Power Allocation in Wireless Multi-User Relay Networks

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***Abstract*—In this paper, we consider an amplify-and-forward wireless relay system where multiple source nodes communicate with their corresponding destination nodes with the help of relay nodes. Conventionally, each relay equally distributes the available resources to its relayed sources. This approach is clearly sub-optimal since each user[1] experiences dissimilar channel conditions, and thus, demands different amount of allocated resources to meet its quality-of-service (QoS) request. Therefore, this paper presents novel power allocation schemes to i) maximize the minimum signal-to-noise ratio among all users; ii) minimize the maximum transmit power over all sources; iii) maximize the network throughput. Moreover, due to limited power, it may be impossible to satisfy the QoS requirement for every user. Consequently, an admission control algorithm should first be carried out to maximize the number of users possibly served. Then, optimal power allocation is performed. Although the joint optimal admission control and power allocation problem is combinatorially hard, we develop an effective heuristic algorithm with significantly reduced complexity. Even though theoretically sub-optimal, it performs remarkably well. The proposed power allocation problems are formulated using geometric programming (GP), a well-studied class of nonlinear and nonconvex optimization. Since a GP problem is readily transformed into an equivalent convex optimization problem, optimal solution can be obtained efficiently. Numerical results demonstrate the effectiveness of our proposed approach.**

***Index Terms*—Power allocation, geometric programming, relay networks.**

I. Introduction

RECENTLY, it has been shown that the operation efficiency and quality-of-service (QoS) of cellular and/or ad-hoc networks can be increased through the use of relay(s) [1], [2]. In such systems, the information from the source to the destination is not only transmitted via a direct link but also forwarded via relays. Although various relay models have been studied, the simple two-hop relay model has attracted extensive research attention due to its implementation practicability [1]–[11]. The performance of a two-hop relay system has been investigated for various channels, i.e., Rayleigh or Nakagami-m, and relay strategies, i.e., amplify-and-forward (AF) or decode-and-forward (DF) [1]–[5]. Note, however, that resource allocation is assumed to be fixed in these works.

A critical issue for improving the performance of wireless networks is the efficient management of available radio resources [12]. Specifically, resource allocation via power control is commonly employed. As a result, numerous works have been conducted to optimally allocate the radio resources, for example power and bandwidth to improve the performance of relay networks [6]-[11]. It is worth mentioning that a *single* source-destination pair is typically considered in the aforementioned papers. In [6], the authors derive closed-form expressions for the optimal and near-optimal relay transmission powers for the cases of single and multiple relays, respectively. The problem of minimizing the transmit power given an achieved target outage probability is tackled in [7]. In [8], by using either the signal-to-noise ratio (SNR) or the outage probability as the performance criteria, different power allocation strategies are developed for three-node AF relay system to exploit the knowledge of channel means. Bounds on the channel capacity are derived for a similar model with Rayleigh fading and channel state information (CSI) is assumed available at transmitter [9]. The bandwidth allocation problem in three-node Gaussian orthogonal relay system is investigated in [10] to maximize a lower bound on the capacity. Two power allocation schemes based on minimization of the outage probability are presented in [11] for the case when the information of the wireless channel responses or statistics is available at transmitter.

It should be noted that very few works have considered the aforementioned two-hop relay model for more practical case of multiple users.[2] Therefore, the above-mentioned analysis is applicable to only a special case of the problem under consideration. Indeed, each relay is usually delegated to assist more than one user, especially when the number of relays is

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[1]Hereafter, the term 'user' refers to a source-destination pair or only the source node depending on the context.

[2]Note that multi-user cooperative network employing orthogonal frequency-division multiple-access (OFDMA) where subscribers can relay information for each other is already considered, for example see [13], [14] and references therein.

(much) smaller than the number of users. A typical example of such scenario is the deployment of few relays in a cell at convenient locations to assist mobile users operating in heavily scattering environment for uplink transmission. Relays can also be used for helping the base station forwarding information to mobile users in downlink mode. Resource allocation in a multi-user system usually has to take into account the fairness issue among users, their relative quality-of-service (QoS) requirements, channel quality and so on. Mathematically, optimizing relay networks with multiple users is a challenging problem, especially when the number of users and relays is large.

In this paper, we develop efficient power allocation schemes for multi-user wireless relay systems. Specifically, we derive optimal power allocation schemes to i) maximize the minimum SNR among all users; ii) minimize the maximum transmit power over all sources; iii) maximize the network throughput. We show that the corresponding optimization problems can be formulated as geometric programming (GP) problems. Therefore, optimal power allocation can be obtained efficiently using convex optimization techniques.[3]

Another issue is that due to limited power resource, achieving QoS requirements for all users may turn out to be impossible. Therefore, some sort of admission control where users are not automatically admitted into the network, with pre-specified objectives should be carried out. Yet, none of the existing works has considered this practical scenario in the context of relay communications. Note, however, that the methodology for joint multiuser downlink beamforming and admission control has been recently developed in [18]. In this paper, we also propose a joint admission control and power allocation algorithm for multi-user relay systems. The proposed algorithm first aims at maximizing the number of users that can be admitted and QoS-guaranteed. Then, the optimal power allocation is performed. Since the aforementioned joint admission control and power allocation problem is combinatorially hard, we develop an effective heuristic approach with significantly reduced complexity. Moreover, the algorithm determines accurately the users to be admitted in most of the simulation examples. As well, its complexity in terms of running time is much smaller than that of the original optimal admission control problem. A preliminary version of this work has been presented in [19]. During the review process for this paper, the authors also became aware of the very recent contributions [20], [21]. In [20], the joint power and admission control problem is solved in the context of traditional cellular networks, while the same problem is considered in [21] in the context of cognitive underlay networks.

The rest of this paper is organized as follows. In Section II, a multi-user wireless relay model with multiple relays is described. Section III contains problem formulations for three power control schemes. The proposed problems are converted into GP problems in Section IV. The problem of joint admission control and power allocation is presented in Section V. The algorithm for solving the joint admission control and power allocation problem is described in Section VI. Numerical examples are presented in Section VII, followed by the conclusions in Section VIII.

II. System Model

Consider a multi-user relay network where M source nodes $S_i,\quad i \in \{1,...M\}$ transmit data to their corresponding destination nodes D_i, $i \in \{1,...M\}$.[4] There are L relay nodes $R_j,\ j \in \{1,...,L\}$ which are employed for forwarding the information from source to destination nodes. The conventional two-stage AF relaying with *orthogonal transmission* through time devision [1], [2], [11] is assumed. Therefore, to increase the throughput (or more precisely, to prohibit decreasing of the throughput), each source S_i is assisted by one relay denoted by R_{S_i}. Single relay assignment for each user also reduces the coordination between relays and/or implementation complexity at the receivers.[5] The set of source nodes which use the relay R_j is denoted by $\mathcal{S}\,(R_j)$, i.e., $\mathcal{S}(R_j) = \{S_i \mid R_{S_i} = R_j\}$.

Let P_{S_i}, $P_{R_{S_i}}$ denote the power transmitted by source S_i and relay R_{S_i} in the link S_i-R_{S_i}-D_i, respectively. Since unit duration time slots are assumed, P_{S_i} and $P_{R_{S_i}}$ correspond also to the average energies consumed by source S_i and relay R_{S_i}. For simplicity, we present the signal model for link S_i-R_{S_i}-D_i only. In the first time slot, source S_i transmits the signal x_i with unit energy to the relay R_{S_i}.[6] The received signal at relay R_{S_i} can be written as

$$r_{S_i R_{S_i}} = \sqrt{P_{S_i}} a_{S_i R_{S_i}} x_i + n_{R_{S_i}}$$

where $a_{S_i R_{S_i}}$ stands for the channel gain for link S_i-R_{S_i}, $n_{R_{S_i}}$ is the additive circularly symmetric white Gaussian noise (AWGN) at the relay R_{S_i} with variance $N_{R_{S_i}}$. The channel gain includes the effects of path loss, shadowing and fading. In the subsequent time slot, assuming the relay R_{S_i} knows the CSI for link S_i-R_{S_i}, it uses the AF protocol, i.e., it normalizes the received signal and retransmits to the destination node D_i. The received signal at the destination node D_i can be expressed as

$$\begin{aligned} r_{D_i} &= \sqrt{P_{R_{S_i}}} a_{R_{S_i} D_i} \frac{r_{S_i R_{S_i}}}{\sqrt{E\{|r_{S_i R_{S_i}}|^2\}}} + n_{D_i} \\ &= \sqrt{\frac{P_{R_{S_i}} P_{S_i}}{P_{S_i}|a_{S_i R_{S_i}}|^2 + N_{R_{S_i}}}} a_{R_{S_i} D_i} a_{S_i R_{S_i}} x_i + \hat{n}_{D_i} \end{aligned}$$

where $E\{\cdot\}$ denotes statistical expectation operator, $a_{R_{S_i} D_i}$ is the channel coefficient for link R_{S_i}-D_i, n_{D_i} is the AWGN at the destination node D_i with variance N_{D_i}, $\hat{n}_{D_i}$ is the modified AWGN noise at D_i with equivalent variance $N_{D_i} + \big(P_{R_{S_i}} |a_{R_{S_i} D_i}|^2 N_{R_{S_i}}\big) / \big(P_{S_i} |a_{S_i R_{S_i}}|^2 + N_{R_{S_i}}\big)$. The

[3]Note that GP has been successfully applied to approximately solve the power allocation problem in traditional cellular and ad hoc networks [15], [16]. The exact solution for the same problem can be obtained using the difference of two convex functions optimization at a price of high complexity [17].

[4]This includes the case of one destination node for all sources, for example, a base station in cellular network, or a central processing unit in a sensor network.

[5]The single relay assignment may be done during the connection setup phase, or done by relay selection process [11].

[6]We consider the case in which the source-to-relay link is (much) stronger than the source-to-destination link, that is usual scenario in practice.

equivalent SNR of the virtual channel between source S_i and destination D_i can be written as [11]

$$\begin{aligned}\gamma_i &= \frac{P_{R_{S_i}} P_{S_i} |a_{R_{S_i} D_i}|^2 |a_{S_i R_{S_i}}|^2}{P_{S_i} |a_{S_i R_{S_i}}|^2 N_{D_i} + P_{R_{S_i}} |a_{R_{S_i} D_i}|^2 N_{R_{S_i}} + N_{D_i} N_{R_{S_i}}} \\ &= \frac{P_{S_i} P_{R_{S_i}}}{\eta_i P_{S_i} + \alpha_i P_{R_{S_i}} + \beta_i}\end{aligned}$$

where $\eta_i = \frac{N_{D_i}}{|a_{R_{S_i} D_i}|^2}$, $\alpha_i = \frac{N_{R_{S_i}}}{|a_{S_i R_{S_i}}|^2}$, $\beta_i = \frac{N_{R_{S_i}} N_{D_i}}{|a_{S_i R_{S_i}}|^2 |a_{R_{S_j} D_i}|^2}$.

It can be seen that for fixed $P_{R_{S_i}}$, γ_i is a concave increasing function of P_{S_i}. However, no matter how large P_{S_i} is, the maximum achievable γ_i can be shown to be equal to $P_{R_{S_i}}/\eta_i$. Vice versa, when P_{S_i} is fixed, γ_i is a concave increasing function of $P_{R_{S_i}}$ and the corresponding maximum achievable γ_i is P_{S_i}/α_i. Moreover, since γ_i is a concave increasing function of P_{S_i}, the incremental change in γ_i is smaller for large P_{S_i}, and γ_i is monotone. Note that monotonicity is a useful property helping to provide some insights into optimization problems at optimality.

In the following sections, we consider efficient power allocation and admission control schemes based on a centralized approach with assumed complete knowledge of channel gains. This assumption involves some timely and accurate channel estimation and feedback techniques which are beyond the scope of this paper.

III. Power Allocation in Multi-user Relay Networks: Problem Formulations

Power control for single user relay networks has been popularly advocated [6]-[11]. In this section, we extend the power allocation framework to multi-user networks. Different power allocation based criteria which are suitable and distinct for multi-user networks are investigated.

A. Max-min SNR Based Allocation

Power control in wireless networks often has to take into account the fairness consideration since the fairness among different users is also a major issue in a QoS policy. In other words, the performance of the worst user(s) is also of concern to the network operator. Note that the traditionally used maximum sum SNR based power allocation favors users with good channel quality. Instead, we consider max-min fair based power allocation problem which aims at maximizing the minimum SNR over all users.[7] This can be mathematically posed as

$$\max_{P_{S_i},\, P_{R_{S_i}}} \quad \min_{i=1,\ldots,M} \gamma_i \tag{1a}$$

$$\text{subject to:} \quad \sum_{S_i \in \mathcal{S}(R_j)} P_{R_{S_i}} \le P_{R_j}^{\max},\ j = 1, \ldots, L \tag{1b}$$

$$\sum_{i=1}^{M} P_{S_i} \le P \tag{1c}$$

$$0 \le P_{S_i} \le P_{S_i}^{\max},\ i = 1, \ldots, M \tag{1d}$$

where $P_{R_j}^{\max}$ is the available power at the relay R_j and P is the maximum total power of all sources. The left-hand side of (1b) is the total power that R_j allocates to its relayed users, and thus, it is limited by the maximum available power of the relay. Constraint (1c) represents the possible limit on the total power of all sources while the constraint (1d) specifies the peak power limit $P_{S_i}^{\max}$ for source S_i. We should emphasize here that in applications when sources are operating independently, it is sufficient to have only limits on the individual source powers indicated by (1d), and (1c) can be effectively removed by simply setting $P \ge \sum_{i=1}^{M} P_{S_i}^{\max}$. In this case, sources S_i, $i = 1, \ldots, M$ would transmit with their maximum power $P_{S_i}^{\max}$. However, there are applications where the total power is of concern, e.g., when the sources share a common power pool as in the case of a base-station (or access point, access node) transmitter, or in an energy-aware system when energy consumption and related emission in the system are more related to *total* power than individual *peak* powers. In such a case, it is possible that $P < \sum_{i=1}^{M} P_{S_i}^{\max}$, and both the constraints (1c) and (1d) are applied in order to control the total power consumed by all sources within a specified target. In other words, the constraint (1c) is included in (1a)–(1d) for the sake of generality. On the other hand, there is no such limit for relay nodes since relays are usually energy-unlimited stations. Note, however, that such constraint for the relays can be included straightforwardly. In terms of system implementation, the constraint (1c) requires the sources to be coordinated in order to share the power resource.

It can be seen that the set of linear inequality constraints with positive variables in the optimization problem (1a)–(1d) is compact and nonempty. Hence, the problem (1a)–(1d) is always feasible. Moreover, since the objective function $\min_{i=1,\ldots,M} \gamma_i$ is an increasing function of the allocated powers P_{S_i} and $P_{R_{S_i}}$, the inequality constraints (1b), (1c) must be met with equality at optimality when $P \le \sum_{i=1}^{M} P_{S_i}^{\max}$. Moreover, when $P > \sum_{i=1}^{M} P_{S_i}^{\max}$, the inequality constraints (1b), (1d) must be met with equality at optimality. It can be observed that while the performance of user i depends only on the allocated powers P_{S_i} and $P_{R_{S_i}}$, the performance of all users interact with each other via shared and limited power resource at the relays and the sources. Therefore, proper power allocation among users is necessary to maximize a specific criterion on the system performance.[8]

B. Power Minimization Based Allocation

In wireless networks, power allocation can help to achieve the minimum QoS and low power consumption for users. Commonly, to improve the link performance, the source can transmit at its maximum available power which causes itself to run out of energy quickly. Fortunately, by taking into consideration the channel qualities, relative QoS requirements of users and optimal power allocation at the relays, sources might not always need to transmit at their largest power. Therefore, sources save their power and prolong its lifetime. Since the relays usually have much less severe energy constraints, resource allocation in relay networks can exploit the

[7] In this way, the minimum data rate among users is also maximized since data rate is a monotonic increasing function of SNR.

[8] Resource allocation in a multi-user network is not as simple as allocating resources for each user individually, albeit orthogonal transmissions are assumed.

available power at the relays to save power at the energy-limited source nodes. One of the most reasonable design objectives is the minimization of the maximum transmit power over all sources. Subject to the SNR requirements for each user, the resulting optimization problem can be posed as

$$\min_{P_{S_i},\, P_{R_{S_i}}} \quad \max_{i=1,\dots,M} P_{S_i} \tag{2a}$$

$$\text{subject to:} \quad \gamma_i \geq \gamma_i^{\min},\ i = 1, \dots, M \tag{2b}$$

$$\text{The constraints (1b), (1d)} \tag{2c}$$

where $\gamma_i^{\min}$ is the threshold SNR for ith user.[9] However, there are applications where the total power is of concern, e.g., when the sources share a common power pool as in the case of a base-station (or access point, access node) transmitter, or in an energy-aware system in which energy consumption and related emission are more related to *total* power than individual *peak* power. In such applications, minimizing the total power, i.e., $\min_{P_{S_i},\, P_{R_{S_i}}} \sum_{i=1}^{M} P_{S_i}$, can be a more appropriate objective since it is expected to provide a solution with lower sum power. Moreover, a weighted sum of powers may be also considered to cover the general case of non-homogeneous users.

It can be observed that at optimality, the inequality constraints (2b) and (1b) in (2c) must be met with equality. This is because γ_i is an increasing function of P_{S_i} and $P_{R_{S_i}}$. In order to minimize P_{S_i}, γ_i and $P_{R_{S_i}}$ must attain their minimum and maximum values, respectively. Note that we have implicitly assumed in (2a)–(2c) that none of the sources needs to transmit more than $P_{S_i}^{\max}$ at optimality.

C. Throughput Maximization Based Allocation

The max-min SNR based allocation improves the system performance by improving the performance of the worst user. On the other hand, it is well-known that the max-min fairness among users is associated with a loss in the network throughput, i.e., the users sum rate. For some applications which require high data rate transmission from any user, it is preferable to allocate power to maximize the network throughput. Users with "good" channel quality can transmit "faster" and users with "bad" channel quality can transmit "slower". Moreover, the network throughput, in the case of perfect CSI and optimal power allocation, defines the upper bound on the system achievable rates. Given the SNR γ_i of user i, the data rate $\mathcal{R}_i$ can be written as a function of γ_i as

$$\mathcal{R}_i = \frac{1}{T}\log_2(1 + K\gamma_i) \approx \frac{1}{T}\log_2(K\gamma_i)$$

where T is the symbol period which is assumed to be equal to 1 for brevity, $K = -\zeta_1/\ln(\zeta_2 \mathrm{BER})$, BER is the target bit error rate, and $\zeta_1,\ \zeta_2$ are constants dependent on the modulation scheme [22]. Note that we have approximated $1+K\gamma_i$ as $K\gamma_i$ which is reasonable when $K\gamma_i$ is much larger than 1. For notational simplicity in the rest of the paper, we set $K = 1$. Then, the aggregate throughput for the system can be written as [16]

$$R = \sum_{i=1}^{M} \mathcal{R}_i \approx \log_2\Big[\prod_{i=1}^{M}\gamma_i\Big].$$

The power allocation problem to maximize the network throughput can be mathematically posed as

$$\max_{P_{S_i},\, P_{R_{S_i}}} \quad \log_2\Big[\prod_{i=1}^{M}\gamma_i\Big] \tag{3a}$$

$$\text{subject to:} \quad \text{The constraints (1b), (1c), (1d).} \tag{3b}$$

Therefore, in the high SNR region, maximizing network throughput can be approximately replaced by maximizing the product of SNRs.[10] Here, we have assumed that there is no lower limit constraint.[11] At optimality, the inequality constraints (1b), (1c) in (3b) of the problem (3a)–(3b) must be met with equality when $P \leq \sum_{i=1}^{M} P_{S_i}^{\max}$. Moreover, when $P > \sum_{i=1}^{M} P_{S_i}^{\max}$, the inequality constraints (1b), (1d) must be met with equality at optimality. Similar to the previous problems, this can be explained using the monotonicity of the objective function (3a).

Note that the throughput maximization based power allocation (3a)–(3b) does not penalize users with "bad" channels and favor users with "good" channels. This is different from the scenario when network throughput maximization is used as a criterion for power allocation in cellular networks where some users are prevented from transmitting data [16]. However in our case, as the SNR γ_i for a particular user i is concave increasing function of allocated powers, the incremental change in SNR is smaller for larger transmit power. In Fig. 1, we plot the SNRs versus allocated power at the relays when source powers are fixed and equal. It can be seen that instead

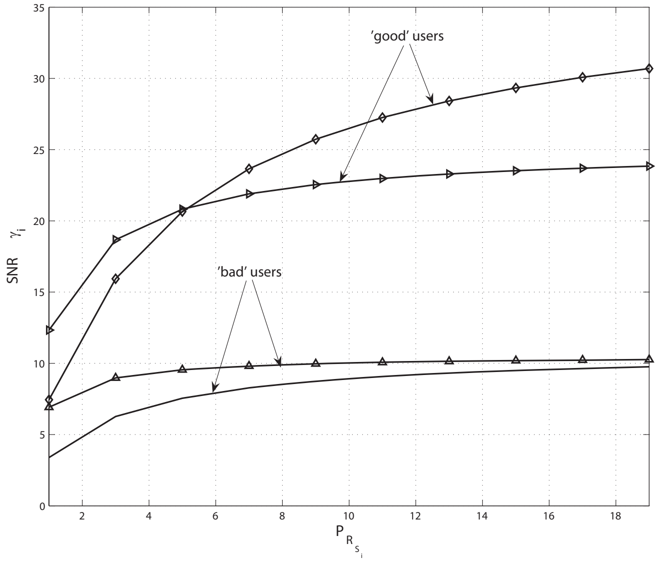

Fig. 1. SNR versus allocated power at the relay node (source powers are fixed and equal).

[9] We assume that the threshold $\gamma_i^{\min}$ is not larger than the maximum achievable SNR for user i as previously discussed.

[10] Note, however, that in the low SNR region, the approximation of $1+\gamma_i$ by γ_i does not hold satisfactorily, and therefore, will not give accurate results.

[11] Such constraint for each user can be, however, easily incorporated in the problem.

of allocating more power to the users with "good" channel conditions at high SNR, the proposed scheme allocates power to the users with "bad" channel conditions at low SNR. It results in better improvement in the sum throughput of the network. This explains why the performance of the users with "bad" channel conditions is not severely affected. This fact is also confirmed in the simulation section.

IV. Power Allocation in Relay Networks via GP

GP is a well-investigated class of nonlinear, nonconvex optimization problems with attractive theoretical and computational properties [15], [16]. Since equivalent convex reformulation is possible for a GP problem, there exist no local optimum points but only global optimum. Moreover, the availability of large-scale software solvers makes GP more appealing.

A. Max-min SNR Based Allocation

Introducing a new variable t, we can equivalently rewrite the optimization problem (1a)–(1d) as follows

$$\min_{P_{S_i},\, P_{R_{S_i}},\, t\geq 0} \quad \frac{1}{t} \tag{4a}$$

$$\text{subject to:} \quad \frac{P_{S_i} P_{R_{S_i}}}{\eta_i P_{S_i} + \alpha_i P_{R_{S_i}} + \beta_i} \geq t, i = 1, \ldots, M \tag{4b}$$

$$\text{The constraints (1b), (1c), (1d).} \tag{4c}$$

The objective function in the problem (4a)–(4c) is a monomial function. Moreover, the constraints in (4b) can be easily converted into posynomial constraints. The constraints (1b), (1c), (1d) are linear on the power variables, and thus, are posynomial constraints. Therefore, the optimization problem (4a)–(4c) is a GP problem.

B. Power Minimization Based Allocation

In this case, by using an extra variable *t*, the objective can be recast as monomial t with monomial constraints $P_{S_i} \leq t$. The constraints can be also written in the form of posynomials. Therefore, the power minimization based allocation is a GP problem.

C. Throughput Maximization Based Allocation

A simple manipulation of the optimization problem (3a)–(3b) gives

$$\min_{P_{S_i},\, P_{R_{S_i}}} \quad \frac{1}{\prod_{i=1}^{M} \gamma_i} \tag{5a}$$

$$\text{subject to:} \quad \text{The constraints (1b), (1c), (1d).} \tag{5b}$$

Each of the terms $1/\gamma_i$ is a posynomial in P_{S_i}, $P_{R_{S_i}}$ and the product of posynomials is also a posynomial. Therefore, the optimization problem (5a)–(5b) belongs to the class of GP problems.[12] As maximizing aggregate throughput can be unfair to some users, a weighted sum of data rates, i.e., $\sum_{i=1}^{M} w_i \mathcal{R}_i$ where w_i is a given weight coefficient for user i, can be used as the objective function to be maximized. Using some manipulations, the resulting optimization problem can be reformulated as a GP problem as well.

We have shown that the three aforementioned power allocation schemes can be reformulated as GP problems. The proposed optimization problems with distinct features of relaying model are mathematically similar to the ones in [16] for conventional cellular network. However, the numerator and denominator of the SNR expression for each user considered in [16] are linear functions of the power variables which is not the case in our work.

[12]Note that the high operating SNR region is assumed. If medium or low SIR regions are assumed, the approximation $1+\gamma_i$ by γ_i may not be accurate. In this case, successive convex approximation method as in [16] can be used. However, it is outside of the scope of this paper.

V. Joint Admission Control and Power Allocation

It is well-known that one of the important resource management issues is the determination of which users to establish connections. Then, radio resources are allocated to the connected users in order to ensure that each connected user has an acceptable signal quality [23]. Since wireless systems are usually resource-limited, they are typically unable to meet users' QoS requirements that need to be satisfied. Consequently, users are not automatically admitted and only certain users can be served. Our admission control algorithm determines which users can be admitted concurrently. Then, the power allocation is used to minimize the transmit power.

A. Revised Power Minimization Based Allocation

The problem formulation (2a)–(2c) can be shown to be feasible as long as $\gamma_i^{\min}$, $i = 1, \ldots, M$ is less than the maximum achievable value. This is because it has been assumed that the sources are able to transmit as much power as possible to increase their SNRs. This approach is impractical for some wireless applications with strictly limited total transmit power, for instance, power limitation of the base station in downlink transmission. The power minimization based problem incorporating the power constraint can be written as

$$\min_{P_{S_i},\, P_{R_{S_i}}} \quad \sum_{i=1}^{M} P_{S_i} \tag{6a}$$

$$\text{subject to:} \quad \sum_{i=1}^{M} P_{S_i} \leq P \tag{6b}$$

$$\text{The constraints (2b), (2c).} \tag{6c}$$

Note that the objective function in the above problem is sufficiently general[13], and it aims at minimizing the overall energy consumed by the group of sources. It requires the cooperation among sources. Such cooperation can be organized in different ways. The simplest example is the presence of only one source (a base station in downlink transmission) with multiple antennas. Also note that in some applications, the constraint (6b) can be effectively excluded from the problem formulation (6a)–(6c) by setting $P \geq \sum_{i=1}^{M} P_{S_i}^{\max}$. Since the objective function is a sum of powers, some sources may need to transmit more power than the others at optimality. Note that

[13]A more general objective function could be the weighted sum, i.e., $\sum_{i=1}^{M} w_i P_{S_i}$ where w_i is a weight coefficient for source i.

for some applications it can be more appropriate to consider the following alternative problem formulation

$$\min_{P_{S_i},\, P_{R_{S_i}}} \quad \max_{i=1,\ldots,M} P_{S_i} \tag{7a}$$
$$\text{subject to:} \quad \text{The constraints (6b), (6c).} \tag{7b}$$

Our methodology can be straightforwardly adapted to cover the above formulation as well. However, due to space limitation, we skip the details here.

There are instances when the optimization problem (6a)–(6c) becomes infeasible. For example, when SNR targets $\gamma_i^{\min}$ are too high, or when the number of users M is large. However, the core reason for infeasibility is the power limits of both the relays and/or the sources. A practical implication of the infeasibility is that it is impossible to serve (admit) all M users at their desired QoS requirements. Some approaches to the infeasible problem can be however used. For instance, some users can be dropped or the SNR targets could be relaxed, i.e., made smaller. We investigate the former scenario and try to maximize the number of users that can be served at their desired QoS.

B. Mathematical Framework for Joint Admission Control and Power Minimization Problem

Following the methodology developed in [18], the joint admission control and power allocation problem can be mathematically stated as a 2-stage optimization problem. All possible sets of admitted users $S_0, S_1, \ldots$ (can be only one or several sets) are found in the first stage by solving the following optimization problem

$$\arg\max_{S\subseteq\{1,\ldots,M\},\, P_{S_i},\, P_{R_{S_i}}} \quad |S| \tag{8a}$$
$$\text{subject to:} \quad \gamma_i \geq \gamma_i^{\min},\ i \in S \tag{8b}$$
$$\text{The constraints (6b), (2c)} \tag{8c}$$

where $|S|$ denotes the cardinality of S. We should note that although the sets $S_0, S_1, \ldots$ contain different users, they have the same cardinality.

Given each set $S_0, S_1, \ldots$ of admitted users, the transmit power is minimized in the second stage. The corresponding optimization problem can be written, for example, for the set S_k as

$$P_k^{\text{opt}} = \arg\min_{P_{S_i},\, P_{R_{S_i}}} \sum_{i\in S_k} P_{S_i} \tag{9a}$$
$$\text{subject to:} \quad \gamma_i \geq \gamma_i^{\min},\ i \in S_k \tag{9b}$$
$$\text{The constraints (6b), (2c).} \tag{9c}$$

The optimal set of admitted users S_k is the one among the sets $S_0, S_1, \ldots$ which requires minimum P_k^{opt}. Alternatively, the joint admission control and power minimization can be regarded as a bilevel programming problem. The admission control problem is combinatorially hard, and therefore, is more difficult. This is because the number of possible sets of admitted users grows exponentially with M. Once the sets of admitted users are determined, the power minimization problem is just the problem (2a)–(2c). Greedy algorithm(s) can be used to solve the first stage. However, it is noted that there may be many sets of admitted users with the same maximal cardinality and deriving optimal greedy algorithm(s) is obviously a difficult problem. Due to its combinatorial hardness, the joint admission control and power allocation problem admits high complexity for practical implementation. In the following section, we propose an efficient algorithm to sub-optimally solve (8a)–(8c) and (9a)–(9c) with significantly reduced complexity.

VI. Proposed Algorithm

A. A Reformulation of Joint Admission Control and Power Allocation Problem

Optimal admission control (8a)–(8c) involves exhaustively solving all subsets of users that is NP-hard. Therefore, a better way of solving the problem of joint admission control and power allocation is highly desirable. The admission control problem (8a)–(8c) can be mathematically recast as follows

$$\max_{s_i\in\{0,1\},\, P_{S_i},\, P_{R_{S_i}}} \quad \sum_{i=1}^{M} s_i \tag{10a}$$
$$\text{subject to:} \quad \gamma_i \geq \gamma_i^{\min} s_i,\ i = 1,\ldots,M \tag{10b}$$
$$\text{The constraints (6b), (2c)} \tag{10c}$$

where the indicator variables $s_i,\ i = 1,\ldots,M$, i.e, $s_i = 0$, $s_i = 1$ means that user i is not admitted, or otherwise, respectively. The following theorem is in order.

Theorem 1: The aforementioned 2-stage optimization problem (8a)–(8c) and (9a)–(9c) is equivalent to the following 1-stage optimization problem

$$\max_{s_i\in\{0,1\},\, P_{S_i},\, P_{R_{S_i}}} \quad \epsilon\sum_{i=1}^{M} s_i - (1-\epsilon)\sum_{i=1}^{M} P_{S_i} \tag{11a}$$
$$\text{subject to:} \quad \gamma_i \geq \gamma_i^{\min} s_i,\ i = 1,\ldots,M \tag{11b}$$
$$\text{The constraints (6b), (2c)} \tag{11c}$$

where ϵ is some constant and is chosen such that $P/(P+1) < \epsilon < 1$.

Proof: The proof is a 2-step process. In the first step, we prove that the solution of the one-stage problem (11a)–(11c) and that of the admission control problem (10a)–(10c) will both give the same maximum number of admitted users. Suppose that $S_0^+,\ P_{S_i}^+,\ P_{R_{S_i}}^+$ is (one of) the optimal solutions of the admission control problem (10a)–(10c) with optimal value $|S_0^+| = n^+$.[14] Similarly, suppose that $S_0^*,\ P_{S_i}^*,\ P_{R_{S_i}}^*$ is the optimal solution of the problem (11a)–(11c) and $|S_0^*| = n^*$. Thus, the optimal value of (11a)–(11c) is $\mathcal{L}^* = \epsilon n^* - (1-\epsilon)\sum_{i=1}^{M} P_{S_i}^*$. We show that $n^* = n^+$ by using contradiction.

Let us suppose that $n^* < n^+$. Since the problems (10a)–(10c) and (11a)–(11c) have the same set of constraints, and thus, the same feasible set, the set $S_0^+,\ P_{S_i}^+,\ P_{R_{S_i}}^+$ is also a feasible solution to (11a)–(11c) with the objective value $\mathcal{L}^+ = \epsilon n^+ - (1-\epsilon)\sum_{i=1}^{M} P_{S_i}^+$. We have

$$\begin{aligned}\mathcal{L}^+ - \mathcal{L}^* &= \epsilon(n^+ - n^*) + (1-\epsilon)\left(\sum_{i=1}^{M} P_{S_i}^* - \sum_{i=1}^{M} P_{S_i}^+\right)\\ &\geq \epsilon - (1-\epsilon)P > 0.\end{aligned} \tag{12}$$

[14] We should note that n^+ is some unknown but it is a fixed number.

The first inequality corresponds to the assumption that $n^+ - n^* \geq 1$ and the fact that

$$\left|\sum_{i=1}^{M} P_{S_i}^* - \sum_{i=1}^{M} P_{S_i}^+\right| \leq P.$$

The latter fact holds true because $\sum_{i=1}^{M} P_{S_i}^* \leq P$ and $\sum_{i=1}^{M} P_{S_i}^+ \leq P$. The second inequality is valid due to the choice of $P/(P+1) < \epsilon < 1$. This obviously contradicts the assumption that $S_0^*, P_{S_i}^*, P_{R_{S_i}}^*$ is the optimal solution of (11a)–(11c). Therefore, we conclude that n^* cannot be less than n^+. On the other hand, we also have $S_0^*, P_{S_i}^*, P_{R_{S_i}}^*$ is a feasible solution of (10a)–(10c). Therefore, the optimal value of (10a)–(10c) is at least equal to $|S_0^*| = n^*$, i.e., $n^+ \geq n^*$. By the virtues of two mentioned facts, we conclude that $n^* = n^+$, or equivalently, the solution of the one-stage optimization problem (11a)–(11c) gives the same number of admitted users as that of the solution of the admission control problem (10a)–(10c).

In the second step, we prove that the user set obtained by solving (11a)–(11c) is the optimal set of admitted users with minimum transmit power. Again, suppose that $S_0^\dagger, P_{S_i}^\dagger, P_{R_{S_i}}^\dagger$ is another feasible solution to (11a)–(11c) such that $|S_0^\dagger| = |S_0^*| = n^*$ with the objective value $\mathcal{L}^\dagger = \epsilon n^* - (1-\epsilon)\sum_{i=1}^{M} P_{S_i}^\dagger$. Since $S_0^*, P_{S_i}^*, P_{R_{S_i}}^*$ is the optimal solution of (11a)–(11c), we must have $\mathcal{L}^\dagger < \mathcal{L}^*$, or equivalently, $\sum_{i=1}^{M} P_{S_i}^* < \sum_{i=1}^{M} P_{S_i}^\dagger$. Therefore, among sets which have the same maximum number of admitted users, the one obtained by solving (11a)–(11c) requires the minimum transmit power. This completes the proof. □

Careful observation reveals some insights into the optimization problem (11a)–(11c) which is in rather similar form as the one in [18]. For example, it is similar to a multi-objective optimization problem, i.e., maximization of the number of admitted users and minimization of the transmit power, with ϵ being the priority for the former criterion. Therefore, it is reasonable to set ϵ large to maximize number of admitted users as a priority. The formulation (11a)–(11c) provides a compact and easy-to-understand mathematical framework for the joint optimal admission control and power allocation. However, as well as the original 2-stage problem, the formulation (11a)–(11c) is NP-hard to solve. Moreover, it is easy to see that the optimization problem (11a)–(11c) is always feasible. This is due to the fact that no users are admitted in the worst case, i.e., $s_i = 0,\ i = 1, \ldots, M$.

To this end, we should mention that the optimization problem (11a)–(11c) is extremely hard, if possible, to solve. It belongs to the class of nonconvex integer optimization problems. Therefore, we next propose a reduced-complexity heuristic algorithm to perform joint admission control and power allocation. Albeit theoretically sub-optimal, its performance is remarkably close to that of the optimal solution for most of the testing instances (see Section VII).

B. Proposed Algorithm

The following heuristic algorithm can be used to solve (11a)–(11c).

- **Step 1.** *Set* $S := \{S_i \mid i = 1, \ldots, M\}$.

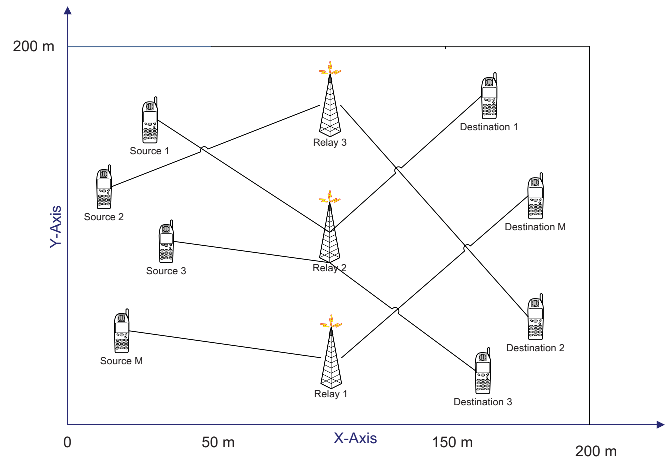

Fig. 2. A wireless relay system.

- **Step 2.** *Solve GP problem (6a)–(6c) without the constraint (6b) for the sources in* S. *Let* $P_{S_i}^*, P_{R_{S_i}}^*$ *denote the resulting power allocation values.*
- **Step 3.** *If* $\sum_{S_i \in S} P_{S_i}^* \leq P$, *then stop and* $P_{S_i}^*, P_{R_{S_i}}^*$ *being power allocation values. Otherwise, user* S_i *with largest required power value, i.e.,* $S_i = \arg\max_{S_i \in S}\{P_{S_i}^*\}$ *is removed from* S *and go to step 2.*

We can see that after each iteration, either the set of admitted users and the corresponding power allocation levels are determined or one user is removed from the list of most possibly admitted users. Since there are M initial users, the complexity is bounded above by that of solving M GP problems of different dimensions. It worths mentioning that the proposed reduced complexity algorithm always returns one solution.

VII. Simulation Results

Consider a wireless relay network as in Fig. 2 with 10 users and 3 relays distributed in a two-dimensional region $200m \times 200m$. The relays are fixed at coordinates (100,50), (100,100), and (100,150). The ten source nodes and their corresponding destination nodes are deployed randomly in the area inside the box areas $[(0,0),(50,200)]$ and $[(150,0),(200,200)]$, respectively. In our simulations, each source is assisted by a random (and then fixed) relay. For simulation simplicity, we assume that there is no microscopic fading and the gain for each transmission link is computed using the path loss model as $a = 1/d$ where d is the Euclidean distance between two transmission ends.[15] The noise power at the receiver ends is assumed to be identical and equals to $N_0 = -50$ dB. Although each relay node may assist different number of users, they are assumed to have the same maximum power level $P_{R_j}^{\max}$. Similarly, all users are assumed to have equal minimum SNR thresholds $\gamma^{\min}$. We have used software package [24] for solving convex programs in our simulations.

[15]If fading is present, the proposed techniques can also be straightforwardly applied assuming that the instantaneous channel fading gains are known and not varied during the time required to compute the solutions. In this case, the average performance computed over a long time interval for different sets of channel fading gains can serve as a performance measure.

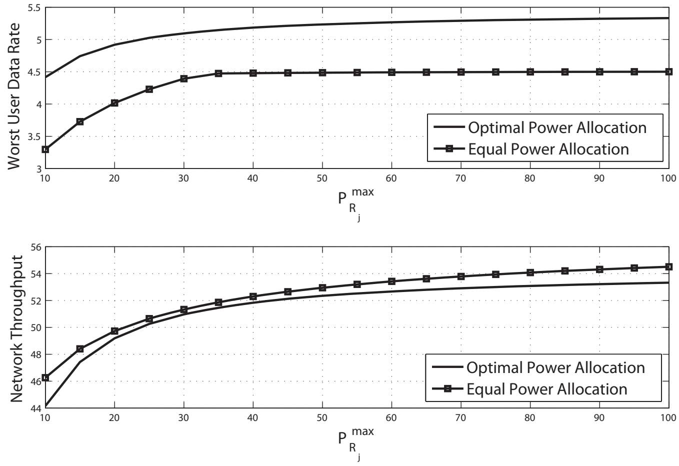

Fig. 3. Max-min SNR based allocation: data rate versus $P_{R_j}^{\max}$.

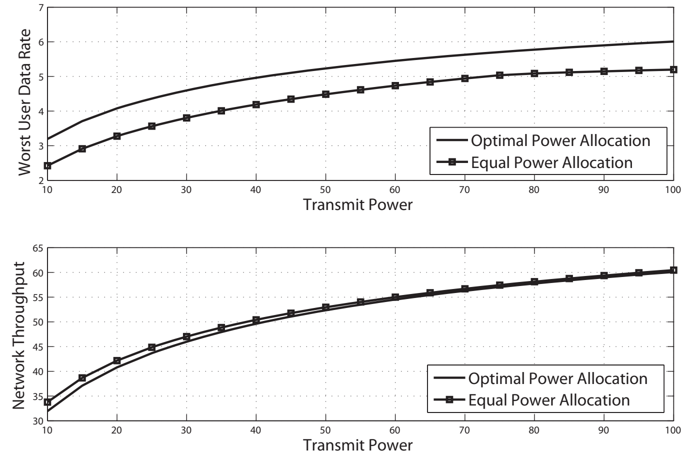

Fig. 4. Max-min SNR based allocation: data rate versus P.

A. Power Allocation without Admission Control

1) Max-min SNR based allocation: Figs. 3 and 4 show the minimum rate of the users and the network throughput when the maximum power levels of the relays $P_{R_j}^{\max}$ and sources P are varied. The performance of the equal power allocation (EPA) scheme is also plotted. In this case, the power is allocated equally among all sources, i.e., $P_{S_i} = P/10$, $\forall S_i$ and each relay distributes power equally among all relayed users. For $P = 50$ (see Fig. 3), the optimal power allocation (OPA) scheme achieves about 0.8 bits performance improvement over the EPA scheme for the worst user data rate. The performance improvement of both schemes is higher when $P_{R_j}^{\max}$ is small (less than 30). The EPA scheme provides a slight performance improvement for the worst user(s) for $P_{R_j}^{\max} \geq 35$. However, the OPA scheme is able to take advantage from larger $P_{R_j}^{\max}$. This demonstrates the effectiveness of OPA scheme in general and our proposed approach in particular. In Fig. 4, we fix $P_{R_j}^{\max} = 50$. It can be seen that the OPA scheme also outperforms the EPA scheme. The improvement is about 0.8 bits and increases when P increases. In both scenarios, it can be seen that since our objective is to improve the performance of the worst user(s), there is a loss in the network throughput. This confirms the well-known fact that achieving max-min fairness among users usually results in performance loss for the whole system.

2) Power minimization based allocation: Figs. 5 and 6 display the total transmit power and the maximum power of all users for two scenarios, where in the first scenario the objective is to attain a minimum SNR $\gamma^{\min}$ with fixed $P_{R_j}^{\max} = 50$, while in the second scenario it is assumed that $P_{R_j}^{\max}$ is varied with fixed $\gamma^{\min} = 10$ dB. We plot the results for both the minimization of the maximum power based power allocation (min-max scheme) and the minimization of sum power based power allocation (minimum sum power scheme).

For the first scenario, the OPA minimum sum power scheme allocates less power than that of the EPA and OPA min-max schemes. Moreover, when $\gamma^{\min} \geq 18$ dB, the EPA scheme can not find a feasible power allocation (in fact, suggests negative power allocation) which is represented by the weird part in the EPA curve. It is because the threshold $\gamma^{\min} \geq 18$ dB exceeds the maximum value of γ_i for some users as discussed in Section II. We can see that by appropriate power distribution at the relays, OPA scheme can find power allocation to achieve larger target SNR $\gamma^{\min}$. This further demonstrates the advantages of our proposed approach over the EPA scheme. Moreover, the OPA min-max scheme needs significantly larger total transmit power than the OPA minimum sum power scheme. Therefore, the latter scheme is preferable when applicable.

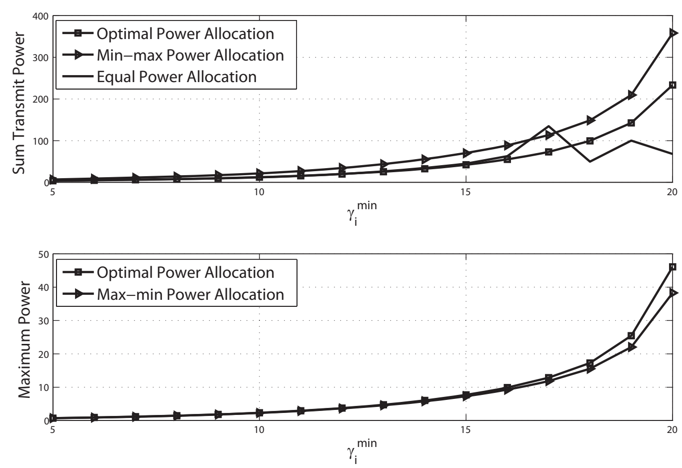

Fig. 5. Power minimization based allocation: transmit power versus $\gamma_i^{\min}$.

For the second scenario, the OPA minimum sum power scheme again requires less total power than that of the EPA and OPA min-max scheme, especially when $P_{R_j}^{\max}$ is small. The transmit power required by the max-min scheme is significantly larger than that required by the other two schemes. It can be observed that as there is more available $P_{R_j}^{\max}$, less sum power is required to achieve a target SNR.

3) Throughput maximization based allocation: In the last example, we use the OPA to maximize the network throughput. Fig. 7 shows the performance of our proposed approach versus $P_{R_j}^{\max}$ when $P = 50$. The OPA scheme outperforms the EPA for all values of $P_{R_j}^{\max}$. It is noticeable that OPA scheme achieves better performance in terms of both worst user data rate and network throughput. Comparing with the results in Figs. 3 and 4, we can see the tradeoff between achieving

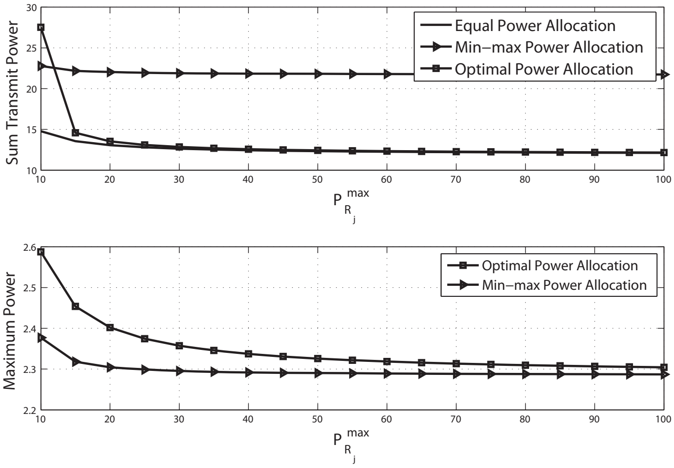

Fig. 6. Power minimization based allocation: transmit power versus $P_{R_j}^{\max}$.

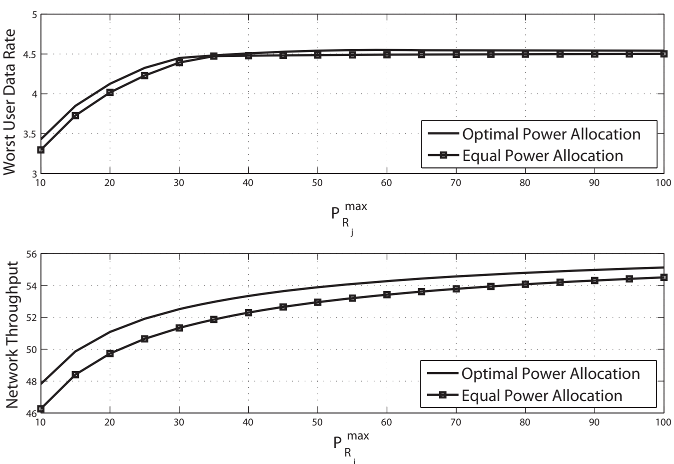

Fig. 7. Throughput maximization based allocation: data rate versus $P_{R_j}^{\max}$.

fairness and sum throughput.

TABLE I

$P = 50$, $P_{R_j}^{\max} = 50$, RUNNING TIME IN SECONDS

	Enumeration	Proposed Algorithm
SNR	17 dB	17 dB
# users served	8	8
Users served	1, 2, 4, 5, 7, 8, 9, 10	1, 2, 4, 5, 7, 8, 9, 10
Transmit power	44.8083	44.8083
Users served	1, 2, 3, 4, 5, 8, 9, 10	-
Transmit power	48.1041	-
Users served	1, 2, 3, 4, 7, 8, 9, 10	-
Transmit power	49.2948	-
Users served	1, 2, 4, 5, 6, 8, 9, 10	-
Transmit power	48.7522	-
Users served	1, 2, 4, 6, 7, 8, 9, 10	-
Transmit power	48.6768	-
Running time	231.68	11.77
SNR	18 dB	18 dB
# users served	7	7
Users served	1, 2, 4, 5, 8, 9, 10	1, 2, 4, 7, 8, 9, 10
Transmit power	47.0270	47.2129
Users served	1, 2, 3, 4, 8, 9, 10	-
Transmit power	49.9589	-
Users served	1, 2, 4, 7, 8, 9, 10	-
Transmit power	47.2129	-
Users served	1, 4, 5, 7, 8, 9, 10	-
Transmit power	48.9124	-
Running time	683.96	14.66
SNR	19 dB	19 dB
# users served	6	6
Users served	1, 2, 4, 8, 9, 10	1, 2, 4, 8, 9, 10
Transmit power	44.9402	44.9402
Users served	1, 4, 7, 8, 9, 10	-
Transmit power	49.4305	-
Running time	1411.23	17.48
SNR	20 dB	20 dB
# users served	5	5
Users served	1, 4, 8, 9, 10	1, 4, 8, 9, 10
Transmit power	44.9199	44.9199
Users served	1, 2, 4, 8, 10	-
Transmit power	46.3774	-
Users served	1, 2, 8, 9, 10	-
Transmit power	46.0823	-
Users served	2, 4, 8, 9, 10	-
Transmit power	46.0185	-
Running time	2170.6	18.95

B. Joint Admission Control and Power Allocation

In this section, we provide several testing instances to demonstrate the performance of the proposed admission control scheme. For such purpose, the performance of the optimal admission control is used as benchmark results.[16] The convenient and informative method of representing results as in [18] is used.

In Tables I and II, $P_{R_j}^{\max}$ are taken to be equal to 50 and 20, respectively while P is fixed at $P = 50$. Different values of $\gamma_i^{\min}$ are used. To gain more insights into the optimal admission control and power allocation problem, all feasible subsets of users which have maximum possible number of users are also provided in Table I.[17] The optimal subset of users is the one which requires the smallest transmit power. The running times required for the optimal exhaustive search based algorithm and the proposed algorithm are also shown. As we can see, our proposed algorithm determines exactly the optimal number of admitted users and the users themselves in all cases except for the case when $P_{R_j}^{\max} = 20$, $\gamma_i^{\min} = 19$ dB. The transmit power required by our proposed algorithm is exactly the same as that required by the optimal admission control using exhaustive search. However, the complexity in terms of running time of the former algorithm is much smaller than that of the latter. This makes the proposed approach attractive for practical implementation. Moreover, it is natural that when $\gamma_i^{\min}$ increases, less users are admitted with a fixed amount of power. For example, when $P_{R_j}^{\max} = 50$, eight users and six users are admitted with SNR $\gamma_i^{\min} = 17$ dB and 19 dB, respectively. Similarly, when more power is available, more users are likely to be admitted for a particular $\gamma_i^{\min}$ threshold. For instance, when $\gamma_i^{\min} = 19$ dB, six and four users are admitted with $P_{R_j}^{\max} = 50$ and 20, respectively.

Table III displays the performance of the proposed algorithm when $P_{R_j}^{\max} = 50$ and $P = 20$. The proposed algorithm is able to decide correctly (optimally) which users should be

[16]Optimal admission control is done by solving the problem (8a)–(8c) for all possible combinations of users.

[17]In Tables II and III, only the optimal set of users and its corresponding transmit power are provided.

TABLE II
$P = 50$, $P_{R_j}^{\max} = 20$

	Enumeration	Proposed Algorithm
SNR	17 dB	17 dB
# users served	7	7
Users served	1, 2, 4, 5, 8, 9, 10	1, 2, 4, 5, 8, 9, 10
Transmit power	42.1896	42.1896
SNR	19 dB	19 dB
# users served	4	3
Users served	1, 4, 8, 10	8, 9, 10
Transmit power	29.6160	19.7388
SNR	21 dB	21 dB
# users served	3	3
Users served	4, 8, 10	8, 9, 10
Transmit power	33.0519	46.0857

TABLE III
$P = 20$, $P_{R_j}^{\max} = 50$

	Enumeration	Proposed Algorithm
SNR	17 dB	17 dB
# users served	4	4
Users served	1, 8, 9, 10	1, 8, 9, 10
Transmit power	14.7282	14.7282
SNR	19 dB	19 dB
# users served	3	3
Users served	8, 9, 10	8, 9, 10
Transmit power	14.9059	14.9059
SNR	21 dB	21 dB
# users served	2	2
Users served	8, 10	8, 10
Transmit power	10.1811	10.1811

admitted and assign an optimal amount of power for each admitted user. As before, less users are admitted when the required SNR threshold is larger. Moreover, as P increases, more users can be admitted. For example, when $P_{R_j}^{\max} = 50$ and $\gamma_i^{\min} = 17$ dB, four and eight users are admitted for $P = 20$ and $P = 50$, respectively.

VIII. Conclusions

In this paper, we have proposed the power allocation schemes for wireless multi-user AF relay networks. Particularly, we have presented three power allocation schemes to i) maximize the minimum SNR among all users; ii) minimize the maximum transmit power over all sources; iii) maximize the network throughput. Although the problem formulations are nonconvex, they were equivalently reformulated as GP problems. Therefore, obtaining optimal power allocation can be done efficiently via convex optimization techniques. Simulation results demonstrate the effectiveness of the proposed approaches over the equal power allocation scheme. Moreover, since it may not be possible to admit all users at their desired QoS demands due to limited power resource, we have proposed a joint admission control and power allocation algorithm which aimed at maximizing the number of users served and minimizing the transmit power. A highly efficient GP heuristic algorithm is developed to solve the proposed nonconvex and combinatorially hard problem. In this paper, the GP problems are solved in a centralized manner using the highly efficient interior point methods. However, whether distributed power allocation via GP is possible is an interesting research area.

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