Robust Joint Frequency Offset and Channel Estimation for OFDM Systems

Tao Cui and Chintha Tellambura
Department of Electrical and Computer Engineering
University of Alberta
Edmonton, AB, Canada T6G 2V4
Email: {taocui, chinta}@ece.ualberta.edu

Abstract— **We consider the joint carrier frequency offset (CFO) and channel estimation for OFDM systems. A novel frequency offset estimator is derived based on maximum likelihood principles by using the pilot symbols embedded in each OFDM symbol. An iterative joint CFO and channel estimator is proposed to further reduce the estimation error and improve the system performance. The joint estimator is initialized with the first CFO estimate and linearly interpolated channel estimates. The CFO and channel response estimates are updated by maximum likelihood (ML) and least squares (LS) algorithms. Simulation results show that the proposed joint estimator is effective and can achieve excellent bit error rates in just one iteration.**

Keywords – Channel estimation, Frequency offset, OFDM, Maximum likelihood

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has received considerable interest in the wireless research community recently [1]. It has been used in European digital audio broadcasting (DAB) systems, high performance radio local area network (HIPERLAN) and 802.11a wireless LAN standards. OFDM uses a large number of orthogonal subcarriers and the bandwidth of each subcarrier is much less than the channel coherence bandwidth, which ensures that each subcarrier experiences flat fading. This results in greatly simplified equalization.

For wideband mobile communication systems, the radio channel is frequency-selective and can be time-variant. Therefore, accurate and fast estimation of channel is essential for high-performance, coherent detection of OFDM symbols. Pilot-symbol-aided channel estimation has proven to be a feasible method for OFDM systems [2]–[5]. Not surprisingly, all existing OFDM standards embed pilot symbols into the data frame for this purpose. However, if a CFO, a Doppler spread or phase noise exists at the receiver, subcarrier orthogonality is lost and this introduces interchannel interference (ICI). If an OFDM system suffers from severe ICI, pilot assisted channel estimation degrades. Consequently, CFO must be estimated and compensated for at the receiver before channel estimation. Several approaches for carrier acquisition and tracking have been published in [6]–[8]. Some of them are blind methods using the statistics of transmitted signals [7], [9]. However, such techniques may not be compatible with the existing OFDM standards which use deterministic pilot symbols.

Many channel estimation (CE) algorithms and CFO estimation algorithms published in the literature treat the two problems separately. The channel is estimated assuming zero CFO or CFO is estimated assuming perfect CE. To the best of our knowledge, few papers deal with joint estimation of channel and frequency offset. Ma, Kobayashi and Schwartz [10] derive an adaptive joint ML algorithm. Larsson, Liu, Li and Giannakis [11] derive a joint symbol timing and channel estimation for IEEE 802.11 WLAN systems. Since many OFDM standards adopt a pilot-embedded frame structure for CE, if the same pilot symbols can also be used to estimate the CFO, significant benefits may be realized. Our technique is motivated by this fact and is fully compatible with the existing standards.

We first derive an ML CFO estimator using the pilot symbols embedded in each data frame. The idea is to exploit the correlation structure induced by the fixed pilot symbols in an OFDM symbol. This first CFO estimate can be used to compensate the pre-DFT samples and the first set of channel estimates are obtained by linear interpolation. To further improve the estimation accuracy, we propose an iterative joint CE and CFO estimation technique using decision feedback. With this, the accuracy of channel estimation, frequency offset estimation and symbol detection is enhanced simultaneously. Since the pilots are used for both channel and frequency offset estimation, the pilot usage efficiency is greatly improved. The simulation results show that our proposed iterative joint channel and frequency offset estimator is effective. Just one iteration of our joint estimator can achieve the same Bit Error Rate (BER) as that of an ideal reference receiver with perfect knowledge of channel response and carrier frequency offset.

The rest of the paper is organized as follows. Section II reviews the basic baseband OFDM system model. Section III introduces the high resolution ML frequency offset estimator. Section IV derives a robust joint channel and frequency offset estimator. Computer simulation results are given in Section V and final conclusions are made in Section VI.

Notation: If x and y are Gaussian random variables (RV) with $E[x] = \mu_x$, $E[y] = \mu_y$ and $E\left[(x-\mu_x)^2\right] = E\left[(y-\mu_y)^2\right] = \sigma^2/2$, the $z = x + jy$ has Complex Gaussian distribution. We write $z \sim \mathcal{CN}(\mu_x + \mu_y, \sigma^2)$ in this case.

II. SYSTEM MODEL

Fig.1 shows block diagram of an OFDM system with pilot symbols. The binary source data are grouped and mapped into d_k, which is selected from a complex signal constellation $\mathcal{Q}$. The complex data are modulated by inverse discrete Fourier transform (IDFT) on N parallel subcarriers. The resulting OFDM symbol comprises N samples given by

$$x(n) = \sum_{k=0}^{N-1} X(k) e^{j(2\pi kn/N)}, \quad n = 0, 1, 2, \cdots, N-1 \tag{1}$$

where

$$X(k) = \begin{cases} d_k & k \in \bar{I}_p \\ p_k & k \in I_p \end{cases} \tag{2}$$

and I_p is the index set of subcarriers reserved for pilot symbols with P elements. A guard interval is typically inserted to prevent inter-frame interference. This includes a cyclic prefix which replicates the end of the IFFT output samples. The number of samples in the guard interval N_g is assumed to be larger than the delay spread of the channel. The signal is transmitted over a frequency selective fading channel which is modelled as [12, p.802]

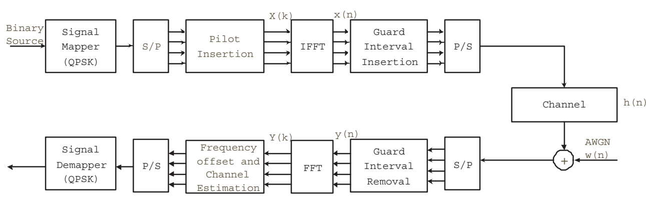

Fig. 1. OFDM system with pilot-symbol-aided channel estimation

$$h(t) = \sum_{n=0}^{L-1} \alpha_n \delta(t - \tau_l) \tag{3}$$

where $\alpha_n \sim \mathcal{CN}(0, \sigma_n^2)$ and τ_l is the delay of lth tap. When a CFO exists, the received signal after sampling is given by

$$y(n) = e^{j2\pi\epsilon\frac{n}{N}} \sum_{l=0}^{L-1} h(l) x(n-l) + w(n) \tag{4}$$

where $w(n)$ is an Additive White Gaussian Noise (AWGN) sample; h_l $(l = 0, \ldots, L-1)$ represents the sampled overall channel impulse response (which comprises the transmit/receive filters and the physical channel $h(t)$); L is the total number of propagation paths. The CFO normalized by the symbol rate is denoted by ϵ. We assume that the receiver performs coarse frequency acquisition so that the range of CFO is half of the frequency separation between adjacent subcarriers, $|\epsilon| < 0.5$. As usual, we assume the channel stays constant within each OFDM symbol. The receiver estimates the CFO, removes the guard interval and DFT demodulates $y(n)$. Let the CFO estimate be $\hat{\epsilon}$, then post-DFT received samples are

$$Y'(k) = \text{DFT}\{e^{-j\frac{2\pi\hat{\epsilon}n}{N}} y(n)\}, \quad k = 0, 1, 2, \cdots, N-1. \tag{5}$$

The channel response at pilot positions is estimated as

$$\hat{H}(k) = \frac{Y'(k)}{p_k} \quad k \in I_p. \tag{6}$$

Linear interpolation or other techniques are used to find $\hat{H}(k)$ for $k \in \bar{I}_p$ from the pilot channel estimates $\hat{H}(k)$ for $k \in I_p$. The transmitted data symbols $\{d_k\}$ can then be recovered using

$$\hat{X}(k) = \frac{Y'(k)}{\hat{H}(k)}, \quad k \in \bar{I}_p. \tag{7}$$

This is known as one-tap equalization.

III. ROBUST CARRIER FREQUENCY OFFSET ESTIMATION

A. Analysis

When a CFO ϵ exists between the transmitter and the receiver, the received post-DFT samples are given by [10]

$$\begin{aligned} Y'(k) = & \frac{\sin \pi\epsilon}{N \sin \frac{\pi\epsilon}{N}} X(k) H_k e^{\pi\frac{(N-1)\epsilon}{N}} \\ & + \text{ICI}(k) + W(k) \end{aligned} \tag{8}$$

where H_k is the channel frequency response at subcarrier k and $H_k = \sum_{l=0}^{L-1} h_l e^{-j2\pi lk/N}$ which is a Complex Gaussian RV. The complex Gaussian noise $W(k) \sim \mathcal{CN}(0, \sigma_n^2)$. The ICI term is given by

$$\text{ICI}(k) = \frac{1}{N} \sum_{i=0}^{N-1} \sum_{m \neq i} X(m) H_m e^{j2\pi\frac{i(m-k+\epsilon)}{N}}. \tag{9}$$

Since ICI (9) is not zero if $\epsilon \neq 0$, it can severely degrade the system performance. In particular, if the samples (8) are used in (6), channel estimates can be severely degraded. CFO must thus be estimated and cancelled before the DFT demodulation and channel estimation.

B. Maximum likelihood frequency offset estimation

We write (4) in vector form as

$$\mathbf{y} = \frac{1}{N} \Gamma(\epsilon) \mathbf{F}^H \mathbf{X} \mathbf{F}_L \mathbf{h} + \mathbf{w} \tag{10}$$

where $\mathbf{y} = [y(0), y(1), \cdots, y(N-1)]^T$, $\mathbf{h} = [h_0, h_1, \cdots, h_{L-1}]$, $\mathbf{w} = [w(0), w(1), \cdots, w(N-1)]$, $\Gamma(\epsilon) = \text{diag}(1, \exp(j2\pi\epsilon/N), \cdots, \exp(j2\pi(N-1)\epsilon/N))$, $\mathbf{X} = \text{diag}(X(0), X(1), \cdots, X(N-1))$ and $\mathbf{F}_L$ is a $N \times L$ submatrix of DFT matrix $\mathbf{F}$. Using Eq.(2), we write $\mathbf{X}$ as the sum of two diagonal matrices:

$$\mathbf{X} = \mathbf{X}_d + \mathbf{X}_p \tag{11}$$

where $\mathbf{X}_d = \text{diag}(s_1, s_2, \cdots, s_{N-1})$, $s_k = d_k$ if $k \in \bar{I}_p$, $s_k = 0$ otherwise. $\mathbf{X}_p = \text{diag}(t_1, t_2, \cdots, t_{N-1})$, $t_k = p_k$ if $k \in I_p$, $t_k = 0$ otherwise. Note here we do not consider virtual subcarriers for simplicity. However, our CFO estimator can also be applied to OFDM systems with virtual subcarriers.

Since the received signal can be modelled as complex Gaussian process, the autocorrelation matrix of the received signal is given by

$$\begin{aligned}\mathbf{R}_y =&E\{yy^H\}\\ =&\frac{1}{N^2}E\{\Gamma(\epsilon)\mathbf{F}^H\mathbf{X}\mathbf{F}_L\mathbf{h}\mathbf{h}^H\mathbf{F}_L^H\mathbf{X}^H\mathbf{F}\Gamma^H(\epsilon)\}+\sigma_n^2\mathbf{I}\\ =&\frac{1}{N^2}E\{\Gamma(\epsilon)\mathbf{F}^H\mathbf{X}\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H\mathbf{X}^H\mathbf{F}\Gamma^H(\epsilon)\}+\sigma_n^2\mathbf{I}\\ =&\frac{1}{N^2}E\{\Gamma(\epsilon)\mathbf{F}^H(\mathbf{X}_d+\mathbf{X}_p)\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H\\ &\times(\mathbf{X}_d+\mathbf{X}_p)^H\mathbf{F}\Gamma^H(\epsilon)\}+\sigma_n^2\mathbf{I}.\end{aligned} \tag{12}$$

It can be readily proved that

$$\begin{aligned}E\{\mathbf{X}_d\} =&0\\ E\{\mathbf{X}_p\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H\mathbf{X}_d^H\} =&0\\ E\{\mathbf{X}_d\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H\mathbf{X}_p^H\} =&0\\ E\{\mathbf{X}_d\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H\mathbf{X}_d^H\} =&\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H E\{\mathbf{X}_d\mathbf{X}_d^H\}\\ =&\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H\mathbf{D}\end{aligned} \tag{13}$$

where $\mathbf{D}=E\{\mathbf{X}_d\mathbf{X}_d^H\}$. Hence

$$\begin{aligned}\mathbf{R}_y =&\frac{1}{N^2}E\{\Gamma(\epsilon)\mathbf{F}^H(\mathbf{X}_d\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H\mathbf{X}_d^H\\ &+\mathbf{X}_p\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H\mathbf{X}_p^H)\mathbf{F}\Gamma^H(\epsilon)\}+\sigma_n^2\mathbf{I}\\ =&\frac{1}{N^2}\Gamma(\epsilon)\mathbf{F}^H(\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H\mathbf{D}\\ &+\mathbf{X}_p\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H\mathbf{X}_p^H)\mathbf{F}\Gamma^H(\epsilon)+\sigma_n^2\mathbf{I}.\end{aligned} \tag{14}$$

The probability density function of $\mathbf{y}$ is

$$p(\mathbf{y}|\epsilon)=(\pi^N\det(\mathbf{R}_y))^{-1}\exp(-\mathbf{y}^H\mathbf{R}_y^{-1}\mathbf{y}). \tag{15}$$

Since the determinant of $\mathbf{R}_y$ can be expressed as

$$\det(\mathbf{R}_y)=\det(\Gamma(\epsilon))\det(\mathbf{R}_{rr})\det(\Gamma^H(\epsilon))=\det(\mathbf{R}_{rr}) \tag{16}$$

where $\mathbf{r}=\mathbf{F}^H\mathbf{X}\mathbf{F}_L\mathbf{h}$. The determinant of $\mathbf{R}_y$ is independent of ϵ. We drop the terms that are independent of ϵ and get the log-likelihood function as

$$\Lambda(\mathbf{y}|\epsilon)=-\mathbf{y}^H\mathbf{R}_y^{-1}\mathbf{y}. \tag{17}$$

Maximizing the log likelihood function is equivalent to minimizing the following cost function

$$\hat{\epsilon}=\arg\min_{\epsilon}\mathbf{y}^H\mathbf{R}_y^{-1}\mathbf{y}. \tag{18}$$

This CFO estimator is based on the presence of the pilot symbols, which will introduce a special correlation structure into the pre-DFT samples. Knowledge of the constant pilot patterns will thus be exploited. The estimator also requires knowledge of the channel correlation and channel noise variance. In practice, these quantities not known perfectly and mismatch conditions exist. These issues will be discussed in a long version of this paper. From (12), if virtual carriers exist, the proposed estimator increases its estimation range. Since coarse frequency offset acquisition is assumed, we do not discuss the estimation with virtual carriers for simplicity.

C. Performance

The performance of the CFO estimation algorithm with different pilot symbols and SNR is of interest. It can be proved that the CFO estimator is unbiased. From (18), our proposed nonlinear estimator involves solving the roots of a nonlinear function. Although this appears difficult, the minimum of the cost function is markedly pronounced and can be readily found. Furthermore, the minimum is insensitive to the additive noise. Fig. 2 illustrates the average normalized mean square error (NMSE) $E\{|\hat{\epsilon}-\epsilon|^2\}/\epsilon^2$ versus ϵ for our proposed frequency estimator. The number of carriers is 64 and the number of pilot symbols is 32. Binary Phase Shift Keying (BPSK) is used.

Fig. 3 gives the NMSE versus SNR with different number of pilot symbols. Our proposed carrier frequency estimator has high resolution. This appears to be better than most existing frequency offset estimators for OFDM. This is probably due to the fact that the pilot symbols provide redundant information for frequency offset estimation. With the decrease of the number of pilot symbols the estimation error increases. Even with 4 pilot symbols the proposed frequency offset estimator also shows robustness to additive noise. This CFO estimate can be used as an initial estimate of ϵ in the following joint ML channel and frequency offset estimator. Later we will show that the joint ML estimator can further reduce the residual frequency offset.

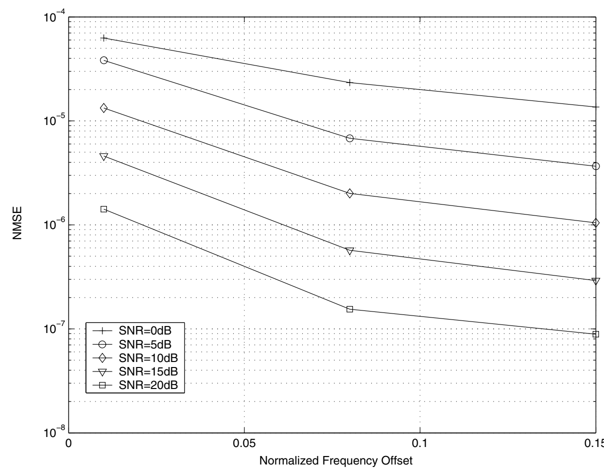

Fig. 2. NMSE versus carrier frequency offset.

IV. JOINT ML ESTIMATION ALGORITHM

A. Initial channel estimation

Once the CFO is estimated, the channel taps can be estimated by using the pilot symbols after removing ICI. Pilot assisted channel estimation for OFDM systems is discussed in [4], [5], [13]. Given the estimate $\hat{\epsilon}$, the CFO can be compensated by using (5). The channel response at pilot subcarriers can be obtained from (6). Typical algorithms include LS estimation, minimum mean square error (MMSE) estimation [4] and linear interpolation [13]. For a slow fading channel, the joint ML estimator performs well with linear interpolation. The channel response at the pilot carrier (H_p) is estimated by using the

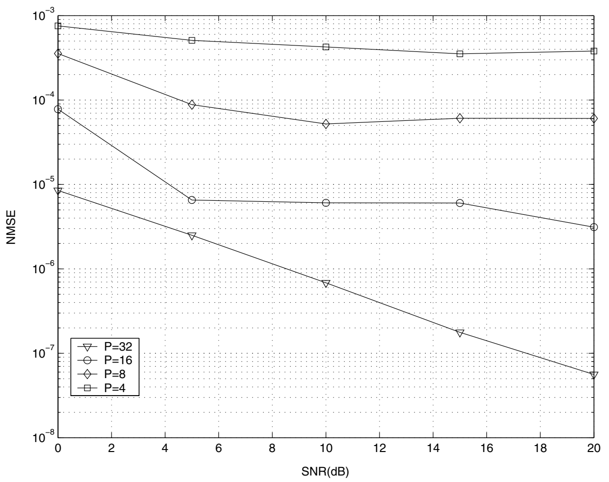

Fig. 3. NMSE versus SNR. P is the number of pilot symbols used in estimation, N=64, BPSK.

corresponding $\hat{Y}_p$ and X_p, and then the channel response at other subcarriers is obtained by using linear interpolation. This channel estimate suffices as an initializer of the decision feedback joint ML estimator.

B. Iterative joint ML estimation

Due to the residual CFO, the initial channel and CFO estimates may not be accurate for good performance. To further reduce the estimation error and improve the BER performance, we propose a decision-feedback joint ML estimator. We estimate channel at each step using a simple estimation procedure by assuming the CFO and data. Assuming that the channel estimation and data are known, we estimate CFO by using an ML algorithm. Finally the detected data symbols are updated by the estimated frequency offset and channel response. This iterative procedure is repeated until convergence. Simulation results show that one iteration is often enough.

In detail, we first estimate $\hat{\epsilon}_{k+1}$ from a ML estimation

$$\begin{aligned}\hat{\epsilon}_{k+1} = & \arg\max_{\epsilon}\left[\mathbf{y} - \frac{1}{N}\Gamma(\epsilon)\mathbf{F}^H\mathbf{X}_k\mathbf{F}_L\mathbf{h}_k\right]^H \\ & \times\left[\mathbf{y} - \frac{1}{N}\Gamma(\epsilon)\mathbf{F}^H\mathbf{X}_k\mathbf{F}_L\mathbf{h}_k\right]\end{aligned} \quad (19)$$

where $\mathbf{X}_k$ is the k-th estimates of the transmitted signal; $\mathbf{h}_k$ is the k-th estimate of the channel response. $\mathbf{h}_1$ is obtained by linear interpolation. The CFO estimate $\hat{\epsilon}_{k+1}$ can be found using the estimate $\hat{\epsilon}_k$. The initial estimate $\hat{\epsilon}_1$ is obtained by the algorithm given in Section III.

Using least-squares (LS) estimation, we get the channel estimate $\hat{\mathbf{h}}_{k+1}$ as

$$\hat{\mathbf{h}}_{k+1} = \left[(\mathbf{X}_k\mathbf{F}_L)^H\mathbf{X}_k\mathbf{F}_L\right]^{-1}(\mathbf{X}_k\mathbf{F}_L)^H\mathbf{F}\Gamma^H(\hat{\epsilon}_{k+1})\mathbf{y} \quad (20)$$

and the signal detection is carried out by simply using division and hard decisions:

$$\mathbf{X}_{k+1} = \frac{\mathbf{F}\Gamma^H(\hat{\epsilon}_{k+1})\mathbf{y}}{\mathbf{F}_L\hat{\mathbf{h}}_{k+1}} \quad (21)$$

where the division is component-wise division of two vectors.

V. Numerical Results

We now present numerical results to illustrate the effectiveness of the proposed joint estimator for a practical OFDM system. We consider a frequency-selective fading channel with three complex Gaussian taps h_l and the mean power $\sigma_l^2 = E[|h_l|^2] = \sigma_0^2 e^{-l/5}$ for $l = 1, \cdots, L$. The channel remains constant during each OFDM data block but varies from one block to another. A normalized CFO of 0.25 considered. The signal to noise ratio (SNR) is E_b/N_0. The OFDM system has N=64 subcarriers and P=32 pilot symbols. For simplicity, both the data bits and pilots are BPSK symbols. The proposed joint estimator is used for data detection and channel and CFO estimation. The lines denotes the simulation results averaged over 1000 Monte Carlo runs. The legend shows the number of iterations performed in the simulation.

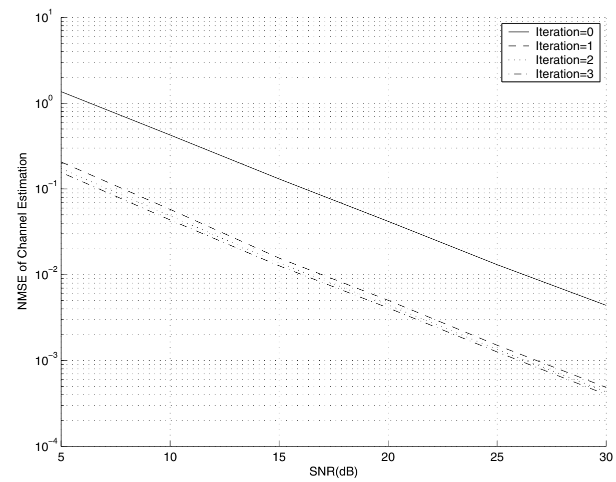

Fig. 4. NMSE of the joint ML estimation of the channel response versus SNR

Figs. 4, 5 and 6 show the normalized mean square error (NMSE) $E\{|\hat{\epsilon} - \epsilon|^2\}/\epsilon^2$ of CFO, NMSE $E\{||\hat{\mathbf{h}} - \mathbf{h}||^2\}/E\{||\mathbf{h}||^2\}$ of channel response and BER of the OFDM system respectively. Fig. 4 shows that the iterative joint estimator improves the performance of channel estimation by about 7-8dB. The performance of the CFO estimator is improved by about 3-4dB in Fig. 5. In Fig. 6, the proposed joint estimator performance almost achieves the bound provided by the perfect knowledge of channel response and carrier frequency offset. Furthermore, only one iteration is enough to achieve the desired BER performance though more iterations can further reduce the channel estimation error and frequency offset error.

VI. Conclusion

We have investigated joint estimation of channel and frequency offset for OFDM systems. We derived a high resolution CFO estimator by using the embedded pilot symbols in each OFDM block. The estimator is robust to additive Gaussian noise. An iterative joint CFO and channel estimator using

0-7803-8521-7/04/$20.00 (C) 2004 IEEE

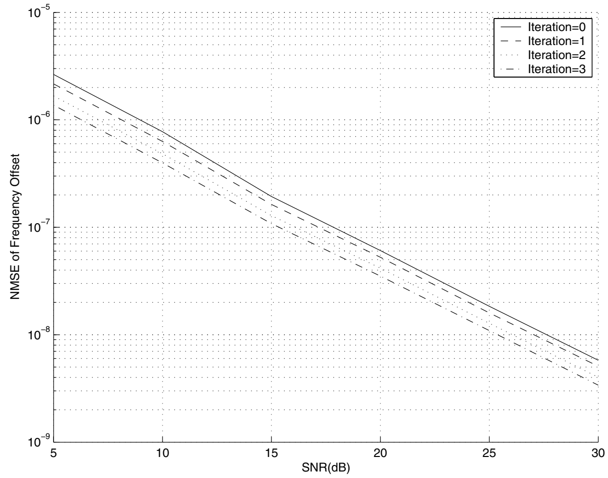

Fig. 5. NMSE of the joint ML estimation of the carrier frequency offset versus SNR

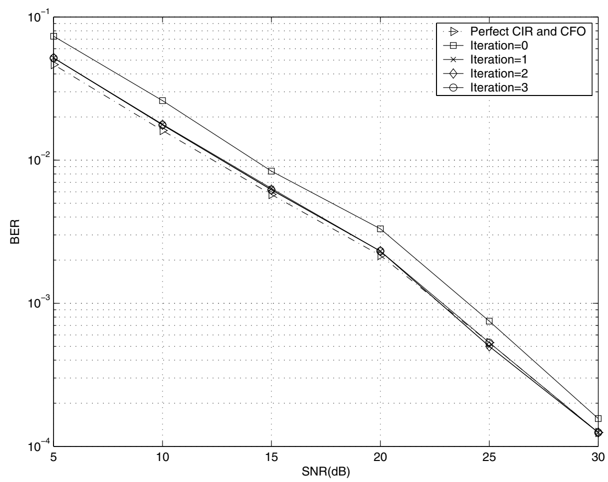

Fig. 6. Bit error rate of the joint ML estimation algorithm versus SNR

decision feedback is also derived to further reduce the estimation error and improve the system performance. The joint estimator is initialized using the CFO estimator (18) and the linear interpolation channel estimator. The channel estimate is updated by the LS method and the CFO estimated by an ML algorithm. Simulation results show that the proposed joint estimator with just one iteration can achieve the same BER as that of an ideal receiver with perfect knowledge of channel and the CFO.

REFERENCES

[1] J. Bingham, "Multicarrier modulation for data transmission: an idea whose time has come," *IEEE Communications Magazine*, vol. 28, no. 5, pp. 5–14, May. 1990.

[2] J. Rinne and M. Renfors, "Pilot spacing in orthogonal frequency division multiplexing systems on practical channels," *IEEE Transactions on Consumer Electronics*, vol. 42, no. 4, pp. 959–962, Nov. 1996.

[3] O. Edfors, M. Sandell, J.-J. van de Beek, S. Wilson, and P. Borjesson, "OFDM channel estimation by singular value decomposition," *IEEE Transactions on Communications*, vol. 46, no. 7, pp. 931–939, Jul. 1998.

[4] Y. Li, "Pilot-symbol-aided channel estimation for OFDM in wireless systems," *IEEE Transactions on Vehicular Technology*, vol. 49, no. 4, pp. 1207–1215, Jul. 2000.

[5] R. Negi and J. Cioffi, "Pilot tone selection for channel estimation in a mobile OFDM system," *IEEE Transactions on Consumer Electronics*, vol. 44, no. 3, pp. 1122–1128, Aug. 1998.

[6] P. Moose, "A technique for orthogonal frequency division multiplexing frequency offset correction," *IEEE Transactions on Communications*, vol. 42, no. 10, pp. 2908–2914, Oct. 1994.

[7] J. van de Beek, M. Sandell, and P. Borjesson, "ML estimation of time and frequency offset in OFDM systems," *IEEE Transactions on Signal Processing*, vol. 45, no. 7, pp. 1800–1805, Jul. 1997.

[8] H. Liu and U. Tureli, "A high-efficiency carrier estimator for OFDM communications," *IEEE Communications Letters*, vol. 2, no. 4, pp. 104–106, Apr. 1998.

[9] F. Daffara and O. Adami, "A new frequency detector for orthogonal multicarrier transmission techniques," in *Vehicular Technology Conference, 1995 IEEE 45th*, vol. 2, Jul. 1995, pp. 804–809.

[10] X. Ma, H. Kobayashi, and S. Schwartz, "Joint frequency offset and channel estimation for OFDM," in *Global Telecommunications Conference, 2003.*, vol. 1, Dec. 2003, pp. 15–19.

[11] E. Larsson, G. Liu, J. Li, and G. Giannakis, "Joint symbol timing and channel estimation for OFDM based WLANs," *IEEE Communications Letters*, vol. 5, no. 8, pp. 325–327, Aug. 2001.

[12] J. G. Proakis, "Digital communications, 4th ed." *New York: McGraw-Hill*, 2001.

[13] S. Coleri, M. Ergen, A. Puri, and A. Bahai, "Channel estimation techniques based on pilot arrangement in OFDM systems," *IEEE Transactions on Broadcasting*, vol. 48, no. 3, pp. 223–229, Sep. 2002.