Channel Feedback Quantization for High Data Rate MIMO Systems

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Abstract

In this work, we study a multiple-input multiple-output (MIMO) wireless system where the channel state information is partially available at the transmitter through a feedback link. Based on singular value decomposition, the MIMO channel is split into independent sub-channels. Effective feedback of the required spatial channel information entails efficient quantization/encoding of a unitary matrix. We propose two schemes for quantizing unitary matrices via Givens rotations and examine the performance for a scenario where the rates allocated to the sub-channels are selected according to their corresponding gains. Numerical results show that the proposed schemes offer a significant performance improvement as compared to that of MIMO systems without feedback, with a negligible increase in the complexity.

Index Terms:MIMO wireless systems, singular value decomposition, Givens decomposition, matrix quantization

This work is financially supported by Communications and Information Technology Ontario (CITO), Nortel Networks, and Natural Sciences and Engineering Research Council of Canada (NSERC). Mehdi Ansari and Amir K. Khandani are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, Ontario, Canada, N2L 3G1 (e-mail: {mehdi, khandani}@cst.uwaterloo.ca) Farshad Lahouti is with the Department of Electrical and Computer Engineering, Tehran University, Tehran, Iran. (e-mail:lahouti@ieee.org)

I. Introduction

In recent years, researchers have examined the transmission strategies for MIMO systems in which the transmitter and/or the receiver have full or partial knowledge of the channel state information (CSI). It is shown that the capacity is substantially improved through even partial CSI at the transmitter [1]. Subject to finite rate feedback, optimal MIMO signaling is studied in [2] [3] to maximize the average channel capacity, and precoder design for MIMO systems with linear receivers is investigated in [4] [5].

Assuming partial CSI is available at the transmitter, the authors in [6] design a codebook of beamformer vectors to minimize the outage probability. Reference [7] addresses the problem of codebook design with partial CSI where the criterion is to maximize the received signal to noise ratio (SNR). A beamforming method is presented in [8] which relies on the method of [9] for the quantization of the channel spatial information (singular vectors of the channel matrix). In [10], the authors use the Givens parameters to represent the singular matrix of the channel in a slowly time-varying environment. The adaptive delta modulation is applied to quantize each parameter with a one-bit quantizer.

The motivation of this work is to design unitary matrix quantizers based on minimizing the interference measure defined later in the paper. Assuming a block fading channel model, we consider the situation in which a MIMO channel is split into several independent sub-channels by means of singular value decomposition (SVD). In this scheme, the spatial information of the channel and the constellation index of each sub-channel is needed at the transmitter. The modulation format is selected to match the SNR on each sub-channel. We use Givens rotation to decompose the spatial information of the channel (a unitary matrix). We develop quantization methods by expressing the distortion function of the unitary matrix in terms of the Givens matrices using the first order approximation. The quantizer design and the optimum bit allocation among the quantizers are achieved based on minimizing the interference measure defined in Section III. The simulation results are presented in Section IV. Finally, Section V concludes the paper.

II. System Model

We consider an independent and identically distributed (*i.i.d.*) block fading channel model. For a MIMO system with M transmit and M receive antennas, the model leads to the following complex baseband representation of the received signal

$$\mathbf{y} = \mathbf{H}\mathbf{W}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{x}$ is the $M \times 1$ vector of the transmitted symbols, $\mathbf{H}$ is the $M \times M$ channel matrix, $\mathbf{W}$ is the $M \times M$ precoder matrix, $\mathbf{n}$ is the $M \times 1$ zero mean Gaussian noise vector with the autocorrelation $\sigma^2\mathbf{I}$ where $\mathbf{I}$ is the identity matrix, and $\mathbf{y}$ is the received signal. Matrix $\mathbf{H}$ consists of circularly symmetric complex Gaussian elements with zero mean and unit variance.

The SVD of $\mathbf{H}$ is defined as $\mathbf{H} = \mathbf{V}\mathbf{\Lambda}\mathbf{U}^*$, where $\mathbf{V}$ and $\mathbf{U}$ are the unitary matrices, $\mathbf{\Lambda}$ is a diagonal matrix [11], and $(.)^*$ denotes the hermitian of $(.)$. We assume that CSI is known to the receiver and a noiseless feedback link from the receiver to the transmitter is available. By the SVD of $\mathbf{H}$ at the receiver, $\mathbf{U}$ is computed, quantized and sent to the transmitter. The transmitter uses the quantized version of $\mathbf{U}$ as a precoder, i.e. $\mathbf{W} = \mathbf{U} + \mathbf{\Delta U}$, where $\mathbf{\Delta U}$ represents the quantization error.

$$\mathbf{y} = \mathbf{H}(\mathbf{U} + \mathbf{\Delta U})\mathbf{x} + \mathbf{n}. \tag{2}$$

The receiver multiplies the received vector $\mathbf{y}$ by $\mathbf{V}^*$,

$$\mathbf{r} = \mathbf{V}^*\mathbf{y} = \mathbf{\Lambda}\mathbf{x} + \mathbf{\Lambda}\mathbf{U}^*\mathbf{\Delta U}\mathbf{x} + \mathbf{n}. \tag{3}$$

Noting $\Pr(\mathbf{n}) = \Pr(\mathbf{V}^*\mathbf{n})$, we have simplified (3). We consider a case in which data is transmitted and received separately in each sub-channel with different rates and with equal energy. The power constraint of the transmitted signal is defined as $E(\mathbf{x}\mathbf{x}^*) = M\mathcal{E}\mathbf{I}$, where $\mathcal{E}$ is the energy per data stream and E represents the expectation. Under the assumption of continuous approximation[1], it

[1] If C is a lattice code of reasonably large size, then the distribution of its points in N dimensional space is well approximated by a uniform continuous distribution over the region bounding the constellation. This is called the continuous approximation [12].

can be shown that the use of equal energy maximizes the rate for a cubical shaping region[2] (subject to a constraint on total energy) [13].

At the receiver, a modulation scheme for each sub-channel is selected such that a target bit error rate (BER), P_b, is achieved. The indices of the corresponding modulation schemes are sent to the transmitter. The received SNR at the kth sub-channel is

$$SNR_k = \frac{\mathcal{E}\lambda_k^2}{\sigma^2 + \widehat{\sigma}_k^2}, \tag{4}$$

where $\widehat{\sigma}_k^2$ is the corresponding noise variance caused by the quantization error in the kth sub-channel. We consider a set of QAM modulation formats. The rate of the kth sub-channel, r_k, is computed such that $r_k = \max_{P(r,SNR_k)\leq P_b} r$, where $P(r, SNR)$ is the BER function of a QAM modulation scheme in terms of the rate r and SNR. An approximation formula for $P(r, SNR)$ is given in [14]. If none of the modulation formats meets the desired BER in a given sub-channel, no data stream is sent over that sub-channel. In fact, the total power is allocated equally among the sub-channels in which the desired BER is met.

III. Feedback Design: Channel Singular Matrix Quantization

Noting that the receiver detects the sub-channels separately, the quantizers are designed to minimize the interference between the sub-channels. The variance of the interference signal is

$$\begin{aligned} E(\|\mathbf{\Lambda}\mathbf{U}^*\mathbf{\Delta}\mathbf{U}\mathbf{x}\|^2) &= \lambda^2 E\mathrm{Tr}(\mathbf{U}^*\mathbf{\Delta}\mathbf{U}\mathbf{x}\mathbf{x}^*\mathbf{\Delta}\mathbf{U}^*\mathbf{U}) \\ &= \lambda^2 E\mathrm{Tr}(\mathbf{\Delta}\mathbf{U}\mathbf{\Delta}\mathbf{U}^*\mathbf{x}\mathbf{x}^*) \\ &= \lambda^2 \mathcal{E} E(\|\mathbf{\Delta}\mathbf{U}\|^2), \end{aligned} \tag{5}$$

where Tr denotes the trace function, $\|\mathbf{S}\|^2 = \mathrm{Tr}(\mathbf{S}\mathbf{S}^*)$ and $E(\mathbf{\Lambda}^2) = \lambda^2\mathbf{I}$ [15]. Note that the singular values are not ordered. In deriving (5), we use the property that the singular values of a Gaussian matrix with *i.i.d.* entries are independent from the corresponding singular vectors [15]. In the following, we develop two methods to quantize a unitary matrix to minimize (5).

[2] N dimensional cubical shaping region is a cube bounded between $-a_i$ and a_i along the ith dimension, $1 \leq i \leq N$.

We consider Givens rotation which decomposes a unitary matrix to the minimum number of parameters ($n^2 - n$ parameters for an $n \times n$ matrix) [11]. An $n \times n$ unitary matrix **U** can be decomposed in terms of the products of Givens matrices [11], i.e.

$$\mathbf{U} = \prod_{k=1}^{n-1} \prod_{i=k+1}^{n} \mathbf{G}(k,i). \tag{6}$$

Each $\mathbf{G}(k,i)$ consists of two parameters, $c_{k,i}$ and $s_{k,i}$, where $c_{k,i}$ is in both the (k,k)th and the (i,i)th positions, $s_{k,i}$ is in the (k,i)th position and $-s^*_{k,i}$ is in the (i,k)th position. The other diagonal elements of the matrix $\mathbf{G}(k,i)$ are 1 and the remaining elements are zero. Since $\mathbf{G}(k,i)$ is a unitary matrix, then $|c_{k,i}|^2 + |s_{k,i}|^2 = 1$. In this work, we assume that the procedure of decomposing the unitary matrix is performed such that $c_{k,i}$ is real and non-negative [13]. If **U** is a singular matrix derived from an $n \times n$ Gaussian matrix with *i.i.d.* entries, the set of Givens matrices $\mathbf{G}(k,i)$, $1 \leq k < i \leq n$, in (6) will be statistically independent of each other and the probability distribution function (PDF) of the elements of $\mathbf{G}(k,i)$ is[3] [16]

$$p_{c_{k,i},\angle s_{k,i}}(c,\angle s) = p_{c_{k,i}}(c) p_{\angle s_{k,i}}(\angle s) = \frac{i-k}{\pi} c^{2(i-k)-1}, 0 \leq c \leq 1, \quad \angle s \in [-\pi, \pi]. \tag{7}$$

Based on the criterion presented for the quantizer design in (5), we define the distortion measure as follows

$$D(\mathbf{Q}, \widehat{\mathbf{Q}}) = \frac{1}{2} E(\|\mathbf{Q} - \widehat{\mathbf{Q}}\|^2), \tag{8}$$

where $\widehat{\mathbf{Q}}$ is the quantized version of **Q**. Using (6) and (8), we can easily derive the first order approximation of the distortion measure of **U** as follows [13]

$$D(\mathbf{U}, \widehat{\mathbf{U}}) \simeq \frac{1}{2} \sum_{k=1}^{n-1} \sum_{i=k+1}^{n} E(\|\mathbf{G}(k,i) - \widehat{\mathbf{G}}(k,i)\|^2). \tag{9}$$

In the following the quantization schemes for an $n \times n$ unitary matrix is presented. The quantization methods can be easily generalized for a non-square unitary matrix [13].

[3] After we completed this work as reported in [16], we became aware of [10] which independently proves a similar result.

1) Method A: The parameters of $\mathbf{G}(k,i)$, $c_{k,i}$ and $\theta_{k,i} = \angle s_{k,i}$, are quantized as $\widehat{c}_{k,i}$ and $\widehat{\theta}_{k,i}$, independently, for $1 \leq k < i \leq n$. The matrix $\widehat{\mathbf{G}}(k,i)$ with the corresponding parameters $\widehat{c}_{k,i}$ and $\widehat{s}_{k,i}$ is constructed at the transmitter, using

$$\widehat{s}_{k,i} = \sqrt{1-\widehat{c}_{k,i}^2}\, e^{j\widehat{\theta}_{k,i}}, \tag{10}$$

which forces $\widehat{\mathbf{G}}(k,i)$ to be a unitary matrix. Alternatively, one can quantize the underlying complex values using polar representation [17]. Using (10) and applying the first order approximation, we have [13]

$$\|\mathbf{G}(k,i) - \widehat{\mathbf{G}}(k,i)\|^2 \simeq \frac{2}{1-c_{k,i}^2}(c_{k,i}-\widehat{c}_{k,i})^2 + 2(1-c_{k,i}^2)(\theta_{k,i}-\widehat{\theta}_{k,i})^2 \quad 1 \leq k < i \leq n \tag{11}$$

Substituting (11) in (9) and using (7), we have

$$D(\mathbf{U},\widehat{\mathbf{U}}) \simeq \sum_{k=1}^{n-1}\sum_{i=k+1}^{n} E\left(\frac{(c_{k,i}-\widehat{c}_{k,i})^2}{1-c_{k,i}^2}\right) + \frac{1}{2(i-k)+1}E(\theta_{k,i}-\widehat{\theta}_{k,i})^2. \tag{12}$$

Noting (12), we design Linde-Buzo-Gray (LBG) quantizers for $c_{k,i}$ and $\theta_{k,i}$ to minimize $E\left(\frac{(c_{k,i}-\widehat{c}_{k,i})^2}{1-c_{k,i}^2}\right)$ and, $E(\theta_{k,i}-\widehat{\theta}_{k,i})^2$, $1 \leq k < i \leq n$, respectively. The quantizer for $\theta_{k,i}$ follows the conventional approach to iterative design of a scalar LBG quantizer [18], while for $c_{k,i}$, the iterative design procedure should use the following reconstruction value, $\widehat{c}_{k,i} = \frac{E(\frac{c_{k,i}}{1-c_{k,i}^2})}{E(\frac{1}{1-c_{k,i}^2})}$, which is easily derived by setting $\frac{\partial}{\partial \widehat{c}}E\left(\frac{(c-\widehat{c})^2}{1-c^2}\right) = 0$.

We utilize dynamic programming to find the optimum bit allocation among the quantizers. First, we design b-bit quantizers for $c_{k,i}$ and $\theta_{k,i}$, $1 \leq k < i \leq n$ and $0 \leq b \leq B$. Then, we calculate $\mu^b(c_{k,i}) = E\left(\frac{(c_{k,i}-\widehat{c}_{k,i})^2}{1-c_{k,i}^2}\right)$, and $\mu^b(\theta_{k,i}) = \frac{1}{2(i-k)+1}E(\theta_{k,i}-\widehat{\theta}_{k,i})^2$, $1 \leq k < i \leq n$ and $0 \leq b \leq B$, using the PDF of $c_{k,i}$ and $\theta_{k,i}$ given in (7). We use a trellis diagram with B states and n^2-n stages to allocate B bits to the quantizers corresponding to $c_{k,i}$ and $\theta_{k,i}$, $1 \leq k < i \leq n$. In the trellis diagram, each branch represents the difference between the number of bits corresponding to the two ending states on the branch. The metric of the branch connecting the lth state at the $(j-1)$th trellis stage to the $(l+b)$th state at the jth trellis stage is $\mu^b(\vartheta_j)$, where ϑ_j is the quantization parameter corresponding to the jth stage. The search through the trellis determines the path with the minimum overall distortion and the corresponding number of bits for each parameter. The overall additive metric along a given trellis path is equal to the overall distortion given in (9). Examples of the bit allocation for the Givens parameters of a 3×3 unitary matrix is provided in Table I.

2) Method B: In this method, we quantize each Givens matrix as one unit. Let us define a new parameterization as follows: $c = \cos(\eta)$ and $s = e^{j\theta} \sin(\eta)$, where $0 \leq \theta \leq 2\pi$ and $0 \leq \eta \leq \pi$. We use the LBG algorithm to determine the regions and centroids of the two-dimensional quantizers corresponding to various (η, θ) for each Givens matrix. Using (8), the distortion function of a Givens matrix is

$$D(\mathbf{G}, \widehat{\mathbf{G}}) = \sum_{m=1}^{T} \int_{R_m} (1 - \cos(\eta)\cos(\eta_m) + \sin(\eta)\sin(\eta_m)\cos(\theta - \theta_m))\, p(\eta, \theta) d\eta d\theta, \qquad (13)$$

where R_m is the mth quantization region and T is the number of quantization partitions. The centroid (η_m, θ_m) is determined iteratively by minimizing the distortion in the region R_m [13]

$$\theta_m = \tan^{-1}\left(\frac{\varsigma_m}{\gamma_m}\right), \qquad (14)$$

$$\eta_m = \tan^{-1}\left(\frac{\sqrt{\varsigma_m^2 + \gamma_m^2}}{\int_{R_m} \cos^{l+1}(\eta)\sin(\eta) d\eta d\theta}\right), \qquad (15)$$

where $\gamma_m = \int_{R_m} \cos^l(\eta)\sin^2(\eta)\cos(\theta) d\eta d\theta$, $\varsigma_m = \int_{R_m} \cos^l(\eta)\sin^2(\eta)\sin(\theta) d\eta d\theta$, and $l = 2(i - k) - 1$, in the case of quantizing $\mathbf{G}(k, i)$ in (6). In this method, similar to the earlier case, a trellis diagram is used for the optimum bit allocation. The trellis diagram contains $\frac{n^2-n}{2}$ stages, each corresponding to a Givens component of an $n \times n$ unitary matrix, and B states where B is the total number of bits.

IV. Performance Evaluation

Fig. 1 shows the average bit rate versus SNR for different MIMO systems with $M = 3$ at the target BER$= 5 \times 10^{-3}$. Method B outperforms method A at the cost of a higher complexity for the codebook search. It is observed that the performance gain, compared to the gain of a 3×3 open loop MIMO system with the ML decoding, is noticeable. We also compare the performance of the proposed system with that of a V-BLAST system which is proposed as a solution to overcome the decoding complexity at the receiver. Note that as the receiver in the proposed method decodes the sub-channels separately, the decoding complexity is similar to that of the V-BLAST. Fig. 1 displays a significant improvement in comparison with the V-BLAST, showing the gain achieved through feedback.

In Fig. 2, we plot the BER of a 3×3 MIMO system using 9 bits to feed back the precoding unitary matrix in each block. The transmitter sends two independent streams of 64-QAM symbols with equal energy over the two sub-channels with the higher gains. The third sub-channel is left empty. The proposed methods and the methods in [9] and [19] are used to quantize the precoding unitary matrix. In [9], the authors use Householder reflections to decompose an $n \times m, m \leq n$ unitary matrix into m unit-norm vectors with different dimensions, $q_1 \in S_n$, $q_2 \in S_{n-1}$, ..., $q_m \in S_{n-m+1}$, where $S_t = \{u \in \mathbb{C}^t :\parallel u \parallel = 1\}$. Then, vector quantization is applied to separately quantize q_1 to q_m. In [4], a method which has been proposed in [19] (to design unitary space-time constellations) is used to directly quantize the precoding unitary matrices. The bit allocation for different methods is shown in Table II, and the codebook search complexity of different quantization methods is compared in Table III. In order to search a codebook in the method proposed in [9], one needs to perform 32 vector multiplications of size 3, 32 norm calculations and 32 comparisons to select the corresponding q_1. Similarly, 16 vector multiplications of size 2, 16 norm calculations and 16 comparisons is needed to select the corresponding q_2. In the method used in [4], one needs to perform exhaustive search among 2^9, 3×3 matrices, requiring 2^9 matrix multiplications and 2^9 trace calculations. Note that the complexity of SVD, Givens rotations and Householder reflection is in the order of n^3 for an $n \times n$ matrix [11]. Although the method in [9] and the unitary space-time constellation design used in [4] outperform our quantization schemes, our proposed methods have a much lower complexity.

V. Conclusion

In this work, we have presented efficient methods for the channel information quantization in a MIMO system. We have developed efficient algorithms for the quantization of the underlying unitary matrices. Compared to the similar methods, our methods provide very low codebook search complexity at the expense of minor performance degradation. Simulation results show a significant improvement as compared to a MIMO system without feedback, at the cost of a low-rate feedback link and a small increase in the computational complexity.

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TABLE I

THE BIT ALLOCATION FOR DIFFERENT GIVENS PARAMETERS.

$\theta_{1,2}$	$\theta_{1,3}$	$\theta_{2,3}$	$c_{1,2}$	$c_{2,3}$	$c_{1,3}$	Total bits
3	3	3	2	2	1	14
4	4	4	3	3	2	20
5	5	5	4	4	4	27

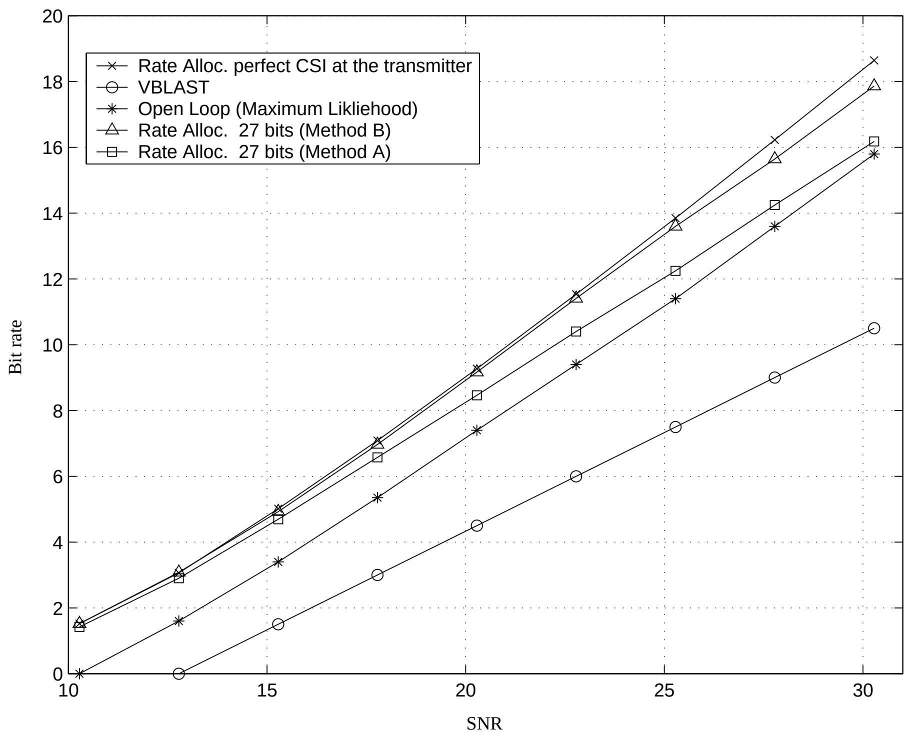

Fig. 1. The average bit rate for different schemes where $M = 3$. The target BER$= 5 \times 10^{-3}$.

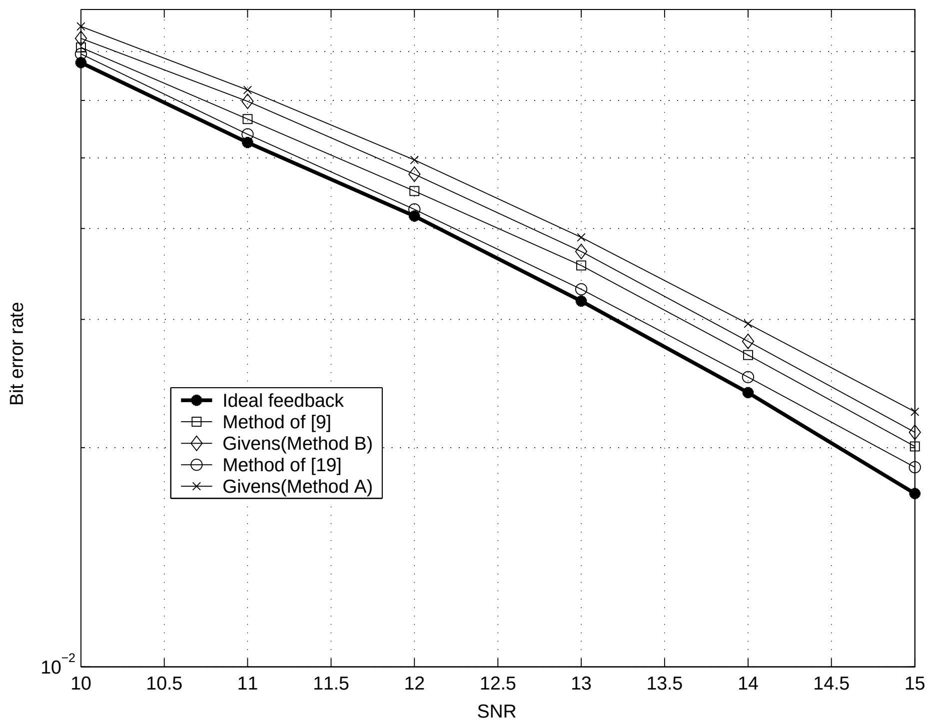

Fig. 2. The bit error rate for different schemes in a 3×3 MIMO system sending 2 64-QAM streams

TABLE II

THE BIT ALLOCATION FOR METHOD A, METHOD B AND HOUSEHOLDER REFLECTION METHOD

$c_{1,2}$	$c_{1,3}$	$c_{2,3}$	$\theta_{1,2}$	$\theta_{1,3}$	$\theta_{2,3}$
1	1	1	2	2	2

$G(1,2)$	$G(1,3)$	$G(2,3)$
3	3	3

q_1	q_2
5	4

TABLE III

THE CODEBOOK SEARCH COMPLEXITY OF DIFFERENT METHODS ARE COMPARED.

	Givens(Method A)	Givens(Method B)	Householder [9]	Space-time Constel. [19]
Multiplications	18	72	768	9216
Additions	0	48	384	8704
Comparisons	18	24	48	512