Analysis of Receiver Algorithms for LTE SC-FDMA Based Uplink MIMO Systems

Zihuai Lin† and Pei Xiao†† and Branka Vucetic
School of Electrical and Information Engineering
University of Sydney, NSW 2006, Australia
The author †† is with the Institute of Electronics,
Communications and Information Technology (ECIT)
Queens University Belfast. (e-mail: pei.xiao@qub.ac.uk).
† the corresponding author
Email: zihuai@ee.usyd.edu.au. Tel:+61 2 9351 4485

Abstract

This letter derives mathematical expressions for the received signal-to-interference-plus-noise ratio (SINR) of uplink Single Carrier (SC) Frequency Division Multiple Access (FDMA) multiuser MIMO systems. An improved frequency domain receiver algorithm is derived for the studied systems, and is shown to be significantly superior to the conventional linear MMSE based receiver in terms of SINR and bit error rate (BER) performance.

Keywords

SC-FDMA, Multiple-Input Multiple-Output (MIMO), 3GPP LTE uplink.

I. INTRODUCTION

Single Carrier (SC) Frequency Division Multiple Access (FDMA) techniques for uplink transmission have attracted appreciable attention because of its low Peak to Average Power Ratio (PAPR) property compared with competitive Orthogonal FDMA (OFDMA) techniques [1–3]. In 3GPP Long Term Evolution (LTE) (also known as Evolved-UMTS Terrestrial Radio Access (E-UTRA or EUTRA)), SC-FDMA has been adopted for uplink transmission, whereas the OFDMA signaling format has been exploited for the downlink transmission [4]. The SC-FDMA signal can be obtained using Discrete Fourier Transform (DFT) spread OFDMA, where DFT is applied to convert time domain input data symbols to the frequency domain before feeding them into an OFDMA modulator.

From user capacity point of view, MIMO technique is preferred due to its capacity enhancement ability. For wide band wireless transmission systems, e.g., LTE OFDMA downlink and SC-FDMA uplink [5, 6], to simply scheduling task, several consecutive subcarriers are usually grouped together for scheduling. A basic scheduling unit is called a Resource Block (RB). The scheduler in a Base Station (BS) may assign single or multiple RBs to a Mobile Station (MS).

Two MIMO schemes for SC-FDMA uplink transmission are being investigated under 3GPP LTE, namely, multi-user MIMO and single user MIMO. For a single user MIMO, the BS only schedules one single user into one RB. For a multi-user MIMO, multiple MSs are allowed to transmit simultaneously on a RB. This paper investigates receiver algorithms for a SC-FDMA based uplink in a multi-user MIMO system. The novelties of this paper are the derivation of the received Signal to Interference plus Noise Ratio (SINR) and the proposal of an improved frequency domain receiver algorithm.

Notations: we use upper bold-face letters to represent matrices and vectors. The (n, k) element of a matrix $\mathbf{A}$ is represented by $[\mathbf{A}]_{n,k}$ and the nth element of a vector $\mathbf{b}$ is denoted by $[\mathbf{b}]_n$. Superscripts $(\cdot)^{\mathcal{H}}$, $(\cdot)^{\mathcal{T}}$ denote the Hermitian transpose and transpose, respectively, $(\cdot)^*$ denotes conjugate.

II. System Model

The cellular multiple access system under study has n_R receive antennas at the BS and a single transmit antenna at the ith user terminal, $i = 1, 2, \cdots, K_T$ where K_T is the total number of users in the system. We consider the multi-user MIMO case with K ($K < K_T$) users being served at each time slot and $K = n_R$. The system model for a SC-FDMA based MIMO transmitter and receiver is shown in Figs. 1 and 2, respectively. On the transmitter side, the user data block containing N symbols is firstly transformed by a N point DFT to a frequency domain representation. The outputs are then mapped to M $(M > N)$ orthogonal subcarriers followed by a M point Inverse Fast Fourier Transform (IFFT) to convert to a time domain complex signal sequence. A Circle Prefix (CP) is inserted into the signal sequence before it is passed to the Radio Frequency (RF) module. On the receiver side, the opposite operating procedures are performed after the noisy signals are received by the receive antennas. A MIMO Frequency Domain Equalizer (FDE) is applied to the frequency domain signals after subcarrier demapping as shown in Fig. 2. For simplicity, we employ a linear Minimum Mean Squared Error (MMSE) receiver, which provides a good tradeoff between the noise enhancement and the multiple stream interference mitigation [7].

In the following, we let $\mathbf{D_{F_M}} = \mathbf{I_K} \otimes \mathbf{F}_M$ and denote by $\mathbf{F}_M$ the $M \times M$ Fourier matrix with the element $[\mathbf{F}_M]_{m,k} = \exp(-j\frac{2\pi}{M}(m-1)(k-1))$ where $k, m \in \{1, \cdots, M\}$ is the sample number and the frequency tone number, respectively. Here $\otimes$ is the Kronecker product, $\mathbf{I_K}$ is the K dimension identity matrix. We denote by $\mathbf{D_{F_M}^{-1}}$ the $KM \times KM$

dimension inverse Fourier matrix defined as $\mathbf{I_K} \otimes \mathbf{F}_M^{-1}$ and $\mathbf{F}_M^{-1}$ is the $M \times M$ inverse Fourier matrix with the element $[\mathbf{F}_M^{-1}]_{m,k} = \frac{1}{M}\exp(j\frac{2\pi}{M}(m-1)(k-1))$. $\mathbf{D_{F_N}}$ and $\mathbf{D_{F_N}^{-1}}$ are defined in the similar way as $\mathbf{D_{F_M}}$ and $\mathbf{D_{F_M}^{-1}}$ with the only difference in the matrix size. Furthermore, we let $\mathcal{F}_n$ represent the subcarrier mapping matrix of size $M \times N$ and $\mathcal{F}_n^{-1}$ is the subcarrier demapping matrix of size $N \times M$.

The received signal after the RF module and removing CP becomes $\tilde{\mathbf{r}} = \tilde{\mathbf{H}}\mathbf{D_{F_M}^{-1}}(\mathbf{I_K} \otimes \mathcal{F}_n)\mathbf{D_{F_N}}\tilde{\mathbf{x}} + \tilde{\mathbf{w}}$, where $\tilde{\mathbf{x}} = [\tilde{\mathbf{x}}_1^{\mathcal{T}}, \cdots, \tilde{\mathbf{x}}_K^{\mathcal{T}}]^T \in \mathbb{C}^{KN\times 1}$ is the data sequence of all K users, and $\tilde{\mathbf{x}}_i \in \mathbb{C}^{N\times 1}$, $i \in \{1, \cdots, K\}$, is the transmitted user data block for the ith user; $\tilde{\mathbf{w}} \in \mathbb{C}^{Mn_R\times 1}$ is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $N_0\mathbf{I} \in \mathbb{R}^{Mn_R\times Mn_R}$, i.e., $\tilde{\mathbf{w}} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$; $\tilde{\mathbf{H}}$ is an $n_R M \times KM$ channel matrix.

With the MIMO FDE, the output time domain signal is given by

$$\begin{aligned} \tilde{\mathbf{z}} &= \mathbf{D_{F_N}^{-1}}\mathbf{A}^{\mathcal{H}}(\mathbf{I_K} \otimes \mathcal{F}_n^{-1})\mathbf{D_{F_M}}\tilde{\mathbf{r}} = \mathbf{D_{F_N}^{-1}}\mathbf{A}^{\mathcal{H}}(\mathbf{I_K} \otimes \mathcal{F}_n^{-1})\mathbf{D_{F_M}}(\tilde{\mathbf{H}}\mathbf{D_{F_M}^{-1}}(\mathbf{I_K} \otimes \mathcal{F}_n)\mathbf{D_{F_N}}\tilde{\mathbf{x}} + \tilde{\mathbf{w}}) \\ &= \mathbf{D_{F_N}^{-1}}\mathbf{A}^{\mathcal{H}}(\mathbf{H}\mathbf{D_{F_N}}\tilde{\mathbf{x}} + \mathbf{w}) = \mathbf{D_{F_N}^{-1}}\mathbf{z}, \end{aligned} \tag{1}$$

where $\mathbf{A}$ is a $KN \times KN$ equalization matrix and $\mathbf{H} = (\mathbf{I_K} \otimes \mathcal{F}_n^{-1})\mathbf{D_{F_M}}\tilde{\mathbf{H}}\mathbf{D_{F_M}^{-1}}(\mathbf{I_K} \otimes \mathcal{F}_n) \in \mathbb{C}^{KN\times KN}$; $\mathbf{w} \in \mathbb{C}^{n_R N\times 1}$ is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $N_0\mathbf{I} \in \mathbb{R}^{n_R N\times n_R N}$, i.e., $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$.

In the frequency domain, $\mathbf{z} = \mathbf{A}^{\mathcal{H}}[\mathbf{H}\mathbf{D_{F_N}}\tilde{\mathbf{x}} + \mathbf{w}]$, where $\tilde{\mathbf{x}}$ can be expressed as $\tilde{\mathbf{x}} = \mathbf{P}\cdot\tilde{\mathbf{s}}$, where $\tilde{\mathbf{s}} = [\tilde{\mathbf{s}}_1^{\mathcal{T}} \cdots \tilde{\mathbf{s}}_K^{\mathcal{T}}]^{\mathcal{T}}$ and $\tilde{\mathbf{s}}_i \in \{\mathbb{C}^{N\times 1}\}$, $i \in \{1, 2, \cdots, K\}$, is the user data block for the ith user, and $\mathrm{E}[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^{\mathcal{H}}] = \mathbf{I}_N$. The power loading matrix $\mathbf{P} \in \mathbb{R}^{KN\times KN}$ is a block diagonal matrix with its ith sub-matrix expressed as $\mathbf{P}_i = \mathrm{diag}\{\sqrt{p_{i,1}}, \sqrt{p_{i,2}}, \cdots, \sqrt{p_{i,N}}\} \in \mathcal{R}^{N\times N}$ and $p_{i,n}$ $(i \in \{1, 2, \cdots, K\})$ is the transmitted power for the ith user at the nth subcarrier; $\tilde{\mathbf{s}} \in \mathbb{C}^{KN\times 1}$ represents the transmitted data symbol vector from different users with $E[\tilde{\mathbf{s}}\tilde{\mathbf{s}}^{\mathcal{H}}] = \mathbf{I}_{KN}$.

In the frequency domain, the received signal can be expressed as

$$\mathbf{r} = \mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{w} = \mathbf{H}\mathbf{P}\mathbf{D}_F\tilde{\mathbf{s}} + \mathbf{w}, \tag{2}$$

where $\mathbf{s} = \mathbf{D}_{F_N}\tilde{\mathbf{s}}$ is the transmitted signal in the frequency domain.

We apply the FDE matrix $\mathbf{A}$ on $\mathbf{r}$ to obtain the equalized signal $\mathbf{z} = \mathbf{A}^{\mathcal{H}}\mathbf{r}$, where $\mathbf{A}$ in the conventional system is derived from the cost function $e = \mathrm{E}[\|\mathbf{z} - \mathbf{s}\|^2] = \mathrm{E}[\|\mathbf{A}^{\mathcal{H}}\mathbf{r} - \mathbf{s}\|^2]$. Minimizing this cost function leads to the optimal matrix of $\mathbf{A}$ as

$$\mathbf{A} = (\mathbf{H}\mathbf{P}\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} + N_0\mathbf{I})^{-1}\mathbf{H}\mathbf{P}. \tag{3}$$

III. Improved frequency domain receiver algorithm

In the previous section, we investigated conventional linear MMSE receiver for the SC-FDMA based uplink MIMO system. It is optimum for systems with proper modulations, such as M-QAM and M-PSK (for which $E[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^{\mathcal{T}}] = \mathbf{0}$). However, for the improper modulation schemes, such as M-ary ASK, OQPSK (for which $E[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^{\mathcal{T}}] = \mathbf{I} \neq \mathbf{0}$), the conventional solution becomes suboptimum as will become evident later on. In (2), let us assume $\tilde{\mathbf{s}}_i \in \mathbb{C}^{N\times 1}$ is an improper signal vector, satisfying the condition $\mathrm{E}[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^{\mathcal{T}}] = \mathrm{E}[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^{\mathcal{H}}] = \mathbf{I}_N$. Since $\mathrm{E}[\mathbf{s}_i\mathbf{s}_i^{\mathcal{H}}] = \mathbf{D}_F\,\mathrm{E}[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^{\mathcal{H}}]\mathbf{D}_F^{\mathcal{T}} = \mathbf{D}_F\mathbf{D}_F^{\mathcal{H}} = \mathbf{I}_N, \mathrm{E}[\mathbf{s}_i\mathbf{s}_i^{\mathcal{T}}] = \mathbf{D}_F\,\mathrm{E}[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^{\mathcal{T}}]\mathbf{D}_F^{\mathcal{T}} = \mathbf{D}_F\mathbf{D}_F^{\mathcal{T}} \neq \mathbf{0}$, we can conclude that $\mathbf{s}_i$ is also an improper signal vector. In order to utilize the improperness of $\mathbf{s}$, we need to apply widely linear processing [8, 9], the principle of which is not only to process $\mathbf{r}$, but also its conjugated version $\mathbf{r}^*$ in order to derive the filter output, i.e.,

$$\mathbf{z} = \boldsymbol{\zeta}\mathbf{r} + \boldsymbol{\eta}\mathbf{r}^* = \boldsymbol{\Omega}^{\mathcal{H}}\mathbf{y}, \tag{4}$$

where $\boldsymbol{\Omega} = \begin{bmatrix}\boldsymbol{\zeta} & \boldsymbol{\eta}\end{bmatrix}^{\mathcal{H}}$ and $\mathbf{y} = \begin{bmatrix}\mathbf{r} & \mathbf{r}^*\end{bmatrix}^{\mathcal{T}}$. It is worth noticing that the conventional linear MMSE receiver is a special case of the one expressed by (4), when $\boldsymbol{\zeta} = \mathbf{A}^{\mathcal{H}}$ and $\boldsymbol{\eta} = \mathbf{0}$. The cost function for deriving the new filter is defined by

$$\begin{aligned}\epsilon^{\mathrm{WL}} &= \mathrm{E}[\|\boldsymbol{\Omega}^{\mathcal{H}}\mathbf{y} - \mathbf{s}\|^2] = \mathrm{E}[(\boldsymbol{\Omega}^{\mathcal{H}}\mathbf{y} - \mathbf{s})(\mathbf{y}^{\mathcal{H}}\boldsymbol{\Omega} - \mathbf{s}^{\mathcal{H}})];\\ &= \boldsymbol{\Omega}^{\mathcal{H}}\mathbf{C}_{\mathbf{yy}}\boldsymbol{\Omega} - \boldsymbol{\Omega}^{\mathcal{H}}\mathbf{C}_{\mathbf{ys}} - \mathbf{C}_{\mathbf{sy}}\boldsymbol{\Omega} + \mathbf{I}_N,\end{aligned} \tag{5}$$

where

$$\mathbf{C}_{\mathbf{yy}} = \mathrm{E}\{\mathbf{y}\mathbf{y}^{\mathcal{H}}\} = \mathrm{E}\left\{\begin{bmatrix}\mathbf{r}\\ \mathbf{r}^*\end{bmatrix}\begin{bmatrix}\mathbf{r}^{\mathcal{H}} & \mathbf{r}^{\mathcal{T}}\end{bmatrix}\right\} = \begin{bmatrix}\mathbf{C}_{\mathbf{rr}} & \tilde{\mathbf{C}}_{\mathbf{rr}}\\ \tilde{\mathbf{C}}_{\mathbf{rr}}^* & \mathbf{C}_{\mathbf{rr}}^*\end{bmatrix}, \tag{6}$$

and

$$\mathbf{C_{rr}} = \mathrm{E}\{\mathbf{r}\mathbf{r}^{\mathcal{H}}\} = \mathrm{E}\{(\mathbf{HPs}+\mathbf{w})(\mathbf{s}^{\mathcal{H}}\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}+\mathbf{w}^{\mathcal{H}})\} = \mathbf{HP}\,\mathrm{E}[\mathbf{s}\mathbf{s}^{\mathcal{H}}]\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} + N_0\mathbf{I} = \mathbf{HPP}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} + N_0\mathbf{I}$$

$$\tilde{\mathbf{C}}_{\mathbf{rr}} = \mathrm{E}\{\mathbf{r}\mathbf{r}^{\mathcal{T}}\} = \mathrm{E}\{(\mathbf{HPs}+\mathbf{w})(\mathbf{s}^{\mathcal{T}}\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}+\mathbf{w}^{\mathcal{T}})\} = \mathbf{HP}\,\mathrm{E}[\mathbf{s}\mathbf{s}^{\mathcal{T}}]\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}} = \mathbf{HPD}_F\mathbf{D}_F^{\mathcal{T}}\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}},$$

$$\mathbf{C_{ys}} = \mathrm{E}\{\mathbf{y}\mathbf{s}^{\mathcal{H}}\} = \mathrm{E}\left\{\begin{bmatrix}\mathbf{r}\\ \mathbf{r}^*\end{bmatrix}\mathbf{s}^{\mathcal{H}}\right\} = \mathrm{E}\left\{\begin{bmatrix}\mathbf{r}\mathbf{s}^{\mathcal{H}}\\ \mathbf{r}^*\mathbf{s}^{\mathcal{H}}\end{bmatrix}\right\} = \begin{bmatrix}\mathbf{HP}\,\mathrm{E}[\mathbf{s}\mathbf{s}^{\mathcal{H}}]\\ \mathbf{H}^*\mathbf{P}\,\mathrm{E}[\mathbf{s}^*\mathbf{s}^{\mathcal{H}}]\end{bmatrix} = \begin{bmatrix}\mathbf{HPD}_F\mathbf{D}_F^{\mathcal{H}}\\ \mathbf{H}^*\mathbf{P}(\mathbf{D}_F\mathbf{D}_F^{\mathcal{T}})^*\end{bmatrix} = \begin{bmatrix}\mathbf{HP}\\ \mathbf{H}^*\mathbf{P}(\mathbf{D}_F\mathbf{D}_F^{\mathcal{T}})^*\end{bmatrix}$$

$$\mathbf{C_{sy}} = \mathrm{E}[\mathbf{s}\mathbf{y}^{\mathcal{H}}] = \mathrm{E}\left\{\mathbf{s}\begin{bmatrix}\mathbf{r}^{\mathcal{H}} & \mathbf{r}^{\mathcal{T}}\end{bmatrix}\right\} = \begin{bmatrix}\mathrm{E}\{\mathbf{s}\mathbf{s}^{\mathcal{H}}\}\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} & \mathrm{E}\{\mathbf{s}\mathbf{s}^{\mathcal{T}}\}\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\end{bmatrix} = \begin{bmatrix}\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} & \mathbf{D}_F\mathbf{D}_F^{\mathcal{T}}\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\end{bmatrix}. \tag{7}$$

Differentiating ϵ^{WL} in (5) with respect to $\mathbf{\Omega}$ results in $\frac{\partial\epsilon}{\partial\mathbf{\Omega}} = (\mathbf{C_{yy}}\mathbf{\Omega})^* - \mathbf{C}_{s\mathbf{y}}^{\mathcal{T}}$, which is set to zero to yield the optimum vector of $\mathbf{\Omega}$

$$\mathbf{\Omega} = \mathbf{C_{yy}^{-1}}\mathbf{C_{sy}^{\mathcal{H}}} = \mathbf{C_{yy}^{-1}}\mathbf{C_{ys}} = \begin{bmatrix}\mathbf{HPP}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} + N_0\mathbf{I} & \mathbf{HPD}_F\mathbf{D}_F^{\mathcal{T}}\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}}\\ \mathbf{H}^*\mathbf{P}^*\mathbf{D}_F^*\mathbf{D}_F^{\mathcal{H}}\mathbf{P}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}} & \mathbf{H}^*\mathbf{P}^*\mathbf{P}^{\mathcal{T}}\mathbf{H}^{\mathcal{T}} + N_0\mathbf{I}\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{HP}\\ \mathbf{H}^*\mathbf{P}(\mathbf{D}_F\mathbf{D}_F^{\mathcal{T}})^*\end{bmatrix}. \tag{8}$$

For the proposed FDE, the augmented autocorrelation matrix $\mathbf{C_{yy}}$ and crosscorrelation matrix $\mathbf{C_{ys}}$ expressed in (7) which give a complete second order description of the received signal are used for deriving the filter coefficient matrix $\mathbf{\Omega}$; whereas for the conventional linear MMSE algorithm, the coefficient matrix $\mathbf{A}$ is calculated using only the autocorrelation of the observation $\mathbf{C_{rr}} = \mathrm{E}[\mathbf{r}\mathbf{r}^{\mathcal{H}}]$ and the crosscorrelation $\mathbf{C_{rs}} = \mathrm{E}[\mathbf{r}\mathbf{s}^{\mathcal{H}}]$. The pseudo-autocorrelation $\tilde{\mathbf{C}}_{\mathbf{rr}} = \mathrm{E}[\mathbf{r}\mathbf{r}^{\mathcal{T}}]$ and pseudo-crosscorrelation $\tilde{\mathbf{C}}_{\mathbf{rs}} = \mathrm{E}[\mathbf{r}\mathbf{s}^{\mathcal{T}}]$ are implicitly assumed to be zero, leading to sub-optimum solutions.

IV. Performance Analysis

A. SINR expression for conventional FDE

The signal vector detected at the receiver in the time domain can be expressed as

$$\tilde{\mathbf{z}} = \mathbf{D_{F_N}^{-1}}\mathbf{A}^{\mathcal{H}}(\mathbf{HD_{F_N}}\tilde{\mathbf{x}} + \mathbf{w}) = \mathbf{D_{F_N}^{-1}}\mathbf{A}^{\mathcal{H}}(\mathbf{HPs} + \mathbf{w}). \tag{9}$$

Let $\mathbf{B} = \mathbf{A}^{\mathcal{H}}\mathbf{H}\mathbf{P}$, $\mathbf{A}$ and $\mathbf{B}$ can be expressed as

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_{11}} & \mathbf{A_{12}} & \cdots & \mathbf{A_{1K}} \\ \mathbf{A_{21}} & \mathbf{A_{22}} & \cdots & \mathbf{A_{2K}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A_{K1}} & \mathbf{A_{K2}} & \cdots & \mathbf{A_{KK}} \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} \mathbf{B_{11}} & \mathbf{B_{12}} & \cdots & \mathbf{B_{1K}} \\ \mathbf{B_{21}} & \mathbf{B_{22}} & \cdots & \mathbf{B_{2K}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B_{K1}} & \mathbf{B_{K2}} & \cdots & \mathbf{B_{KK}} \end{bmatrix}, \tag{10}$$

where $\mathbf{A_{ij}} \in \mathbb{C}^{N\times N}$ is the equalization matrix between the jth transmitter and the ith receiver antenna. $\mathbf{B}_{ij}$ is defined similarly. The signal vector detected at the receiver for the ith user, $i \in \{1, 2, \cdots, K\}$, in the time domain can be expressed as

$$\tilde{\mathbf{z}}_i = \sum_{j=1, j\neq i}^{K} \mathbf{F}_N^{-1}\mathbf{B}_{ij}\mathbf{F}_N\tilde{s}_j + \mathbf{F}_N^{-1}\mathbf{B}_{ii}\mathbf{F}_N\tilde{\mathbf{s}}_i + \sum_{j=1}^{K}\mathbf{F}_N^{-1}\mathbf{A}_{ij}\mathbf{w}_j. \tag{11}$$

The kth symbol, $k \in \{1, 2, \cdots, N\}$, of $\tilde{\mathbf{z}}_i$ can be expressed as

$$\begin{aligned} \tilde{\mathbf{z}}_i(k) \quad &= \quad \mathbf{F}_N^{-1}(k,:)\mathbf{B}_{ii}\mathbf{F}_N(:,k)\tilde{\mathbf{s}}_i(k) + \sum_{j=1, j\neq k}^{N}\mathbf{F}_N^{-1}(k,:)\mathbf{B}_{ii}\mathbf{F}_N(:,j)\tilde{\mathbf{s}}_i(j) \\ &+ \quad \sum_{j=1, j\neq i}^{K}\mathbf{F}_N^{-1}(k,:)\mathbf{B}_{ij}\mathbf{F}_N\tilde{\mathbf{s}}_j + \sum_{j=1}^{K}\mathbf{F}_N^{-1}(k,:)\mathbf{A}_{ij}\mathbf{w}_j. \end{aligned} \tag{12}$$

The first term on the right hand side of (12) represents the desired signal, the second term is the intersymbol interferences from the same substream, the third term is the interference from the other substreams, and the fourth one is the noise. The power of the received desired signal is then $P_d^i(k) = \mathbf{F}_N^{-1}(k,:)\mathbf{B}_{ii}\mathbf{F}_N(:,k)\mathbf{F}_N(:,k)^{\mathcal{H}}\mathbf{B}_{ii}^{\mathcal{H}}\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}}$. The total power of the received signal can be expressed as,

$$P_t^i(k) = \sum_{j=1}^{K}\mathbf{F}_N^{-1}(k,:)\mathbf{B}_{ij}\mathbf{B}_{ij}^{\mathcal{H}}\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}}. \tag{13}$$

The power of the noise is

$$P_n^i(k) = N_0\sum_{j=1}^{K}\mathbf{F}_N^{-1}(k,:)\mathbf{A}_{ij}\mathbf{A}_{ij}^{\mathcal{H}}\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}}. \tag{14}$$

The received SINR for the kth symbol of the ith user is thus

$$\begin{aligned}\gamma^i_{con}(k) &= \left[\frac{P^i_t(k)+P^i_n(k)}{P^i_d(k)}-1\right]^{-1}\\ &= \left[\frac{\sum_{j=1}^{K}\mathbf{F}_N^{-1}(k,:)\mathbf{B}_{ij}\mathbf{B}_{ij}^{\mathcal{H}}\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}}+N_0\sum_{j=1}^{K}\mathbf{F}_N^{-1}(k,:)\mathbf{A}_{ij}\mathbf{A}_{ij}^{\mathcal{H}}\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}}}{\mathbf{F}_N^{-1}(k,:)\mathbf{B}_{ii}\mathbf{F}_N(:,k)\mathbf{F}_N(:,k)^{\mathcal{H}}\mathbf{B}_{ii}^{\mathcal{H}}\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}}}-1\right]^{-1}. \qquad (15)\end{aligned}$$

B. SINR expression for improved FDE

With the improved FDE, the frequency domain signal is given by (4) as $\mathbf{z}=\boldsymbol{\zeta}\mathbf{r}+\boldsymbol{\eta}\mathbf{r}^*$. The corresponding time domain representation is

$$\begin{aligned}\tilde{\mathbf{z}} &= \mathbf{D}^{-\mathbf{1}}_{\mathbf{F_N}}\mathbf{z}=\mathbf{D}^{-\mathbf{1}}_{\mathbf{F_N}}(\boldsymbol{\zeta}(\mathbf{HPs}+\mathbf{w})+\boldsymbol{\eta}(\mathbf{HPs}+\mathbf{w})^*)\\ &= \mathbf{D}^{-\mathbf{1}}_{\mathbf{F_N}}(\boldsymbol{\zeta}(\mathbf{HPD}_F\tilde{\mathbf{s}}+\mathbf{w})+\boldsymbol{\eta}(\mathbf{HPD}_F\tilde{\mathbf{s}}+\mathbf{w})^*) \qquad (16)\end{aligned}$$

Let $\mathbf{C}=\boldsymbol{\zeta}\mathbf{HP}$ and $\mathbf{Q}=\boldsymbol{\eta}\mathbf{H}^*\mathbf{P}$ and decompose $\mathbf{C}$ and $\mathbf{Q}$ into the block matrices $\mathbf{C}_{ij}$ and $\mathbf{Q}_{ij}$, respectively, in the similar way as for decomposing matrix $\mathbf{A}$ (see eq. (10)). The time domain received signal for the ith user is then

$$\tilde{\mathbf{z}}_i=\sum_{j=1,j\neq i}^{K}\mathbf{F}_N^{-1}\mathbf{C}_{ij}\mathbf{F}_N\tilde{s}_j+\mathbf{F}_N^{-1}\mathbf{C}_{ii}\mathbf{F}_N\tilde{\mathbf{s}}_i+\sum_{j=1}^{K}\mathbf{F}_N^{-1}\boldsymbol{\zeta}_{ij}\mathbf{w}_j+\sum_{j=1,j\neq i}^{K}\mathbf{F}_N^{-1}\mathbf{Q}_{ij}\mathbf{F}_N^*\tilde{s}_j^*+\mathbf{F}_N^{-1}\mathbf{Q}_{ii}\mathbf{F}_N^*\tilde{\mathbf{s}}_i^*+\sum_{j=1}^{K}\mathbf{F}_N^{-1}\boldsymbol{\eta}_{ij}\mathbf{w}_j^* \qquad (17)$$

The kth symbol of $\tilde{\mathbf{z}}_i$ can be expressed as

$$\begin{aligned}\tilde{\mathbf{z}}_i(k) &= \mathbf{F}_N^{-1}(k,:)\left(\mathbf{C}_{ii}\mathbf{F}_N(:,k)\tilde{\mathbf{s}}_i(k)+\mathbf{Q}_{ii}\mathbf{F}_N(:,k)^*\tilde{\mathbf{s}}_i(k)^*\right)\\ &+ \sum_{j=1,j\neq k}^{N}\mathbf{F}_N^{-1}(k,:)\left(\mathbf{C}_{ii}\mathbf{F}_N(:,j)\tilde{\mathbf{s}}_i(j)+\mathbf{Q}_{ii}\mathbf{F}_N(:,j)^*\tilde{\mathbf{s}}_i(j)^*\right)\\ &+ \sum_{j=1,j\neq i}^{K}\mathbf{F}_N^{-1}(k,:)(\mathbf{C}_{ij}\mathbf{F}_N\tilde{\mathbf{s}}_j+\mathbf{Q}_{ij}\mathbf{F}_N^*\tilde{\mathbf{s}}_j^*)+\sum_{j=1}^{K}\mathbf{F}_N^{-1}(k,:)(\boldsymbol{\zeta}_{ij}\mathbf{w}_j+\boldsymbol{\eta}_{ij}\mathbf{w}_j^*) \qquad (18)\end{aligned}$$

The first term on the right hand side of (18) represents the desired signal, the second term is the intersymbol interferences from the same substream, the third term is the interference from the other substreams, and the fourth one is the noise.

The power of the received desired signal is then

$$\begin{aligned} P_d^i(k) &= \mathbf{F}_N^{-1}(k,:)\mathbf{C}_{ii}\mathbf{F}_N(:,k)\mathbf{F}_N(:,k)^{\mathcal{H}}\mathbf{C}_{ii}^{\mathcal{H}}\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}} + \mathbf{F}_N^{-1}(k,:)\mathbf{Q}_{ii}\mathbf{F}_N(:,k)^*[\mathbf{F}_N(:,k)^*]^{\mathcal{H}}\mathbf{Q}_{ii}^{\mathcal{H}}\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}} \\ &+ \mathbf{F}_N^{-1}(k,:)\mathbf{Q}_{ii}\mathbf{F}_N(:,k)^*\mathbf{F}_N(:,k)^{\mathcal{H}}\mathbf{C}_{ii}^{\mathcal{H}}\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}} + \mathbf{F}_N^{-1}(k,:)\mathbf{C}_{ii}\mathbf{F}_N(:,k)\mathbf{F}_N(:,k)^{\mathcal{T}}\mathbf{Q}_{ii}^{\mathcal{H}}\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}}. \end{aligned} \quad (19)$$

Eq. (19) holds since $\mathrm{E}[\tilde{\mathbf{s}}_i(k)^*\tilde{\mathbf{s}}_i(k)^{\mathcal{H}}] = \mathrm{E}[\tilde{\mathbf{s}}_i(k)\tilde{\mathbf{s}}_i(k)^{\mathcal{T}}] = \mathbf{I}_N$ for improper signal vector $\tilde{\mathbf{s}}_i(k)$.

The total power of the received signal can be expressed as

$$\begin{aligned} P_t^i(k) &= \mathrm{E}[\sum_{j=1}^{K}\mathbf{F}_N^{-1}(k,:)(\mathbf{C}_{ij}\mathbf{F}_N\tilde{\mathbf{s}}_j + \mathbf{Q}_{ij}\mathbf{F}_N^*\tilde{\mathbf{s}}_j^*)(\sum_{j=1}^{K}\mathbf{F}_N^{-1}(k,:)(\mathbf{C}_{ij}\mathbf{F}_N\tilde{\mathbf{s}}_j + \mathbf{Q}_{ij}\mathbf{F}_N^*\tilde{\mathbf{s}}_j^*))^{\mathcal{H}}] \\ &= \sum_{j=1}^{K}\mathbf{F}_N^{-1}(k,:)(\mathbf{C}_{ij}\mathbf{C}_{ij}^{\mathcal{H}} + \mathbf{Q}_{ij}\mathbf{Q}_{ij}^{\mathcal{H}} + \mathbf{Q}_{ij}\mathbf{F}_N^*\mathbf{F}_N^{\mathcal{H}}\mathbf{C}_{ij}^{\mathcal{H}} + \mathbf{C}_{ij}\mathbf{F}_N\mathbf{F}_N^{\mathcal{T}}\mathbf{Q}_{ij}^{\mathcal{H}})\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}}. \end{aligned} \quad (20)$$

The power of the noise is

$$\begin{aligned} P_n^i(k) &= N_0\sum_{j=1}^{K}\mathbf{F}_N^{-1}(k,:)(\boldsymbol{\zeta}_{ij}\boldsymbol{\zeta}_{ij}^{\mathcal{H}} + \boldsymbol{\eta}_{ij}\boldsymbol{\eta}_{ij}^{\mathcal{H}})\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}} + \sum_{j=1}^{K}\mathbf{F}_N^{-1}(k,:)(\boldsymbol{\eta}_{ij}\,\mathrm{E}[\mathbf{w}_j^*\mathbf{w}_j^{\mathcal{H}}]\boldsymbol{\zeta}_{ij}^{\mathcal{H}} + \boldsymbol{\zeta}_{ij}\,\mathrm{E}[\mathbf{w}_j\mathbf{w}_j^{T}]\boldsymbol{\eta}_{ij}^{\mathcal{H}})\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}} \\ &= N_0\sum_{j=1}^{K}\mathbf{F}_N^{-1}(k,:)(\boldsymbol{\zeta}_{ij}\boldsymbol{\zeta}_{ij}^{\mathcal{H}} + \boldsymbol{\eta}_{ij}\boldsymbol{\eta}_{ij}^{\mathcal{H}})\mathbf{F}_N^{-1}(k,:)^{\mathcal{H}}. \end{aligned} \quad (21)$$

The second equality in (21) follows from the fact that $\mathrm{E}[\mathbf{w}_j^*\mathbf{w}_j^{\mathcal{H}}] = \mathrm{E}[\mathbf{w}_j\mathbf{w}_j^{T}] = \mathbf{0}$. The received SINR for the ith symbol at time interval k is then

$$\gamma_{imp}^i(k) = \left[\frac{P_t^i(k) + P_n^i(k)}{P_d^i(k)} - 1\right]^{-1}, \quad (22)$$

where $P_d^i(k)$, $P_t^i(k)$ and $P_n^i(k)$ are given by (19), (20) and (21), respectively.

Note that in [10], SINR expression for SC-FDMA with linear MMSE frequency domain receiver was derived for single antenna case. The analysis derived in this paper is for multiple antennas and can be considered as a generalization of the one derived in [10] for the conventional receiver as well as for the newly proposed receiver for SC-FDMA systems employing improper signals.

V. Analytical and Simulation Results

We consider 3GPP LTE baseline antenna configuration, in which two MSs are grouped together and synchronized to form a virtual MIMO channel between BS and MSs. The channel fading coefficients are assumed to be highly correlated within one sub-frame and are independent among different sub-frames. The entries of the channel matrix are modeled as independent identically distributed (i.i.d) complex Gaussian samples, with σ_k^2 as the variance for the kth column of the channel matrix, and σ_k^2 is uniformly distributed in $[0, 1]$. The different variance in each column reflects the variation in average power gains between different users. The block size of the user data is 12, which is also the number of subcarriers in a resource block.

Fig. 3 shows the BER performance comparison between the conventional and the improved receivers for 4PAM and OQPSK systems. The improved receiver scheme significantly outperforms its conventional counterpart, especially at high SNRs. The gap can be over 10 dB. The plot for QPSK system with the conventional receiver is also provided for a baseline comparison. Although its performance is superior to the 4PAM system with the conventional receiver, however, it is much inferior to th 4PAM system with the improved receiver.

Fig. 4 shows the analytical results of the SINR distribution of the 4PAM and QPSK systems with both the conventional linear MMSE and the improved receiver when the transmitted SNR is equal to 20 dB. The curves are obtained by evaluating Eqs. (15) and (22) derived in Section IV. One can see that the SINR distribution of the 4PAM system with improved receiver is significantly better than the 4PAM and QPSK systems with conventional MMSE equalizer. Both BER and SINR performance analyses justify the use of improper signals in conjunction with the proposed frequency domain receiver algorithm in LTE SC-FDMA based uplink MIMO systems.

VI. Conclusion

In this correspondence, we derived an improved frequency domain receiver algorithm for the SC-FDMA based uplink MIMO system with improper signal constellation. Mathematical expressions of the received SINR for the studied MIMO systems have been derived. Both simulation and analytical results reveal that the proposed scheme has superior BER and SINR performance to the conventional linear MMSE receiver for SC-FDMA MIMO uplink systems. This work provides a valuable reference for the future version of the LTE standard and a useful source of information for the practical implementation of the LTE systems.

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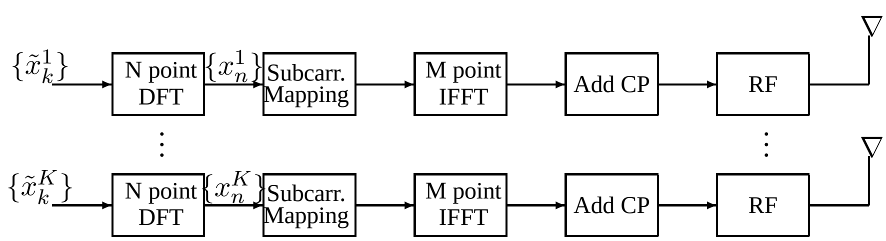

Fig. 1. SC-FDMA based MIMO Transmitter.

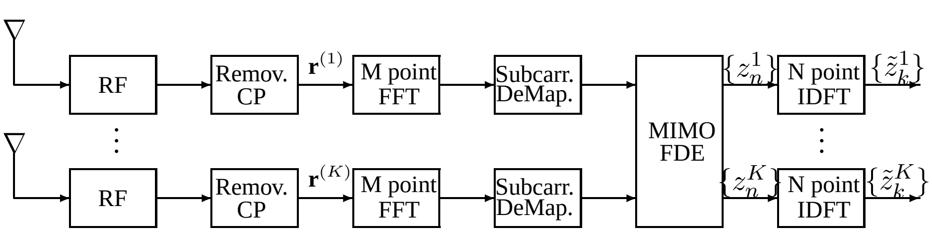

Fig. 2. SC-FDMA based MIMO Receiver.

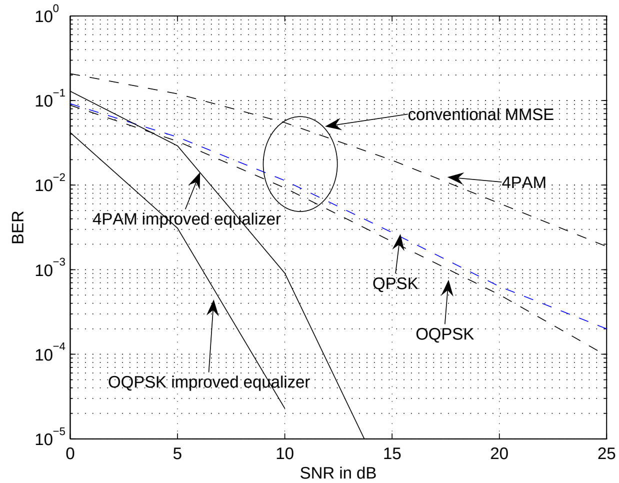

Fig. 3. BER performance for SC-FDMA uplink 2 by 2 MIMO system with conventional MMSE equalizer and the improved FDE equalizer.

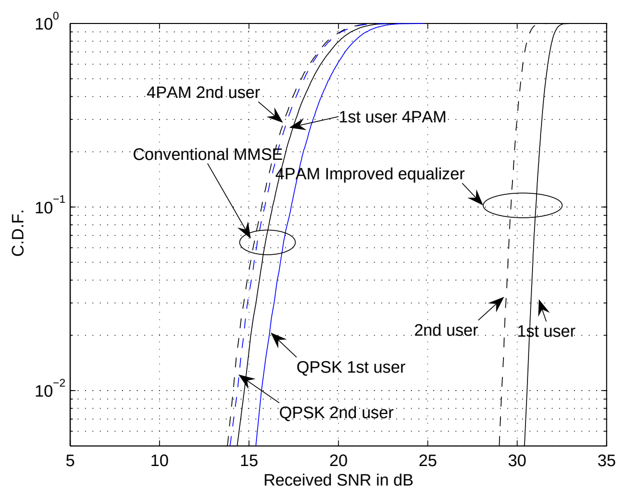

Fig. 4. SINR distribution for SC-FDMA uplink 2 by 2 MIMO system with conventional MMSE equalizer and the improved FDE equalizer.