An Iterative Groupwise Multiuser Detector

for Overloaded MIMO Applications

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Abstract

We consider a narrowband multiuser system in which several transmitting users are received through a synchronous, flat fading channel at an antenna array. The paper introduces a multiuser detection technique that combines groupwise detection with iteration. It is well suited to overloaded conditions, where there are more transmitter antennas than receive antennas. The soft decisions it uses internally also make it suitable as a detector in a concatenated structure. Its performance and computation can be traded off through selection of group size. A variant of the algorithm achieves further computation reduction by incorporating a soft sphere detection core.

Keywords: Group detection, iterative detection, MAP estimation, MIMO systems, Sphere Decoding, soft interference cancellation, multiuser channels, multiuser detection.

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I. INTRODUCTION

We present a multiuser detection (MUD) technique for a narrowband multi-input, multi-output (MIMO) system, with several independently transmitting users and a receiver equipped with a diversity array. In the challenging overloaded configuration, where there are fewer receive antennas than the total number of transmit antennas, the algorithm provides significantly better performance than previously reported suboptimal (i.e., reduced computation) methods.

Most existing MUD techniques perform poorly in overloaded (i.e., underdetermined) conditions. Zero forcing (ZF) solutions [1] cannot be obtained because of matrix singularity, and minimum mean squared error (MMSE) solutions yield poor performance. The original probabilistic data association (PDA) algorithm [2, 3] rests on a preliminary pseudo-inverse solution, which cannot be obtained in overloaded conditions. For the same reason (matrix singularity), the original V-BLAST algorithm [4] does not apply. In group detection (GD) [5], all interferers not in the group being detected are nulled by ZF, so that the number of "excess users" (the number of transmit antennas minus the number of receive antennas) cannot exceed the group size minus one. Sphere detection (SD) [6] does reach the maximum likelihood (ML) solution, but with complexity that is exponential in the number of excess users. A modified PDA algorithm [7] removes the pre-filter to allow for overloaded systems; it can be shown to be equivalent to the technique presented here in the special case of single-user groups.

On the other hand, optimum MUD methods, such as joint maximum likelihood (JML) [8] and joint maximum *a posteriori* probability (JMAP) detection (MAP bit detection in [9]) do operate well in overloaded conditions, and cost just a small fraction of a dB per additional user [8]. Unfortunately, their complexity grows exponentially with the number of users and they quickly become impractical.

Although the motivation for reduced complexity suboptimal algorithms for overloaded conditions is clear, there has been little reported work to date. V-BLAST, modified to use

MMSE suppression of undetected users per the suggestion in [4] (which we will term MMSE V-BLAST) is one candidate. Another is a variation of group detection, with MMSE suppression of undetected groups combined with hard cancellation of detected users [10], which we will term GD.

In this paper, we introduce a MUD technique (iterative MUD, or IMUD) that combines group detection with *a posteriori* probability (APP) extraction [9]. The APPs enable the other features of the method: soft cancellation of detected users, iteration to improve the quality of the symbol estimates, and concatenation with coded transmission of the users' data. Its performance is excellent, making even heavily overloaded conditions accessible. The novelty of this work lies in the selection of component techniques suitable to overloaded conditions and the exploration of resulting performance. As a by-product, we also present new results showing the surprising resilience of MMSE V-BLAST to overload.

II. THE IMUD ALGORITHM

A. Description of Signals

We address the basic multiuser interference problem addressed in [1]-[5]. Several users transmit independent data in symbol-synchronous fashion with identical pulse shapes through independently flat-fading channels. For simplicity of discussion, we identify the number of users (or data streams) N with the number of transmit antennas. The signals are received at an M-element antenna array with the same mean SNR at each antenna. Variations such as asynchronous transmission, signature pulse shapes, or delay spread would enhance performance, since they add measurement dimensionality. Conversely, correlated antennas would reduce the diversity. However, we chose to deal with the simplest system to uncover the fundamental performance limits of IMUD.

In such conditions, the matched filter outputs at the receiver can be represented by the length-M vector

$$\mathbf{y} = \mathbf{H}\,\mathbf{b} + \mathbf{n}\,, \tag{1}$$

where $\mathbf{H}$ is the $M \times N$ matrix of channel gains, $\mathbf{b}$ is the length-N vector of user data and $\mathbf{n}$ is the length-M vector of spatially white Gaussian noise. The elements of $\mathbf{H}$ are independent, complex Gaussian variates with zero mean and variance ½ in both the real and imaginary components. For simplicity, all users will employ the BPSK constellation ±1, although this is not essential; higher density constellations will increase the complexity of the group probability extraction (Section II*B*), particularly during creation of the joint probabilities. The elements of $\mathbf{n}$ have zero mean and variance $1/(2\Gamma)$ in both the real and imaginary components, where Γ is the mean SNR per symbol at each antenna. Perfect channel estimation is assumed in the present investigation. The effect of imperfect estimates on performance is the subject of future work.

B. Group Detection

IMUD group detection breaks the N users into N_G groups of size G, and estimates the user symbols of each group in succession, using the group APP extraction (GAPPE) described in Section II*C*. Its complexity is exponential only in the group size G, rather than the total number of users N. The price is suboptimal detection. By varying the group size, computation load can be traded against detection performance.

The measurement vector on which the symbol estimates are based is modified after each group is processed (Section II*D*). For the detection of group j, we write it as

$$\mathbf{y}^{(j)} = \sum_{i=1}^{j-1} \mathbf{r}_i + \mathbf{H}_j \mathbf{b}_j + \sum_{i=j+1}^{N_G} \mathbf{H}_i \mathbf{b}_i + \mathbf{n} = \mathbf{H}_j \mathbf{b}_j + \mathbf{u}^{(j)}\,, \tag{2}$$

where $\mathbf{u}^{(j)}$ represents the undesired components of the signal and $\mathbf{r}_i$ is the zero-mean residual error after soft cancellation (Section II*D*) of the earlier group i. The residual errors $\mathbf{r}_i$ are approximated as being uncorrelated with each other and with the other components of $\mathbf{u}^{(j)}$.

C. Group APP Extraction

The core of IMUD is group APP extraction (GAPPE). The resulting soft estimates of individual data symbols enable both iteration and soft cancellation of interference. The IMUD framework accommodates a variety of GAPPE techniques, provided that they are soft-input/soft-output. The optimal GAPPE is a brute force marginalization over all user data combinations; it is a group-wise variation on the MAP bit detection technique in [9]. The soft SD methods provide a lower complexity alternative. They marginalize only over the points (data vectors) in a sphere around the received vector $\mathbf{y}^{(j)}$ rather than over all data vectors. In the simulations of Section III, we use the soft SD introduced in [11] in its Approximation B variant.

The key approximation for optimum GAPPE is that $\mathbf{u}^{(j)}$ is Gaussian, as in [12], which makes the conditional probability density of the measurements equal to $\mathrm{N}_{\mathbf{y}^{(j)}}\left(\mathbf{H}_j\mathbf{b}_j, \mathbf{R}_{\mathbf{u}}^{(j)}\right)$, where $\mathrm{N}_{\mathbf{y}^{(j)}}\left(\boldsymbol{\mu}, \boldsymbol{\sigma}^2\right)$ is the Gaussian pdf of $\mathbf{y}^{(j)}$ with mean $\boldsymbol{\mu}$ and variance $\boldsymbol{\sigma}^2$. The *a priori* probabilities of the components b_{nj} of $\mathbf{b}_j$ are denoted $\Pr^{(a)}(b_{nj})$ and the components are assumed to be statistically independent. Initially, all *a priori* probabilities are set to ½. The APPs $\Pr^{(o)}(b_{nj}) = \Pr(b_{nj} \mid \mathbf{y}^{(j)})$ may then be computed by marginalizing over the joint probability

$$\Pr\left(\mathbf{y}^{(j)}, b_{nj} = k\right) = \sum_{\mathbf{b}_j : b_{nj} = k} \left(\Pr\left(\mathbf{y}^{(j)} \mid \mathbf{b}_j\right) \prod_{m=1}^{G} \Pr^{(a)}(b_{mj}) \right). \tag{3}$$

Such computations may be simplified by use of log-likelihood ratios (LLRs), to represent the symbol probabilities, and the log-MAP algorithm [13], to compute (3).

D. Soft Cancellation

After processing a group, the APPs are used to improve performance and convergence. In [2, 3, 7], the technique of probabilistic data association (PDA) uses soft outputs to allow convergence. Both IMUD and PDA make the assumption that all undesired users are Gaussian distributed interferers, with IMUD using group structure to exploit the correlation between users.

We have the *a posteriori* mean and error variance of b_{nj} as

$$\mu_{nj}^{(o)} = \Pr^{(o)}(b_{nj} = 1) - \Pr^{(o)}(b_{nj} = -1). \tag{4}$$

$$\sigma_{nj}^2 = 1 - \mu_{nj}^{(o)2} \tag{5}$$

The error vector $\boldsymbol{\phi}_j = \mathbf{b}_j - \boldsymbol{\mu}_j^{(o)}$ has zero mean, where $\boldsymbol{\mu}_j^{(o)}$ is a vector of means for the j^{th} group. To perform a soft cancellation, the measurement vector is updated using $\boldsymbol{\mu}_j^{(o)}$ by

$$\mathbf{y}^{(j+1)} = \mathbf{y}^{(j)} - \mathbf{H}_j \boldsymbol{\mu}_j^{(o)}, \tag{6}$$

in preparation for processing the next group. The errors are approximated as being uncorrelated between and within groups, so that the covariance matrix of the detection error $\boldsymbol{\phi}_j$ of group *j* is

$$\mathbf{R}_{\phi j} = E\left[\boldsymbol{\phi}_j \boldsymbol{\phi}_j^\dagger\right] = diag\left(\sigma_{1j}^2, \ldots, \sigma_{Gj}^2\right). \tag{7}$$

The covariance matrix of the unwanted components can then be written in update form as

$$\mathbf{R}_{\mathbf{u}}^{(j+1)} = \mathbf{R}_{\mathbf{u}}^{(j)} - \sum_{i=1}^{G} \left(1 - \sigma_{ij}^2\right) \mathbf{h}_j^{(i)} \mathbf{h}_j^{(i)\dagger}. \tag{8}$$

Inversion of (8) is required for the next group to be detected. An alternative to explicit inversion, based on repeated application of the matrix inversion lemma [14] to the second line of (8), may be computationally attractive if the group size *G* is much less than the number of antennas *M*.

E. Detection Order

Determination of the best ordering in group detection is more complex than in single detection (V-BLAST), because interactions within the group and between the group and the remaining users must be considered. The problem of optimum grouping for a method based on ZF and hard cancellation was considered in [15]. An alternative in [10] is to assign users arbitrarily to groups, then to order the groups.

We found it most effective to assign users to groups on the basis of error variance minimization (EVM) [16]. Consider formation of group *j*. For simplicity, we consider the residual interferences $\mathbf{r}_i$, $i < j$ to be zero; i.e., perfect DF. Then, MMSE estimation of the remaining $\left(N_G - j + 1\right)G$ user symbols from the measurement vector $\mathbf{y}^{(j)}$ has error variances equal to the diagonal entries of

$$\boldsymbol{\Sigma}_{\mathbf{e}}^2 = \mathbf{I} - \mathbf{H}^{(j)\dagger}\left(\mathbf{H}^{(j)}\mathbf{H}^{(j)\dagger} + \left(1/2\Gamma\right)\mathbf{I}\right)^{-1}\mathbf{H}^{(j)}, \tag{9}$$

where the $M \times \left(N_G - j + 1\right)G$ matrix $\mathbf{H}^{(j)}$ denotes the columns of $\mathbf{H}$ remaining after removal of previously formed column groups $\mathbf{H}_1, \ldots, \mathbf{H}_{j-1}$. The *G* users with the lowest error variances in

(9) are then selected to compose group j, and their columns are removed from $\mathbf{H}^{(j)}$ to form $\mathbf{H}^{(j+1)}$. The process repeats until there are no remaining users.

F. Iteration and Complexity

In Sections II*B*-II*E*, we have the initial formation of groups by EVM and, for each group, the processes of GAPPE and soft cancellation in preparation for the next group. After a complete pass through all groups, IMUD iterates the process. The second and subsequent iterations are similar to the first, with two exceptions: first, the *a priori* probabilities in GAPPE are set to the APP values calculated in the prior iteration (Section III*D*); and second, users are assigned to groups randomly, rather than by EVM (Section III*B*). The purpose of randomization is to break up intra-group dependencies caused by the set of gain vectors in each $\mathbf{H}_j$, as well as dependencies caused by the initial detection order. It is analogous to interleaving in iterative detection of turbo codes [17].

The overall complexity of IMUD using optimal GAPPE is linearly dependent on the number of iterations and exponentially dependent on the group size. For IMUD using the soft SD GAPPE, the dependence on group size is much weaker. With the right pre-sorting technique, IMUD can outperform an MMSE V-BLAST system with a comparable complexity in a fast fading system due to the interference covariance matrix inverses.

III. PERFORMANCE SIMULATIONS

In a series of simulations, we compared the performances of IMUD, the group ML with MMSE suppression of [10] (denoted GD in the results), simple MMSE and MMSE V-BLAST [18]. Throughout this section, the notation (M, N_G, G), for number of antennas, number of groups and size of groups, will refer to the system configuration. The number of users is then $N_G G$. Two of the methods (MMSE and V-BLAST) do not perform grouping of the users. The other group method, GD, employs a different sorting technique, uses hard cancellation as it progresses through the groups, and does not iterate.

A. Loading and Grouping

The number of users, relative to the number of antennas, has a significant effect. We can distinguish three cases: underloaded arrays $(N_G G < M)$, critically loaded arrays $(N_G G = M)$ and overloaded arrays $(N_G G > M)$. We have not presented results for underloaded arrays since, being accessible to a variety of simple methods, they are of less interest for IMUD use.

Fig. 1 illustrates the performance of the various detection methods on a critically loaded $(12, 2, 6)$ system. For reference, it also shows the union bound [8] on the error rate of true JML. Although this bound is tight for high SNR, it is clearly rather loose at low SNR values. The notation IMUD(*I*) indicates IMUD with *I* iterations. For a BER of 10^{-4}, 10^5 independent trials of data, channel gains and noises were used. With two iterations, IMUD almost matches the performance of true JML, but requires far less computation. At an error rate of10^{-4}, IMUD(2) is almost 3 dB better than GD and MMSE V-BLAST. Further iteration produced no perceptible improvement in IMUD.

To stress IMUD further, we recast the critically loaded 12 antenna system as $(12, 6, 2)$. Fig. 2 shows that IMUD(2) degrades by less than 1 dB at a BER of 10^{-4} compared with $(12, 2, 6)$. IMUD's soft cancellation mitigates the effects of the decrease in group size. This is important, since it means that we can run IMUD with relatively small groups without a large drop in performance. We also see that the GD method degrades significantly, and provides much poorer performance than MMSE V-BLAST. The various groupings allow IMUD to provide a flexible tradeoff of computation and performance, from true JML down to a variant of PDA. Other configurations are reported in [16, 19].

Next, we examine the more interesting case of overloaded arrays. In Fig. 3, we reduce the number of antennas to eight, while keeping twelve users, organized in $(8, 6, 2)$ and $(8, 2, 6)$ configurations. Like MMSE, GD shows an error floor. IMUD(2) is about 5.5 dB better than MMSE-VBLAST at a BER of 10^{-3}, and neither shows a floor. In a more extreme example, shown in Fig. 4, we used only six antennas to detect twelve users in a $(6, 2, 6)$ configuration.

Both MMSE-VBLAST and GD show evidence of an error floor at a BER of 10^{-2}. In contrast, IMUD with two iterations provides floor-free performance with a diversity order of about two. This good performance in overloaded conditions is one of the most compelling arguments for use of IMUD.

In the critically loaded situations presented in this paper, simulations have demonstrated that the performance of the soft SD GAPPE is indistinguishable from that of JML. In the IMUD(2) SD (8,2,6) curve in Fig. 3, a limitation in the overloaded case is observed. The first iteration has no degradation over the optimal GAPPE, while the second shifts away at higher SNRs.

One interesting feature of Figs. 1-4 is the relatively good performance of MMSE-VBLAST, even in moderately overloaded conditions, despite its attractively low computation. This feature of the algorithm does not seem to have been observed in previous publications.

B. Effect of Detection Order

IMUD detects users in EVM order for the first iteration, and in randomized order for subsequent iterations. Fig. 5 shows the benefits of this configuration compared to others for the first two iterations in a $(12,3,4)$ configuration. Not surprisingly, randomized ordering provides the worst performance for the first iteration; EVM ordering provides the best performance. It is interesting that the use of EVM ordering in the second iteration is poorer than random ordering in the second iteration. IMUD(2) EVM/EVM is not shown in Fig. 5 since its performance is nearly identical to that of IMUD(1) EVM. We believe this to be due to the fact that a second EVM sort makes little change in the order; consequently, randomization is required to break up the intra-group statistical dependencies. Ordering by the original ZF V-BLAST method [4] (not shown) is about 1 dB poorer than EVM on the first iteration, but only 0.1 dB poorer on the second. The LLR-based sorting of [20] (also not shown) had performance almost indistinguishable from EVM on both iterations.

C. Interference Cancellation and Number of Iterations

IC is employed in two places: from group to group within an iteration (inter-group), and from one iteration to the next. To assess their relative contributions quantitatively, we tested them separately. We compared performance with and without inter-group cancellation when there was no interference cancellation between iterations. For a (12,3,4) configuration, inter-group soft cancellation produced a 4 dB improvement compared with no cancellation at a BER of 10^{-3}. Next, we compared inter-group soft cancellation with inter-group hard cancellation. This is the principal difference between IMUD(1) and GD, so the importance of soft cancellation can be gauged from Figs. 1-3. In overloaded situations, soft cancellation is critical to success. Next, we compared the IMUD in Section II to IMUD with no cancellation between iterations. The number of iterations to convergence was affected. Fig. 6 shows that after 5 iterations, there is around a 0.25dB gain when using soft cancellation between iterations.

D. APP Propagation and Interference Cancellation

In iterative detection of signals defined by trellises and interleavers [17], it is important to distinguish between *a posteriori* probabilities and extrinsic information. Normally, only extrinsic information is propagated from one iteration to the next in order to prevent overemphasis of the *a priori* probabilities and consequent slow convergence or mis-convergence. IMUD addresses a basic MIMO configuration that does not require a trellis for signal description, and randomized detection order is a poor substitute for trellis constraints and interleaving. Nevertheless, it is of interest to explore the effect of propagating only extrinsic information between iterations. To start, we note that the soft IC information (4)-(8) is equivalent to the full APPs. Propagation of extrinsic-only information between iterations therefore requires removal of *a priori* information obtained from the previous iteration and elimination of IC. In [16], it was demonstrated that utilizing the extrinsic APPs in an overloaded (4,3,2) configuration results in a dramatic loss of BER performance, as well as a significant loss of diversity order. However, use of either soft IC or even partial *a priori* information provided the same performance as the combined APP propagation and use of IC.

IV. CONCLUSION

In this paper, a new iterative groupwise MUD technique was introduced. The structure of the group APP extraction, soft cancellation and iteration was derived. Some plots were generated as a comparison of existing techniques and the new iterative algorithms.

We have demonstrated that IMUD can outperform previous techniques, including groupwise detection and the ordered cancellation in V-BLAST. In an overloaded situation, IMUD's performance is exceptional; the error floor compared to other IC based detectors is greatly reduced. In a simulated system with 8 receive antennas and 12 users, IMUD outperforms MMSE V-BLAST with a 5dB or greater decrease in SNR at an error rate of 10^{-3}. Also, it seems that the most performance gain due to iteration is achieved by the second iteration.

As a new multiuser detection technique, IMUD is very promising. One of its biggest advantages is that it is iterative in nature, and uses soft decisions. This makes it a primary candidate for inclusion in iterative detection of multiple coded users, using IMUD in serial concatenation with trellis-based codes, whether space-time codes or conventional codes. There also may be applications in macrodiversity systems, where the soft decisions can be sent as *a priori*s to other base stations within a co-channel interference area.

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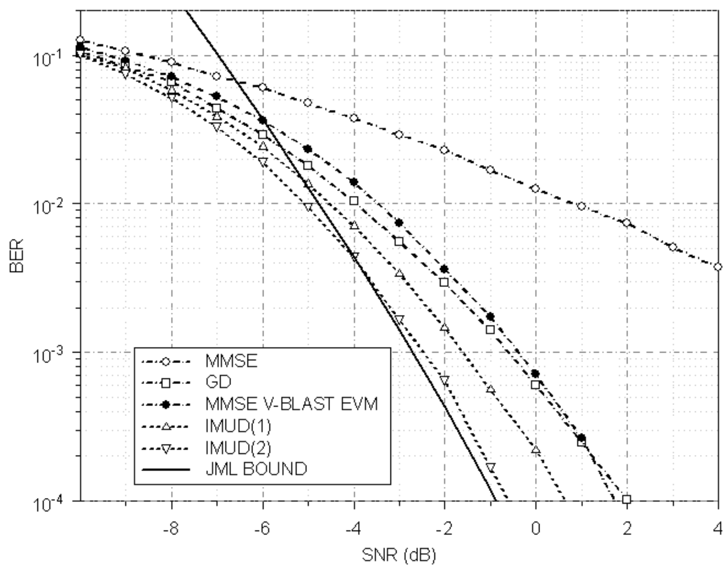

Fig. 1: (12,2,6) system (critically loaded).

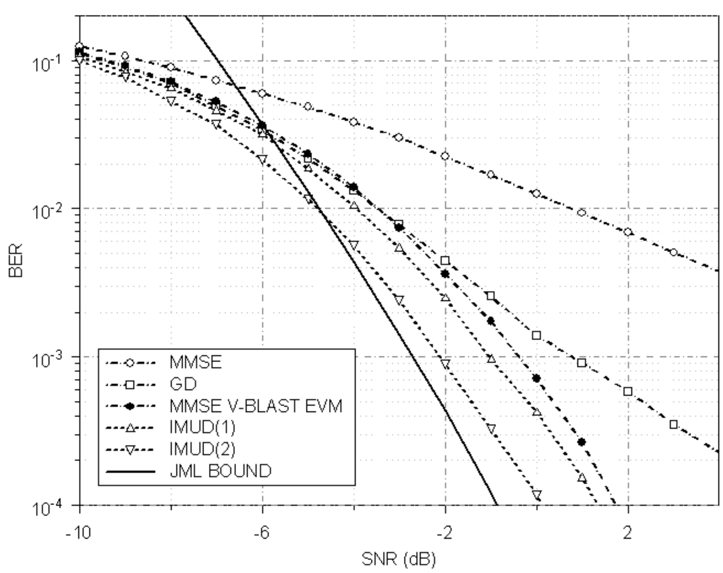

Fig. 2: (12, 6, 2) system (critically loaded). The effect of the smaller group size is evident in the group techniques of IMUD and GD.

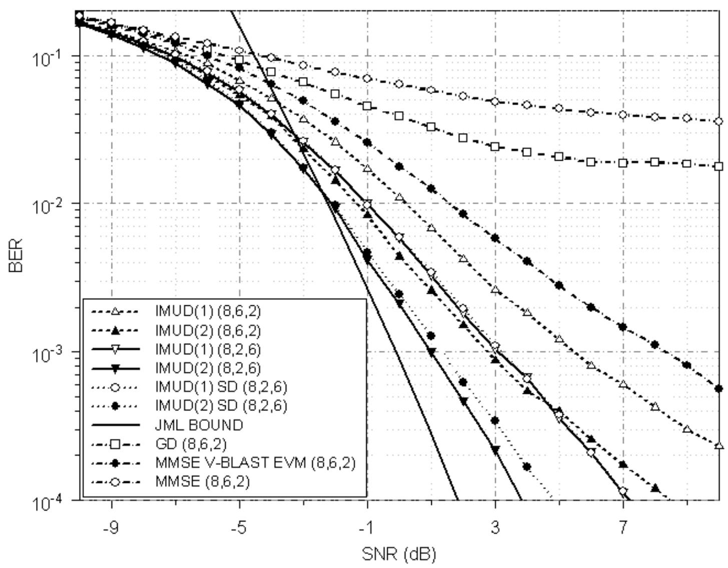

Fig. 3. (8,2,6) and (8,6,2) systems (overloaded).

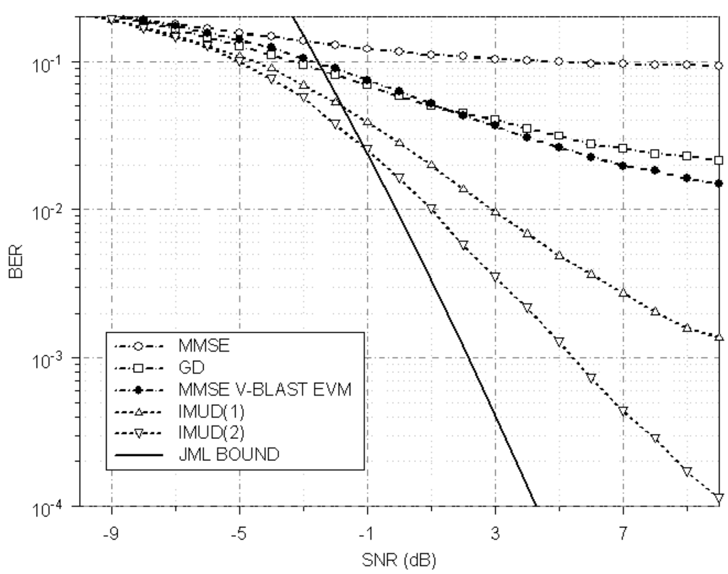

Fig. 4: (6,2,6) system (overloaded). The reduction in the error floor of IMUD is striking when compared to MMSE V-BLAST and straight group detection.

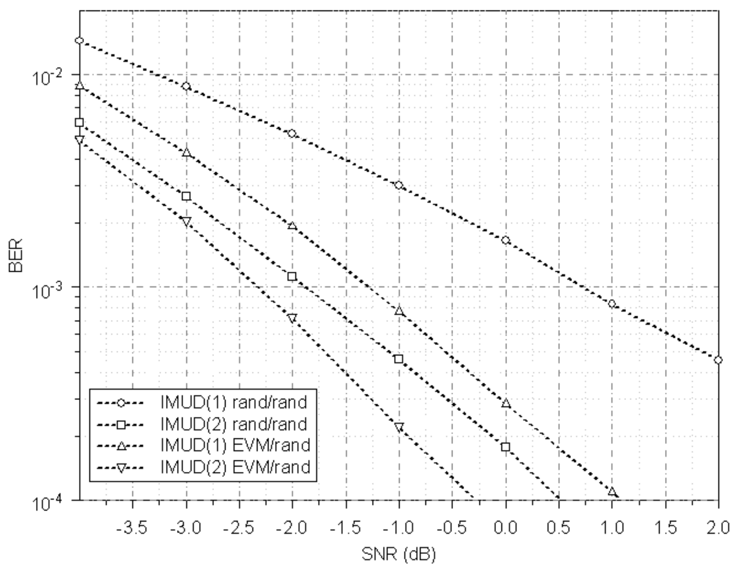

Fig. 5: Effect of detection order on a (12,3,4) configuration.

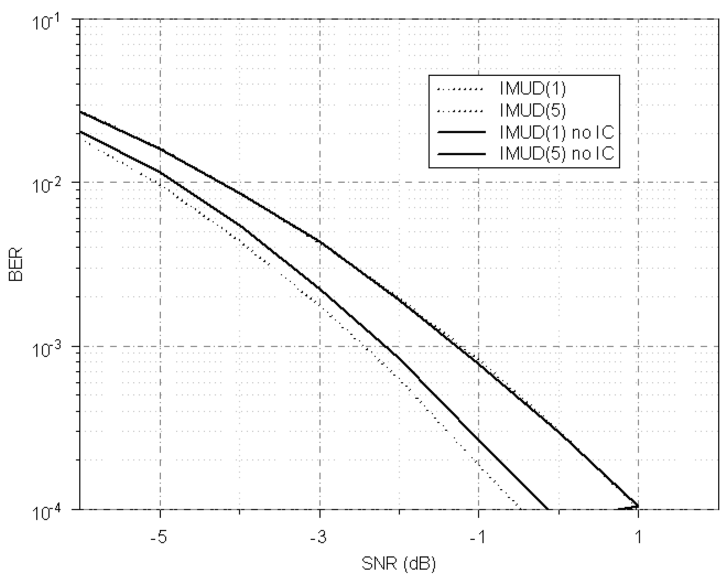

Fig. 6. Interference cancellation between iterations affects the number of iterations to convergence in a (12,3,4) configuration.