A New Compact Multichannel Receiver for Underwater Wireless Communication Networks

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***Abstract*—In this letter it is shown that by taking advantage of the particle velocity, in addition to the pressure, multichannel reception can be accomplished in underwater acoustic wireless channels. Theoretical formulation and Monte Carlo simulations are provided for a vector sensor equalizer that measures the pressure and the velocity at a single point in space. These results demonstrate the usefulness of small-size vector sensors as multichannel equalizers for underwater acoustic wireless systems and sensor networks.**

***Index Terms*—Underwater communication, vector sensors, multichannel equalization, particle velocity.**

I. Introduction

A vector sensor is capable of measuring important non-scalar components of the acoustic field such as the particle velocity, which cannot be obtained by a single scalar pressure sensor. In the past few decades, extensive research has been conducted on the theory and design of vector sensors (see, for example, [1] and [2]). They have been mainly used for underwater target localization and SONAR applications.

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On the other hand, underwater acoustic communication systems have used scalar sensors only, which measure the pressure of the acoustic field. The new idea of this paper is to take advantage of the vector components of the acoustic field, such as the particle velocity, sensed by a vector sensor at the receiver, for detecting the transmitted data. This reduces the size of the receiver since a vector sensor measures the scalar and vector components of the acoustic field at a single point in space. In other words, multiple channels in the proposed receiver are co-located. This is different from conventional multichannel underwater acoustic receivers, which employ spatially separated pressure-only sensors that may result in large arrays. The small size of the proposed receiver makes it particularly useful for small unmanned underwater vehicles and some underwater wireless sensor networks [4].

II. SYSTEM EQUATIONS IN A VECTOR SENSOR RECEIVER

In general, there are two types of vector sensors: inertial and gradient [1] [2]. Inertial sensors truly measure the velocity by responding to the acoustic particle motion. Gradient sensors, however, implement a finite-difference approximation to estimate the gradients of the acoustic field such as the velocity. Each sensor type has its own advantages and disadvantages [3]. However, the proposed idea, to take advantage of the vector components of the field at the receiver, is not limited to a particular sensor type. Of course the input dynamic range, bandwidth, sensitivity, and other characteristics of a vector sensor affect the reception performance, but the principles, models, and concepts developed in the sequel remain the same. Depending on the application, system cost, required precision, etc., one can choose the proper sensor type and technology.

In this section we derive basic system equations for data detection via a vector sensor. To demonstrate the basic concepts of how both the vector and scalar components of the acoustic field can be utilized for data reception, we consider a simple system in a two-dimensional (2D) depth-range underwater channel. As shown in Fig. 1, there is one transmit pressure sensor, shown by a black dot, whereas for reception we use a vector sensor, shown by a black square, which measures the pressure and the *y* and *z* components of the particle velocity. This is basically a 1×3 single-input multiple-output (SIMO) system.

A. *Pressure and Velocity Channels and Noises*

There are three channels in Fig. 1: the pressure channel p, represented by a straight dashed line, and two *pressure-equivalent* velocity channels p^z and p^y, shown by curved dashed lines. To define p^z and p^y, we need to define the two velocity channels v^z and v^y, the vertical and horizontal components of the particle velocity, respectively. According to the linearized equation for time-harmonic waves [2] [5], the *z* and *y* components of the velocity at the frequency f_0 are given by

$$v^z = -(j\rho_0\omega_0)^{-1}\partial p/\partial z,\ v^y = -(j\rho_0\omega_0)^{-1}\partial p/\partial y\,. \tag{1}$$

In the above equations, ρ_0 is the density of the fluid, $j^2 = -1$ and $\omega_0 = 2\pi f_0$. Eq. (1) states that the velocity in a certain direction is proportional to the spatial pressure gradient in that direction [5]. The velocity channels in (1) are then multiplied by $-\rho_0 c$, the negative of the acoustic impedance, where c is the speed of sound. This gives the associated pressure-equivalent velocity channels as $p^z = -\rho_0 c v^z$ and $p^y = -\rho_0 c v^y$. With λ as the wavelength and $k = 2\pi/\lambda = \omega_0/c$ as the wavenumber we finally obtain

$$p^z = (jk)^{-1}\partial p/\partial z,\ \ p^y = (jk)^{-1}\partial p/\partial y\,. \tag{2}$$

The additive ambient noise pressure at the receiver is shown by n in Fig. 1. At the same location, the *z* and *y* components of the ambient noise velocity, sensed by the vector sensor are $\alpha^z = -(j\rho_0\omega_0)^{-1}\partial n/\partial z$ and $\alpha^y = -(j\rho_0\omega_0)^{-1}\partial n/\partial y$, respectively, derived in the same manner as (1). So, the vertical and horizontal pressure-equivalent ambient noise velocities are

$n^z = -\rho_0 c\ \alpha^z = (jk)^{-1}\partial n/\partial z$ and $n^y = -\rho_0 c\ \alpha^y = (jk)^{-1}\partial n/\partial y$, respectively, which resemble (2).

B. Input-Output System Equations

According to Fig. 1, the received pressure signal *r* at *Rx* in response to the signal *s* transmitted from *Tx* can be written as $r = p \oplus s + n$. Here $\oplus$ stands for convolution in time and *p* is the pressure channel impulse response at *Rx*. We also define the *z* and *y* components of the pressure-equivalent received velocity signals as $r^z = (jk)^{-1}\partial r/\partial z$ and $r^y = (jk)^{-1}\partial r/\partial y$, respectively. Based on (2) and by taking the spatial gradient of r with respect to *z* and *y* we easily obtain the key system equations

$$r = p \oplus s + n,\ \ r^y = p^y \oplus s + n^y,\ \ r^z = p^z \oplus s + n^z. \tag{3}$$

It is noteworthy that the three output signals r, r^y and r^z are measured at a single point in space.

C. Pressure and Velocity Noise Correlations

We define the spatial pressure noise correlation between the two locations $(y+\ell_y, z+\ell_z)$ and (y,z) as $q_n(\ell_y,\ell_z) = E[n(y+\ell_y, z+\ell_z)n^*(y,z)]$, where * is the complex conjugate and ℓ_y and ℓ_z are real numbers. Using the correlation properties of a differentiator in p. 326 of [6], at the location (y,z) one can show $E[n\{n^y\}^*] = (jk)^{-1}\partial q_n/\partial \ell_y$, $E[n\{n^z\}^*] = (jk)^{-1}\partial q_n/\partial \ell_z$, and $E[n^z\{n^y\}^*] = -k^{-2}\partial^2 q_n/\partial \ell_z \partial \ell_y$, all calculated for $(\ell_y,\ell_z) = (0,0)$. For an isotropic noise field in the *y*-*z* plane we have $q_n(\ell_y,\ell_z) = J_0(k(\ell_y^2+\ell_z^2)^{1/2})$ [7], with $J_m(.)$ as the *m*-order Bessel function of the first kind. Using the properties of the Bessel functions and their derivatives [8], it is easy to verify that $E[n\{n^y\}^*] = E[n\{n^z\}^*] = E[n^z\{n^y\}^*] = 0$, i.e., all the noise terms in (3) are uncorrelated.

D. Pressure and Velocity Average Powers

1) Noise Powers: Using the statistical properties of a differentiator in p. 326 of [6], the powers of the *y* and *z* components of the pressure-equivalent noise velocity at (y,z) can be obtained as

$\Omega_n^y = E[|n^y|^2] = -k^{-2}\partial^2 q_n / \partial \ell_y^2$ and $\Omega_n^z = E[|n^z|^2] = -k^{-2}\partial^2 q_n / \partial \ell_z^2$, respectively, both calculated at $(\ell_y, \ell_z) = (0,0)$. Based on the q_n of the 2D isotropic noise model described previously, one can show $\Omega_n^y = \Omega_n^z = 1/2$. Note that the noise pressure power in this model is $\Omega_n = E[|n|^2] = q_n(0,0) = 1$. This means $\Omega_n = \Omega_n^y + \Omega_n^z$.

2) Channel Powers: The ambient noise is a superposition of several components coming from different angle of arrivals (AOAs) [7]. In multipath environments such as shallow waters, the channel is also a superposition of multiple subchannels. Based on this analogy between *n* and *p*, as well as their spatial gradients, one can obtain $\Omega_p = \Omega_p^y + \Omega_p^z$, where $\Omega_p = E[|p|^2]$, $\Omega_p^y = E[|p^y|^2]$ and $\Omega_p^z = E[|p^z|^2]$. Note that in the 2D isotropic noise model the distribution of AOA is uniform over $[0, 2\pi)$ [7], which results in $\Omega_n^y = \Omega_n^z = \Omega_n / 2$. However, this is not necessarily the case in multipath channels such shallow waters, which means Ω_p^y and Ω_p^z are not equal in general.

III. Multichannel Equalization with a Vector Sensor

In the previous section, signal and noise characteristics in a vector sensor receiver were discussed. In this section, we demonstrate the feasibility of multichannel intersymbol interference removal with a vector sensor receiver. Among the different types of equalizers [9], we choose the basic zero forcing equalizer to verify the concept. For the system equation we have

$$\mathbf{R} = \mathbf{HS} + \mathbf{N}, \text{ such that } \mathbf{R} = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \\ \mathbf{R}_3 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \mathbf{H}_3 \end{bmatrix}, \text{ and } \mathbf{N} = \begin{bmatrix} \mathbf{N}_1 \\ \mathbf{N}_2 \\ \mathbf{N}_3 \end{bmatrix}. \tag{4}$$

In (4) $\mathbf{S} = [s_0 \ldots s_{K-1}]^T$ includes *K* transmitted symbols and ^{T} is the transpose. With *M* as the number of channel taps, the same for all *l*, $l = 1,2,3$, $\mathbf{R}_l = [r_l(0) \ldots r_l(K+M-2)]^T$ and

$\mathbf{N}_l = [n_l(0) \ldots n_l(K+M-2)]^T$ are the l-th $(K+M-1)\times 1$ received signal and noise vectors, respectively. Also the l-th $(K+M-1)\times K$ banded channel matrix is

$$\mathbf{H}_l = \begin{bmatrix} h_l(0) & & \\ \vdots & \ddots & h_l(0) \\ h_l(M-1) & \ddots & \vdots \\ & & h_l(M-1) \end{bmatrix}. \quad (5)$$

Note that for a vector sensor receiver, the channel indices 1, 2 and 3 in (4) represent the pressure, pressure-equivalent horizontal velocity and pressure-equivalent vertical velocity, respectively. So, based on (3), for an arbitrary discrete time index t we have $r_1(t) = r(t)$, $r_2(t) = r^y(t)$, $r_3(t) = r^z(t)$, $h_1(t) = p(t)$, $h_2(t) = p^y(t)$, $h_3(t) = p^z(t)$, $n_1(t) = n(t)$, $n_2(t) = n^y(t)$ and $n_3(t) = n^z(t)$. Furthermore, according to (5), the channel convolution matrices $\mathbf{H}_1$, $\mathbf{H}_2$, and $\mathbf{H}_3$ in (4) for a vector sensor receiver are given by

$$\mathbf{H}_1 = \begin{bmatrix} p(0) & & \\ \vdots & \ddots & p(0) \\ p(M-1) & \ddots & \vdots \\ & & p(M-1) \end{bmatrix}, \mathbf{H}_2 = \begin{bmatrix} p^y(0) & & \\ \vdots & \ddots & p^y(0) \\ p^y(M-1) & \ddots & \vdots \\ & & p^y(M-1) \end{bmatrix}, \mathbf{H}_3 = \begin{bmatrix} p^z(0) & & \\ \vdots & \ddots & p^z(0) \\ p^z(M-1) & \ddots & \vdots \\ & & p^z(M-1) \end{bmatrix}. \quad (6)$$

Assuming perfect channel knowledge at the receiver, the zero forcing equalizer is given by

$$\hat{\mathbf{S}} = (\mathbf{H}^\dagger \mathbf{\Sigma}^{-1} \mathbf{H})^{-1} \mathbf{H}^\dagger \mathbf{\Sigma}^{-1} \mathbf{R}. \quad (7)$$

In this equation $\hat{\mathbf{S}}$ is the minimum variance unbiased estimate of $\mathbf{S}$ [10], † is the transpose conjugate and $\mathbf{\Sigma} = E[\mathbf{N}\mathbf{N}^\dagger]$ is the covariance matrix of the noise vector $\mathbf{N}$ in (4). The simulations of Section IV show the performance of (7). The effect of imperfect channel knowledge, i.e., channel estimation error is discussed in [3].

IV. Simulation Set up and Performance Comparison

Here we compare the performance of the vector sensor equalizer in (7) with a vertical three-element pressure-only uniform linear array (ULA) that performs the zero forcing equalization. The ULA equations and equalizer are the same as (4) and (7), respectively, where the three channels represent three vertically separated pressure channels. The noise vectors $\mathbf{N}_1, \mathbf{N}_2$ and $\mathbf{N}_3$ in both receivers are considered to be complex Gaussians with white temporal correlations. For the isotropic noise model discussed in subsection II-C, the noise vectors $\mathbf{N}_1, \mathbf{N}_2$ and $\mathbf{N}_3$ are uncorrelated in the vector sensor receiver. Therefore its noise covariance matrix $\mathbf{\Sigma}_{\text{vector sensor}} = E[\mathbf{N}_{\text{vector sensor}} \mathbf{N}^{\dagger}_{\text{vector sensor}}]$ is given by

$$\mathbf{\Sigma}_{\text{vector sensor}} = \begin{bmatrix} \Omega_n \mathbf{I}_{K+M-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (\Omega_n/2)\mathbf{I}_{K+M-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (\Omega_n/2)\mathbf{I}_{K+M-1} \end{bmatrix}, \tag{8}$$

where $\mathbf{I}_m$ is an $m \times m$ identity matrix and $\mathbf{0}$ is a matrix whose elements are all zero. For the pressure-only ULA with the element spacing of L, there are some pressure correlations of $J_0(kL)$ and $J_0(2kL)$ for the separations of L and $2L$, respectively. This means that the noise covariance matrix $\mathbf{\Sigma}_{\text{pressure-only ULA}} = E[\mathbf{N}_{\text{pressure-only ULA}} \mathbf{N}^{\dagger}_{\text{pressure-only ULA}}]$ can be written as

$$\mathbf{\Sigma}_{\text{pressure-only ULA}} = \begin{bmatrix} \Omega_n \mathbf{I}_{K+M-1} & J_0(kL)\mathbf{I}_{K+M-1} & J_0(2kL)\mathbf{I}_{K+M-1} \\ J_0(kL)\mathbf{I}_{K+M-1} & \Omega_n \mathbf{I}_{K+M-1} & J_0(kL)\mathbf{I}_{K+M-1} \\ J_0(2kL)\mathbf{I}_{K+M-1} & J_0(kL)\mathbf{I}_{K+M-1} & \Omega_n \mathbf{I}_{K+M-1} \end{bmatrix}. \tag{9}$$

To calculate the velocity channel impulse responses (IRs) p^y and p^z in simulations using the p channel IR generated by Bellhop [11], each spatial gradient in (2) is approximated by a finite difference. Therefore at location (y,z) we have $\partial p(y,z)/\partial z \approx [\,p(y,z+0.2\lambda) - p(y,z)]/(0.2\lambda)$ and

$\partial p(y,z)/\partial y \approx [p(y+0.2\lambda,z)-p(y,z)]/(0.2\lambda)$.

In simulations the **S** vector includes $K=200$ equi-probable ± 1 BPSK symbols, and the entries of the channel matrix **H** are calculated as described above. The noise vector **N** is generated by pre-multiplying a simulated white complex Gaussian vector with the Cholesky factor of $\mathbf{\Sigma}$, the noise covariance matrix of interest. After calculating the received vector **R** according to (4), **S** is estimated using (7), and the bit error rate (BER) is shown in Fig. 2. The water depth for the shallow channel of Fig. 2 is 81.1 m, where the *Tx* and *Rx* are 5 km apart. The *Tx* and *Rx* are 25 m and 63 m below the water surface, respectively. A coarse silt bottom is considered, with $f_0 = 12\,\mathrm{kHz}$ and a bit rate of 2400 bits/sec. The sound speed profile we used was measured during the underwater communication experiments conducted on May 10, 2002, in waters off San Diego, CA [12], and is shown in Fig. 3.

To define the average signal-to-noise ratio (SNR) per channel in Fig. 2, let $\mathbf{p}=[p(0)\ldots p(M-1)]^T$, $\mathbf{p}^y=[p^y(0)\ldots p^y(M-1)]^T$ and $\mathbf{p}^z=[p^z(0)\ldots p^z(M-1)]^T$ be the taps of the pressure, *y*- and *z*-velocity IRs, respectively. Then the pressure, *y*- and *z*-velocity SNRs are $\zeta_p=\Omega_p/\Omega_n$, $\zeta_p^y=\Omega_p^y/\Omega_n^y$ and $\zeta_p^z=\Omega_p^z/\Omega_n^z$, respectively, such that $\Omega_p=\mathbf{p}^\dagger\mathbf{p}$, $\Omega_p^y=(\mathbf{p}^y)^\dagger\mathbf{p}^y$ and $\Omega_p^z=(\mathbf{p}^z)^\dagger\mathbf{p}^z$. The average SNR per channel for the vector sensor receiver is $\bar{\zeta}=(\zeta_p+\zeta_p^y+\zeta_p^z)/3$ by definition. Also **p** is normalized such that $\Omega_p=1$. Based on subsection II-D, this implies that $\Omega_p^y+\Omega_p^z=1$ in our simulations. Since $\Omega_n^y=\Omega_n^z=\Omega_n/2$ in a 2D isotropic noise model, we finally obtain $\bar{\zeta}=1/\Omega_n$, which is the same as the SNR of a unit-power pressure channel ζ_p.

V. Discussion and Conclusion

The performance of the compact vector sensor receiver is shown in Fig. 2, which is significantly better than a single pressure sensor receiver. Performance of the three-element pressure-only array depends on its element spacing L, as shown in Fig. 2. As L increases, the noise spatial correlation decreases and also the three pressure channels become less correlated. These both result in a reduction in BER, as L increases. According to Fig. 2, the pressure-only array receiver with $L = 50\lambda$ outperforms the vector sensor receiver. By changing the simulation scenario, for example the bottom type, the pressure-only array may outperform the vector sensor receiver with an element spacing smaller than 50λ. However, the general picture does not change, i.e., both the vector sensor and pressure-only array receivers are much better than a single pressure sensor receiver. The advantage of the vector sensor receiver is it smaller size, compared to the pressure-only array.

To confirm the accuracy of the BER results shown in Fig. 2, one can look at the level of error in symbol estimates, which are obtained using the equalizer in (7). The covariance matrix of the symbol estimation error vector $\hat{\mathbf{S}} - \mathbf{S}$ can be shown to be [10]

$$E[(\hat{\mathbf{S}} - \mathbf{S})(\hat{\mathbf{S}} - \mathbf{S})^\dagger] = (\mathbf{H}^\dagger \mathbf{\Sigma}^{-1} \mathbf{H})^{-1}, \tag{10}$$

where $\mathbf{H}$ and $\mathbf{\Sigma}$ are the channel matrix and the noise covariance matrix, respectively. In Fig. 4 the square root of the sorted diagonal elements of $(\mathbf{H}^\dagger \mathbf{\Sigma}^{-1} \mathbf{H})^{-1}$ in (10) are plotted, which are the standard deviations of the symbol estimation errors. As expected, the estimation error standard deviations of the pressure-only array decrease as L increases. Furthermore, the estimation error standard deviations of the vector sensor are much smaller than those of a single pressure sensor and pressure-only arrays with small element spacings. All these are consistent with the BER results of Fig. 2 and demonstrate the usefulness of a vector sensor as a compact multichannel receiver.

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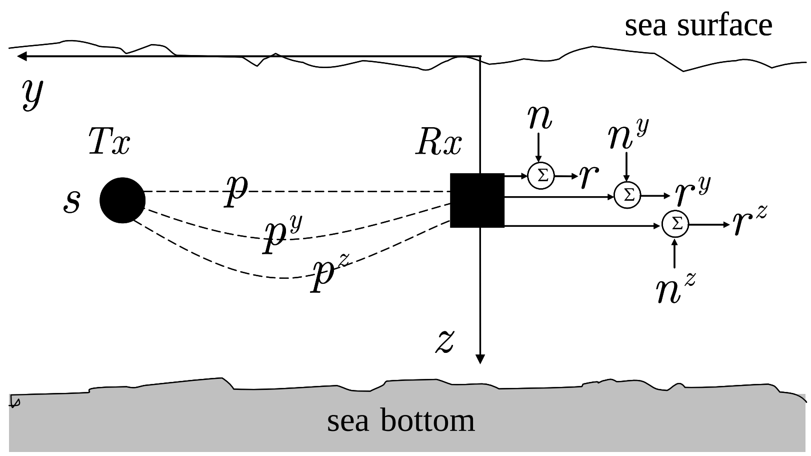

Fig. 1. A 1×3 vector sensor communication system, with one pressure transmitter and one vector sensor receiver. The vector sensor measures the pressure, as well as the *y* and *z* components of the acoustic particle velocity, all in a single point at the receive side. The system equations are given in (3).

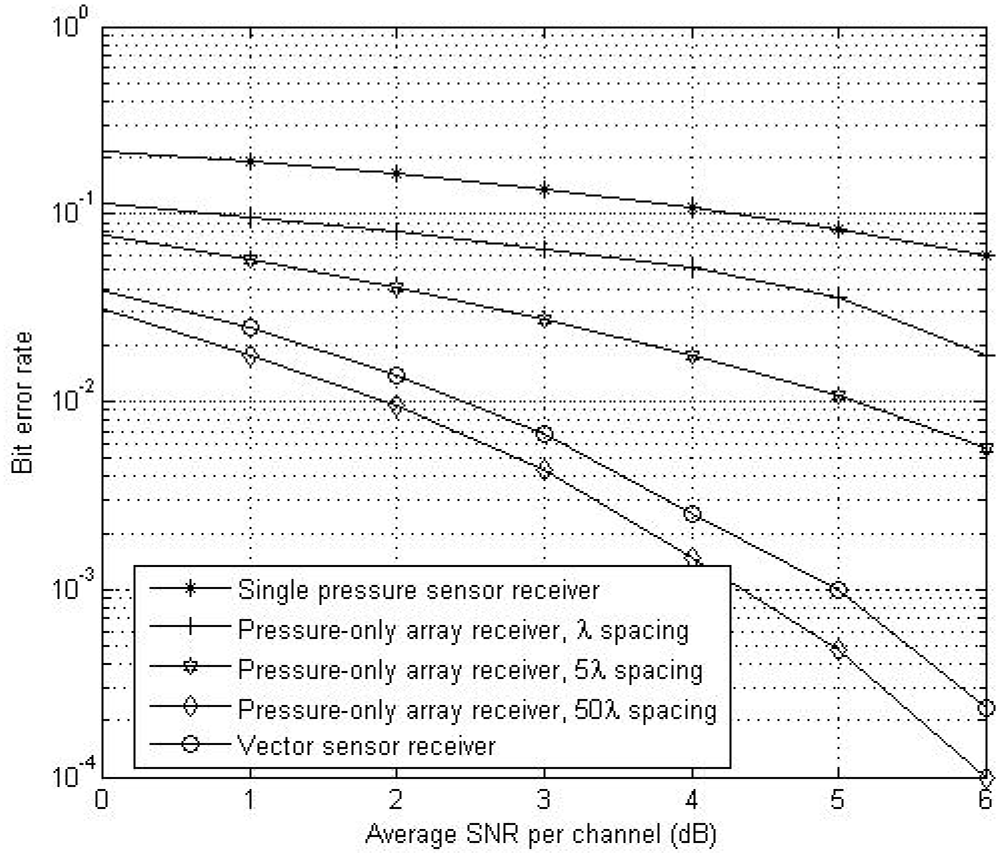

Fig. 2. Performance comparison of a vector sensor receiver, a single pressure sensor receiver, and a uniform linear array receiver with three pressure sensors and different element spacings $L = \lambda, 5\lambda, 50\lambda$.

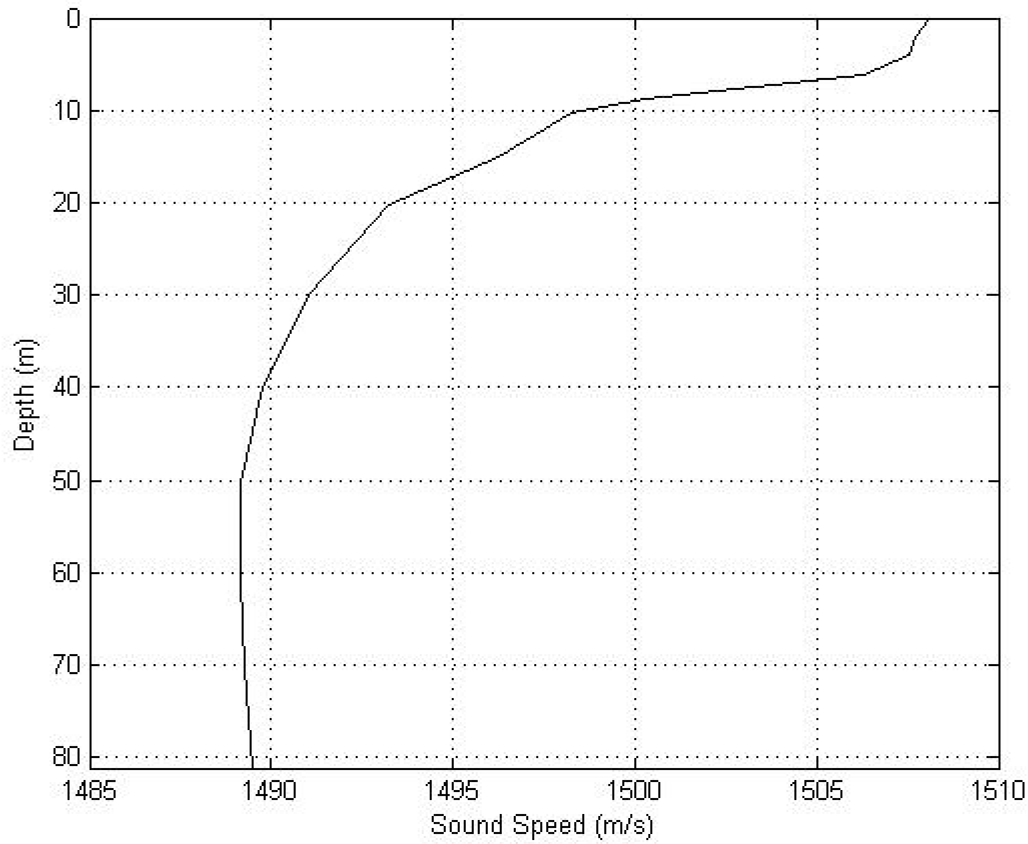

Fig. 3. Sound speed versus the water depth.

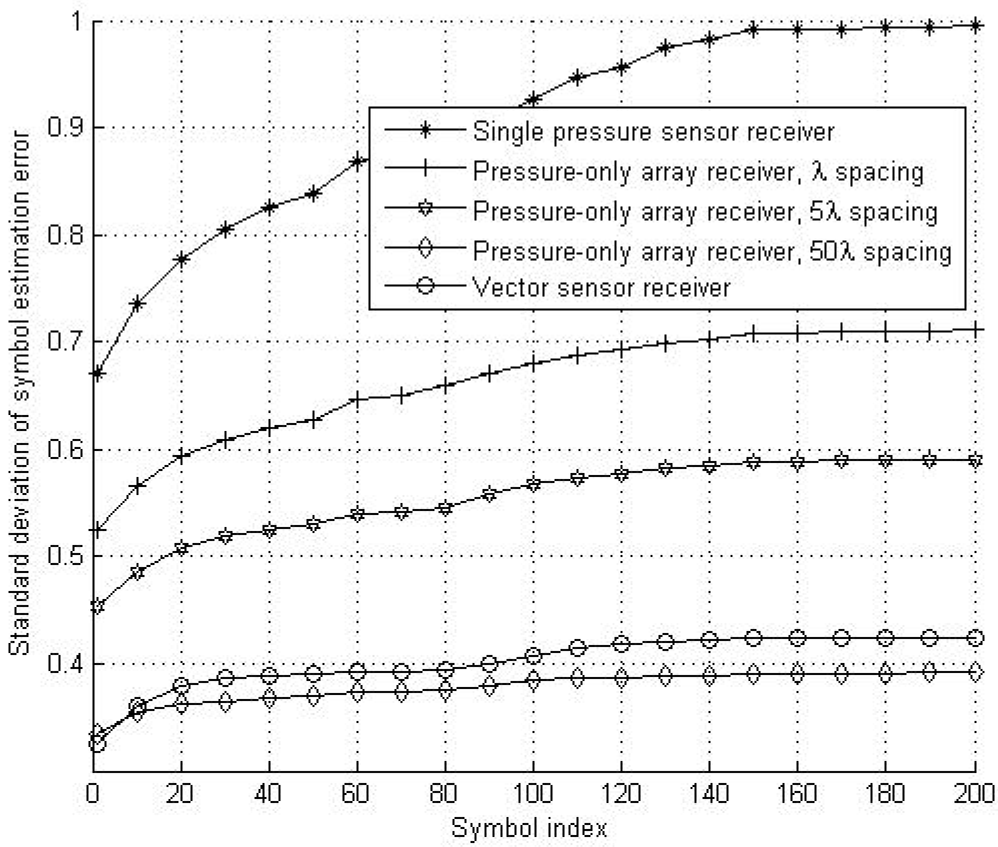

Fig. 4. Square root of the sorted diagonal elements of the symbol estimation error covariance matrix, given in (10). The receivers are a vector sensor, a single pressure sensor and a uniform linear array with three pressure sensors and different element spacings $L = \lambda, 5\lambda, 50\lambda$. The average SNR per channel for each receiver is 6 dB.