Robust Transceiver Design for *K*-Pairs Quasi-Static MIMO Interference Channels via Semi-Definite Relaxation

Eddy Chiu, Student Member, IEEE, Vincent K. N. Lau, Senior Member, IEEE, Huang Huang, Student Member, IEEE, Tao Wu and Sheng Liu

Abstract—In this paper, we propose a robust transceiver design for the K-pair quasi-static MIMO interference channel. Each transmitter is equipped with M antennas, each receiver is equipped with N antennas, and the k^{th} transmitter sends L_k independent data streams to the desired receiver. In the literature, there exist a variety of theoretically promising transceiver designs for the interference channel such as interference alignment-based schemes, which have feasibility and practical limitations. In order to address practical system issues and requirements, we consider a transceiver design that enforces robustness against imperfect channel state information (CSI) as well as fair performance among the users in the interference channel. Specifically, we formulate the transceiver design as an optimization problem to maximize the worst-case signal-to-interference-plus-noise ratio among all users. We devise a low complexity iterative algorithm based on alternative optimization and semi-definite relaxation techniques. Numerical results verify the advantages of incorporating into transceiver design for the interference channel important practical issues such as CSI uncertainty and fairness performance.

Index Terms—Interference channel, robust transceiver, imperfect CSI, precoder design, decorrelator design, max-min fair, alternative optimization, semi-definite relaxation.

I. INTRODUCTION

In many wireless network scenarios, the channel is shared among multiple systems. The coexisting systems create mutual interference, which poses great challenges for communication systems design. Conventionally, interference is either treated as noise in the weak interference case [1] or canceled at the receiver in the strong interference case [2], [3]. In the past decade, various schemes are proposed to utilize multiple signaling dimensions for interference avoidance and mitigation. In particular, in the recent breakthrough work [4], the authors show that the paradigm of interference alignment (IA) can be exploited to confine mutual interference to some lower dimensional subspace, so that desired signals can be transmitted on interference-free subspace. It is shown that this IA

T. Wu and S. Liu are with Huawei Technologies, Co. Ltd., China (e-mail: walnut@huawei.com and martin.liu@huawei.com).

scheme, if feasible, is optimal in the degree-of-freedom (DoF) sense. The results of [4] has triggered a number of extensions [5], [6] and related works [7], [8]. These IA-based schemes, albeit theoretically promising, have various limitations. First, IA-based schemes require ideal conditions to be feasible such as perfect channel state information (CSI) and very large dimensions on the signal space. For example, the conventional IA scheme [4] requires time or frequency extensions to have feasible solutions. For K-pairs quasi-static MIMO interference channels where time / frequency extensions are not viable, the IA scheme [4] is only feasible for $K \leq 3$ (cf. [9]). Second, while IA-based schemes have promising DoF performance which is an asymptotic performance measure for very high signal-to-noise ratio (SNR) - they are not optimal at medium SNR that correspond to practical applications. When designing practical communication systems for the interference channel, a number of technical issues shall be considered. Specifically, in practice only imperfect CSI is available and there are limited signaling dimensions. Moreover, it is important to ensure satisfactory performance among all the systems in the network.

In this paper, we consider the problem of robust transceiver design for the K-pair quasi-static MIMO interference channel with fairness considerations. Specifically, 1) we apply robust design principles to provide resilience against CSI uncertainties; and 2) we formulate the transceiver design as a precoder-decorrelator optimization problem to maximize the worst-case signal-to-interference-plus-noise ratio (SINR) among all users in the interference channel. In the literature, precoder-decorrelator optimization for worst-case SINR are proposed for broadcast and point-to-point systems [10]–[13]. Specifically, in [10], [13] the authors consider precoding design for the worst-case SINR in MISO broadcast channel, where it is shown that the precoder optimization problem is always convex. In [12] the authors consider precoderdecorrelator design for the worst-case SINR MIMO broadcast channel using an iterative algorithm based on solving convex subproblems. On the other hand, in [11] the authors consider a space-time coding scheme for the point-to-point channel with imperfect channel knowledge. However, these existing works cannot be extended to robust transceiver design for the MIMO interference channel, which presents the following key technical challenges.

The Precoder-Decorrelator Optimization Problem is NP-Hard: The precoder-decorrelator optimization problem for the interference channel involves solving a separable homoge-

Manuscript received December 28, 2009; revised May 28, 2010 and September 8, 2010; accepted September 9, 2010. The associate editor coordinating the review of this paper and approving it for publication was O. Simeone. The paper was presented in part at the Asilomar Conference on Signals, Systems, and Computing, Pacific Grove, CA, November 2010.

E. Chiu, V. K. N. Lau, and H. Huang are with the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Hong Kong (e-mail: echiua@ieee.org, eeknlau@ust.hk, and huang@ust.hk).

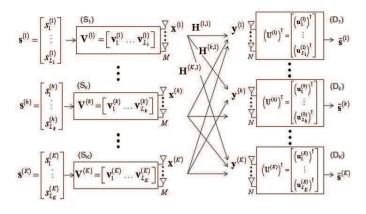


Fig. 1. System model. There are K source-destination pairs where each source node is equipped with M antennas and each destination node is equipped with N antennas. The k^{th} transmitter sends L_k independent data streams to the desired receiver.

neous quadratically constrained quadratic program (QCQP), which is NP-hard in general [14], [15]. One approach to facilitate solving this class of problems is to apply semidefinite relaxation (SDR) by relaxing rank constraints; this method was applied in precoding design for MISO broadcast channel [16], [17] and for MISO multicast channel [18], [19]. Although the resultant semidefinite program (SDP) may be solvable, the optimization in general does not always have the desired rank profile.

Convergence of Alternative Optimization Algorithm: Our proposed solution is based on alternative optimization (AO). The method of AO was proposed in [20], [21] for precoder and decorrelator optimization for multi-user MIMO broadcast channels. However, coupled with the rank constrained SDP issues as well as the absence of uplink-downlink duality (as in the case of broadcast channels) [22], [23], establishing the convergence proof of the AO algorithm in the interference channel is non-trivial [24] and traditional convergence proof [20], [21] cannot be applied to our situations.

Notation: In the sequel, we adopt the following notations. $\mathbb{R}^{M \times N}$, $\mathbb{C}^{M \times N}$ and $\mathbb{Z}^{M \times N}$ denote the set of real, complex and integer $M \times N$ matrices, respectively; \mathbb{R}_+ denotes the set of positive real numbers; upper and lower case letters denote matrices and vectors, respectively; \mathbb{H}^N denotes the set of $N \times N$ Hermitian matrices; $\mathbf{X} \succeq 0$ denotes that \mathbf{X} is a positive semi-definite matrix; $(\cdot)^T$ and $(\cdot)^{\dagger}$ denote transpose and Hermitian transpose, respectively; $\operatorname{rank}(\cdot)$ and $\operatorname{Tr}(\cdot)$ denote matrix rank and trace, respectively; $[\mathbf{X}]_{(a,b)}$ denotes the $(a, b)^{\text{th}}$ element of \mathbf{X} ; $|| \cdot ||$ denotes the Frobenius norm; $\mathcal{I}(\cdot)$ denotes the indicate function; \mathcal{K} denotes the index set $\{1, \ldots, K\}$ and \mathcal{L}_k denotes the index set $\{1, \ldots, L_k\}$; $\mathbf{0}_N$ denotes an $N \times 1$ vector of zeros and \mathbf{I}_N denotes an $N \times N$ identity matrix; $\mathbb{E}[\cdot]$ denotes expectation; and $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Phi})$ denotes complex Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Phi}$.

II. SYSTEM MODEL AND REVIEW OF PRIOR WORKS

A. System Model

We consider a MIMO interference channel consisting of K source-destination pairs where each source node is equipped with M antennas and each destination node is equipped with

N antennas as shown in Fig. 1. For ease of exposition, we focus on the k^{th} user referring to source node S_k and destination node D_k ; nevertheless, the same model applies to all other source-destination pairs. Specifically, S_k transmits L_k data streams $\mathbf{s}^{(k)} = [\mathbf{s}_1^{(k)} \dots \mathbf{s}_{L_k}^{(k)}]^T$ to D_k , which performs linear detection. The received signal of D_k is interfered by the transmitted signals of all other users. To mitigate the impact of mutual interference, prior to transmission S_k precodes the data streams $\mathbf{s}^{(k)}$ using the precoder matrix $\mathbf{V}^{(k)} = [\mathbf{v}_1^{(k)} \dots \mathbf{v}_{L_k}^{(k)}] \in \mathbb{C}^{M \times L_k}$ and D_k decorrelates the received signal using the decorrelator matrix $\mathbf{U}^{(k)} = [\mathbf{u}_1^{(k)} \dots \mathbf{u}_{L_k}^{(k)}] \in \mathbb{C}^{N \times L_k}$. It follows that the transmitted signal of S_k is given by

$$\mathbf{x}^{(k)} = \mathbf{V}^{(k)} \mathbf{s}^{(k)} = \sum_{l=1}^{L_k} \mathbf{v}_l^{(k)} s_l^{(k)}, \qquad (1)$$

the received signal of D_k is given by

$$\mathbf{y}^{(k)} = \sum_{j=1}^{K} \mathbf{H}^{(k,j)} \mathbf{x}^{(j)} + \mathbf{n}^{(k)}$$

= $\mathbf{H}^{(k,k)} \mathbf{x}^{(k)} + \underbrace{\sum_{\substack{j=1\\j \neq k}}^{K} \mathbf{H}^{(k,j)} \mathbf{x}^{(j)}}_{\text{interference}} + \mathbf{n}^{(k)}, \qquad (2)$

and the decorrelator output of D_k is given by

$$\widetilde{\mathbf{s}}^{(k)} = (\mathbf{U}^{(k)})^{\dagger} \mathbf{y}^{(k)}$$

$$= \underbrace{(\mathbf{U}^{(k)})^{\dagger} \mathbf{H}^{(k,k)} \mathbf{V}^{(k)} \mathbf{s}^{(k)}}_{\text{desired signals}}$$

$$+ \underbrace{\sum_{j=1}^{K} (\mathbf{U}^{(k)})^{\dagger} \mathbf{H}^{(k,j)} \mathbf{V}^{(j)} \mathbf{s}^{(j)}}_{\text{leakage interference}} + (\mathbf{U}^{(k)})^{\dagger} \mathbf{n}^{(k)},$$
(3)

where $\mathbf{H}^{(k,j)} \in \mathbb{C}^{N \times M}$ is the fading channel from S_j to D_k and $\mathbf{n}^{(k)} \sim \mathcal{CN}(\mathbf{0}_N, N_0 \mathbf{I}_N)$ is the AWGN. As per (1)–(3), the estimate of data stream $s_l^{(k)}$ is given by

$$\widetilde{s}_{l}^{(k)} = \underbrace{(\mathbf{u}_{l}^{(k)})^{\dagger} \mathbf{H}^{(k,k)} \mathbf{v}_{l}^{(k)} s_{l}^{(k)}}_{\text{desired signal}} + \underbrace{\sum_{\substack{m=1\\m\neq l}}^{L_{k}} (\mathbf{u}_{l}^{(k)})^{\dagger} \mathbf{H}^{(k,k)} \mathbf{v}_{m}^{(k)} s_{m}^{(k)}}_{\text{inter-stream interference}} + \underbrace{\sum_{\substack{j=1\\j\neq k}}^{K} \sum_{m=1}^{L_{j}} (\mathbf{u}_{l}^{(k)})^{\dagger} \mathbf{H}^{(k,j)} \mathbf{v}_{m}^{(j)} s_{m}^{(j)}}_{\text{leakage interference}} + (\mathbf{u}_{l}^{(k)})^{\dagger} \mathbf{n}^{(k)}, \quad (4)$$

where the severity of the inter-stream and leakage interference terms depend on the transceiver processing and CSI assumption. Considering practical systems, we make the following assumptions towards designing effective precoders and decorrelators.

Assumption 1 (Transmit power constraint): We assume the data streams are independent and have unit power, i.e. $\mathbb{E}[(\mathbf{s}^{(k)})^{\dagger}\mathbf{s}^{(k)}] = \mathbf{I}_{L_k}$. Furthermore, we assume the maximum transmit power of the k^{th} source node is P_k so the precoders shall satisfy the power constraint $\mathbb{E}[(\mathbf{x}^{(k)})^{\dagger}\mathbf{x}^{(k)}] = \sum_{l=1}^{L_k} (\mathbf{v}_l^{(k)})^{\dagger}\mathbf{v}_l^{(k)} \leq P_k$.

Assumption 2 (Fading model): We assume quasi-static fading so the fading channels $\mathbf{H}^{(k,j)}$ remain unchanged during a fading block. In addition, we assume rank $(\mathbf{H}^{(k,j)}) = \min(M, N)$.

Assumption 3 (CSI model): We assume perfect CSI is available at the receivers (i.e. perfect CSIR), and only imperfect CSI is available at the transmitters (i.e. imperfect CSIT) for designing the precoders and decorrelators. Specifically, we model channel estimates at the transmitters as

$$\widehat{\mathbf{H}}^{(k,j)} = \mathbf{H}^{(k,j)} - \mathbf{\Delta}^{(k,j)}, \forall j, k \in \mathcal{K},$$
(5)

where $\mathbf{\Delta}^{(k,j)}$ is the CSI error [10], [11], [25]. Specifically, we assume $||\mathbf{\Delta}^{(k,j)}||^2 \leq \varepsilon$, which implies that the actual channel $\mathbf{H}^{(k,j)}$ belongs to a spherical uncertainty region centered at $\mathbf{\widehat{H}}^{(k,j)}$ with radius ε . For notational convenience, we denote $\mathcal{H} = {\{\mathbf{H}^{(k,j)}\}_{j,k=1}^{K}} = {\{\mathbf{\widehat{H}}^{(k,j)} + \mathbf{\Delta}^{(k,j)}\}_{j,k=1}^{K}}$ and $\widehat{\mathcal{H}} = {\{\mathbf{\widehat{H}}^{(k,j)}\}_{j,k=1}^{K}}$.

Remark 1 (Interpretation of the CSI error model): The imperfect CSIT model (5) encapsulates the following scenarios.

- Quantized CSI in FDD Systems [10, Section II-B]: For FDD systems, the transmitters are provided with quantized CSI via feedback. Using uniform quantizers, the quantization cells in the interior of the quantization region can be approximated by spherical regions of radius equal to the quantization step size. As a result, the imperfect CSIT model corresponds to quantized CSI obtained using a uniform vector quantizer with quantization step size $\sqrt{\varepsilon}$.
- Estimated CSI in TDD Systems [11, Section IV-A]: For TDD systems, the transmitters can estimate the channels from the sounding signals received in the reverse link. The imperfectness of the CSIT in this case comes from the estimation noise as well as delay. Using MMSE channel prediction, the CSI estimate $\widehat{\mathbf{H}}^{(k,j)}$ is unbiased, whereas the CSI error $\mathbf{\Delta}^{(k,j)}$ is Gaussian distributed and independent from the CSI estimate $\widehat{\mathbf{H}}^{(k,j)}$. As a result, $\mathbf{\Delta}^{(k,j)}$ is a jointly Gaussian matrix and $||\mathbf{\Delta}^{(k,j)}||^2 \leq \varepsilon$ corresponds to "equal probability contour" on the probability space of $\mathbf{\Delta}^{(k,j)}$. In other words, the probability of the event $||\mathbf{\Delta}^{(k,j)}||^2 \leq \varepsilon$ depends on ε only. Accordingly, we could find an ε such that $\Pr[||\mathbf{\Delta}^{(k,j)}||^2 \leq \varepsilon] = 0.99$ (for example).

By Assumptions 1 to 3, the data stream estimate $\tilde{s}_l^{(k)}$ in (4) can be *equivalently* expressed as

$$\widetilde{s}_{l}^{(k)} = (\mathbf{u}_{l}^{(k)})^{\dagger} (\widehat{\mathbf{H}}^{(k,k)} + \boldsymbol{\Delta}^{(k,k)}) \mathbf{v}_{l}^{(k)} s_{l}^{(k)} + \sum_{\substack{m=1\\m\neq l}}^{L_{k}} (\mathbf{u}_{l}^{(k)})^{\dagger} (\widehat{\mathbf{H}}^{(k,k)} + \boldsymbol{\Delta}^{(k,k)}) \mathbf{v}_{m}^{(k)} s_{m}^{(k)}$$

$$(6)$$

+
$$\sum_{\substack{j=1\\j\neq k}}^{K} \sum_{m=1}^{L_j} (\mathbf{u}_l^{(k)})^{\dagger} (\widehat{\mathbf{H}}^{(k,j)} + \boldsymbol{\Delta}^{(k,j)}) \mathbf{v}_m^{(j)} s_m^{(j)} + (\mathbf{u}_l^{(k)})^{\dagger} \mathbf{n}^{(k)}$$

The actual SINR of $\tilde{s}_l^{(k)}$ at the k^{th} receiver is given by (7), whereby the instantaneous mutual information between data stream $s_l^{(k)}$ and estimate $\tilde{s}_l^{(k)}$ can be expressed as

$$C_{l}^{(k)}(\mathcal{H}, \{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \mathbf{u}_{l}^{(k)})$$

$$= \log_{2}(1 + \gamma_{l}^{(k)}(\mathcal{H}, \{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \mathbf{u}_{l}^{(k)})).$$
(8)

B. Review of Prominent Transceiver Designs for MIMO Interference Channels

In the following, we review the motivations and issues of prominent transceiver designs for MIMO interference channels in the literature. 1) Interference Alignment in Quasi-Static MIMO Signal Space: In [4], [9] the authors exploited IA in quasi-static MIMO signal space for precoder-decorrelator design. Specifically, assuming perfect CSI, we could obtain precoders and decorrelators that confine the interference on each destination node to a lower dimension subspace, such that interference can be more effectively removed. Note that IA is only feasible with sufficiently large number of signaling dimensions. For the K-pair quasi-static $N \times M$ MIMO interference channel, IA could achieve a DoF of $K \frac{\min(M,N)}{2}$ for $K \leq 3$ but might not be feasible for K > 3. Moreover, IA is not optimal in general at medium SNR. For example, consider the data stream estimate $\tilde{s}_l^{(k)}$ in (6); suppose IA is feasible then

$$(\mathbf{u}_l^{(k)})^{\dagger} \widehat{\mathbf{H}}^{(k,j)} \mathbf{v}_m^{(j)} = 0, j \neq k \text{ or } l \neq m,$$

and the actual SINR of the l^{th} data stream at k^{th} receiver is given by

$$\gamma_{l}^{(k)}(\mathcal{H},\{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K},\mathbf{u}_{l}^{(k)}) = \frac{||(\mathbf{u}_{l}^{(k)})^{\dagger}(\widehat{\mathbf{H}}^{(k,k)}+\mathbf{\Delta}^{(k,k)})\mathbf{v}_{l}^{(k)}||^{2}}{\begin{pmatrix}\sum_{m=1}^{L_{j}}||(\mathbf{u}_{l}^{(k)})^{\dagger}\mathbf{\Delta}^{(k,k)}\mathbf{v}_{m}^{(k)}||^{2}\\m \neq l\\+\sum_{j=1}^{K}\sum_{m=1}^{L_{j}}||(\mathbf{u}_{l}^{(k)})^{\dagger}\mathbf{\Delta}^{(k,j)}\mathbf{v}_{m}^{(j)}||^{2}+N_{0}||\mathbf{u}_{l}^{(k)}||^{2}\end{pmatrix}}.$$
(9)

As per (9), the presence of CSI error $\Delta^{(k,j)}$ creates persistent residual interference. Even when the residual interference is negligible, i.e.

$$\gamma_l^{(k)}(\mathcal{H}, \{\{\mathbf{v}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K, \mathbf{u}_l^{(k)}) \approx \frac{||(\mathbf{u}_l^{(k)})^{\dagger}(\widehat{\mathbf{H}}^{(k,k)} + \mathbf{\Delta}^{(k,k)})\mathbf{v}_l^{(k)}||^2}{N_0 ||\mathbf{u}_l^{(k)}||^2},$$

the conventional IA scheme [4], [9] makes no attempt to optimize SINR performance.

2) Interference Alignment in Real Fading Channels: In [7], [8] the authors consider IA along the real line by creating fictitious signaling dimensions. Specifically, assuming perfect CSI, we could design the leakage interference terms at each destination node to have the same scaling factor (or pseudo direction), such that interference can be effectively removed. For example, consider the received signal in (2); for the purpose of illustration let M = N = 1 and $H^{(k,j)} \in \mathbb{R}$ so

$$\begin{split} y^{(k)} &= H^{(k,k)} x^{(k)} + \sum_{\substack{j=1\\j\neq k}}^{K} H^{(k,j)} x^{(j)} + n^{(k)} \\ &\stackrel{(a)}{=} \sum_{l=1}^{L_j} (\hat{H}^{(k,k)} + \Delta^{(k,k)}) v_l^{(k)} s_l^{(k)} \\ &+ \sum_{\substack{j=1\\j\neq k}}^{K} \sum_{l=1}^{L_j} \underbrace{(\hat{H}^{(k,j)} + \Delta^{(k,j)}) v_l^{(j)} s_l^{(j)}}_{\text{leakage interference}} + n^{(k)}, \end{split}$$

where (a) follows from (1) and (5). To facilitate IA along the real line, the data streams shall belong to the set of integers (i.e. $s_l^{(k)} \in \mathbb{Z}$) and we shall choose the precoders such that $\hat{H}^{(k,j)}v_l^{(j)} = \hat{H}^{(k,m)}v_l^{(m)}$ for $j \neq m$. It is shown in [7], [8] that, if ideally CSI error is negligible (i.e. $\Delta^{(k,j)} \approx 0$), this scheme could theoretically achieve a DoF of $K \frac{MN}{M+N}$. However, this scheme would require infinite SNR and cannot be implemented in practice.

$$\gamma_{l}^{(k)}(\mathcal{H},\{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K},\mathbf{u}_{l}^{(k)}) = \frac{||(\mathbf{u}_{l}^{(k)})^{\dagger}(\widehat{\mathbf{H}}^{(k,k)}+\mathbf{\Delta}^{(k,k)})\mathbf{v}_{l}^{(k)}||^{2}}{\sum_{\substack{m=1\\m\neq l}}^{L_{k}}||(\mathbf{u}_{l}^{(k)})^{\dagger}(\widehat{\mathbf{H}}^{(k,k)}+\mathbf{\Delta}^{(k,k)})\mathbf{v}_{m}^{(k)}||^{2}+\sum_{\substack{m=1\\j\neq k}}^{L_{j}}\sum_{m=1}^{L_{j}}||(\mathbf{u}_{l}^{(k)})^{\dagger}(\widehat{\mathbf{H}}^{(k,j)}+\mathbf{\Delta}^{(k,j)})\mathbf{v}_{m}^{(j)}||^{2}+N_{0}||\mathbf{u}_{l}^{(k)}||^{2}}$$
(7)

3) Iterative Algorithms to Minimize Leakage Interference / Maximize SINR: In [5], [6] the authors exploit uplink-downlink duality and propose iterative algorithms for precoder-decorrelator design. Specifically, the algorithms in [6, Algorithm 1], [5] are established with the objective of sequentially minimizing the aggregate leakage interference induced by each data stream, whereas the algorithm in [6, Algorithm 2] is established with the objective to sequentially maximize the SINR of each data stream. Note that the aforementioned algorithms neglect the presence of CSI error, which could have significant performance impacts. Moreover, these algorithms neglect individual user performance and fairness. This is undesirable because for practical systems it is important to ensure all users have satisfactory performance.

III. PROBLEM FORMULATION: ROBUST TRANSCEIVER DESIGN WITH FAIRNESS CONSIDERATIONS

In this section, we formulate a transceiver design for the K-pair quasi-static MIMO interference channel that is robust against CSI uncertainties and with the objective of enforcing fairness among all users' data streams. Specifically, to provide the best resilience against CSI error, we adopt a worst-case design approach. On the other hand, the fairness aspect is motivated by the practical system consideration to ensure all users in the network can have satisfactory performance. As such, we formulate the precoder-decorrelator design with imperfect CSIT as an optimization problem to maximize the *worst-case* SINR among all users' data streams, subject to the maximum transmit power per source node.

A. Optimization Problem

The robust and fair transceiver optimization problem for the K-pair $N \times M$ MIMO interference channel consists of the following components.

- **Optimization Variables:** The optimization variables include the set of precoders $\{\{\mathbf{v}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K$ and the set of decorrelators $\{\{\mathbf{u}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K$. These variables are adaptive with respect to imperfect CSIT $\hat{\mathcal{H}} = \{\widehat{\mathbf{H}}^{(k,j)}\}_{j,k=1}^K$.
- **Optimization Objective**: The optimization objective is to maximize, with imperfect CSIT, the minimum worst-case SINR among¹ all users' data streams (perceived by the transmitter) given by (cf. (7) and Assumption 3)

$$\min_{\substack{\forall k \in \mathcal{K} \\ \forall l \in \mathcal{L}_k}} \min_{\|\boldsymbol{\Delta}^{(k,j)}\|^2 \le \varepsilon} \gamma_l^{(k)} (\mathcal{H}, \{\{\mathbf{v}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K, \mathbf{u}_l^{(k)}).$$
(10)

• **Optimization Constraints**: The optimization constraints are the maximum transmit power for each source node

 P_1, \ldots, P_K , which give the precoder power constraints $\sum_{l=1}^{L_k} (\mathbf{v}_l^{(k)})^{\dagger} \mathbf{v}_l^{(k)} \leq P_k, \forall k \in \mathcal{K} \text{ (cf. Assumption 1).}$

Accordingly, the optimization problem can be formally written as Problem 1.

Problem 1: (Robust Max-Min Fair Precoder-Decorrelator Design):

$$\{\{\{(\mathbf{v}_{m}^{(j)})^{*}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \{\{(\mathbf{u}_{m}^{(j)})^{*}\}_{m=1}^{L_{j}}\}_{j=1}^{K}\} = \mathcal{P}(P_{1}, \dots, P_{K})$$

$$\underset{\mathbf{v}_{m}^{(j)} \in \mathbb{C}^{M \times 1}}{\operatorname{sym}} \underset{\forall l \in \mathcal{L}_{k}}{\min} \underset{\forall l \in \mathcal{L}_{k}}{\min} ||\mathbf{\Delta}^{(k,j)}||^{2} \leq \varepsilon}{\operatorname{sym}} \gamma_{l}^{(k)} (\mathcal{H}, \{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \mathbf{u}_{l}^{(k)}) (11a)$$

 $\sum_{l=1}^{L_k} (\mathbf{v}_l^{(k)})^{\dagger} \mathbf{v}_l^{(k)} \leq P_k, \forall k \in \mathcal{K}.$ (11b)

In (11a), the worst-case SINR with imperfect CSIT is given by the following proposition.

s. t.

Proposition 1 (Worst-Case SINR with Imperfect CSIT): Given CSI estimates $\widehat{\mathcal{H}} = {\{\widehat{\mathbf{H}}^{(k,j)}\}_{j,k=1}^{K}}$ at the transmitter with error $||\Delta^{(k,j)}||^2 \leq \varepsilon$, the worst-case SINR of data stream estimate $\widetilde{s}_l^{(k)}$ perceived by the transmitter can be expressed as (12).

Proof: Please refer to Appendix A. Using Proposition 1 and let $\tilde{P} = \min(P_1, \ldots, P_K)$ and $\rho_k = P_k/\tilde{P}$, we can recast Problem \mathcal{P} as

$$\{\gamma^{\star}, \{\{(\mathbf{v}_{m}^{(j)})^{\star}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \{\{(\mathbf{u}_{m}^{(j)})^{\star}\}_{m=1}^{L_{j}}\}_{j=1}^{K}\} = \mathcal{P}(\widetilde{P})$$

$$\min_{\mathbf{v}_{m}^{(j)} \in \mathbb{C}^{N \times 1}} -\gamma$$

$$u_{m}^{(j)} \in \mathbb{C}^{N \times 1}$$

$$\gamma \in \mathbb{R}_{+}$$

$$\widetilde{C}^{(k)} \in \widehat{\Omega} \in \mathcal{G}_{+} (j) \setminus L_{j} \to K$$

$$(k) = \sum_{m=1}^{N} \sum_{m=1}^{N} \sum_{j=1}^{N} \{(\mathbf{u}_{m}^{(j)})^{\star}\}_{m=1}^{L_{j}} \} = \mathcal{P}(\widetilde{P})$$

$$(13a)$$

s. t.
$$\widetilde{\gamma}_{l}^{(k)}(\widehat{\mathcal{H}}, \{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \mathbf{u}_{l}^{(k)}) \geq \gamma, \forall l \in \mathcal{L}_{k}, \forall k \in \mathcal{K}, (13b)$$

$$\sum_{l=1}^{L_{k}} (\mathbf{v}_{l}^{(k)})^{\dagger} \mathbf{v}_{l}^{(k)} \leq \rho_{k} \widetilde{P}, \forall k \in \mathcal{K}.$$
(13c)

B. Properties of the Optimization Problem

Note that it is not trivial to solve Problem \mathcal{P} since it is non-convex and NP-hard in general as we elaborate below. In Section IV, we shall propose a low complexity iterative algorithm for solving Problem \mathcal{P} .

1) Problem \mathcal{P} is a non-convex problem: The minimum SINR constraints in (13b) can be rearranged as

$$\begin{aligned} (1+\gamma) ||(\mathbf{u}_{l}^{(k)})^{\dagger} \widehat{\mathbf{H}}^{(k,k)} \mathbf{v}_{l}^{(k)}||^{2} + (\gamma - 1)\varepsilon ||\mathbf{u}_{l}^{(k)}||^{2} ||\mathbf{v}_{l}^{(k)}||^{2} \\ -\gamma N_{0} ||\mathbf{u}_{l}^{(k)}||^{2} - \gamma \sum_{j=1}^{K} \sum_{m=1}^{L_{j}} ||(\mathbf{u}_{l}^{(k)})^{\dagger} \widehat{\mathbf{H}}^{(k,j)} \mathbf{v}_{m}^{(j)}||^{2} \\ -\gamma \varepsilon ||\mathbf{u}_{l}^{(k)}||^{2} \sum_{j=1}^{K} \sum_{m=1}^{L_{j}} ||\mathbf{v}_{m}^{(j)}||^{2} \ge 0, \end{aligned}$$

which are non-convex inequalities consisting of non-positive linear combinations of norms. Therefore, Problem \mathcal{P} is a non-convex problem.

2) Problem \mathcal{P} is NP-hard in general: To illustrate that Problem \mathcal{P} is NP-hard in general, we consider the *inverse* problem of jointly minimizing the transmit powers of all source nodes subject to a minimum SINR constraint for all

¹Note that (10) is the worst-case SINR perceived by the transmitter based on imperfect CSIT $\hat{\mathcal{H}} = \{\widehat{\mathbf{H}}^{(k,j)}\}_{j,k=1}^{K}$. We choose the worst-case SINR perceived by the transmitter in order to incorporate robustness against CSI error $\mathbf{\Delta}^{(k,j)}$.

$$\begin{aligned} & \min_{\substack{||\boldsymbol{\Delta}^{(k,j)}||^{2} \leq \varepsilon}} \gamma_{l}^{(k)} (\mathcal{H}, \{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \mathbf{u}_{l}^{(k)}) \\ &= \frac{||(\mathbf{u}_{l}^{(k)})^{\dagger} \widehat{\mathbf{H}}^{(k,k)} \mathbf{v}_{l}^{(k)}||^{2} - \varepsilon ||\mathbf{u}_{l}^{(k)}||^{2} ||\mathbf{v}_{l}^{(k)}||^{2}}{\sum_{j=1}^{L_{j}} \sum_{m=1}^{L_{j}} ||(\mathbf{u}_{l}^{(k)})^{\dagger} \widehat{\mathbf{H}}^{(k,j)} \mathbf{v}_{m}^{(j)}||^{2} + \varepsilon ||\mathbf{u}_{l}^{(k)}||^{2} \sum_{j=1}^{K} \sum_{m=1}^{L_{j}} ||\mathbf{v}_{m}^{(j)}||^{2} - ||(\mathbf{u}_{l}^{(k)})^{\dagger} \widehat{\mathbf{H}}^{(k,k)} \mathbf{v}_{l}^{(k)}||^{2} - \varepsilon ||\mathbf{u}_{l}^{(k)}||^{2} + N_{0}||\mathbf{u}_{l}^{(k)}||^{2} + N_{0}||\mathbf{u}_{l}^{(k)}||^{2} \\ &\triangleq \widetilde{\gamma}_{l}^{(k)} (\widehat{\mathcal{H}}, \{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}, \mathbf{u}_{l}^{(k)}). \end{aligned} \tag{12}$$

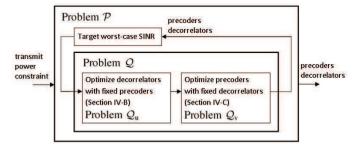


Fig. 2. Interrelationship among the optimization problems.

users' data streams². In Section IV, we shall propose an algorithm for solving Problem \mathcal{P} *facilitated* by solving the inverse problem³ that consists of the following components.

- **Optimization Variables**: The optimization variables include the set of precoders $\{\{\mathbf{v}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K$ and the set of decorrelators $\{\{\mathbf{u}_m^{(j)}\}_{m=1}^{L_j}\}_{i=1}^K$.
- **Optimization Objective**: The optimization objective is to minimize the required transmit power of all source nodes, by means of minimizing the precoder powers $\sum_{l=1}^{L_k} (\mathbf{v}_l^{(k)})^{\dagger} \mathbf{v}_l^{(k)}$, $\forall k \in \mathcal{K}$.
- **Optimization Constraints**: The optimization constraint is for all users' data streams to meet the prescribed minimum SINR γ , i.e. $\tilde{\gamma}_l^{(k)}(\hat{\mathcal{H}}, \{\{\mathbf{v}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K, \mathbf{u}_l^{(k)}) \geq \gamma$.

Accordingly, the inverse problem can be formally written as Problem 2.

Problem 2 (Power Minimization Precoder-Decorrelator Design):

$$\{\beta^{\star}, \{\{(\mathbf{v}_{m}^{(j)})^{\star}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \{\{(\mathbf{u}_{m}^{(j)})^{\star}\}_{m=1}^{L_{j}}\}_{j=1}^{K}\} = \mathcal{Q}(\gamma)$$

$$\min_{\substack{\mathbf{v}_{m}^{(j)} \in \mathbb{C}^{M \times 1} \\ \beta \in \mathbb{R}_{+}}} \beta$$
(14a)

s. t.
$$\sum_{l=1}^{L_k} (\mathbf{v}_l^{(k)})^{\dagger} \mathbf{v}_l^{(k)} \leq \rho_k \beta, \quad \forall k \in \mathcal{K},$$
(14b)
$$\widetilde{\gamma}_l^{(k)} (\widehat{\mathcal{H}}, \{\{\mathbf{v}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K, \mathbf{u}_l^{(k)}) \geq \gamma, \forall l \in \mathcal{L}_k, \forall k \in \mathcal{K}.$$
(14c)

Consider an instance of Problem Q with minimum SINR constraint $\tilde{\gamma}$, i.e.

$$\{\widetilde{\beta}, \{\{\widetilde{\mathbf{v}}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \{\{\widetilde{\mathbf{u}}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}\} = \mathcal{Q}(\widetilde{\gamma}), \quad (15)$$

and the required transmit power of the k^{th} source node is $\rho_k \tilde{\beta}$. It can be shown that

$$\{\widetilde{\gamma}, \{\{\widetilde{\mathbf{v}}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \{\{\widetilde{\mathbf{u}}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}\} = \mathcal{P}(\widetilde{\beta})$$
(16)

³The inverse problem will be utilized in Section IV-C.

so we can solve Problem Q to obtain a corresponding solution for Problem \mathcal{P} , and vice-versa. Since Problem Q is NP-hard in general, Problem \mathcal{P} is also NP-hard. Specifically, we define the *special case* of Problem Q with *fixed* decorrelators as

Problem 3 (Power Minimization Precoder Design with Fixed Decorrelators):

$$\{\xi^{\star}, \{\{(\mathbf{v}_{m}^{(j)})^{\star}\}_{m=1}^{L_{j}}\}_{j=1}^{K}\} = \mathcal{Q}_{\mathsf{v}}(\gamma, \{\{\mathbf{u}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K})$$

$$\min_{\substack{\mathbf{v}_{m}^{(j)} \in \mathbb{C}^{M \times 1}\\ \xi \in \mathbb{R}_{+}}} \xi$$
(17a)

s. t.
$$\sum_{l=1}^{L_k} (\mathbf{v}_l^{(k)})^{\dagger} \mathbf{v}_l^{(k)} \leq \rho_k \xi, \forall k \in \mathcal{K},$$
(17b)
$$\widetilde{\gamma}_l^{(k)} (\widehat{\mathcal{H}}, \{\{\mathbf{v}_m^{(j)}\}_{m=1}^{L_j}\}_{l=1}^K, \mathbf{u}_l^{(k)}) \geq \gamma, \forall l \in \mathcal{L}_k, \forall k \in \mathcal{K}.$$
(17c)

Note that Problem Q_v belongs to the class of separable homogenous QCQP, which is NP-hard in general [14], [15]. This implies that Problem Q, which contains Problem Q_v as special case, is also NP-hard in general⁴.

IV. LOW COMPLEXITY ITERATIVE SOLUTION

In this section, we propose a low complexity iterative algorithm for solving the robust and fair transceiver optimization problem \mathcal{P} . In particular, the proposed algorithm is facilitated by solving the inverse Problem \mathcal{Q} , whereby we exploit the structure of Problem \mathcal{Q} to apply effective optimization techniques.

A. Overview of Algorithm

The proposed algorithm for solving Problem \mathcal{P} is facilitated by solving Problem \mathcal{Q} as illustrated in Fig. 2, which is also detailed in Algorithm 1. Specifically, we iteratively refine the decorrelators and precoders to monotonically improve the minimum SINR. Each iteration consists of two stages:

• (Steps 1-3 of Algorithm 1) First, given the *status quo* minimum SINR $\tilde{\gamma}$ achieved with the k^{th} source node transmitting at power P_k , we solve Problem Q to optimize the precoders and decorrelators for minimizing the transmit powers, i.e.

$$\{\widetilde{\beta}, \{\{\widetilde{\mathbf{v}}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \{\{\widetilde{\mathbf{u}}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}\} = \mathcal{Q}(\widetilde{\gamma}), \quad (18)$$

such that the minimum SINR $\tilde{\gamma}$ is achieved with the k^{th} source node transmitting at a *reduced* power of $\sum_{l=1}^{L_k} (\widetilde{\mathbf{v}}_l^{(k)})^{\dagger} \widetilde{\mathbf{v}}_l^{(k)} = \rho_k \tilde{\beta} \leq P_k.$

• (Steps 4-5 of Algorithm 1) Second, we improve the minimum SINR by up-scaling the transmit precoding power⁵ of the k^{th} user to the power constraint P_k , i.e. $\widetilde{\mathbf{v}}_m^{(j)} = \sqrt{\frac{P_j}{(\rho_j \widetilde{\beta})}} \widetilde{\mathbf{v}}_m^{(j)}$.

⁴Problem Q_v will be utilized in Section IV-C.

 5 We show in (31) that up-scaling the precoding powers improve the minimum SINR.

²Please refer to [16]–[18], [26] and references therein for discussions on the inverse relationship between max-min fair and minimum power precoder design problems for MISO *broadcast* and *multicast* channels.

We repeat the iteration until the minimum SINR converges to a maximum. However, it is not trivial to solve the iteration step as per (18) since Problem Q is NP-hard in general as shown in Section III-B2. As such, we shall solve Problem Qbased on alternative optimization between the decorrelators and the precoders, i.e. we present the algorithm for optimizing the decorrelators with *fixed* precoders in Section IV-B, and introduce the algorithm for optimizing the precoders with *fixed* decorrelators in Section IV-C. The top-level detail steps of the optimization algorithm is summarized below (Algorithm 1) and illustrated in Fig 3. The convergence proof for Algorithm 1 is provided in Appendix D.

Algorithm 1 (Top-Level Algorithm):

Inputs: maximum transmit power for each source node P_1, \ldots, P_K

Outputs: precoders $\{\{(\mathbf{v}_{m}^{(j)})^{*}\}_{m=1}^{L_{j}}\}_{j=1}^{K}$ and decorrelators $\{\{(\mathbf{u}_{m}^{(j)})^{*}\}_{m=1}^{L_{j}}\}_{j=1}^{K}$

• Step 0: Initialize decorrelators $\{\{\widetilde{\mathbf{u}}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K$ and precoders $\{\{\widetilde{\mathbf{v}}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K$, where the transmit power for the j^{th} source node is $\sum_{m=1}^{L_j} (\widetilde{\mathbf{v}}_m^{(j)})^{\dagger} \widetilde{\mathbf{v}}_m^{(j)} = P_j$.

Repeat

• **Step 1**: Optimize the decorrelators with fixed precoders (cf. Section IV-B)

$$\{\{\widetilde{\mathbf{u}}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K} = \mathcal{Q}_{u}(\{\{\widetilde{\mathbf{v}}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K})$$

Update the candidate decorrelators $(\mathbf{u}_m^{(j)})^{\star} = \widetilde{\mathbf{u}}_m^{(j)}$.

• **Step 2**: Evaluate the minimum SINR

$$\min_{\substack{\forall k \in \mathcal{K} \\ l \in \mathcal{L}_k}} \widetilde{\gamma}_l^{(k)} (\widehat{\mathcal{H}}, \{\{\widetilde{\mathbf{v}}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K, \widetilde{\mathbf{u}}_l^{(k)}) = \widehat{\gamma}.$$

Update the target SINR $\tilde{\gamma} = \hat{\gamma}$.

• **Step 3**: Optimize the precoders with fixed decorrelators (cf. Section IV-C)

$$\{ \{ \widetilde{\mathbf{v}}_{m}^{(j)} \}_{m=1}^{L_{j}} \}_{j=1}^{K} \} = \mathcal{Q}_{\mathbf{v}}(\widetilde{\gamma}, \{ \{ \widetilde{\mathbf{u}}_{m}^{(j)} \}_{m=1}^{L_{j}} \}_{j=1}^{K}).$$

- Step 4: Evaluate the required transmit power of each source node $\rho_j \tilde{\beta} = \sum_{m=1}^{L_j} (\tilde{\mathbf{v}}_m^{(j)})^{\dagger} \tilde{\mathbf{v}}_m^{(j)}$.
- Step 5: Evaluate the minimum SINR with up-scaled precoders

$$\min_{\substack{\forall k \in \mathcal{K} \\ \forall l \in \mathcal{L}_k}} \widetilde{\gamma}_l^{(k)}(\widehat{\mathcal{H}}, \{\{\sqrt{P_j/(\rho_j \widetilde{\beta})}) \widetilde{\mathbf{v}}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K, \widetilde{\mathbf{u}}_l^{(k)}) = \widehat{\gamma}.$$

Update the target SINR $\tilde{\gamma} = \hat{\gamma}$ and candidate precoders $(\mathbf{v}_m^{(j)})^{\star} = \sqrt{P_j/(\rho_j \tilde{\beta})} \tilde{\mathbf{v}}_m^{(j)}$.

Until the minimum SINR $\tilde{\gamma}$ converges. Return precoders $\{\{(\mathbf{v}_m^{(j)})^{\star}\}_{m=1}^{L_j}\}_{j=1}^K$ and decorrelators $\{\{(\mathbf{u}_m^{(j)})^{\star}\}_{m=1}^{L_j}\}_{j=1}^K$.

B. Decorrelator Optimization with Fixed Precoders

We define the decorrelator optimization problem with fixed precoders to maximize the minimum SINR among all users' data streams as

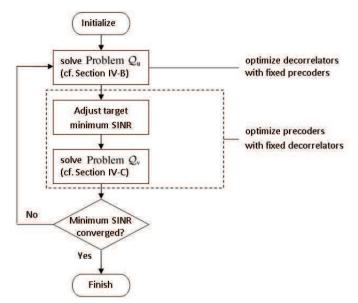


Fig. 3. Illustration of overall algorithm.

Problem 4 (Maximum SINR Decorrelator Design with Fixed Precoders):

$$\{\{(\mathbf{u}_{m}^{(j)})^{\star}\}_{m=1}^{L_{j}}\}_{j=1}^{K} = \mathcal{Q}_{\mathbf{u}}(\{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{m=1}^{K})$$

$$\underset{\mathbf{u}_{m}^{(j)} \in \mathbb{C}^{N \times 1}}{\operatorname{sym}} \underset{\forall k \in \mathcal{K}}{\forall k \in \mathcal{K}} \widetilde{\gamma}_{l}^{(k)}(\widehat{\mathcal{H}}, \{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \mathbf{u}_{l}^{(k)}).$$

$$(19)$$

As per (19), the worst-case SINR of data stream estimate $\tilde{s}_l^{(k)}$ only depends on decorrelator $\mathbf{u}_l^{(k)}$. Therefore, we can independently optimize each decorrelator, i.e.

$$\begin{aligned} & (\mathbf{u}_{l}^{(k)})^{\star} = \mathcal{Q}_{\mathbf{u}}^{(k,l)}(\{\{\mathbf{v}_{m}^{(j)}\}_{l=1}^{L_{j}}\}_{j=1}^{K}) \\ & \underset{\mathbf{u}_{l}^{(k)} \in \mathbb{C}^{N \times 1}}{\operatorname{smax}} \quad \widetilde{\gamma}_{l}^{(k)}(\widehat{\mathcal{H}},\{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K},\mathbf{u}_{l}^{(k)}), \end{aligned}$$
(20)

and the optimal decorrelator is given by Theorem 1.

$$\begin{split} & \text{Theorem 1 (Optimal Decorrelator with Fixed Precoders):} \\ & \text{Given precoders } \{\{\mathbf{v}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K, \text{ the optimal decorrelator for data stream estimate } \widetilde{s}_l^{(k)} \text{ is given by } \\ & (\mathbf{u}_l^{(k)})^\star = \frac{(\mathbf{F}_l^{(k)})^{-\frac{1}{2}}(\mathbf{w}_l^{(k)})^\star}{||(\mathbf{F}_l^{(k)})^{-\frac{1}{2}}(\mathbf{w}_l^{(k)})^\star||}, \text{ where} \\ & \mathbf{F}_l^{(k)} = \sum_{j=1}^K \sum_{m=1}^{L_j} \widehat{\mathbf{H}}^{(k,j)} \mathbf{v}_m^{(j)} (\mathbf{v}_m^{(j)})^\dagger (\widehat{\mathbf{H}}^{(k,j)})^\dagger \\ & + \varepsilon \sum_{j=1}^K \sum_{m=1}^{L_j} ||\mathbf{v}_m^{(j)}||^2 \mathbf{I}_N - \widehat{\mathbf{H}}^{(k,k)} \mathbf{v}_l^{(k)} (\mathbf{v}_l^{(k)})^\dagger (\widehat{\mathbf{H}}^{(k,k)})^\dagger \\ & - \varepsilon ||\mathbf{v}_l^{(k)}||^2 \mathbf{I}_N + N_0 \mathbf{I}_N, \end{aligned}$$

and
$$\mathbf{E}_{l}^{(k)} = \widehat{\mathbf{H}}^{(k,k)} \mathbf{v}_{l}^{(k)} (\mathbf{v}_{l}^{(k)})^{\dagger} (\widehat{\mathbf{H}}^{(k,k)})^{\dagger} - \varepsilon ||\mathbf{v}_{l}^{(k)}||^{2} \mathbf{I}_{N}.$$

Proof: Please refer to Appendix B.

C. Precoder Optimization with Fixed Decorrelators

In Section III-B2, we defined the precoder optimization problem with fixed decorrelators, Problem Q_v (cf. (17a)– (17c)). Since Problem Q_v belongs to the class of separable homogenous QCQP, it is NP-hard in general. In the literature, some authors consider instances of this class of problems for MISO *broadcast* channel that are always solvable (cf. [17], [26] and references therein), whereas some authors consider problems for MISO multicast channel that are always NPhard (cf. [18] and references therein). For the interference channel model considered herein, we provide an algorithm for obtaining the optimal solution for Problem $Q_{\rm v}$.

One effective approach for solving separable homogenous QCQP is to apply semidefinite relaxation (SDR) techniques. Let $\mathbf{V}_l^{(k)} = \mathbf{v}_l^{(k)} (\mathbf{v}_l^{(k)})^{\dagger}$. From (12), the worst-case SINR of data stream estimate $s_1^{(k)}$ can be expressed as (21). It follows that we can *equivalently* express the precoder optimization problem with fixed decorrelators as

$$\{ \Xi^{\star}, \{ \{ (\mathbf{V}_{m}^{(j)})^{\star} \}_{m=1}^{L_{j}} \}_{j=1}^{K} \} = \widetilde{\mathcal{Q}}_{\mathbf{v}}(\gamma, \{ \{ \mathbf{u}_{m}^{(j)} \}_{m=1}^{L_{j}} \}_{j=1}^{K})$$

$$\min_{\Xi \in \mathbb{R}_{+}} \Xi$$
(22a)

s. t.
$$\sum_{l=1}^{L_k} \operatorname{Tr}(\mathbf{V}_l^{(k)}) \le \rho_k \Xi, \forall k \in \mathcal{K},$$
(22b)
$$\widetilde{r}^{(k)}(\widehat{q}) \in \operatorname{Cr}^{(j)}(L_k) \times K \quad (k) \ge 1 \quad \text{(22b)}$$

$$\Gamma_{l}^{(k)}(\mathcal{H},\{\{\mathbf{V}_{m}^{(j)}\}_{m=1}^{\mathcal{L}_{j}}\}_{j=1}^{\mathcal{K}},\mathbf{u}_{l}^{(k)}) \geq \gamma, \forall l \in \mathcal{L}_{k}, \forall k \in \mathcal{K}(22c)$$

$$\mathbf{W}_{k}^{(k)} = 0 \quad \forall l \in \mathcal{L} \setminus \mathcal{U}_{k} = \mathcal{L}$$

(22d) $\mathbf{V}_{l}^{(n)} \succeq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \succeq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \geq 0, \forall k \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \in \mathcal{L}_{k}, \\ (\mathbf{v}_{l}^{(k)}) \in \mathcal{L}_{k}, \\$

$$\operatorname{rank}(\mathbf{V}_{l}^{(k)}) = 1, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k},$$
(22e)

where (22d) and (22e) follow from the definition of $\mathbf{V}_{l}^{(k)}$, (22b) are power constraints, and (22c) are SINR constraints. Note that we could obtain the optimal precoder $(\mathbf{v}_m^{(j)})^*$ from the eigenvector of $(\mathbf{V}_m^{(j)})^*$ corresponding to the only non-zero eigenvalue.

Comparing between Problem $\tilde{\mathcal{Q}}_{v}$ and Problem \mathcal{Q}_{v} , the SINR constraints of Problem Q_v (22c) are convex inequalities, i.e.

$$(1+\gamma) \operatorname{Tr}((\widehat{\mathbf{H}}^{(k,k)})^{\dagger} \mathbf{u}_{l}^{(k)}(\mathbf{u}_{l}^{(k)})^{\dagger} \widehat{\mathbf{H}}^{(k,k)} \mathbf{V}_{l}^{(k)}) - \gamma N_{0} ||\mathbf{u}_{l}^{(k)}||^{2} + (\gamma-1)\varepsilon ||\mathbf{u}_{l}^{(k)}||^{2} \operatorname{Tr}(\mathbf{V}_{l}^{(k)}) - \gamma \varepsilon ||\mathbf{u}_{l}^{(k)}||^{2} \sum_{j=1}^{K} \sum_{m=1}^{L_{j}} \operatorname{Tr}(\mathbf{V}_{m}^{(j)}) - \gamma \sum_{j=1}^{K} \sum_{m=1}^{L_{j}} \operatorname{Tr}((\widehat{\mathbf{H}}^{(k,j)})^{\dagger} \mathbf{u}_{l}^{(k)}(\mathbf{u}_{l}^{(k)})^{\dagger} \widehat{\mathbf{H}}^{(k,j)} \mathbf{V}_{m}^{(j)}) \ge 0,$$

but Problem $\hat{\mathcal{Q}}_{v}$ is still a non-convex problem due to the rank constraints (22e). By means of SDR, we neglect the rank constraints and Problem Q_v degenerates into an SDP that can be solved efficiently [27]. In general, the resultant solution $\{\Xi^*, \{\{(\mathbf{V}_m^{(j)})^*\}_{m=1}^{L_j}\}_{j=1}^K\}$ could have arbitrary rank. If $\operatorname{rank}((\mathbf{V}_m^{(j)})^*) = 1, \forall m \in \mathcal{L}_j \text{ and } \forall j \in \mathcal{K}$, then constraints (22e) are intrinsically satisfied and $\{\{(\mathbf{V}_m^{(j)})^{\star}\}_{m=1}^{L_j}\}_{j=1}^K$ are optimal. The following theorem summarizes the optimality of the SDR solution in (22a)–(22e).

Theorem 2 (Optimality of the SDR Solution): The SDR solution of Problem \hat{Q}_v will always give rank 1 solutions (i.e. $\operatorname{rank}((\mathbf{V}_m^{(j)})^*) = 1)$ and hence, the SDR solution is optimal for \mathcal{Q}_{v} .

Proof: Please refer to Appendix C.

V. SIMULATION RESULTS AND DISCUSSIONS

In this section, we evaluate the proposed robust transceiver design via numerical simulations. In particular, we compare the performance of the proposed scheme against four baseline schemes:

- **Baseline 1** the conventional IA scheme [4];
- **Baseline 2** the SINR maximization scheme [6, Algorithm 2];

Baseline 4 a naive max-min SINR scheme adopted from [26].

As discussed in Section II-B, baselines 1 and 2 are theoretically promising schemes for the interference channel but neglect important practical issues such as CSI uncertainty and fairness among users. On the other hand, baselines 3 and 4 are adopted from existing max-min SINR schemes that are originally designed for the broadcast channel (i.e. there is only a single transmitter and multiple receivers). Without loss of generality, we assume independent and identically distributed (iid) Rayleigh fading channels, i.e. $[\mathbf{H}^{(k,j)}]_{(a,b)} \sim \mathcal{CN}(0,1)$, $\forall j,k \in \mathcal{K}, \forall a \in [1,N], \text{ and } \forall b \in [1,M].$ For the purpose of illustration, we consider the scenario where all users have the same power constraint $P_1 = \ldots = P_K = P$. In Fig. 4 to Fig. 6 we present simulation results for the average data rates⁶ versus SNR⁷ with different number of users and levels of CSI uncertainty.

A. Fairness Performance

In Fig. 4 and Fig. 5, we compare the average data rates of the proposed and baseline schemes. For the purpose of illustration, we consider the three-user 4×4 MIMO interference channel, where each user transmits L = 2 data streams and the precoders are designed with imperfect CSIT with $\varepsilon = \{0.1, 0.15\},\$ whereas the receivers have perfect CSIR. It can be observed that the proposed scheme achieves much higher average worstcase data rate per user than all the baseline schemes, and thus provides better minimum performance. For example, at CSI error $\varepsilon = 0.15$, the proposed scheme has 5dB SNR gain over the SINR maximization algorithm (baseline 2) at providing a worst case data rate of 6 b/s/Hz and the conventional IA scheme (baseline 1) cannot provide worst-case data rate of 6 b/s/Hz. The superior performances of the proposed scheme is accountable to both the SDR approach as well as a suitably chosen utility function (optimizing the worst case performance). Specifically, the chosen utility function 1) provide resilience against CSI uncertainties as well as 2) achieve fair performance among users. On the other hand, the SDR approach also contributes to obtaining a good solution for solving the optimization problem.

B. Total Sum Data Rate Performance

In Fig. 5, we compare the average total sum data rates of the proposed and baseline schemes for K = 3, N = M = 4, L = 2, and CSI error $\varepsilon = \{0.1, 0.15\}$. It can be observed that the proposed scheme not only achieves better worst-case data rate but also achieves higher total sum data rate than all the baseline schemes. In particular, due to the presence of

 $^{6}\mbox{The}$ average data rate is defined as the average goodput (i.e. the bits/s/Hz successfully delivered to the receiver). Specifically, the goodput of data stream $s_l^{(k)}$ is given by $r_l^{(k)}\mathcal{I}(r_l^{(k)} \leq C_l^{(k)})$, where $r_l^{(k)} = \log_2(1 + \gamma_l^{(k)}(\hat{\mathcal{H}}, \{\{\mathbf{v}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^{K}, \mathbf{u}_l^{(k)}))$ is the scheduled data rate based on the SINR perceived with respect to imperfect CSIT $\hat{\mathcal{H}} = \{\widehat{\mathbf{H}}^{(k,j)}\}_{j,k=1}^{K}$ and $C_l^{(k)} = \log_2(1 + \gamma_l^{(k)}(\mathcal{H}, \{\{\mathbf{v}_m^{(j)}\}_{m=1}^{L_j}\}_{j=1}^K, \mathbf{u}_l^{(k)}))$ is the actual instantaneous mutual information. ⁷The SNR is defined as $\frac{P}{N_0}$, where N_0 is the AWGN variance.

$$\widetilde{\gamma}_{l}^{(k)}(\widehat{\mathcal{H}}, \{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \mathbf{u}_{l}^{(k)}) = \frac{\operatorname{Tr}((\widehat{\mathbf{h}}^{(k,k)})^{\dagger}\mathbf{u}_{l}^{(k)}(\mathbf{u}_{l}^{(k)})^{\dagger}\widehat{\mathbf{h}}^{(k,k)}\mathbf{V}_{l}^{(k)}) - \varepsilon ||\mathbf{u}_{l}^{(k)}||^{2}\operatorname{Tr}(\mathbf{V}_{l}^{(k)})}{\left(\sum_{j=1}^{L_{j}}\sum_{m=1}^{L_{j}}\operatorname{Tr}((\widehat{\mathbf{h}}^{(k,j)})^{\dagger}\mathbf{u}_{l}^{(k)}(\mathbf{u}_{l}^{(k)})^{\dagger}\widehat{\mathbf{h}}^{(k,j)}\mathbf{V}_{m}^{(j)}) + \varepsilon ||\mathbf{u}_{l}^{(k)}||^{2}\sum_{j=1}^{L_{j}}\sum_{m=1}^{L_{j}}\operatorname{Tr}(\mathbf{V}_{m}^{(j)})}{-\operatorname{Tr}((\widehat{\mathbf{h}}^{(k,k)})^{\dagger}\mathbf{u}_{l}^{(k)}(\mathbf{u}_{l}^{(k)})^{\dagger}\widehat{\mathbf{h}}^{(k,k)}\mathbf{V}_{l}^{(k)}) - \varepsilon ||\mathbf{u}_{l}^{(k)}||^{2}\operatorname{Tr}(\mathbf{V}_{l}^{(k)}) + N_{0}||\mathbf{u}_{l}^{(k)}||^{2}}) \\ \triangleq \widetilde{\Gamma}_{l}^{(k)}(\widehat{\mathcal{H}}, \{\{\mathbf{V}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{L_{j}}, \mathbf{u}_{l}^{(k)}).$$

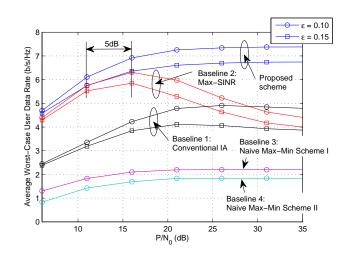


Fig. 4. Average data rate of the worst-case user versus SNR. K = 3, N = M = 4, L = 2 and CSI error $\varepsilon = \{0.1, 0.15\}$.

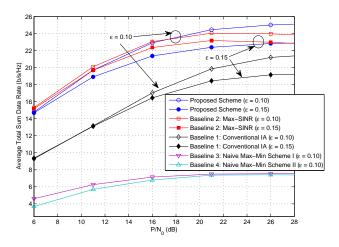


Fig. 5. Average total sum data rate versus SNR. K = 3, N = M = 4, L = 2 and CSI error $\varepsilon = \{0.1, 0.15\}$.

CSI error, the total sum rate of the conventional IA scheme (baseline 1) does not scale linearly with the SNR anymore. Comparing Fig. 5 with Fig. 4, it can be observed that the proposed scheme achieves the performance gain on fairness without sacrificing the total sum data rate.

C. Robustness to CSI Errors

In Fig. 6, we show the average worst-case data rates of the proposed and baseline schemes for different levels of CSI uncertainty. It can be observed that the proposed scheme always achieves higher average worst-case data rate than the baseline schemes. For example, the SINR maximization

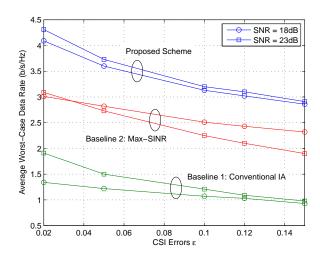


Fig. 6. Average worst-case data rate versus CSI errors. $K=3,\,N=M=4,\,L=2$ and SNR 18dB and 23dB.

algorithm (baseline 2) is designed assuming perfect CSI; its performance degrades rapidly for CSI error $\varepsilon > 0.02$ and it can be observed that the achieved data rate could decrease with increasing SNR. On the other hand, the proposed scheme achieves a robust degradation with respect to CSI errors.

VI. CONCLUSIONS

In this paper, we proposed a robust transceiver design for the *K*-pair quasi-static MIMO interference channel with fairness considerations. Specifically, we formulated the precoderdecorrelator design as an optimization problem to maximize the worst-case SINR among all users. We devised a low complexity iterative algorithm based on AO and SDR techniques. Numerical results verify the advantages of incorporating into transceiver design for the interference channel important practical issues such as CSI uncertainty and fairness performance.

APPENDIX A

PROOF: WORST-CASE SINR WITH IMPERFECT CSIT

Given CSI estimates $\hat{\mathcal{H}} = {\{\widehat{\mathbf{H}}^{(k,j)}\}_{j,k=1}^{K}}$ at the transmitter, the worst-case SINR for each data stream estimate can be expressed as follows. Consider $\tilde{s}_{l}^{(k)}$ whose SINR $\gamma_{l}^{(k)}(\mathcal{H}, {\{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \mathbf{u}_{l}^{(k)})}$ is given by (23a). First, by the triangle inequality

$$\begin{split} ||(\mathbf{u}_{l}^{(k)})^{\dagger}(\widehat{\mathbf{H}}^{(k,j)} + \mathbf{\Delta}^{(k,j)})\mathbf{v}_{m}^{(j)}||^{2} \\ &\geq ||(\mathbf{u}_{l}^{(k)})^{\dagger}\widehat{\mathbf{H}}^{(k,j)}\mathbf{v}_{m}^{(j)}||^{2} - ||(\mathbf{u}_{l}^{(k)})^{\dagger}\mathbf{\Delta}^{(k,j)}\mathbf{v}_{m}^{(j)}||^{2}, \\ ||(\mathbf{u}_{l}^{(k)})^{\dagger}(\widehat{\mathbf{H}}^{(k,j)} + \mathbf{\Delta}^{(k,j)})\mathbf{v}_{m}^{(j)}||^{2} \\ &\leq ||(\mathbf{u}_{l}^{(k)})^{\dagger}\widehat{\mathbf{H}}^{(k,j)}\mathbf{v}_{m}^{(j)}||^{2} + ||(\mathbf{u}_{l}^{(k)})^{\dagger}\mathbf{\Delta}^{(k,j)}\mathbf{v}_{m}^{(j)}||^{2}, \end{split}$$

$$\begin{split} \gamma_{l}^{(k)}(\mathcal{H}, \{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{L}, \mathbf{u}_{l}^{(k)}) \\ &= \frac{||(\mathbf{u}_{l}^{(k)})^{\dagger}(\widehat{\mathbf{H}}^{(k,k)} + \mathbf{\Delta}^{(k,k)})\mathbf{v}_{l}^{(k)}||^{2} + \sum_{j=1}^{K} \sum_{m=1}^{L_{j}} ||(\mathbf{u}_{l}^{(k)})^{\dagger}(\widehat{\mathbf{H}}^{(k,k)} + \mathbf{\Delta}^{(k,k)})\mathbf{v}_{m}^{(k)}||^{2} + \sum_{j\neq k}^{K} \sum_{m=1}^{L_{j}} ||(\mathbf{u}_{l}^{(k)})^{\dagger}(\widehat{\mathbf{H}}^{(k,k)} + \mathbf{\Delta}^{(k,k)})\mathbf{v}_{m}^{(k)}||^{2} + \sum_{j\neq k}^{K} \sum_{m=1}^{L_{j}} ||(\mathbf{u}_{l}^{(k)})^{\dagger}(\widehat{\mathbf{H}}^{(k,k)} + \mathbf{\Delta}^{(k,k)})\mathbf{v}_{m}^{(k)}||^{2} + \sum_{j\neq k}^{K} ||^{2} - ||(\mathbf{u}_{l}^{(k)})^{\dagger}\mathbf{\Delta}^{(k,k)}\mathbf{v}_{l}^{(k)}||^{2} + ||(\mathbf{u}_{l}^{(k)})^{\dagger}\mathbf{\Delta}^{(k,k)}\mathbf{v}_{l}^{(k)}||^{2} \\ &\geq \frac{||(\mathbf{u}_{l}^{(k)})^{\dagger}\widehat{\mathbf{H}}^{(k,j)}\mathbf{v}_{m}^{(j)}||^{2} + ||(\mathbf{u}_{l}^{(k)})^{\dagger}\mathbf{\Delta}^{(k,j)}\mathbf{v}_{m}^{(j)}||^{2} - ||(\mathbf{u}_{l}^{(k)})^{\dagger}\widehat{\mathbf{H}}^{(k,k)}\mathbf{v}_{l}^{(k)}||^{2} + ||(\mathbf{u}_{l}^{(k)})^{\dagger}\mathbf{\Delta}^{(k,k)}\mathbf{v}_{l}^{(k)}||^{2} - \varepsilon||\mathbf{u}_{l}^{(k)}||^{2}||^{2} + \varepsilon||\mathbf{u}_{l}^{(k)}||^{2} + \varepsilon||\mathbf{u}_{l}^$$

and so the SINR is lowered bounded as (23b). Second, with CSI error $||\mathbf{\Delta}^{(k,j)}||^2 \leq \varepsilon$,

$$\begin{split} &||(\mathbf{u}_{l}^{(k)})^{\dagger} \mathbf{\Delta}^{(k,j)} \mathbf{v}_{m}^{(j)}||^{2} \\ &= \mathrm{Tr}((\mathbf{u}_{l}^{(k)})^{\dagger} \mathbf{\Delta}^{(k,j)} \mathbf{v}_{m}^{(j)} (\mathbf{v}_{m}^{(j)})^{\dagger} (\mathbf{\Delta}^{(k,j)})^{\dagger} \mathbf{u}_{l}^{(k)}) \\ \stackrel{(a)}{\leq} \mathrm{Tr}(\mathbf{u}_{l}^{(k)} (\mathbf{u}_{l}^{(k)})^{\dagger}) \mathrm{Tr}(\mathbf{\Delta}^{(k,j)} \mathbf{v}_{m}^{(j)} (\mathbf{v}_{m}^{(j)})^{\dagger} (\mathbf{\Delta}^{(k,j)})^{\dagger}) \\ \stackrel{(b)}{\leq} \mathrm{Tr}(\mathbf{u}_{l}^{(k)} (\mathbf{u}_{l}^{(k)})^{\dagger}) \underbrace{\mathrm{Tr}((\mathbf{\Delta}^{(k,j)})^{\dagger} \mathbf{\Delta}^{(k,j)})}_{=||\mathbf{\Delta}^{(k,j)}||^{2}} \mathrm{Tr}(\mathbf{v}_{m}^{(j)} (\mathbf{v}_{m}^{(j)})^{\dagger}) \\ &= \varepsilon ||\mathbf{u}_{l}^{(k)}||^{2} ||\mathbf{v}_{m}^{(j)}||^{2}, \end{split}$$

where (a) and (b) follow from the properties that $\operatorname{Tr}(\mathbf{AB}) = \operatorname{Tr}(\mathbf{BA})$ for $\mathbf{A} \in \mathbb{C}^{M \times N}$ and $\mathbf{B} \in \mathbb{C}^{N \times M}$ and $\operatorname{Tr}(\mathbf{CD}) \leq$ $Tr(\mathbf{C})Tr(\mathbf{D})$ for positive semi-definite $\mathbf{C}, \mathbf{D} \in \mathbb{C}^{N \times N}$. Thus, the worst-case SINR perceived by the transmitter can be expressed as (23c).

APPENDIX B PROOF: OPTIMAL DECORRELATOR WITH FIXED PRECODERS

From (12), the worst-case SINR of data stream estimate $\tilde{s}_{l}^{(k)}$ can be expressed as

$$\widetilde{\gamma}_{l}^{(k)}(\widehat{\mathcal{H}}, \{\{\mathbf{v}_{m}^{(j)}\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \mathbf{u}_{l}^{(k)}) = \frac{(\mathbf{u}_{l}^{(k)})^{\dagger} \mathbf{E}_{l}^{(k)} \mathbf{u}_{l}^{(k)}}{(\mathbf{u}_{l}^{(k)})^{\dagger} \mathbf{F}_{l}^{(k)} \mathbf{u}_{l}^{(k)}}, \qquad (24)$$

where

$$\begin{split} \mathbf{F}_{l}^{(k)} &= \sum_{j=1}^{K} \sum_{m=1}^{L_{j}} \widehat{\mathbf{H}}^{(k,j)} \mathbf{v}_{m}^{(j)} (\mathbf{v}_{m}^{(j)})^{\dagger} (\widehat{\mathbf{H}}^{(k,j)})^{\dagger} \\ &+ \varepsilon \sum_{j=1}^{K} \sum_{m=1}^{L_{j}} ||\mathbf{v}_{m}^{(j)}||^{2} \mathbf{I}_{N} - \widehat{\mathbf{H}}^{(k,k)} \mathbf{v}_{l}^{(k)} (\mathbf{v}_{l}^{(k)})^{\dagger} (\widehat{\mathbf{H}}^{(k,k)})^{\dagger} \\ &- \varepsilon ||\mathbf{v}_{l}^{(k)}||^{2} \mathbf{I}_{N} + N_{0} \mathbf{I}_{N}, \end{split}$$

which is a Hermitian and positive definite matrix, and

$$\mathbf{E}_{l}^{(k)} = \widehat{\mathbf{H}}^{(k,k)} \mathbf{v}_{l}^{(k)} (\mathbf{v}_{l}^{(k)})^{\dagger} (\widehat{\mathbf{H}}^{(k,k)})^{\dagger} - \varepsilon ||\mathbf{v}_{l}^{(k)}||^{2} \mathbf{I}_{N}$$

which is a non-negative definite⁸ Hermitian matrix. Without loss of generality, let $\mathbf{u}_l^{(k)} = c(\mathbf{F}_l^{(k)})^{-\frac{1}{2}}\mathbf{w}_l^{(k)}$ for arbitrary scaling factor $c \in \mathbb{C}$. We can equivalently expressed (24) as

$$\frac{(\mathbf{u}_{l}^{(k)})^{\dagger} \mathbf{E}_{l}^{(k)} \mathbf{u}_{l}^{(k)}}{(\mathbf{u}_{l}^{(k)})^{\dagger} \mathbf{F}_{l}^{(k)} \mathbf{u}_{l}^{(k)}} = \frac{(\mathbf{w}_{l}^{(k)})^{\dagger} (\mathbf{F}_{l}^{(k)})^{-\frac{1}{2}} \mathbf{E}_{l}^{(k)} (\mathbf{F}_{l}^{(k)})^{-\frac{1}{2}} \mathbf{w}_{l}^{(k)}}{(\mathbf{w}_{l}^{(k)})^{\dagger} \mathbf{w}_{l}^{(k)}} \qquad (25)$$
$$= \frac{(\mathbf{w}_{l}^{(k)})^{\dagger} \mathbf{Q} \mathbf{A} \mathbf{Q}^{\dagger} \mathbf{w}_{l}^{(k)}}{(\mathbf{w}_{l}^{(k)})^{\dagger} \mathbf{w}_{l}^{(k)}},$$

⁸If $\mathbf{E}_l^{(k)}$ is negative definite, then the CSI error ε is too high. Without loss of generality, we assume ε is sufficiently small.

denotes the eigen-decomposition where $\mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^{\dagger}$ of ($\mathbf{F}_{l}^{(k)}$)^{- $\frac{1}{2}$} $\mathbf{E}_{l}^{(k)}$ ($\mathbf{F}_{l}^{(k)}$)^{- $\frac{1}{2}$}. It can be shown that⁹ (25) is maximized with ($\mathbf{w}_{l}^{(k)}$)^{*} being the principal eigenvector¹⁰ of ($\mathbf{F}_{l}^{(k)}$)^{- $\frac{1}{2}$} $\mathbf{E}_{l}^{(k)}$ ($\mathbf{F}_{l}^{(k)}$)^{- $\frac{1}{2}$}. In turn, the optimal unit norm decorrelator is given by $(\mathbf{u}_l^{(k)})^* = \frac{(\mathbf{F}_l^{(k)})^{-\frac{1}{2}}(\mathbf{w}_l^{(k)})^*}{||(\mathbf{F}_l^{(k)})^{-\frac{1}{2}}(\mathbf{w}_l^{(k)})^*||}$

APPENDIX C PROOF: OPTIMALITY OF THE SDR SOLUTION FOR Problem Q_v

By using SDR, we solve the following SDP problem with complex-valued parameters:

$$\min_{\mathbf{V}_{(i)}^{(j)},\Xi} \Xi$$
(26a)

s. t.
$$\sum_{m=1}^{L_j} \operatorname{Tr}(\mathbf{V}_m^{(j)}) \le \rho_j \Xi, \forall j \in \mathcal{K}$$
$$\sum_{j=1}^{K} \sum_{m=1}^{L_j} \operatorname{Tr}(\mathbf{A}_{(l,m)}^{(k,j)} \mathbf{V}_m^{(j)}) \ge b_l^{(k)}, \forall l \in \mathcal{L}_k, \forall k \in \mathcal{K}_k (26b)$$
$$\Xi \ge 0, \tag{26c}$$

$$\mathbf{V}_{m}^{(j)} \succeq 0, \forall m \in \mathcal{L}_{j}, \forall j \in \mathcal{K},$$
(26d)

where $\mathbf{A}_{(l,m)}^{(k,j)} \in \mathbb{H}^M$ is given by

$$\mathbf{A}_{(l,m)}^{(k,j)} = \begin{cases} (\widehat{\mathbf{H}}^{(k,k)})^{\dagger} \mathbf{u}_{l}^{(k)} (\mathbf{u}_{l}^{(k)})^{\dagger} \widehat{\mathbf{H}}^{(k,k)} - \varepsilon ||\mathbf{u}_{l}^{(k)}||^{2} \mathbf{I} & \text{if } j = k \\ \text{and } m = l \\ -\gamma ((\widehat{\mathbf{H}}^{(k,j)})^{\dagger} \mathbf{u}_{l}^{(k)} (\mathbf{u}_{l}^{(k)})^{\dagger} \widehat{\mathbf{H}}^{(k,j)} + \varepsilon ||\mathbf{u}_{l}^{(k)}||^{2} \mathbf{I}) & \text{otherwise} \end{cases}$$

and $b_l^{(k)} = \gamma N_0 ||\mathbf{u}_l^{(k)}||^2 > 0$. The corresponding dual problem is given by the following SDP:

$$\max_{\substack{y_l^{(k)}\\x^{(j)}}} \sum_{k=1}^{K} \sum_{l=1}^{L_k} y_l^{(k)} b_l^{(k)}$$
(27a)

s. t.
$$\underbrace{x^{(j)}\mathbf{I} - \sum_{k=1}^{K} \sum_{l=1}^{L_k} y_l^{(k)} \rho_j \mathbf{A}_{(l,m)}^{(k,j)}}_{=\mathbf{Z}_m^{(j)}} \succeq 0, \forall m \in \mathcal{L}_j, \forall j \in \mathcal{K}_27b)$$

$$1 - \sum_{j=1}^{K} x^{(j)} \ge 0, \tag{27c}$$

$$y_l^{(k)} \ge 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k,$$
(27d)

$$x^{(j)} \ge 0, \forall j \in \mathcal{K},\tag{27e}$$

⁹Please refer to [28, Appendix E]. ¹⁰As per [29, Theorem 7.6.3] the principle eigenvalue of $(\mathbf{F}_{l}^{(k)})^{-\frac{1}{2}}\mathbf{E}_{l}^{(k)}(\mathbf{F}_{l}^{(k)})^{-\frac{1}{2}}$ is always positive.

$$\mathbf{Z}_{m}^{(j)} = x^{(j)}\mathbf{I} - \sum_{k=1}^{K} \sum_{l=1}^{L_{k}} y_{l}^{(k)} \rho_{j} \mathbf{A}_{(l,m)}^{(k,j)} \\
= \underbrace{x^{(j)}\mathbf{I} + \sum_{k=1}^{K} \sum_{l=1}^{L_{k}} y_{l}^{(k)} \rho_{j} \gamma \left((\widehat{\mathbf{H}}^{(k,j)})^{\dagger} \mathbf{u}_{l}^{(k)} (\mathbf{u}_{l}^{(k)})^{\dagger} \widehat{\mathbf{H}}^{(k,j)} + \varepsilon ||\mathbf{u}_{l}^{(k)}||^{2} \mathbf{I} \right) \mathcal{I}\{k \neq j \& l \neq m\} + y_{m}^{(j)} \rho_{j} \varepsilon ||\mathbf{u}_{m}^{(j)}||^{2} \mathbf{I}}_{rank M} \\
- \underbrace{y_{m}^{(j)} \rho_{j} (\widehat{\mathbf{H}}^{(j,j)})^{\dagger} \mathbf{u}_{m}^{(j)} (\mathbf{u}_{m}^{(j)})^{\dagger} \widehat{\mathbf{H}}^{(j,j)}}_{rank 1}.$$
(29)

Note that $(\mathbf{V}_m^{(j)})^* \neq \mathbf{0}, \forall j \in \mathcal{K}, \forall m \in \mathcal{L}_j$, and from the complementary conditions for the primal and dual SDP:

$$\operatorname{Tr}(\mathbf{Z}_m^{(j)}(\mathbf{V}_m^{(j)})^*) = 0, \forall j \in \mathcal{K}, \forall m \in \mathcal{L}_j$$
(28)

we can infer that $\mathbf{Z}_m^{(j)} \not\succ \mathbf{0}$. Suppose that one of the optimal values $\{\{(y_l^{(k)})^\star\}_{l=1}^{L_k}\}_{k=1}^K$ for the dual problem, say $(y_1^{(1)})^\star = 0$, then

$$\mathbf{Z}_{1}^{(1)} = x^{(1)}\mathbf{I} + \sum_{k=1}^{K} \sum_{l=1}^{L_{k}} y_{l}^{(k)} (-\rho_{1}\mathbf{A}_{(l,1)}^{(k,1)}) \mathcal{I}\{k \neq 1 \& l \neq 1\} \succ \mathbf{0}$$

It contradicts the fact $\mathbf{Z}_{1}^{(1)} \not\geq \mathbf{0}$, and hence $(y_{l}^{(k)})^{\star} > 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_{k}$. From (27b) and (29), $\operatorname{rank}(\mathbf{Z}_{m}^{(j)}) \geq M - 1$. On the other hand, from (28), since $\mathbf{Z}_{m}^{(j)} \not\geq \mathbf{0}$ so $\operatorname{rank}(\mathbf{Z}_{m}^{(j)}) < M$. It follows that $\operatorname{rank}(\mathbf{Z}_{m}^{(j)}) = M - 1$. Moreover, due to (28) the optimal solution $\{\{(\mathbf{V}_{m}^{(j)})^{\star}\}_{m=1}^{L_{j}}\}_{j=1}^{K}$ of primal problem (26) must be of rank one. In other words, there will be zero duality gap between the primal non-convex problem $\widetilde{\mathcal{Q}}_{v}$ and the dual problem obtained by relaxing the rank constraint given by (27).

APPENDIX D PROOF: CONVERGENCE OF ALGORITHM 1

At the n^{th} iteration of Algorithm 1, we denote the precoders as $\{\{\widetilde{\mathbf{v}}_{m}^{(j)}[n]\}_{m=1}^{L_{j}}\}_{j=1}^{K}$, the decorrelators as $\{\{\widetilde{\mathbf{u}}_{m}^{(j)}[n]\}_{m=1}^{L_{j}}\}_{j=1}^{K}$, the minimum SINR as $\widetilde{\gamma}[n]$, and the transmit power scaling factor as $\widetilde{\beta}[n]$.

Upon initialization, we define the minimum SINR as $\tilde{\gamma}[0] = 0$ and start with arbitrary precoders $\{\{\widetilde{\mathbf{v}}_{m}^{(j)}[0]\}_{m=1}^{L_{j}}\}_{j=1}^{K}$, where the transmit power of the j^{th} source node is $\sum_{m=1}^{L_{j}} (\widetilde{\mathbf{v}}_{m}^{(j)}[0])^{\dagger} \widetilde{\mathbf{v}}_{m}^{(j)}[0] = P_{j}$, and the transmit power scaling factor is $\beta[0] = \min(P_{1}, \ldots, P_{K})$.

In the following, we show that each iteration of Algorithm 1 increases the minimum SINR, i.e. $\tilde{\gamma}[n] \geq \tilde{\gamma}[n-1]$, so Algorithm 1 must converge.

In Step 1, given the precoders $\{\{\widetilde{\mathbf{v}}_m^{(j)}[n-1]\}_{m=1}^{L_j}\}_{j=1}^K$, the decorrelators $\{\{\widetilde{\mathbf{u}}_m^{(j)}[n]\}_{m=1}^{L_j}\}_{j=1}^K$ are optimized to increase the minimum SINR, i.e.

$$\begin{split} \widehat{\gamma} &= \min_{\substack{l \in \mathcal{L}_k \\ k \in \mathcal{K}}} \widetilde{\gamma}_l^{(k)}(\widehat{\mathcal{H}}, \{\{\widetilde{\mathbf{v}}_m^{(j)}[n-1]\}_{m=1}^{L_j}\}_{j=1}^K, \widetilde{\mathbf{u}}_l^{(k)}[n]) \\ &\geq \min_{\substack{l \in \mathcal{L}_k \\ k \in \mathcal{K}}} \widetilde{\gamma}_l^{(k)}(\widehat{\mathcal{H}}, \{\{\widetilde{\mathbf{v}}_m^{(j)}[n-1]\}_{m=1}^{L_j}\}_{j=1}^K, \widetilde{\mathbf{u}}_l^{(k)}[n-1]) \\ &= \widetilde{\gamma}[n-1]. \end{split}$$
(30)

In Step 3, given the decorrelators $\{\{\widetilde{\mathbf{u}}_{m}^{(j)}[n]\}_{m=1}^{L_{j}}\}_{j=1}^{K}$ and the minimum SINR constraint $\widehat{\gamma}$, the precoders $\{\{\widetilde{\mathbf{v}}_{m}^{(j)}[n]\}_{m=1}^{L_{j}}\}_{j=1}^{K}$ are optimized to jointly reduce the transmit powers of all nodes, i.e. the minimum SINR is unchanged

$$\begin{split} \widehat{\gamma} &= \min_{\substack{l \in \mathcal{L}_k \\ k \in \mathcal{K}}} \widetilde{\gamma}_l^{(k)} (\widehat{\mathcal{H}}, \{\{\widetilde{\mathbf{v}}_m^{(j)}[n]\}_{m=1}^{L_j}\}_{j=1}^K, \widetilde{\mathbf{u}}_l^{(k)}[n]) \\ &= \min_{\substack{l \in \mathcal{L}_k \\ k \in \mathcal{K}}} \widetilde{\gamma}_l^{(k)} (\widehat{\mathcal{H}}, \{\{\widetilde{\mathbf{v}}_m^{(j)}[n-1]\}_{m=1}^{L_j}\}_{j=1}^K, \widetilde{\mathbf{u}}_l^{(k)}[n]) \end{split}$$

whereas the transmit powers of all source nodes are reduced

$$\begin{split} \rho_j \widetilde{\beta}[n] &= \sum_{m=1}^{L_j} (\widetilde{\mathbf{v}}_m^{(j)}[n])^{\dagger} \widetilde{\mathbf{v}}_m^{(j)}[n] \\ &\leq \sum_{m=1}^{L_j} (\widetilde{\mathbf{v}}_m^{(j)}[n-1])^{\dagger} \widetilde{\mathbf{v}}_m^{(j)}[n-1] \\ &= P_j. \end{split}$$

In Step 5, the precoders are up-scaled to the power constraint, i.e. $\mathbf{v}_m^{(j)}[n] = \sqrt{P_j/(\rho_j \tilde{\beta}[n])} \mathbf{v}_m^{(j)}[n]$, where by definition $P_1/\rho_1 = \ldots = P_K/\rho_K$. As such, the minimum SINR is increased according to

$$\begin{split} \widehat{\gamma} &= \min_{\substack{l \in \mathcal{L}_{k} \\ k \in \mathcal{K}}} \widetilde{\gamma}_{l}^{(k)} (\widehat{\mathcal{H}}, \{\{\mathbf{v}_{m}^{(j)}[n]\}_{m=1}^{L_{j}}\}_{j=1}^{K}, \mathbf{u}_{l}^{(k)}[n]) \\ &= \min_{\substack{l \in \mathcal{L}_{k} \\ k \in \mathcal{K}}} \frac{||(\mathbf{u}_{l}^{(k)}[n])^{\dagger} \widehat{\mathbf{H}}^{(k,k)} \mathbf{v}_{l}^{(k)}[n]||^{2} - \varepsilon ||\mathbf{u}_{l}^{(k)}[n]||^{2} ||\mathbf{v}_{l}^{(k)}[n]||^{2}}{\left(\sum_{j=1}^{K} \sum_{m=1}^{L_{j}} ||(\mathbf{u}_{l}^{(k)}[n])^{\dagger} \widehat{\mathbf{H}}^{(k,j)} \mathbf{v}_{m}^{(j)}[n]||^{2} - \varepsilon ||\mathbf{u}_{l}^{(k)}[n]||^{2} ||\mathbf{v}_{l}^{(k)}[n]||^{2}}{\left(+\varepsilon ||\mathbf{u}_{l}^{(k)}[n]|^{\dagger} \widehat{\mathbf{L}}_{j=1} \sum_{m=1}^{L_{j}} ||\mathbf{v}_{m}^{(j)}[n]||^{2} + N_{0}||\mathbf{u}_{l}^{(k)}[n]||^{2}||\mathbf{v}_{l}^{(k)}[n]||^{2}}\right) \\ &< \min_{\substack{l \in \mathcal{L}_{k} \\ k \in \mathcal{K}}} \widetilde{\gamma}_{l}^{(k)} (\widehat{\mathcal{H}}, \{\{\sqrt{P_{K}/(\rho_{K}\widetilde{\beta}[n])} \mathbf{v}_{m}^{(j)}[n]\}_{m=1}^{L}\}_{j=1}^{K}, \mathbf{u}_{l}^{(k)}[n]) \\ &= \min_{\substack{l \in \mathcal{L}_{k} \\ k \in \mathcal{K}}} \frac{||(\mathbf{u}_{l}^{(k)}[n])^{\dagger} \widehat{\mathbf{H}}^{(k,k)} \mathbf{v}_{l}^{(k)}[n]||^{2} - \varepsilon ||\mathbf{u}_{l}^{(k)}[n]||^{2}||\mathbf{v}_{l}^{(k)}[n]||^{2}}{\left(\sum_{j=1}^{K} \sum_{j=1}^{L_{j}} \sum_{m=1}^{L} ||(\mathbf{u}_{l}^{(k)}[n])||^{2} - \varepsilon ||\mathbf{u}_{l}^{(k)}[n]||^{2}||\mathbf{v}_{l}^{(k)}[n]||^{2}}{\left(\sum_{i=1}^{K} \sum_{j=1}^{L_{j}} \sum_{m=1}^{L_{j}} ||\mathbf{v}_{m}^{(j)}[n]||^{2} - \varepsilon ||\mathbf{u}_{l}^{(k)}[n]||^{2}||\mathbf{v}_{l}^{(k)}[n]||^{2}}{\left(\sum_{i=1}^{K} \sum_{j=1}^{L_{j}} \sum_{j=1}^{L_{j}} \sum_{m=1}^{L_{j}} ||\mathbf{v}_{m}^{(j)}[n]||^{2} - \varepsilon ||\mathbf{u}_{l}^{(k)}[n]||^{2}||\mathbf{v}_{l}^{(k)}[n]||^{2}}{\left(\sum_{i=1}^{K} \sum_{j=1}^{L_{j}} \sum_{j=1}^{L_{j}} \sum_{m=1}^{L_{j}} ||\mathbf{v}_{m}^{(j)}[n]||^{2} - \varepsilon ||\mathbf{u}_{l}^{(k)}[n]||^{2}||\mathbf{v}_{l}^{(k)}[n]||^{2}}{\left(-||(\mathbf{u}_{l}^{(k)}[n])|^{\dagger} \widehat{\mathbf{H}}^{(k,k)} \mathbf{v}_{l}^{(k)}[n]||^{2} + \frac{N_{0}}{(P_{K}/\rho_{K})(1/\widetilde{\beta}[n])} ||\mathbf{u}_{l}^{(k)}[n]||^{2}}{\left(-||(\mathbf{u}_{l}^{(k)}[n])|^{\dagger} \widehat{\mathbf{H}}^{(k,k)} \mathbf{v}_{l}^{(k)}[n]||^{2} + \frac{N_{0}}{(P_{K}/\rho_{K})(1/\widetilde{\beta}[n])} ||\mathbf{u}_{l}^{(k)}[n]||^{2}}\right)} \\ = \widetilde{\gamma}[n]. \end{split}$$

It follows from (30) and (31) that the minimum SINR increases with each iteration, i.e. $\tilde{\gamma}[n] \geq \hat{\gamma} \geq \tilde{\gamma}[n-1]$, and Algorithm 1 must converge.

REFERENCES

- R. H. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, pp. 5534– 5562, Dec. 2008.
- [2] A. Carleial, "A case where interference does not reduce capacity," *IEEE Trans. Inf. Theory*, vol. 21, pp. 569–570, Sep. 1975.
- [3] D. Tse and P. Viswanath, Fundamentals of Wireless Communication. Cambridge, UK: Cambridge University Press, 2005.
- [4] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, pp. 3425–3441, Aug. 2008.
- [5] S. W. Peters and R. W. Heath, Jr., "Interference alignment via alternating minimization," in *Proc. IEEE ICASSP'09*, Apr. 2009.

- [6] K. S. Gomadam, V. R. Cadambe, and S. A. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment." [Online]. Available: http://newport.eecs.uci.edu/~syed/papers/dist.pdf
- [7] A. S. Motahari, S. O. Gharan, M. Maddaha-Ali, and A. K. Khandani, "Real interference alignment: Exploiting the potential of single antenna systems," 2009. [Online]. Available: http://arxiv.org/abs/0908.2282
- [8] A. Ghasemi, A. S. Motahari, and A. K. Khandani, "Interference alignment for the K user MIMO interference channel," 2009. [Online]. Available: http://arxiv.org/abs/0909.4604
- [9] C. M. Yetis, T. Gou, S. A. Jafar, and A. H. Kayran, "Feasibility conditions for interference alignment," in *Proc. IEEE GLOBECOM*'09, Nov. 2009.
- [10] M. Botros and T. N. Davidson, "Convex conic formulations of robust downlink precoder designs with quality of service constraints," *IEEE J. Sel. Areas Signal Process.*, vol. 1, pp. 714–724, Dec. 2007.
- [11] A. Pascual-Iserte, D. P. Palomar, A. I. Prez-Neira, and M. A. Lagunas, "A robust maximin approach for MIMO communications with partial channel state information based on convex optimization," *IEEE Trans. Signal Process.*, vol. 54, pp. 346–360, Jan. 2006.
- [12] N. Vucic, H. Boche, and S. Shi, "Robust transceiver optimization in downlink multiuser MIMO systems with channel uncertainty," in *Proc. IEEE ICC'08*, 2008.
- [13] M. Payaro, A. Pascual-Iserte, and M. A. Lagunas, "Robust power allocation designs for multiuser and multiantenna downlink communication systems through convex optimization," *IEEE J. Sel. Areas Commun.*, vol. 25, pp. 1392–1401, Sep. 2007.
- [14] Z.-Q. Luo and T.-H. Chang, "SDP relaxation of homogeneous quadratic optimization: approximation bounds and applications," in *Convex Optimization in Signal Processing and Communications*, D. P. Palomar and Y. C. Eldar, Eds. Cambridge, UK: Cambridge University Press, 2009.
- [15] Y. Huang, A. D. Maio, and S. Zhang, "Semidefinite programming, matrix decomposition, and radar code design," in *Convex Optimization in Signal Processing and Communications*, D. P. Palomar and Y. C. Eldar, Eds. Cambridge, UK: Cambridge University Press, 2009.
- [16] Y. Huang and D. P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," IEEE Trans. Signal Process., to be published.
- [17] D. Hammarwall, M. Bengtsson, and B. Ottersten, "On downlink beamforming with indefinite shaping constraints," *IEEE Trans. Signal Process.*, vol. 54, pp. 3566–3580, Sep. 2006.
- [18] E. Karipidis, N. D. Sidiropoulos, and Z.-Q. Luo, "Quality of service and max-min fair transmit beamforming to multiple cochannel multicast groups," *IEEE Trans. Signal Process.*, vol. 56, pp. 1268–1279, Mar. 2008.
- [19] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Trans. Signal Process.*, vol. 54, pp. 2239–2251, Jun. 2006.
- [20] C. B. Chae, "Multiuser/multi-cell MIMO transmission with coordinated beamforming," in *Proc. IEEE CTW'09*, May 2009.
- [21] C.-B. Chae and R. W. Heath, "On the optimality of linear multiuser MIMO beamforming for a two-user two-input multiple-output broadcast system," *IEEE Signal Process. Lett.*, vol. 16, pp. 117–120, Feb. 2009.
- [22] F. Rashid-Farrokhi, K. J. R. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE J. Sel. Areas Commun.*, vol. 16, pp. 1437–1450, Oct. 1998.
- [23] H. Boche and M. Schubert, "A general duality theory for uplink and downlink beamforming," in *Proc. IEEE VTC'02*, 2002.
- [24] J. Gorski, F. Pfeuffer, and K. Klamroth, "Biconvex sets and optimization with biconvex functions – a survey and extensions," *Mathematical Methods of Operations Research*, vol. 66, pp. 373–408, Dec. 2007.
- [25] J. Wang and D. P. Palomar, "Worst-case robust MIMO transmission with imperfect channel knowledge," *IEEE Trans. Signal Process.*, vol. 57, pp. 3086–3100, Aug. 2009.
- [26] A. Wiesel, Y. C. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, pp. 161–176, Jan. 2006.
- [27] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.
- [28] S. Haykin, *Adaptive Filter Theory*, 4th ed. Upper Saddle River: Prentice-Hall, 2002.
- [29] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, UK: Cambridge University Press, 1985.



Eddy Chiu received the B.A.Sc. (Honors) and M.A.Sc. degrees from Simon Fraser University, Canada, in 2003 and 2006, respectively, both in Electrical Engineering. Currently, he is working towards the Ph.D. degree at the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology. His research interests include MIMO communications with limited feedback, relay-assisted communications, and interference mitigation techniques.



Vincent K. N. Lau obtained B.Eng (Distinction 1st Hons) from the University of Hong Kong in 1992 and Ph.D. from Cambridge University in 1997. He was with PCCW as system engineer from 1992-1995 and Bell Labs - Lucent Technologies as member of technical staff from 1997-2003. He then joined the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology as Professor. His current research interests include robust and delay-sensitive cross-layer scheduling of MIMO/OFDM wireless systems with

imperfect channel state information, cooperative and cognitive communications as well as stochastic approximation and Markov Decision Process.



Huang Huang received the B.Eng. and M.Eng. (Gold medal) from the Harbin Institute of Technology (HIT) in 2005 and 2007 respectively, all in Electrical Engineering. He is currently a PhD student at the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology. His recent research interests include cross layer design, interference management in interference network, and embedded system design.

Tao Wu

Sheng Liu