, the decorrelator and the linear MMSE detector.

#### 1) Synchronous Transmission: Matched Filter

The SIR condition for an arbitrary node i using a matched filter receiver in a network with random, normalized spreading sequences can be expressed as:

$$SIR_{i} = \frac{P_{t}h_{i}}{\sigma^{2} + \frac{1}{L}\sum_{j=1, j\neq i}^{N} P_{t}h_{j}}$$
$$= \frac{h_{i}}{SNR^{-1} + \frac{1}{L}\sum_{j=1, j\neq i}^{N} h_{j}} \ge \gamma.$$
(11)

Denoting by  $\alpha$  the fixed ratio N/L and letting the number of nodes and the spreading gain go to infinity, by using the law of large numbers [17], it follows that:  $\frac{1}{L} \sum_{j=1, j \neq i}^{N} h_j \rightarrow \alpha E_H$ , with  $E_H$  computed as in (9).

According to our framework, we derive the link probability p, such that physical layer QoS requirements are met, i.e., the link meets its target SIR with probability p:

$$P(H \ge \gamma SNR^{-1} + \alpha \gamma E_H) = P(H \ge T_{MF}) = p. \quad (12)$$

Using the notation  $T_{MF} = \gamma SNR^{-1} + \alpha \gamma E_H$ , the link probability condition renders an SNR condition

$$\gamma SNR^{-1} + \alpha \gamma E_H = T_{MF} \Rightarrow SNR = \frac{\gamma}{T_{MF} - \alpha \gamma E_H},$$
(13)

where  $T_{MF}$  can be derived using (7) as follows

$$p = 1 - F_H(T_{MF}) = 1 - \exp\left(-C\frac{1}{T_{MF}}\right);$$
 (14)

or

$$T_{MF} = \frac{C}{\log\left(\frac{1}{1-p}\right)}.$$
(15)

Equation (13) implies that a positive power solution exists if and only if

$$\alpha_{MF} < \frac{T_{MF}}{\gamma E_H} = \frac{\frac{C}{\log(\frac{1}{1-p})}}{\gamma E_H}.$$
 (16)

For ad hoc networks, it is most likely that the mobile nodes are energy limited such that we assume that a maximum power transmission limit  $\bar{P}_t$  is imposed. Denoting  $SNR_c = \bar{P}_t/\sigma^2$ , the ad hoc network capacity becomes:

$$\alpha_{MF} \le \frac{T_{MF}}{\gamma E_H} - \frac{1}{E_H SNR_c} = \frac{\frac{C}{\log(\frac{1}{1-p})}}{\gamma E_H} - \frac{1}{E_H SNR_c}.$$
 (17)

#### Decorrelator

According to results presented in [14], the SIR of an arbitrary node in an asymptotically large network using decorrelating receivers can be expressed as:

$$SIR_d = \begin{cases} \frac{P_t h(1-\alpha)}{\sigma^2}, & \alpha < 1, \\ 0 & \alpha \ge 1 \end{cases}$$
(18)

Thus, if no power constraints are imposed, the network capacity region is

$$\alpha_d < 1. \tag{19}$$

If power constraints are imposed, and  $SNR \leq SNR_c$  ( $SNR_c$  is the maximum SNR allowed), the physical layer constraint can be expressed as:

$$P\left(H \ge \frac{\gamma}{SNR(1-\alpha)}\right) = p.$$
 (20)

If we define  $T_d = \frac{\gamma}{SNR(1-\alpha)}$ , the feasibility condition becomes

$$SNR = \frac{\gamma}{T_d(1-\alpha)} \le SNR_c.$$
(21)

Imposing a network constraint on the  $T_d$  value,  $T_d = \frac{C}{\log(\frac{1}{1-p})}$ , the asymptotic capacity region for a network using decorrelating receivers and having transmission power constraints is given as

$$\alpha_d \le 1 - \frac{\gamma}{T_d SNR_c} = 1 - \frac{\gamma}{\frac{C}{\log\left(\frac{1}{1-p}\right)}SNR_c}.$$
 (22)

#### **MMSE Detector**

To derive the asymptotic ad hoc network capacity we first express the SIR ratio for an arbitrary node i in a large network using MMSE receivers, as in [14]:

$$SIR_{i} = \frac{P_{t}h_{i}}{\sigma^{2} + \frac{1}{L}\sum_{j=1, j\neq i}^{N} \frac{P_{t}h_{i}P_{t}h_{j}}{P_{t}h_{i} + P_{t}h_{j}SIR_{i}}} = \frac{h_{i}}{SNR^{-1} + \frac{1}{L}\sum_{j=1, j\neq i}^{N} \frac{h_{i}h_{j}}{h_{i} + h_{j}SIR_{i}}}.$$
 (23)

Imposing the QoS condition:  $SIR_i \ge \gamma$ ,  $\forall i = 1, 2, ..., N$ , (where  $\gamma$  is the target SIR), we have

$$SIR_i \ge \frac{h_i}{SNR^{-1} + \frac{1}{L}\sum_{j=1, j \neq i}^N \frac{h_i h_j}{h_i + h_j \gamma}} = \gamma.$$
(24)

Denoting  $\alpha = N/L$  and letting the spreading gain and the number of nodes go to infinity we can apply the law of large numbers, such that,

$$\frac{1}{L}\sum_{j=1,\ j\neq i}^{N}\frac{h_ih_j}{h_i+h_j\gamma} = \alpha \frac{1}{N}\sum_{j=1,\ j\neq i}^{N}\frac{h_ih_j}{h_i+h_j\gamma} \to \alpha E[H|h_i],$$

where we used the notation  $E[H|h_i]$  to denote the normalized conditional average interference (normalized to the number of nodes per dimension). It is shown in the Appendix that  $E[H|h_i]$  can be expressed as:

$$E[H|h_i] =$$

$$= C \exp\left(\frac{C\gamma}{h_i}\right) \left[ E_1\left(\delta_m^2 C + \frac{C\gamma}{h_i}\right) - E_1\left(\delta_M^2 C + \frac{C\gamma}{h_i}\right) \right].$$
(25)

Thus, the link probability constraint becomes

1

$$P\left(H \ge \gamma SNR^{-1} + \alpha \gamma E[H|h]\right) = p.$$
<sup>(26)</sup>

We define the function  $f(h) = h - \gamma SNR^{-1} - \alpha \gamma E[H|h]$  and we plot it in Fig. 5. We observe that f(h) is a monotonically increasing function of h for the region of interest, and thus we can express the condition (26) as

$$P(H \ge T_{MMSE}) = p. \tag{27}$$



Fig. 5. SIR condition monotonicity

Equation (27) has the same solution as in the previously analyzed cases, and the physical layer constraint becomes

$$SNR = \frac{\gamma}{T_{MMSE} - \alpha \gamma E[H|h = T_{MMSE}]}.$$
 (28)

A positive transmitting power solution exists if and only if

$$\alpha_{MMSE} < \frac{T_{MMSE}}{\gamma E[H|h = T_{MMSE}]};$$
(29)

or equivalently,

$$\alpha_{MMSE} < \frac{\frac{C}{\log\left(\frac{1}{1-p}\right)}}{\gamma C\left(\frac{1}{1-p}\right)^{\gamma} \left[E_1\left(\zeta_m\right) - E_1\left(\zeta_M\right)\right]},\tag{30}$$

where 
$$\zeta_m = \delta_m^2 C + \gamma \log\left(\frac{1}{1-p}\right)$$
, and  $\zeta_M = \delta_M^2 C + \gamma \log\left(\frac{1}{1-p}\right)$ .

If power constraints are imposed, the capacity region becomes

$$\alpha_{MMSE} \le \frac{T_{MMSE}}{\gamma E[H|h = T_{MMSE}]} - \frac{1}{E[H|h = T_{MMSE}]SNR_c};$$
(31)

or equivalently,

$$\alpha_{MMSE} \leq \frac{\frac{1}{\log\left(\frac{1}{1-p}\right)}}{\gamma C\left(\frac{1}{1-p}\right)^{\gamma} \left[E_{1}\left(\zeta_{m}\right) - E_{1}\left(\zeta_{M}\right)\right]} - (32)$$
$$-\frac{1}{C\left(\frac{1}{1-p}\right)^{\gamma} \left[E_{1}\left(\zeta_{m}\right) - E_{1}\left(\zeta_{M}\right)\right] SNR_{c}}.$$

Figure 6 illustrates the physical layer capacity as a function of the link probability constraint for the three receivers considered, and with or without power constraints. For the power-constrained case, a maximum transmission power of  $\bar{P}_t = 10^4 \sigma^2$  is considered for this example. A target SIR  $\gamma = 5$  is imposed.



Fig. 6. Physical layer capacity for given link probability constraint: synchronous transmission

From Figure 6 we can observe that there is a significant capacity advantage if multiuser receivers are used, and conversely, for given capacity requirements, substantial power savings can be achieved by networks using multiuser receivers. As expected, the MMSE receiver performs the best due to its property of maximizing the SIR. For higher transmission rates and lower delay requirements (translated into a higher link probability constraint) using the matched filter is not feasible.

2) Asynchronous Transmission: Although often not a practical scenario, the above synchronous analysis is very useful as a performance benchmark. Moreover, the analysis can usually be extended to the asynchronous case by considering an equivalent synchronous system with more interferers. To extend our results to asynchronous ad hoc networks, we rely on SIR convergence properties developed in [6]. According to results presented in [6], the matched filter performance in the asynchrounous case has the same asymptotic performance as for the synchronous case. Also, if the observation window is infinite, the decorrelator and the MMSE performance does not change either. However, for the "one-shot" detection approach, the achieved SIR for both the decorrelator and the MMSE detector degrades for the asynchronous case. Although exact capacity values are difficult to derive, in [6] the authors present very tight lower bounds on the achievable SIR for both the decorrelator and the MMSE detector in the asynchronous case, under the simplifying assumption that the nodes are chipsynchronous. They also showed by means of simulations that the chip-synchronous scenario provides conservative estimates for a truly asynchronous system.

#### Decorrelator

According to [6], the SIR for the decorrelator can be approximated as

$$SIR_d = \begin{cases} \frac{P_t h(1-2\alpha)}{\sigma^2}, & \alpha < 1/2, \\ 0 & \alpha \ge 1/2 \end{cases}$$
(33)

Therefore, the capacity results from the previous subsection can be extended straightforwardly to

$$\alpha_d < 1/2, \tag{34}$$

when no power constraints are imposed.

If power constraints are imposed  $(SNR \leq SNR_c)$ , we derive the capacity region as

$$\alpha_d \le \frac{1}{2} - \frac{\gamma}{\frac{2C}{\log\left(\frac{1}{1-\tau}\right)}SNR_c}.$$
(35)

#### **MMSE Detector**

To characterize the capacity of an asynchronous ad hoc network using MMSE receivers, we rely on the lower bound obtained for the achievable SIR in an asymptotic system for symmetric delay distributions [6]:

$$SIR_{i} = \frac{P_{i}}{\sigma^{2} + \alpha E_{P} E_{\tau} \{ I(\tau P, P_{i}, SIR_{i}) + I((1-\tau)P, P_{i}, SIR_{i}) \}}$$
(36)

where  $\tau$  is a random variable that characterizes the delay associated with an arbitrary node. Since the received power P can be expressed as  $P = P_t h$ , for equal transmit powers for all nodes, (36) becomes

$$SIR_i =$$

$$=\frac{h_i}{SNR^{-1} + \alpha E_H E_\tau \{I_\tau + I_{(1-\tau)}\}},$$
(37)

where we used the notations:  $I_{\tau} = I(\tau P_t h, P_t h_i, SIR_i)$ , and  $I_{(1-\tau)} = I((1-\tau)P_t h, P_t h_i, SIR_i)$ .

It is straightforward to see (using a similar derivation as in the appendix) that  $\alpha E_H E_\tau \{I(\tau P_t h, P_t h_i, SIR_i) + I((1 - \tau)P_t h, P_t h_i, SIR_i)\}$  can be expressed as

$$\alpha E_{\tau} \left[ E[H|h_i, \tau] \right] = \alpha E_{\tau} \left[ C\tau \exp\left(\frac{C\gamma\tau}{h_i}\right) \left[ E_1\left(\xi_m^{\tau}\right) - E_1\left(\xi_M^{\tau}\right) \right] + \right]$$

$$+C(1-\tau)\exp\left(\frac{C\gamma(1-\tau)}{h_i}\right)\left[E_1\left(\xi_m^{(1-\tau)}\right) - E_1\left(\xi_M^{(1-\tau)}\right)\right]\right]$$
(38)

where we used the notations  $\xi_m^{(\tau)} = \delta_m^2 C + \frac{C\gamma\tau}{h_i}, \ \xi_M^{(\tau)} = \delta_M^2 C + \frac{C\gamma\tau}{h_i}, \ \xi_M^{(\tau)} = \delta_m^2 C + \frac{C\gamma(1-\tau)}{h_i}, \ \text{and} \ \xi_M^{(1-\tau)} = \delta_M^2 C + \frac{C\gamma(1-\tau)}{h_i}.$ 

 $E_{\tau} \{E[H|h_i, \tau]\}$  can be determined using numerical integration. For our example, we have considered  $\tau$  to be a uniform random variable taking values in the interval [0, 1].

Using an identical derivation for the network capacity as for the synchronous case, all the capacity formulas hold with  $E[H|h_i]$ , replaced by  $E_{\tau} \{E[H|h_i, \tau]\}$ . In Figure 7, we illustrate capacity comparisons between networks using MMSE receivers in the synchronous and the asynchronous cases.

#### B. Network Layer Performance

The overall network capacity is determined such that both physical layer and network layer QoS requirements can be met. In the previous section we have determined the maximum number of active nodes that can be supported simultaneously



Fig. 7. Capacity comparisons for ad hoc networks with MMSE receivers: synchronous versus asynchronous transmission

by the network, as a function of a link probability constraint p. In this section, we use geometric arguments to determine the dependence of the link probability on the network diameter constraint (which is a measure of the delay constraint).

We consider the asymptotic case, in which we have an infinite number of nodes in the considered square area. The number of nodes is uniformly distributed, and we ignore the edge effects: the square area can be considered to be a part of a multiple cell layout. It can be seen in Figure 8 that the worst case distance is obtained when the source and destination nodes are on the opposite vertices of the square.



Fig. 8. Network diameter constraint

Consider now a diameter restriction of D = 2, as in Figure 8. In order to be able to transmit from the source node (SN) to the destination node (DN) using only one intermediate node (IN), the reliable transmission range  $d_r$  should be equal to the distance between the source node and intermediate node, and also between the intermediate node and the destination node, respectively:

$$d_r = \frac{b\sqrt{2}}{2}.$$
 (39)

For a generic value of D, the network QoS constraint becomes

$$d \le d_r = \frac{b\sqrt{2}}{D}.\tag{40}$$

Thus, a link may be used for routing with probability

$$p = F_d(d_r) = 1 - \exp\left(-\frac{C}{\lambda^2}\frac{2b^2}{D^2}\right).$$
 (41)

Figure 9 illustrates the link probability values required for various network diameter constraints. The case D = 1 is trivial, as  $p \approx 1$  (the approximation is due to approximations in the derivation of distributions for link distances).



Fig. 9. Link probability requirement

We note that the assumption that we will always find a relaying node at the required distance is optimistic for finite networks, whereas the assumption the the link length is equal to the diagonal is a pessimistic assumption. Simulaton results will be presented to illustrate how accurate this analysis is for finite networks.

#### C. User Capacity

In the previous sections, we derived link probability (p) expressions to reflect both physical layer and network layer QoS constraints. Based on these expressions, we can now derive the user capacity for the network (see Figure 3).

More specifically, (41) gives the network probability constraint p to be substituted into all of the user capacity equations derived in Section IV-A.

Furthermore, we can see that the network QoS condition imposes that a reliable transmission is possible within a radius  $d_r$  of the transmitting node:

$$p = P(H \ge T) = P\left(d_r \le \frac{\lambda}{\sqrt{T}}\right),$$
 (42)

where the threshold T depends on the particular receiver structure used.

Based on this observation, an alternate simple way to derive the capacity is to substitute  $h = \lambda^2/d_r^2$  in the physical layer capacity conditions and solve for the number of users. Alternatively, from (42) and (40), we can obtain a threshold requirement of

$$T = \frac{\lambda^2 D^2}{2b^2},\tag{43}$$

which can then be used to determine the network capacity.

Figures 10 (a) and (b) illustrate the network capacity for a network diameter constraint of D = 2 and D = 3, respectively. Figure 10 shows the number of users per dimension

that can be supported in an ad hoc network for a given delay constraint, as a function of the maximum transmission power requirement,  $SNR_c = \bar{P}_t/\sigma^2$ . It can be seen that, using multiuser receivers, almost cellular capacity (obtained for the case with multiuser receivers) can be obtained even for very stringent delay (D = 2, D = 3) and power requirements (transmission power  $\bar{P}_t = 10^5 \sigma^2$ ).



Fig. 10. Ad hoc network capacity for delay sensitive traffic: (a) D=2; (b) D=3.

#### D. Network Throughput

As we have seen in the previous subsections, ad hoc network capacity is greatly enhanced by using a CDMA access method and separating the users using multiuser detectors. Tight power and delay constraints can thus be met in such networks. We will show now that using multiuser detectors in CDMA ad hoc networks also improves the overall throughput of the network. To see this, we compare the network throughput that can be achieved for our analysis by the MMSE receiver, with the scenario described in [5], in which random access is used. No delay constraints are enforced, and very similar network models are used for comparisons: all nodes are randomly located and independently and uniformly distributed in a unit area (disc for [5], square in our analysis), each node transmits traffic to a randomly chosen destination, all nodes transmit with the same power and the transmission rate is R. Both synchronous and asynchronous transmission cases are considered for the CDMA network and MMSE receivers are considered.

For the random access scenario, the order of the average throughput capacity has been shown in [5] to be l(N) =. For the CDMA network we approximate  $\theta$  $\sqrt{Nlog(N)}$ the network throughput based on (3):  $l(N) \approx \frac{R}{D(N)}$ , where R = W/L. The dependence of the network diameter on the number of nodes can be easily determined using (43) and the appropriate formula for T given the type of receiver used (see Section IV-A). We compare the network throughput for the Gupta-Kumar analysis (G-K) [5], with both a synchronous and an asynchronous CDMA network using MMSE receivers. The same numerical values as before are selected for the example plotted in Figure 11, which shows the normalized network throughput as a function of the number of nodes per unit area. The spreading gain is chosen to be L = 32.



Fig. 11. Network throughput comparison

We can see that, although the CDMA ad hoc network capacity also decreases with the increase of the number of nodes per unit area, its capacity is significantly higher than the random access network (G-K). Also, the use of MMSE receivers yields unreduced throughputs for the network for a fairly large network (approx. 40 nodes per unit area for synchronous transmission). Of course, this advantage comes at the price of an increased implementation complexity in acquiring the signature sequences for all users and dynamically adjusting the receivers.

#### V. CAPACITY FOR FINITE NETWORKS: SIMULATIONS

The capacity results obtained in the previous section are asymptotic in nature, thus requiring validation through simulations for practical finite networks. Since we showed in the previous section that the network using matched filters performs poorly compared with a system using multiuser detectors, the emphasis is on networks using multiuser receivers, and the results are only validated for the matched filter case. All the experiments consider unlimited power transmission for the MMSE case, and maximum power constraints for the decorrelator,  $\bar{P}_t = 10^4 \sigma^2$  (the case of the decorrelator with unlimited transmission power is trivial:  $\alpha < 1$ ). For implementation simplicity, all numerical results are obtained for synchronous transmission, and using b = 6,  $\lambda = 0.1 m$ and  $\gamma = 5$ . Our experiments consist of selecting a finite (variable) number of nodes and randomly generating their locations uniformly across a square area. Then, the link gains, and consequently the achieved SIRs are computed for all pairs of nodes, using Eqs. (11), (18), and (23), respectively. We note that the simulations do not consider the SIR formulas' accuracy for finite systems, as this issue has already been studied in [15], where it was shown that the standard deviation for the achieved SIR goes to zero as  $1/\sqrt{N}$ .

If the computed SIR is greater than or equal to the target SIR, the link is feasible. The adjacency matrix is then constructed, and based on it, the network diameter is determined. The computation of the network diameter uses Dijkstra's algorithm [2], as a Matlab function from the Bayes Net Toolbox package [1]. The experiment is repeated 100 times and the probabilities associated with a range of network diameters are determined. An infinite network diameter means that the network is disconnected.

We also determine the probability p of a feasible link and we compare it with our theoretical results. Some simulation examples are presented in Tables I, II, and III, for different values of L and N, selected such that we will get a range of values for  $\alpha$ . It can be seen that both the physical layer capacity results, reflected in the achievable link probability p, as well as the network performance results (i.e., the achieved network diameter) are very close to the asymptotic ones, especially for larger numbers of nodes in the network cell (the considered square area).

#### VI. CONCLUSIONS

In this paper we have analyzed the asymptotic capacity for delay sensitive traffic in ad hoc networks. While previous results have focused on enhancing the network capacity at the expense of increased transmission delay, our approach is to exploit advanced signal processing techniques, such as multiuser detection, to enhance capacity when tight delay constraints are enforced. We have analyzed three different network scenarios for a DS-CDMA air interface in which the users have matched filters, decorrelating or MMSE receivers. We combined physical layer requirements (signal to interference ratio) with network layer QoS constraints (transmission delay). The maximum network transmission delay has been expressed in terms of the maximum number of hops for any arbitrarily selected source-destination pair of nodes. We then have characterized the network delay using geometric arguments for the asymptotic case. Since all derivations in this paper are asymptotic in nature, simulation results have been presented for performance validation with finite systems. Both analysis and simulations have shown significant network capacity gains for ad hoc networks employing multiuser detectors, compared with those using matched filters, as well as very good performance even under tight delay and power constraints.

#### REFERENCES

- Bayes net toolbox for Matlab. www.cs.berkeley.edu/ murphyk/Bayes/bnt.html.
- [2] D. Bertsekas and R. Gallager. *Data Networks*. Prentice Hall, Upper Saddle River, NJ, 1992.
- [3] M. Grossglauser and D. Tse. Mobility increases the capacity of ad-hoc wireless networks. *IEEE/ACM Transactions on Networking*, to appear.

- [4] N. Gupta and S. R. Das. A capacity and utilization study of mobile ad hoc networks. In *Proceedings of the 26th Annual Conference on Local Computer Networks*, pages 576–583, November 2001.
- [5] P. Gupta and P. R. Kumar. The capacity of wireless networks. *IEEE Transactions on Information Theory*, 46(2):388–404, March 2000.
- [6] Kiran and D. Tse. Effective bandwidths and effective interference for linear multiuser receivers in asynchronous systems. *IEEE Transactions* on *Information Theory*, 46(4):1426–1447, 2000.
- [7] G. Mergen and L. Tong. Receiver controlled medium access in multihop ad hoc networks with multipacket reception. In *Proceedings of the IEEE Military Communications Conference (MILCOM)*, volume 2, pages 1014 –1018, 2001.
- [8] L. E. Miller. Distribution of link distances in a wireless network. Journal of Research of the National Institute of Standards and Technology, 106(2):401–412, March-April 2001.
- [9] N. Mostofa, K. Howlader, and B. D. Woerner. System architecture for implementing multiuser detector within an ad-hoc network. In *Proceedings of the IEEE Military Communications Conference (MILCOM)*, volume 2, pages 1119 –1123, 2001.
- [10] V. Rodoplu and T. H. Meng. Position based CDMA with multiuser detection (P-CDMA/MUD) for wireless ad hoc networks. In *IEEE Sixth International Symposium on Spread Spectrum Techniques and Applications*, volume 1, pages 336 –340, 2000.
- [11] C. Sankaran and A. Ephremides. The use of multiuser detectors for multicasting in wireless ad hoc CDMA networks. *IEEE Transactions* on *Information Theory*, pages 2873 –2887, November 2002.
- [12] E. S. Sousa and J. A. Silvester. Optimum transmission ranges in a directsequence spread-spectrum multihop packet radio network. *IEEE Journal* on Selected Areas in Communications, 8(5):762 –771, June 1990.
- [13] S. Toumpis and A. J. Goldsmith. Capacity regions for wireless ad hoc networks. *IEEE Transactions on Wireless Communications*, September 2001. submitted.
- [14] D. Tse and S. Hanly. Linear multiuser receivers: Effective interference, effective bandwidth and user capacity. *IEEE Transactions on Information Theory*, 45(2), March 1999.
- [15] D. Tse and O. Zeitouni. Performance of linear multiuser receivers in random environments. *IEEE Transactions on Information Theory*, 46(1):171–178, January 2000.
- [16] X. Wang and H. V. Poor. Blind multiuser detection: A subspace approach. *IEEE Transactions on Information Theory*, 44(2):677–690, March 1998.
- [17] R. D. Yates and D. J. Goodman. Probability and Stochastic Processes: A Friendly Introduction for Electrical and Computer Engineers. John Wiley & Sons, New York, 1999.

### Appendix

## Normalized conditional average interference derivation for MMSE networks:

Given the fact that the link gain h takes values in the interval  $[\lambda^2/\delta_m^2, \lambda^2/\delta_M^2]$  with high probability, the normalized conditional average interference derivation for MMSE networks can be approximated as:

$$E[H|h_i] \approx C \int_{1/\delta_M^2}^{1/\delta_m^2} \frac{h_i}{h(h_i + h\gamma)} \exp\left(-\frac{C}{h}\right) dh.$$

Denoting x = C/h, we have

$$E[H|h_i] \approx C \int_{\delta_m^2 C}^{\delta_M^2 C} \frac{h_i}{h_i x + C\gamma} \exp(-x) dx.$$

Again, denoting  $y = h_i x + C\gamma$ , we further have

$$E[H|h_i] \approx C \exp\left(\frac{C\gamma}{h_i}\right) \int_{h_i \delta_m^2 C + C\gamma}^{h_i \delta_M^2 C + C\gamma} \frac{1}{y} \exp\left(-\frac{y}{h_i}\right) dy.$$

On making a further change of variable  $z = y/h_i$ , we arrive at

$$E[H|h_i] \approx C \exp\left(\frac{C\gamma}{h_i}\right) \int_{\delta_m^2 C + (C\gamma)/h_i}^{\delta_M^2 C + (C\gamma)/h_i} \frac{1}{z} \exp\left(-z\right) dz,$$

Receiver	L	Ν	p (analysis)	<i>p</i> (sim.)	D (asymptotic)	D (sim.)
Decorrelator	512	60	p= 0.7773	p=0.7472	$D \approx 2$	P 0.5 0 1 2 3 4 6 6 7 8 km
Decorrelator	1024	120	p=0.7773	p= 0.7510	$D \approx 2$	P 0.5
Decorrelator	64	28	p=0.6160	p=0.5670	$D \approx 3$	P as
Decorrelator	128	92	p=0.3803	p=0.3392	$D \approx 4$	P 0.5
Decorrelator	128	96	p=0.3464	p=0.3074	$D \approx 4$	p 0.5 0 1 2 3 4 5 6 7 8 Hd
Decorrelator	128	100	p=0.3107	p=0.2764	$D \approx 4$	
Decorrelator	64	57	p=0.1698	p=0.1515	$D \approx 7$	P 0.5 0 2 4 6 8 let D

### TABLE I Simulation results: Decorrelator

which yields

$$\begin{split} E[H|h_i] &\approx C \exp\left(\frac{C\gamma}{h_i}\right) \left[E_1\left(\delta_m^2 C + \frac{C\gamma}{h_i}\right) - E_1\left(\delta_M^2 C + \frac{C\gamma}{h_i}\right)\right],\\ \text{where } E_1(x) &= \int_x^\infty \frac{1}{t} \exp(-t) dt \text{ is the exponential integral.} \end{split}$$

# TABLE IISimulation results: MMSE

Receiver	L	Ν	p (analysis)	p (sim.)	D (asymptotic)	D (sim.)
MMSE	20	20	-0.6056	-0.7401		p 0.5
MMSE	32	38	p=0.6056	p=0.7491	$D \approx 2$	
MMSE	32	39	p=0.5415	p=0.4886	$D \approx 3$	P 0.5 0 1 2 3 4 5 6 7 8 Inf D
	-		1	I ·····		
MMSE	32	42	p=0.4024	p=0.433	$D \approx 3/4$	p 0.5 0 1 2 3 4 5 6 7 8 Inf D
MMSE	20	45	a_0.2127	-0.2260		p 0.5
MMSE	32	45	p=0.3137	p=0.3260	$D \approx 4$	
MMSE	32	46	p=0.2913	p=0.2983	$D \approx 4$	p 0.5 0 1 2 3 4 5 6 7 8 Inf D
MMSE	32	48	p=0.2537	p=0.2590	$D \approx 5$	p 0.5
			1	1		
MMSE	32	57	p=0.1546	p=0.1584	$D \approx 7$	p 0.5 0 1 2 3 4 5 6 7 8 Inf D
MMSE	64	78	p=0.5415	p=0.5490	$D \approx 3$	P 0.5 0 1 2 3 4 5 6 7 8 Inf D
MMSE	64	74	p=0.6814	p=0.7482	$D \approx 2$	p 0.5

Receiver	L	Ν	p (analysis)	p (sim.)	D (asymptotic)	D (sim.)
MF	1024	44	p=0.5117	p=0.6107	$D \approx 3$	p 0.5
MF	256	31	p=0.2246	p=0.3093	$D \approx 5$	p 0.5
MF	512	144	p=0.1037	p=0.1127	$D \approx 8$	p 0.5 0 1 2 3 4 5 6 7 8 Inf D

TABLE IIISimulation results: MF