# Multi-Relay Selection Design and Analysis for Multi-Stream Cooperative Communications

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## Abstract

In this paper, we consider the problem of multi-relay selection for multi-stream cooperative MIMO systems with M relay nodes. Traditionally, relay selection approaches are primarily focused on selecting one relay node to improve the transmission reliability given a single-antenna destination node. As such, in the cooperative phase whereby both the source and the selected relay nodes transmit to the destination node, it is only feasible to exploit cooperative spatial diversity (for example by means of distributed space time coding). For wireless systems with a multi-antenna destination node, in the cooperative phase it is possible to opportunistically transmit multiple data streams to the destination node by utilizing multiple relay nodes. Therefore, we propose a low overhead multi-relay selection protocol to support multi-stream cooperative communications. In addition, we derive the asymptotic performance results at high SNR for the proposed scheme and discuss the diversity-multiplexing tradeoff as well as the throughput-reliability tradeoff. From these results, we show that the proposed multi-stream cooperative communication scheme achieves lower outage probability compared to existing baseline schemes.

#### I. INTRODUCTION

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Cooperative communications for wireless systems has recently attracted enormous attention. By utilizing cooperation among different users, spatial diversity can be created and this is referred to as *cooperative diversity* [1]-[3]. In [1], the authors considered the case where multiple relay nodes are available to assist the communication between the source and the destination nodes using the decode-and-forward (DF) protocol, and they showed that a diversity gain of M(1-2r) can be achieved with M relay nodes and a multiplexing gain of r. However, the advantages of utilizing more relay nodes is coupled with the consumption of additional system resources and power, so it is impractical (or even infeasible) to activate many relay nodes in resource- or power-constrained systems. As a result, various relay selection protocols have been considered in the literature. For example, in [4]-[7] the authors considered cooperative MIMO systems with a single-antenna destination node and the selection of one relay node in the cooperative phase. In [4] the authors proposed an opportunistic relaying protocol and showed that this scheme can achieve the same diversity-multiplexing tradeoff (DMT) as systems that activate all the relay nodes to perform distributed space-time coding. In [5] the authors applied fountain code to facilitate exploiting spatial diversity. In [6], [7] the authors proposed a dynamic decode-and-forward (DDF) protocol which allows the selected relay node to start transmitting as soon as it successfully decodes the source message. On the other hand, there are a number of works [8]-[10] that studied the capacity bounds and asymptotic performance (e.g. DMT relation) for single-stream cooperative MIMO systems with a single-antenna destination node. In [8] the authors derived the DMT relation with multiple full-duplex relays and showed that the DMT relation is the same as the DMT upper bound for point-to-point MISO channels. In [9] the authors studied the network scaling law based on the amplify-and-forward (AF) relaying protocol. In all the above works, the destination node is assumed to have single receive antenna and hence, only one data stream is involved in the cooperative phase. When the destination node has multiple receive antennas, the system could support multiple data streams in the cooperative phase and this could lead to a higher spectral efficiency.

In this paper, we design a relay selection scheme for multi-stream cooperative systems and analyze the resultant system performance. In order to effectively implement multi-stream cooperative systems, there are several technical challenges that require further investigations.

- How to select multiple relay nodes to support multi-stream cooperation in the cooperative phase? Most of the existing relay selection schemes are designed with respect to having a single-antenna destination node and supporting cooperative spatial diversity. For example, in [11] the authors considered a rateless-coded system and proposed to select relay nodes for supporting cooperative spatial diversity based on the criterion of maximizing the received SNR. In order to support multi-stream cooperation, the relay selection metric should represent the *holistic* channel condition between all the selected relay nodes and the destination node, but this property cannot be addressed by the existing relay selection schemes.
- How much additional benefit can multi-stream cooperation achieve? The spectral efficiency of the cooperative phase can be substantially increased with multi-relay multi-stream cooperation compared to conventional schemes. However, the

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system performance may be bottlenecked by the source-relay links. Therefore, we characterize the advantage of multi-stream cooperation in terms of the end-to-end performance gain over conventional schemes that are based on cooperative spatial diversity.

We propose a multi-stream cooperative relay protocol (DF-MSC-opt) for cooperative systems with a multi-antenna destination node. We consider optimized node selection in which a set of relay nodes is selected for *multi-stream cooperation*. Based on the optimal relay selection criterion, we compare the outage capacity, the diversity-multiplexing tradeoff (DMT), and the throughput-reliability tradeoff (TRT) of the proposed DF-MSC-opt scheme against traditional reference baselines.

Notation: In the sequel, we adopt the following notations.  $\mathbb{C}^{M \times N}$  denotes the set of complex  $M \times N$  matrices;  $\mathbb{Z}$  denotes the set of integers; upper and lower case letters denote matrices and vectors, respectively;  $(\cdot)^T$  denotes matrix transpose;  $\operatorname{Tr}(\cdot)$  denotes matrix trace; diag $(x_1, \ldots, x_L)$  is a diagonal matrix with entries  $x_1, \ldots, x_L$ ;  $\mathcal{I}(\cdot)$  denotes the indicator function;  $\operatorname{Pr}(X)$  denotes the probability of event X;  $\mathbf{I}_N$  denotes the  $N \times N$  identity matrix;  $\lceil \cdot \rceil$  and  $\lfloor \cdot \rfloor$  denote the ceiling and floor operations, respectively;  $(\cdot)^+ = \max(\cdot, 0)$ ;  $\doteq$ ,  $\leq$  and  $\geq$  denote exponential equality and inequalities where, for example,  $f(\rho) \doteq \rho^b$  if  $\lim_{\rho \to \infty} \frac{\log(f(\rho))}{\log(\rho)} = b$ ;  $\mathbb{E}_Y[\cdot]$  denotes expectation over Y; and F(x, k) denotes the  $\chi^2$  cumulative distribution function (CDF) for value x and degrees-of-freedom k.

#### II. RELAY CHANNEL MODEL

We consider a system consisting of a single-antenna source node, M half-duplex single-antenna relay nodes, and a destination node with  $N_r$  antennas. For notational convenience, we denote the source node as the 0<sup>th</sup> node and the M relay nodes as the  $\{1, 2, ..., M\}$ -th node. We focus on block fading channels such that the channel coefficients for all links remain constant throughout the transmission of a source message.

We divide the transmission of a source message that requires N channel uses into two phases, namely the *listening phase* and *cooperative phase*. In the listening phase, all the relay nodes listen to the signals transmitted by the source node until K out of the M relay nodes can decode the source message<sup>1</sup>. In the cooperative phase, the destination node chooses  $N_r$  nodes from amongst the source node and the successfully decoding relay nodes to transmit multiple data streams to the destination node. Specifically, let  $\mathbf{x} = [x(1), x(2), \ldots, x(N)]^T \in \mathbb{C}^{N \times 1}$  denote the signals transmitted by the source node over N channel uses. Similarly, let  $\mathbf{x}_m = [x_m(1), x_m(2), \ldots, x_m(N)]^T \in \mathbb{C}^{N \times 1}$ ,  $m = 1, \ldots, M$ , denote the signals transmitted by the  $m^{\text{th}}$  relay node over N channel uses. The signals received by the  $m^{\text{th}}$  relay node is given by  $\mathbf{y}_m = [y_m(1), y_m(2), \ldots, y_m(N)]^T \in \mathbb{C}^{N \times 1}$ ,  $m = 1, \ldots, M$ , where

$$y_m(n) = h_{SR,m} x(n) + z_m(n),$$
 (1)

 $h_{SR,m}$  is the fading channel coefficient between the source node and  $m^{th}$  relay node,  $\mathbf{H}_{SR} = [h_{SR,1}, h_{SR,2}, \dots, h_{SR,M}]^T \in \mathbb{C}^{M \times 1}$  is a vector containing the channel coefficients between the source and the M relay nodes, and  $z_m(n)$  is the additive noise with power normalized to unity. Each relay node attempts to decode the source message with each received signal observation until it can successfully decode the message. The listening phase ends and the cooperative phase begins after K relay nodes successfully decodes the source message. In the cooperative phase, let  $\tilde{x}_k(n)$ ,  $k = 1, \dots, K$ , denote the signal relayed by the  $k^{th}$  successfully decoding relay node and the aggregate signal transmitted in the cooperative phase can be expressed as  $\mathbf{x}_D(n) = [x(n), \tilde{x}_1(n), \tilde{x}_2(n), \dots, \tilde{x}_K(n)]^T \in \mathbb{C}^{(K+1) \times 1}$ . Accordingly, the received signals at the destination node are given by  $\mathbf{Y} = [\mathbf{y}(1), \mathbf{y}(2), \dots, \mathbf{y}(N)]^T \in \mathbb{C}^{N \times N_r}$ , where

$$\mathbf{y}(n) = \begin{cases} \mathbf{H}_{SD} x(n) + \mathbf{z}(n) & \text{for the listening phase} \\ \mathbf{H}_D(\mathcal{D}) \mathbf{V} \mathbf{x}_D(n) + \mathbf{z}(n) & \text{for the cooperative phase} \end{cases}$$

 $\mathbf{H}_{SD} \in \mathbb{C}^{N_r \times 1}$  represents the fading channel coefficients between the source and the destination nodes,  $\mathcal{D}$  is the set representing the *K* successfully decoding relay nodes,  $\mathbf{H}_D(\mathcal{D}) = [\mathbf{H}_{SD}, \tilde{\mathbf{H}}_{RD,1}, \tilde{\mathbf{H}}_{RD,2}, \dots, \tilde{\mathbf{H}}_{RD,K}] \in \mathbb{C}^{N_r \times (K+1)}$  represents the aggregate channel in the cooperative phase with  $\tilde{\mathbf{H}}_{RD,k} \in \mathbb{C}^{N_r \times 1}$  being the fading channel coefficients between  $k^{th}$  successfully decoding relay and the destination node,  $\mathbf{z}(n) \in \mathbb{C}^{N_r \times 1}$  is the additive noise with power normalized to unity, and  $\mathbf{V}$  is the *node selection matrix*. The selection matrix is defined as  $\mathbf{V} = \text{diag}(v_0, v_1, v_2, \dots, v_K)$ , where  $v_k = 1$  if the  $k^{\text{th}}$  node  $(k \in \{0, \dots, K\})$  is selected to transmit in the cooperative phase and  $v_k = 0$  if the node is not selected.

The following assumptions are made throughout the rest of the paper.

Assumption 1 (Half-duplex relay model): The half-duplex relay nodes can either transmit or receive during a given time interval but not both.

Assumption 2 (Fading model): We assume block fading channels such that the channel coefficients  $\mathbf{H}_{SR}$  and  $\mathbf{H}_D(\mathcal{D})$  remain unchanged within a fading block (i.e., N channel uses). Moreover, we assume the fading channel coefficients of the source-to-relay (S-R) links, relay-to-destination (R-D) links, and source to destination (S-D) links are independent and identically distributed (i.i.d.) complex symmetric random Gaussian variables with zero-mean and variance  $\sigma_{SR}^2$ ,  $\sigma_{RD}^2$  and  $\sigma_{SD}^2$ , respectively.

<sup>1</sup>The relay system cannot enter the cooperative phase if less than K relay nodes can decode the source message within N channel uses.

Assumption 3 (CSI model): Each relay node has perfect channel state information (CSI) of the link between the source node and itself. The destination node has perfect CSI of the S-D link and all R-D links. For notational convenience, we denote the aggregate CSI as  $\mathbf{H} = (\mathbf{H}_{SR}, \mathbf{H}_D(\mathcal{D}))$ .

Assumption 4 (Transmit power constraints): The transmit power of the source node is limited to  $\rho_S$ . The transmit power of the  $k^{th}$  relay node is limited to  $\rho_k$ .

#### **III. PROBLEM FORMULATION**

In this section, we first present the encoding-decoding scheme and transmission protocol of the proposed DF-MSC-opt scheme. Based on that, we formulate the multi-relay selection problem as a combinatorial optimization problem.

## A. Encoding and Decoding Scheme

The proposed multi-stream cooperation system is facilitated by random coding and maximum-likelihood (ML) decoding [12]. At the source node, an *R*-bit message W, drawn from the index set  $\{1, 2, ..., 2^R\}$ , is encoded through an encoding function  $\mathcal{X}^N : \{1, 2, ..., 2^R\} \to \mathbf{X}^N$ . The encoding function at the source node can be characterized by a vector codebook  $\mathcal{C} = \{\mathbf{X}^N(1), \mathbf{X}^N(2), ..., \mathbf{X}^N(2^R)\} \in \mathbb{C}^{N_r \times N}$ . The m<sup>th</sup> codeword of codebook  $\mathcal{C}$  is defined as

$$\mathbf{X}^{N}(m) = \begin{bmatrix} x_{1}^{(m)}(1) & \dots & x_{1}^{(m)}(N_{1}) \\ \vdots \\ x_{N_{r}}^{(m)}(1) & \dots & x_{N_{r}}^{(m)}(N_{1}) \end{bmatrix} \begin{bmatrix} x_{1}^{(m)}(N_{1}+1) & \dots & x_{1}^{(m)}(N) \\ \vdots \\ x_{N_{r}}^{(m)}(N_{1}) & \dots & x_{N_{r}}^{(m)}(N_{1}) \end{bmatrix}, m = 1, \dots, 2^{R},$$
(2)

which consists of N vector symbols of dimension  $N_r \times 1$ , and  $x_k^{(m)}(n)$  is the symbol to be transmitted by the  $k^{\text{th}}$  antenna during the  $n^{\text{th}}$  channel use. The vector codebook C is known to all the M relay nodes (for decoding and re-encoding) as well as the destination node for decoding. At the receiver side (relay node or destination node), the receiver decodes the R-bit message based on the observations, the CSI, and a decoding function  $\mathcal{Y}^N : (\mathbf{Y}^N \times \mathbf{H}) \to \{1, 2, \dots, 2^R\}$ . We assume ML detection in the decoding process. The detailed operation of the source node and the relay nodes in the listening and cooperative phases are elaborated in the next subsection.

#### B. Transmission Protocol for Multi-Stream Cooperation

The proposed transmission protocol is illustrated in Fig. 1, and the flow charts for the processing by the source, relay, and destination nodes are shown in Fig. 2. Specifically, the N-symbol source message codeword (cf. (2)) is transmitted to the destination node over two phases; the listening phase that spans the first  $N_1$  channel uses and the cooperative phase that spans the remainder  $N_2 = N - N_1$  channel uses.

1) Listening phase: In the listening phase, the single-antenna source node transmits the first row<sup>2</sup> of the message codeword (i.e.  $[x_1^{(m)}(1) \dots x_1^{(m)}(N_1)]$  as per (2)) to the relay and destination nodes. Each relay node attempts to decode the source message with each received signal observation. Although the source node transmits only the first row of the source message codeword, the relay nodes can still detect the source message using standard random codebook and ML decoding argument. Effectively, we can visualize a virtual system with a multi-antenna source node as shown in Fig. 3, and the missing rows in the message codeword transmitted by the source node is equivalent to channel erasure in a virtual MISO source-relay channel. Once a relay node successfully decodes the source message, it sends an acknowledgement  $ACK_{RD}$  to the destination node through a dedicated zero-delay error free feedback link.

2) Control phase and signaling scheme: Without loss of generality, we assume that K relay nodes can successfully decode the source message with  $N_1$  received signal observations. Upon receiving the acknowledgement from the K successfully decoding relay nodes, the destination node enters the *control* phase and selects  $N_r$  nodes to participate in the multi-stream cooperation phase (cf. Section III-C). Specifically, the destination node indicates to the source and relay nodes the transition to the cooperative phase as well as the node selection decisions via an (M + 1)-bit feedback pattern. The first bit of the feedback pattern is used to index the 0<sup>th</sup> node (the source node) and the last M bits are used to index the M relay nodes. The feedback pattern contains  $N_r$  bits that are set to 1; the m<sup>th</sup> bit of the feedback pattern is set to 1 if the corresponding node is selected to participate in the cooperative phase, whereas the bit is set to 0 if the node is not selected. Note that the total number of feedback bits required by the proposed multi-stream cooperation scheme is  $K ACK_{RD}$  plus one feedback pattern with M + 1bits, which is less than 2 bits per relay node.

<sup>&</sup>lt;sup>2</sup>The source node has a single transmit antenna and hence, could only transmit one row of the vector codeword  $X^{N}(m)$  during the listening phase.

3) Cooperative phase: In the cooperative phase, the  $N_r$  selected nodes cooperate to transmit the  $(N_1 + 1)^{\text{th}}$  to the  $N^{\text{th}}$  columns of the source message codeword (cf. (2)) to the destination node to assist it with decoding the source message. Specifically, for  $k = 1, \ldots, N_r$ , the node corresponding to the  $k^{th}$  bit that is set to 1 in the feedback pattern transmits the  $k^{\text{th}}$  row of the message codeword (i.e.  $[x_k^{(m)}(N_1 + 1) \ldots x_k^{(m)}(N)]$  as per (2)) to the destination node.

To better illustrate the proposed transmission protocol, we show in Fig. 1 an example of the proposed system with a destination node with  $N_r = 2$  antennas. Suppose the feedback pattern is given as follows:

This corresponds to selecting the first relay node  $(R_1)$  and the  $M^{\text{th}}$  relay node  $(R_M)$  to participate in the cooperative phase. In the listening phase, the source node transmits the first row of the codeword  $X^N(m)$  given by  $[x_1^{(m)}(1) \dots x_1^{(m)}(N_1)]$ . In the cooperative phase,  $R_1$  and  $R_M$  transmit  $[x_1^{(m)}(N_1+1) \dots x_1^{(m)}(N)]$  and  $[x_2^{(m)}(N_1+1) \dots x_2^{(m)}(N)]$ , respectively. Therefore, the effective transmitted codeword can be expressed as

$$\mathbf{X}^{N}(m) = \begin{bmatrix} x_{1}^{(m)}(1) & \dots & x_{1}^{(m)}(N_{1}) \\ \hline x_{1}^{(m)}(1) & \dots & x_{1}^{(m)}(N_{1}) \\ \hline x_{2}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{1}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{2}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{1}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{2}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{1}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{2}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{1}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{2}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{2}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{1}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{2}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{1}^{(m)}(N_{1}+1) & \dots & x_{2}^{(m)}(N) \\ \hline x_{1}^{($$

### C. Problem Formulation for Multi-stream Cooperation

We focus on node selection by the destination node for optimizing outage performance (cf. Section III-B2), where the destination node only has CSI of the S-D and R-D links  $H_D(D)$  (cf. Assumption 3). The proposed Multi-stream Cooperation Scheme with Optimized node selection is called DF-MSC-opt and we define the outage event as follows:

Definition 1 (Outage): Outage refers to the event when the total instantaneous mutual information between the source and the destination nodes is less than the target transmission rate R. Mathematically, outage can be expressed as

$$\mathcal{I}(I_{\text{DF-MSC-opt}}(\mathbf{H}, \mathbf{V}) < R), \tag{3}$$

where  $\mathbf{H} = (\mathbf{H}_{SR}, \mathbf{H}_D(\mathcal{D}))$  is the aggregate channel realization and  $\mathbf{V}$  is the node selection action given  $\mathbf{H}_D(\mathcal{D})$ . The total instantaneous mutual information of the multi-stream cooperative system in (3) is given by the following theorem.

Theorem 1: (Instantaneous Mutual Information of Multi-Stream Cooperative System) Given the aggregate channel realization  $\mathbf{H} = (\mathbf{H}_{SR}, \mathbf{H}_D(\mathcal{D}))$ , the instantaneous mutual information  $I_{DF-MSC-opt}(\mathbf{H}, \mathbf{V})$  (bits/second/channel use) between the source and destination nodes for multi-stream cooperative system can be expressed as

$$I_{\text{DF-MSC-opt}}(\mathbf{H}, \mathbf{V}) = \frac{1}{N} \left\{ N_1 \log(1 + \rho_S |\mathbf{H}_{SD}|^2) + N_2 \log \det(\mathbf{I}_{N_r} + \mathbf{H}_D(\mathcal{D}) \mathbf{V} \mathbf{\Gamma} \mathbf{V}^H \mathbf{H}_D(\mathcal{D})^H) \right\},\tag{4}$$

where  $N_1$  and  $N_2$  are the numbers of channel uses of the listening phase and the cooperative phase, respectively, and  $\Gamma = \text{diag}(\rho_S, \rho_1, \rho_2, \dots, \rho_M)$  is the transmit power matrix of the source node and the M relay nodes. Note that  $N_1$  and  $N_2$  are random variables that depend on the realization of the S-R links  $\mathbf{H}_{SR}$ .

The proof of Theorem 1 can be extended from [12], [13] by applying random Gaussian codebook argument. Note that the first term in the mutual information in (4) corresponds to the contribution from the source transmission<sup>3</sup>. By virtue of the proposed DF-MSC-opt scheme, multiple data streams are cooperatively transmitted in the cooperative phase (unlike traditional schemes wherein only a single stream is transmitted). Hence, we can fully exploit the spatial channels created from the multi-antenna destination node and therefore achieving higher mutual information in the second term of (4).

By Definition 1, the average outage probability is given by

$$\mathcal{P}_{out}(\mathbf{V}) = \mathbb{E}_{\mathbf{H}} \left[ \mathcal{I}(I_{\text{DF-MSC-opt}}(\mathbf{H}, \mathbf{V}) < R) \right]$$

which is a function of the node selection policy V. It follows that, given the channel realization **H**, the optimal node selection policy is given by  $\mathbf{V}^{\star} = \arg \min_{\mathbf{V} \in \Omega} \mathcal{I}(I_{\text{DF-MSC-opt}}(\mathbf{H}, \mathbf{V}) < R)$ , where the set of all feasible node selection actions is defined as

$$\mathbf{\Omega} = \{ \mathbf{\Lambda} \in \{0, 1\}^{(M+1) \times (M+1)} | \mathbf{\Lambda} \text{ is diagonal and } \operatorname{Tr}(\mathbf{\Lambda}) = Nr \}.$$
(5)

Equivalently, for any transmission rate R, the optimal node selection policy is given by

$$\mathbf{V}^{\star} = \arg \max_{\mathbf{V} \in \Omega} I_{\text{DF-MSC-opt}}(\mathbf{H}, \mathbf{V}).$$
(6)

 $^{3}$ The mutual information contributed by the source node can also be obtained from the mutual information of the virtual multi-antenna source model in Fig. 3.

# IV. OUTAGE ANALYSIS FOR THE DF-MSC-OPT SCHEME

In this section, we derive the asymptotic outage probability of the DF-MSC-opt scheme. For simplicity, we assume  $\sigma_{SD}^2 = \sigma_{RD}^2 = \sigma_D^2$  and the power constraints of all the nodes are scaled up in the same order<sup>4</sup>, i.e.,  $\lim_{\rho_S \to \infty} \frac{\rho_m}{\rho_S} = 1$  for all  $m \in \{1, \ldots, M\}$ . The outage probability can be expressed as

$$\mathcal{P}_{out}^{\star} = \mathbb{E}_{\mathbf{H}} \left[ \mathcal{I} \left( I_{\text{DF-MSC-opt}}(\mathbf{H}, \mathbf{V}^{\star}) < R \right) \right] \stackrel{(a)}{=} \mathbb{E}_{\mathbf{H}_{SR}} \left[ \Pr \left( I_{\text{DF-MSC-opt}}(\mathbf{H}, \mathbf{V}^{\star}) < R | \mathbf{H}_{SR} \right) \right], \tag{7}$$

where in (a) the randomness of  $I_{\text{DF-MSC-opt}}(\mathbf{H}, \mathbf{V}^{\star})$  is introduced by the aggregate channel gain in the cooperative phase  $\mathbf{H}_D(\mathcal{D})$  and the optimal node selection policy  $\mathbf{V}^{\star}$ . Let  $\mathcal{H}_{N_1}$  denote the collection of all realizations of the S-R links  $\mathbf{H}_{SR}$ such that K out of M relay nodes can successfully decode the source message, and the listening phase ends at the  $(N_1)^{\text{th}}$ channel use. We define the outage probability given  $\mathbf{H}_{SR}$  and the duration of the listening phase  $N_1$  as  $P_{SR}(\mathbf{H}_{SR}, N_1) =$  $\Pr(I_{\text{DF-MSC-opt}}(\mathbf{H}, \mathbf{V}^{\star}) < R | \mathbf{H}_{SR} \in \mathcal{H}_{N_1})$ , which includes the following cases.

• Case 1: The listening phase ends within N channel uses. In this case,  $N_1 < N$ ,  $N_2 = N - N_1$ , and the outage probability  $P_{SR}(\mathbf{H}_{SR}, N_1)$  can be expressed as

$$P_{SR}^{\text{Case 1}}(\mathbf{H}_{SR}, N_1) = \Pr\left[\left(\frac{N_1}{N}\log(1+\rho_S|\mathbf{H}_{SD}|^2) + \frac{N-N_1}{N}g(\mathbf{H}, \mathbf{V}^{\star})\right) < R\right],\tag{8}$$

where  $g(\mathbf{H}, \mathbf{V}^{\star}) = \max_{\mathbf{V} \in \mathbf{\Omega}} \log \det \left( \mathbf{I}_{N_r} + \rho_S \mathbf{H}_D(\mathcal{D}) \mathbf{V} \mathbf{V}^H \mathbf{H}_D(\mathcal{D})^H \right)$  for  $K \ge N_r$ , and  $g(\mathbf{H}, \mathbf{V}^{\star}) = \log \det \left( \mathbf{I}_{N_r} + \rho_S \mathbf{H}_D(\mathcal{D}) \mathbf{H}_D(\mathcal{D})^H \right)$ for  $K < N_r$ .

• Case 2: The listening phase lasts for more than N channel uses. In this case, the outage event is only contributed by the direct transmission between the source and the destination nodes. The outage probability  $P_{SR}(\mathbf{H}_{SR}, N_1)$  can be expressed as

$$P_{SR}^{\text{Case 2}}(\mathbf{H}_{SR}, N_1) = \Pr\left(\log(1 + \rho_S |\mathbf{H}_{SD}|^2) < R\right).$$
(9)

Substituting (8) and (9) into (7), the outage probability is given by

$$\mathcal{P}_{out}^{\star} = \sum_{l=1}^{N} P_{SR}^{\text{Case 1}}(\mathbf{H}_{SR}, l) \Pr(\mathbf{H}_{SR} \in \mathcal{H}_l) + \sum_{l>N} P_{SR}^{\text{Case 2}}(\mathbf{H}_{SR}, l) \Pr(\mathbf{H}_{SR} \in \mathcal{H}_l)$$

$$= \sum_{l=1}^{N} P_{SR}^{\text{Case 1}}(\mathbf{H}_{SR}, l) \Pr(\mathbf{H}_{SR} \in \mathcal{H}_l) + \Pr(\log(1 + \rho_S |\mathbf{H}_{SD}|^2) < R) \Pr(\mathbf{H}_{SR} \in \cup_{l>N} \mathcal{H}_l),$$
(10)

where  $Pr(\mathbf{H}_{SR} \in \mathcal{H}_l)$  is the probability that K out of M relay nodes can successfully decode the message at the  $l^{th}$  channel use. To evaluate the outage probability, we calculate each term in equation (10) as follows. First, it can be shown that

$$\Pr(\mathbf{H}_{SR} \in \mathcal{H}_{N_1}) = \Pr\left(\mathbf{H}_{SR} \in \bigcup_{l > N_1 - 1} \mathcal{H}_l\right) - \Pr\left(\mathbf{H}_{SR} \in \bigcup_{l > N_1} \mathcal{H}_l\right) = \Phi_{N_1 - 1} - \Phi_{N_1}$$
(11)

where

$$\Phi_{l} = \sum_{i=0}^{K-1} {\binom{M}{i}} \left(1 - \exp\left(-\frac{2^{NR/l}-1}{\rho_{S}\sigma_{SR}^{2}}\right)\right)^{M-i} \left(\exp\left(-\frac{2^{NR/l}-1}{\rho_{S}\sigma_{SR}^{2}}\right)\right)^{i}, \ l = 1, \dots, N,$$
(12)

denotes the probability that less than K relay nodes can successfully decode the source message at the  $l^{th}$  channel use. Second,  $P_{SR}^{\text{Case 1}}(\mathbf{H}_{SR}, l)$  can be upper-bounded<sup>5</sup> as shown in the following lemma.

Lemma 1: The outage probability for the DF-MSC-opt scheme given that K relay nodes can successfully decode the source message at the l<sup>th</sup> channel use can be upper bounded as

$$P_{SR}^{\text{Case I}}(\mathbf{H}_{SR}, l) \le \Pr\left(f(\alpha_l, \mathbf{H}, \mathbf{V}^{\star}) < R\right), \ l = 1, \dots, N,$$
(13)

where  $\alpha_l = l/N$ ,  $f(\alpha_l, \mathbf{H}, \mathbf{V}^*) = \alpha_l \log(1 + \rho_S |\mathbf{H}_{SD}|^2) + (1 - \alpha_l) \sum_{i=1}^{L_T} \log_2 (1 + \rho_S \sigma_D^2 \kappa(i))$  with  $L_T = \min(N_r, K + 1)$  denoting the number of transmitted streams in the cooperative phase, and  $\kappa(i)$ ,  $i = 1, \dots, L_T$ , are  $(L_T \text{ out of } K + 1)$  ordered  $\chi^2$ -distributed variables with  $2N_r$  degrees of freedom.

Proof: Refer to Appendix A for the proof.

Third, when fewer than K relay nodes can successfully decode the source message within N channel uses, the source node transmits to the destination node with the direct link only; it follows that  $Pr(\mathbf{H}_{SR} \in \bigcup_{l>N} \mathcal{H}_l) = \Phi_N$  and  $Pr(\log(1 + \log(1 + \log($  $\rho_S |\mathbf{H}_{SD}|^2 < R = F\left(\frac{2^R - 1}{\rho_S \sigma_D^2}; N_r\right)$ . Therefore, the outage probability (cf. (10)) is given by

$$\mathcal{P}_{out}^{\star} = \Phi F\left(\frac{2^{R}-1}{\rho_{S}\sigma_{D}^{2}}; N_{r}\right) + \sum_{\alpha_{l}=1/N}^{1} (\Phi_{l-1} - \Phi_{l}) P_{SR}^{\text{Case 1}}(\mathbf{H}_{SR}, l)$$

$$\leq \Phi F\left(\frac{2^{R}-1}{\rho_{S}\sigma_{D}^{2}}; N_{r}\right) + \sum_{\alpha_{l}=1/N}^{1} (\Phi_{l-1} - \Phi_{l}) \Pr\left(f(\alpha_{l}, \mathbf{H}, \mathbf{V}^{\star}) < R\right)$$

$$= \Phi F\left(\frac{2^{R}-1}{\rho_{S}\sigma_{D}^{2}}; N_{r}\right) - \Phi_{l'}^{\prime} \Pr\left(f(\alpha_{l}^{\prime}, \mathbf{H}, \mathbf{V}^{\star}) < R\right), \qquad (14)$$

<sup>4</sup>As such, there is no loss of generality to assume  $\Gamma = \rho_S \mathbf{I}_{M+1}$  when studying high SNR analysis. <sup>5</sup>It is non-trivial to evaluate  $P_{SR}^{\text{Case 1}}(\mathbf{H}_{SR}, l)$  exactly due to the dynamics of the optimal nodes selection.

where  $\Phi'_l$  and  $\Phi'_{l'}$  denote the first order derivative of  $\Phi_l$  with respect to  $\alpha_l$  and evaluated at  $l = \alpha_l N$  and  $l' = \alpha'_l N$ , respectively. Note that in the last step of (14) we apply the *Mean Value Theorem* [14] which guarantees the existence of the point  $\alpha'_l \in (0, 1)$ . Moreover, the outage probability upper bound of the DF-MSC-opt scheme is given by the following theorem.

Theorem 2: For sufficiently large N, the outage probability upper bound of the DF-MSC-opt scheme is given by

$$\mathcal{P}_{out}^{\star} \leq \underbrace{\Phi F\left(\frac{2^{R}-1}{\rho_{S}\sigma_{D}^{2}}; N_{r}\right)}_{\text{Direct Transmission}} - \underbrace{\Phi_{l'}^{\prime} F\left(\frac{2^{\frac{R}{\alpha_{l}^{\prime}}-1}}{\rho_{S}\sigma_{D}^{2}}; N_{r}\right) F\left(\frac{2^{\frac{1}{1-\alpha_{l}^{\prime}}-1}}{\rho_{S}\sigma_{D}^{2}}; N_{r}\right)^{K+1}}_{\text{Relav-Assisted Transmission}}.$$
(15)

*Proof:* Refer to Appendix B for the proof.

For systems without relays, the outage probability is given by  $\mathcal{P}_{out,pp} = F\left(\frac{2^R-1}{N_r\rho_S\sigma_D^2}; N_r\right)$ . Note that the DF-MSC-opt scheme can achieve lower outage probability by taking advantage of multi-stream cooperative transmission. We can interpret  $-\Phi'_{l'}$  as the *cooperative level*, which quantifies the probability that the relay nodes can *assist* the direct transmission. For example,  $-\Phi'_{l'} = 1$  implies that the DF-MSC-opt scheme can be fully utilized, whereas  $-\Phi'_{l'} = 0$  implies that the source node transmits to the destination node with the direct link only. As per (14), the outage probability  $\mathcal{P}^*_{out}$  is a decreasing function with respect to  $^6-\Phi'_{l'}$ , whereas  $-\Phi'_{l'}$  is a decreasing function with respect to the strength of the S-R links  $\sigma_{SR}^2$ . As shown in Fig. 4, when the strength of the S-R links increases from 30dB to 40dB, the cooperative level increases and hence the outage probability decreases. On the other hand, as shown in (15) and as illustrated in Fig. 5, the outage probability decreases with increasing number of receive antennas at the destination node.

#### V. DMT AND TRT ANALYSES FOR THE DF-MSC-OPT SCHEME

In this section, we focus on characterizing the performance of the DF-MSC-opt scheme in the high SNR regime. Specifically, we perform DMT and TRT analyses based on the outage probability  $\mathcal{P}_{out}^{\star}$  as shown in (14). There are two fundamental reasons for the performance advantage of the proposed DF-MSC-opt scheme, namely multi-stream cooperation and optimal node selection  $\mathbf{V}^{\star}$  (cf. (6)). To illustrate the contribution of the first factor, we shall compare the performance of DF-MSC-opt with the following baselines:

- *DF-SDiv* (*Baseline 1*): *The DF relay protocol for cooperative spatial diversity*. The listening phase and the cooperative phase have fixed durations, i.e., each phase consists of N/2 channel uses. The source node and *all* the successfully decoding relay nodes cooperate using distributed space-time coding. At the destination node, maximum ratio combining (MRC) is used to combine the observations from the different receive antennas.
- *AF-SDiv* (*Baseline 2*): *The AF relay protocol for cooperative spatial diversity*. All the relay nodes transmit a scaled version of their soft observations in the cooperative phase. At the destination node, MRC is used to combine the observations from different receive antennas.
- *DDF* (*Baseline 3*): The *dynamic DF protocol* [6]. Once a relay node successfully decodes the source message, it immediately joins the transmission using distributed space-time coding. At the destination node, MRC is used to combine the observations from different receive antennas.

On the other hand, to illustrate the contribution of optimal node selection, we shall compare the performance of the DF-MSCopt scheme with the DF-MSC-rand scheme (Baseline 4), which corresponds to a similar multi-stream cooperation scheme but with randomized node selection V randomly generated from the node selection space  $\Omega$  (cf. (5)). Table I summarizes the major differences among the DF-MSC-opt scheme and the baseline schemes. We illustrate in Fig. 6 the outage capacity versus SNR (with  $\mathcal{P}_{out} = 0.01$ ) of the cooperative systems with  $N_r = 3$ , K = 3, M = 15. Note that the proposed DF-MSC-opt scheme can achieve a gain of more than 1 bit/channel use over the four baseline schemes.

## A. DMT Analysis

In order to analyze the DMT relation of the DF-MSC-opt scheme, we first derive the relation between the outage probability and the multiplexing gain. In the outage probability expression (14), for a sufficiently large  $\rho_S$ , the asymptotic expression of the first term is given by

$$\Phi F\left(\frac{2^{R}-1}{\rho_{S}\sigma_{D}^{2}};N_{r}\right) \doteq \sum_{i=0}^{K-1} \binom{M}{i} \rho_{S}^{-(M-i)(1-r)^{+}} \rho_{S}^{-N_{r}(1-r)^{+}} \doteq \rho_{S}^{-(M-K+1+N_{r})(1-r)^{+}},\tag{16}$$

and the asymptotic expression of  $\Phi'_{l'}$  is given by

₫

$$\begin{aligned}
\mathcal{P}'_{l'} &= \sum_{i=0}^{K-1} \binom{M}{i} (1-\phi_{l'})^{M-i-1} (\phi_{l'})^{i-1} ((M-i)(1-\phi_{l'})'\phi_{l'} + i(1-\phi_{l'})(\phi_{l'})') \\
&\doteq \sum_{i=0}^{K-1} \binom{M}{i} \rho_S^{-(M-i)\left(1-\frac{r}{\alpha_l'}\right)^+} \stackrel{-\min_i(M-i)\left(1-\frac{r}{\alpha_l'}\right)^+}{\leq \rho_S} &= \rho_S 
\end{aligned} \tag{17}$$

<sup>6</sup>For a sufficiently large SNR  $\rho_S$ ,  $F\left(\frac{2\frac{R}{\alpha_l'-1}}{\rho_S\sigma_D^2}; N_r\right)F\left(\frac{2\frac{R}{L_T(1-\alpha_l')}}{\rho_S\sigma_D^2}; N_r\right)^{K+1}$  is much smaller than  $F\left(\frac{2^R-1}{N_r\rho_S\sigma_D^2}; N_r\right)$ .

where  $\phi_{l'} = \exp\left(-\frac{2^{\frac{r\log\rho_S}{\alpha_{l'}}-1}}{\rho_S\sigma_{SR}^2}\right)$ . Moreover, we can express the term  $\Pr\left(f(\alpha'_l, \mathbf{H}, \mathbf{V}) < R\right)$  in (14) as  $\Pr\left(f(\alpha'_l, \mathbf{H}, \mathbf{V}) < R\right) \doteq \Pr\left(\rho_S^{\alpha'_l(1-\delta)^+} \prod_{i=1}^{L_T} \rho_S^{(1-\alpha'_l)(1-\beta_i)^+} < \rho_S^r\right)$ 

$$\Pr\left(j(\alpha_l, \mathbf{n}, \mathbf{v}) < \kappa\right) = \Pr(\rho_S + \prod_{i=1}^{l} \rho_S + (\gamma_i) < \gamma_i) = \sum_{j=1}^{l} \rho_j + \sum_{i=1}^{L_T} (1 - \alpha_l')(1 - \beta_i)^+ < r\right) = \iint_{\mathcal{B}} p(\delta, \beta) d\delta d\beta,$$
(18)

where  $-\delta$  is the exponential order of  $\sigma_{SD}^2$ ,  $-\beta_i$  is the exponential order of  $\gamma(i)$ ,  $\mathcal{B} = \{\delta, \beta : \alpha'_l(1-\delta)^+ + (1-\alpha'_l)\sum_i (1-\beta_i)^+ < r\}$ , and  $p(\delta, \beta)$  is the joint probability density function of  $\delta$  and  $\beta$ . Substituting (16)-(18) into (14), the outage probability can be expressed as

$$\mathcal{P}_{out}^{\star} \stackrel{i}{\leq} \rho_S^{-(M-K+1+N_r)(1-r)^+} + \rho_S^{-(M-K+1)(1-\frac{r}{\alpha_l})^+} \iint_{\mathcal{B}} p(\delta,\beta) \mathrm{d}\delta \mathrm{d}\beta, \tag{19}$$

and the DMT relation for the DF-MSC-opt scheme is given by the following theorem.

*Theorem 3:* The DMT relation for the DF-MSC-opt scheme can be expressed as

$$d(r, K)_{\text{DF-MSC-opt}} = \min(d_1, d_2 + d_3),$$
 (20)

where  $d_1 = (M - K + 1 + N_r)(1 - r)^+$ ,  $d_2 = (M - K + 1)(\frac{1 - 2r}{1 - r})^+$ , and

$$d_{3} = \begin{cases} N_{r} + (N_{r} - 1)K & 0 \le r < 1/2\\ \min\left(\frac{N_{r} + (N_{r} - \theta)(K+1-\theta)}{-(N_{r} + K - 2\theta)(\frac{r}{1-r} - \theta)}, \frac{(N_{r} - \theta)(K+1-\theta)}{+N_{r}\theta(\frac{1-r}{r})}\right) & \frac{\theta}{\theta+1} \le r < \frac{\theta+1}{\theta+2}\\ N_{r}L_{T}(\frac{1-r}{r}) & \frac{L_{T}}{L_{T}+1} \le r \le 1 \end{cases}$$
(21)

with  $\theta = 1, 2, ..., L_T$  and  $L_T = \min(N_r, K + 1)$ .

*Proof:* Refer to Appendix C for the proof.

We could further optimize the parameter K in the DF-MSC-opt scheme and the resulting DMT relation is given by

$$d(r)_{\text{DF-MSC-opt}}^{\star} = \max_{K \in [1,2,\dots,M]} d(r,K)_{\text{DF-MSC-opt}} = \max\left(d(r,\lfloor K^{\star} \rfloor)_{\text{DF-MSC-opt}}, d(r,\lceil K^{\star} \rceil)_{\text{DF-MSC-opt}}\right),$$

where

$$K^{\star} = \begin{cases} \frac{(M+1)(2-4r+r^2)+N_r(2-3r+r^2)}{(N_r-1)(1-r)+r^2} & 0 \le r \le 1/2\\ \frac{(M+1)(1-r)^2-N_r(3r-r^2-(1-r)2\theta)-\theta(3\theta(1-r)-1-r)}{N_r(1-r)-r+(1-r)^2} & \frac{\theta}{\theta+1} < r \le \frac{\theta+1}{\theta+2} \end{cases}$$
(22)

is the optimal number of successfully decoding relay nodes to wait for in the listening phase.

In the following, we show that the DF-MSC-opt scheme achieves superior DMT performance than the traditional cooperative diversity schemes. Specifically, as per [15], the AF-SDiv and DF-SDiv schemes have identical DMT relations given by  $d(r)_{AF/DF-SDiv} = M(1-2r)^+ + N_r(1-r)^+$  for  $0 \le r \le 1$ . For the DDF protocol, the DMT relation is given by the following lemma.

Lemma 2: The DMT relation for the DDF protocol with  $N_r$  receive antennas at the destination node can be expressed as

$$d(r)_{\text{DDF}} = \begin{cases} (M+N_r)(1-r) & 0 \le r \le \frac{N_r}{M+N_r} \\ N_r + M(\frac{1-2r}{1-r})^+ & \frac{N_r}{M+N_r} \le r \le \frac{1}{2} \\ N_r(\frac{1-r}{r}) & \frac{1}{2} \le r \le 1 \end{cases}$$
(23)

*Proof:* Refer to Appendix D for the proof.

In Fig. 7, we compare the DMT relations for the DF-MSC-opt scheme and the baseline schemes. For a system with M = 15 relay nodes and a destination node with  $N_r = 3$  antennas. Note that since the DF-MSC-opt scheme (as well as the DF-MSC-rand scheme) exploits multi-stream cooperation, it can achieve high diversity gain than the traditional cooperative diversity schemes (i.e. AF-SDiv, DF-SDiv, and DDF) that exploit single-stream cooperation. Moreover, since the DF-MSC-opt scheme optimizes the node selection policy and the number of decoding relay nodes K, it can achieve better diversity gain than the DF-MSC-rand scheme.

# B. TRT Analysis

The DMT analysis alone cannot completely characterize the fundamental tradeoff relation in the high SNR regime. Specifically, the multiplexing gain gives the asymptotic growth rate of the transmission rate R at high SNR  $\rho$  and is only applicable to scenarios where R scales *linearly* with  $\log \rho$ . Hence, there are many unique transmission rates that correspond to the same multiplexing gain, and the DMT analysis only gives a first order comparison of the performance tradeoff at high SNR when we have different multiplexing gains. In order to have a clearer picture on the tradeoff relations, in the following we study the SNR shift in the outage probability  $\mathcal{P}_{out}$  as we increase the transmission rate R by  $\Delta R$ . We quantify the more detailed relations among the three parameters  $(R, \log \rho, \mathcal{P}_{out}(R, \rho))$  by analyzing the TRT<sup>7</sup> relation [16].

Consider the outage probability expression for the DF-MSC-opt scheme (14). In order to facilitate studying the TRT, we choose the parameter K such that the outage events are dominated by relay-assisted transmissions, i.e.  $K \ge \lceil K^* \rceil$  (where  $K^*$  is given by (22)). The following theorem characterizes the asymptotic relationships among R,  $\rho_S$ , and  $\mathcal{P}_{out}^*$  for the  $r > \frac{1}{2}$  case<sup>8</sup>.

Theorem 4: The TRT relation for the DF-MSC-opt scheme under  $K \ge \lceil K^* \rceil$  and  $r > \frac{1}{2}$  is given by

$$\lim_{\rho_S \to \infty, (R, \rho_S) \in \mathcal{R}(z)} \frac{\log \mathcal{P}_{out}^* - c_{\text{DF-MSC-opt}}(z)R}{\log \rho_S} = -g_{\text{DF-MSC-opt}}(z)$$
(24)

where

$$\mathcal{R}(z) \triangleq \left\{ (R, \rho_S) | z + 1 > \frac{R}{(1-r)\log \rho_S} > z \right\} \text{ for } z \in \mathbb{Z}, 0 \le z < L_T,$$
(25)

denotes the  $z^{\text{th}}$  operating region,  $c_{\text{DF-MSC-opt}}(z) \triangleq K + 1 + N_r - (2z + 1)$ , and  $g_{\text{DF-MSC-opt}}(z) \triangleq (K + 1)N_r - z(z + 1)$ . Note that  $g_{\text{DF-MSC-opt}}(z)$  is defined as the reliability gain coefficient and  $t_{\text{DF-MSC-opt}}(z) \triangleq g_{\text{DF-MSC-opt}}(z)/c_{\text{DF-MSC-opt}}(z)$  is defined as the throughput gain coefficient.

*Proof:* Refer to Appendix E for the proof.

By applying Theorem 4, the SNR shift between two outage curves with a  $\Delta R$  rate difference is  $3\Delta R/t(z)$  dB. Fig. 8 shows the outage curves corresponding to  $\Delta R = 2$  bits/channel use for  $N_r = 3$ , K = 3 and M = 15. This scenario corresponds to the region  $\mathcal{R}(1)$  and the TRT relation for the DF-MSC-opt scheme can be expressed as  $g_{\text{DF-MSC-opt}}(1) = 10$  and  $t_{\text{DF-MSC-opt}}(1) = \frac{5}{2}$ . As we can see from the simulation results, the SNR shift is 2.4 dB for 2 bits/channel use increase in the transmission rate, which matches with our analysis.

#### VI. CONCLUSION

In this paper, we proposed a multi-stream cooperative scheme (DF-MSC-opt) for multi-relay network. Optimal multi-relay selection is considered and we derived the associated outage capacity as well as the DMT and TRT relations. The proposed design has significant gains in both the outage capacity as well as the DMT relation due to (1) multi-stream transmissions in the cooperative phase and (2) optimized relay selection.

#### APPENDIX A

#### **PROOF OF LEMMA 1**

To find the outage probability given l, we characterize the function of  $g(\mathbf{H}, \mathbf{V}^{\star})$  under different conditions.

- Condition 1  $K < N_r$ : Under this condition, we allow all the successfully decoding relays to transmit in the cooperative phase. Thus, the communication links can be regarded as a conditional MIMO link [17] and  $g(\mathbf{H}, \mathbf{V}) = \log \det (\mathbf{I}_{N_r} + \rho_S \mathbf{H}_D(\mathcal{D})\mathbf{H}_D(\mathcal{D})^H) \leq \sum_{i=1}^{K+1} \log_2(1 + \rho_S \sigma_D^2 \kappa(i))$ , where  $\kappa(i)$ ,  $i = 1, \ldots, K+1$ , are  $\chi^2$ -distributed variables with  $2N_r$  degrees of freedom.
- Condition 2  $K \ge N_r$ : Under this condition, we select  $N_r$  nodes out of the source and the successfully decoding relay nodes from the node selection space  $\Omega$  to participate in cooperative transmission. The analytical solution for the term  $g(\mathbf{H}, \mathbf{V})$  is in general not trivial [18]. Since in the proposed DF-MSC-opt scheme we choose the *best*  $N_r$  out of K + 1 transmit nodes, the upper bound can be obtained similar to [18, (6)]. Thus, the capacity bound with transmit node selection is  $g(\mathbf{H}, \mathbf{V}) = \max_{\mathbf{V} \in \Omega} \log \det (\mathbf{I}_{N_r} + \rho_S \mathbf{H}_D(\mathcal{D}) \mathbf{V} \mathbf{V}^H \mathbf{H}_D(\mathcal{D})^H) \le \sum_{i=1}^{N_r} \log_2(1 + \rho_S \sigma_D^2 \kappa(i))$ , where the  $\kappa(i)$ ,  $i = 1, \ldots, N_r$ , are ordered  $\chi^2$ -distributed variables with  $2N_r$  degrees of freedom.

Combining the two conditions above, we obtain the general form as shown in Lemma 1.

# APPENDIX B

## **PROOF OF THEOREM 2**

From (14), the only unknown part is given by

$$\Pr(f(\alpha'_{l}, \mathbf{H}, \mathbf{V}) < R) = \Pr[\alpha'_{l} \log(1 + \rho_{S} |\mathbf{H}_{SD}|^{2}) + (1 - \alpha'_{l}) \sum_{i=1}^{L_{T}} \log_{2}(1 + \rho_{S} \sigma_{D}^{2} \kappa(i)) < R],$$

<sup>7</sup>TRT analysis allows for investigating more general scenarios where the transmission rate R does not scale linearly with  $\log \rho$  and helps to study the SNR gain of the outage probability vs SNR curve when we increase the transmission rate R by  $\Delta R$ .

<sup>&</sup>lt;sup>8</sup>For the  $r \leq \frac{1}{2}$  case, the MIMO channel formed by the multiple relay nodes to the destination node can only operate with a multiplexing gain of 1 and the TRT relation is trivial.

which can be relaxed as

$$\begin{aligned} &\Pr[\alpha_{l}' \log(1 + \rho_{S} |\mathbf{H}_{SD}|^{2}) + (1 - \alpha_{l}') \sum_{i=1}^{L_{T}} \log_{2}(1 + \rho_{S} \sigma_{D}^{2} \kappa(i)) < R] \\ &\leq \Pr(\alpha_{l}' \log(1 + \rho_{S} |\mathbf{H}_{SD}|^{2}) < R) \Pr((1 - \alpha_{l}') \sum_{i=1}^{N_{r}} \log_{2}(1 + \rho_{S} \sigma_{D}^{2} \kappa(i)) < R) \\ &= F\left(\frac{2^{\frac{R}{\alpha_{l}'}} - 1}{\rho_{S} \sigma_{D}^{2}}; N_{r}\right) \Pr\left(\sum_{i=1}^{L_{T}} \log_{2}\left(1 + \rho_{S} \sigma_{D}^{2} \kappa(i)\right) < \frac{R}{1 - \alpha_{l}'}\right). \end{aligned}$$

Moreover, the following relation holds for  $\alpha'_l > 0$ 

$$\Pr\left(\sum_{i=1}^{L_T} \log_2\left(1 + \rho_S \sigma_D^2 \kappa(i)\right) < \frac{R}{1 - \alpha_l'}\right) \le \Pr\left(\log_2\left(1 + \rho_S \sigma_D^2 \kappa(1)\right) < \frac{R}{1 - \alpha_l'}\right)$$
$$= \Pr\left(\kappa(1) < \frac{2^{\frac{R}{1 - \alpha_l'} - 1}}{\rho_S \sigma_D^2}\right) = F\left(\frac{2^{\frac{R}{1 - \alpha_l'} - 1}}{\rho_S \sigma_D^2}; N_r\right)^{K+1}$$
(26)

where in the last step we applied the results of order statistics [19]. Substituting (26) into (14), we have Theorem 2.

#### APPENDIX C

## PROOF OF THEOREM 3

The asymptotic expression of the outage probability  $\mathcal{P}_{out}^{\star}$  is given by (19) with  $\mathcal{B} = \{\delta, \beta : \alpha'_l(1-\delta)^+ + (1-\alpha'_l)\sum_i (1-\beta_i)^+ < r\}$ .

From the first term in (19), we can obtain  $d_1 = (M - K + 1 + N_r)(1 - r)^+$ .

Since the source node is equipped with a single antenna, the maximum multiplexing gains for the S-R links as well as the S-D and R-D links is always less than 1. Specifically, the multiplexing gain in the listening phase is  $\frac{r}{\alpha'_l} \leq 1$  and the multiplexing gain in the cooperative phase is  $\frac{r}{1-\alpha'_l} \leq 1$  or equivalently,  $\alpha'_l \geq \max(r, 1-r)$ . Thus, we can evaluate the second term through  $d_2 = (M - K + 1)(1 - \frac{r}{1-r})$  for r < 1/2 and 0 otherwise.

We evaluate the third term under the following cases. When r < 1/2,  $\alpha'_l \ge 1 - r$  and  $\mathcal{B} \subseteq \{\delta, \beta : r((1-\delta)^+ + \sum_i (1-\beta_i)^+) < r\}$ . Equivalently,

$$\iint_{\mathcal{B}} p(\delta,\beta) \mathrm{d}\delta \mathrm{d}\beta \leq \iint_{(1-\delta)^{+} + \sum_{i}(1-\beta_{i})^{+} \leq 1} p(\delta,\beta) \mathrm{d}\delta \mathrm{d}\beta$$
$$\doteq \rho_{S}^{-\inf_{r' \in [0,1]} \left( N_{r}(K+1) - (N_{r}+K)r' + N_{r}r' \right)} = \rho_{S}^{-(N_{r}-1)K-N_{r}}.$$
(27)

As a result,  $d_3 = (N_r - 1)K + N_r$  for r < 1/2. When  $1/2 \le r < 1$ ,  $\alpha'_l \ge r$  and the corresponding  $\mathcal{B} = \{\delta, \beta : r(1-\delta)^+ + (1-r)\sum_i (1-\beta_i)^+ r < r\}$ . Equivalently,

$$\iint_{\mathcal{B}} p(\delta,\beta) \mathrm{d}\delta \mathrm{d}\beta \leq \iint_{(1-\delta)^{+} + \frac{1-r}{r} \sum_{i} (1-\beta_{i})^{+} \leq 1} p(\delta,\beta) \mathrm{d}\delta \mathrm{d}\beta}$$
$$\stackrel{-}{=} \rho_{S}^{-\inf_{r' \in [0,1]} \left( (N_{r} - \frac{rr'}{1-r})(K+1 - \frac{rr'}{1-r}) - N_{r}r' \right)}$$
(28)

for  $\frac{rr'}{1-r} \in \{1, \ldots, L_T\}$ . Thus, we can write the general form as  $d_3 = \inf_{0 \le r' \le 1} \left(N_r r' + (N_r - \theta)(K + 1 - \theta) - (N_r + K - 2\theta)(\frac{rr'}{1-r} - \theta)\right)$  for  $\theta = \{1, 2, \ldots, L_T\}$ . Since  $d_3$  is linear with respect to r', we can write  $d_3 = \min \left\{N_r + (N_r - \theta)(K + 1 - \theta) - (N_r + K - 2\theta)(\frac{r}{1-r} - \theta), N_r \theta(\frac{1-r}{r}) + (N_r - \theta)(K + 1 - \theta)\right\}$ . Moreover, when  $\frac{r}{1-r} \ge L_T$ , we have  $\frac{rr'}{1-r} - L_T = 0$  and  $d_3 = N_r L_T(\frac{1-r}{r})$ .

Combining the three terms above, we have Theorem 3.

## APPENDIX D

# PROOF OF LEMMA 2

The main steps of the proof are based on the work by [6]. Denote  $g_{k,j}$  to be the channel coefficient between the  $j^{th}$  node and the  $k^{th}$  receive antenna at the destination node, and the received signal at the  $k^{th}$  antenna is given by  $y_k = g_{k,j}x_j + z_k$ . Following the same argument in the proof of Theorem 6 in [6], we can find that destinations ML error probability assuming *error-free* decoding at all of the relays, provides a lower bound on the diversity gain achieved by the protocol and the corresponding PEP for  $k^{th}$  antenna is upper bounded by  $P_k \leq \prod_{j=1}^{M+1} \left[ 1 + \left( \sum_{i=1}^j |g_{k,i}|^2 \right) \rho \right]^{-n_j}$ , where  $n_j$  is the number of symbol intervals in the codeword during which a total of j nodes are transmitting, so that, the total number of codeword length  $\sum_{j=1}^{M+1} n_j = N$ . Hence, the PEP for the destination to received the information is upper bounded by  $P \leq \prod_{k=1}^{N_r} P_k$ . Note that, the upper bound is not tight since we do not consider the joint decoding among the receive antennas, but it is sufficient to derive the order-wise relation. By changing the order of k and j, we have  $P \leq \prod_{j=1}^{M+1} \left[ \prod_{k=1}^{N_r} \left( 1 + \left( \sum_{i=1}^j |g_{k,i}|^2 \right) \rho \right) \right]^{-n_j}$ .  $\sum_{j=1}^p n_j$  is the number of symbol intervals that relay p has to wait, before the mutual information between its received signal and the signals that the source and other relays transmit exceeds NR. Thus, we have  $\sum_{j=1}^p n_j \leq \min\left\{ N, \frac{NR}{\log(1+|h_{p+1,1}|^2\rho)} \right\}$ , where  $h_{p+1,1}$  denotes the channel condition between the relay node p and the source node.

Define  $v_{k,i}$  and  $u_{j,i}$  as the exponential orders of  $g_{k,i}$  and  $h_{j,i}$ . We have the following relation,

$$P \le \rho^{-\sum_{j=1}^{M} n_j \left(1 - \min\{\sum_{k=1}^{N_r} v_{k,1}, \dots, \sum_{k=1}^{N_r} v_{k,j}\}\right)^+} = \rho^{-\{\sum_{j=1}^{M} n_j (1 - N_r \min\{v_1, \dots, v_j\})^+},$$

where  $v_i = \frac{\sum_{k=1}^{N_r} v_{k,i}}{N_r}$ . Choose the rate  $R = r \log \rho$  and the PEP bound for rate R is given by

$$P \le \rho^{-\sum_{j=1}^{M} n_j \left(1 - \min\{\sum_{k=1}^{N_r} v_{k,1}, \dots, \sum_{k=1}^{N_r} v_{k,j}\}\right)^+} = \rho^{-N\left[\{\sum_{j=1}^{M} \frac{n_j}{N} (1 - N_r \min\{v_1, \dots, v_j\})^+ - r\right]}$$

Hence, the set of channel realizations that satisfy  $\{\sum_{j=1}^{M} \frac{n_j}{N} (1 - N_r \min\{v_1, \dots, v_j\})^+ \le r \text{ results in the outage event.}$ Define  $\bar{v}_j = \min\{v_1, \dots, v_j\}$  and we can separate the discussion in the following three cases. (Naturally, we have  $\bar{v}_1 \ge 1$  $\bar{v_2} \geq \ldots \geq \bar{v_M}$ ).

- Case 1:  $\bar{v_1} \leq 1$ . In this case, we can follow the discussion from (62) to (71) in [6] and show that the diversity order d is
- Case 1:  $v_1 \leq 1$ : If this case, we can below the  $d \geq \begin{cases} (N_r + M)(1 r), & \frac{N_r}{N_r + M} \geq r \geq 0\\ N_r + M(\frac{1 2r}{1 r}), & \frac{1}{2} \geq r > \frac{N_r}{N_r + M} \\ N_r(\frac{1 r}{r}), & 1 \geq r > \frac{1}{2} \end{cases}$  Case 2:  $v_M \geq 1$ . It's trivial and d = M.

• Case 2.  $v_M \ge 1$ . It is drived and u = M. • Case 3:  $\bar{v}_i > 1 \ge v_{i+1}^-$ . Follow the same discussion from (72) to (82) in [6], we conclude that  $d \ge \begin{cases} N_r + M(\frac{1-2r}{1-r}), & \frac{1}{2} \ge r \ge 0\\ N_r(\frac{1-r}{r}), & 1 \ge r > \frac{1}{2} \end{cases}$ .

Combining the above cases, we have Lemma 2.

### APPENDIX E **PROOF OF THEOREM 4**

Given  $r > \frac{1}{2}$  and  $K \ge \lceil K^* \rceil$ , (14) can be simplified as  $\mathcal{P}_{out}^* \le -\Phi'_{l'} \Pr(f(\alpha'_l, \mathbf{H}, \mathbf{V}^*) < R)$ . Substituting the expression of  $f(\alpha'_{1}, \mathbf{H}, \mathbf{V})$ , we have

$$\mathcal{P}_{out}^{\star} \leq \Pr(\alpha_l' \log(1 + \rho_S |\mathbf{H}_{SD}|^2) + (1 - \alpha_l') \sum_{i=1}^{L_T} \log_2(1 + \rho_S \sigma_D^2 \gamma(i)) < R) 
 \leq \Pr((1 - r) \sum_{i=1}^{L_T} \log_2(1 + \rho_S \sigma_D^2 \gamma(i)) < R).$$
(29)

Based on the above results and following the same lines as the proof of [16, Theorem 2], we have the result of Theorem 4.

## REFERENCES

- [1] J. N. Laneman and G. W. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," IEEE Trans. Inf. Theory, vol. 49, pp. 2415 - 2425, Oct. 2003.
- A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperative diversity part I: System description," IEEE Trans. Commun., vol. 51, pp. 1927 1938, [2] Nov. 2003.
- [3] A. Nosratinia, T. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," IEEE Commun. Mag., vol. 42, pp. 68 73, Oct. 2004.
- [4] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," IEEE J. Sel. Areas Commun., vol. 24, pp. 659 - 672, Mar. 2006.
- [5] A. F. Molisch, N. B. Mehta, J. S. Yedidia, and J. Zhang, "Cooperative relay networks using fountain codes," in Proc. IEEE GLOBECOM'06, Nov. 2006.
- [6] K. Azarian, H. E. Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," IEEE Trans. Inf. Theory, vol. 51, pp. 4152 - 4172, Dec. 2005.
- [7] K. R. Kumar and G. Caire, "Coding and decoding for the dynamic decode and forward relay protocol," submitted to IEEE Trans. Inf. Theory, Jan. 2008.
- [8] M. Yuksel and E. Erkip, "Multiple-antenna cooperative wireless systems: A diversity-multiplexing tradeoff perspective," IEEE Trans. Inf. Theory, vol. 53, pp. 3371 - 3393, Oct. 2007.
- [9] H. Bölcskei, R. U. Nabar, O. Oyman, and A. J. Paulraj, "Capacity scaling laws in MIMO relay networks," IEEE Trans. Wireless Commun., vol. 5, pp. 1433 - 1444, Jun. 2006.
- [10] S. Borade, L. Zheng, and R. Gallager, "Amplify-and-forward in wireless relay networks: Rate, diversity, and network size," IEEE Trans. Inf. Theory, vol. 53, pp. 3302 - 3318, Oct. 2007.
- [11] R. Nikjah and N. C. Beaulieu, "Novel rateless coded selection cooperation in dual-hop relaying systems," in Proc. IEEE GLOBECOM'08, Nov. 2008, pp. 1 – 6.
- [12] T. M. Cover and A. J. Thomas, Elements of Information Theory. Wiley, 1991.
- [13] M. A. Khojastepour, A. Sabharwal, and B. Aazhang, "On capacity of Gaussian cheap relay channel," in Proc. IEEE GLOBECOM'03, Dec. 2003, pp. 1776 - 1780
- [14] W. Rudin, Principles of Mathematical Analysis. McGraw-Hill, 1976.
- [15] A. Tajer and A. Nosratinia, "Opportunistic cooperation via relay selection with minimal information exchange," in Proc. IEEE ISIT'07, 2007, pp. 1926 - 1930.
- [16] K. Azarian and H. E. Gamal, "The throughput-reliability tradeoff in block-fading MIMO channels," IEEE Trans. Inf. Theory, vol. 53, pp. 488 501, Feb. 2007.
- [17] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple antenna channels," IEEE Trans. Inf. Theory, vol. 49, pp. 1073 - 1096, May 2003.
- A. F. Molisch, M. Z. Win, Y.-S. Choi, and J. H. Winters, "Capacity of MIMO systems with antenna selection," IEEE Trans. Wireless Commun., vol. 4, pp. 1759 - 1772, Jul. 2005.
- [19] M. Ahsanullah and V. B. Nevzorov, Ordered random variables. Nova Science Publishers, 2001.



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Relay protocol	AF-SDiv	DF-SDiv	DDF	DF-MSC-rand	DF-MSC-opt
No. of transmitted streams in the cooperative phase	1	1	1	$N_r$	$N_r$
No. of relay nodes in the coop- erative phase	M	depends on S-R links	depends on S-R links	$N_r$	$N_r$ chosen from $K$
Receiver structure	MRC	MRC	MRC	ML	ML

 TABLE I

 Comparisons among the DF-MSC-opt scheme and the baseline schemes.

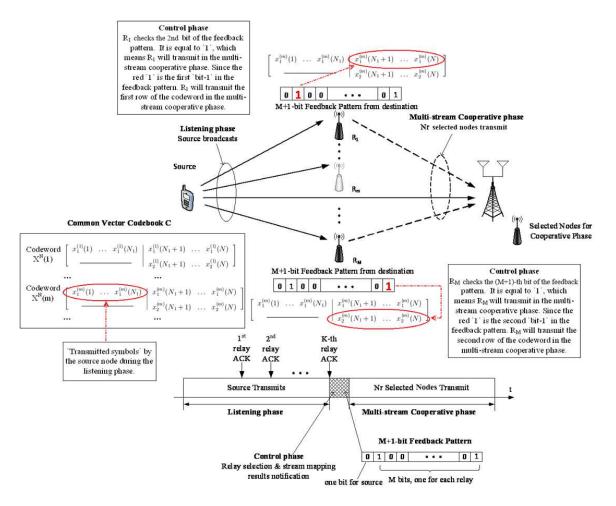


Fig. 1. Illustration of timing diagram and an example of feedback pattern design. In this example, two relay nodes  $R_1$  and  $R_M$  are selected to transmit in the cooperative phase. The selected relay nodes will re-encode the same codeword (codeword  $\mathbf{X}^N(m)$ ) in this example) selected from the same common vector codebook C and the *R*-bit message received in the listening phase.  $R_1$  will send out the "first" row of codeword  $\mathbf{X}^N(m)$  and  $R_M$  will send out the "second" row.

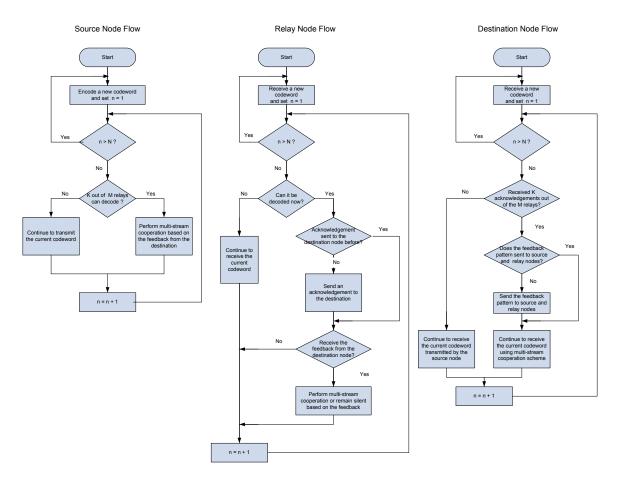


Fig. 2. Flow chart of the protocols at the source node, the relay node and the destination node.

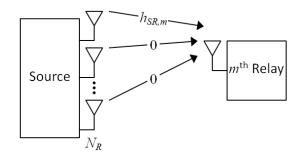


Fig. 3. The single-antenna source-relay channel is equivalent to a multi-antenna virtual MISO channel.

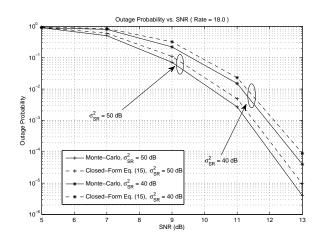


Fig. 4. Outage probability vs. SNR of the DF-MSC-opt scheme for different  $\sigma_{SR}^2$  under  $N_r = 3$ , K = 6 and M = 15.

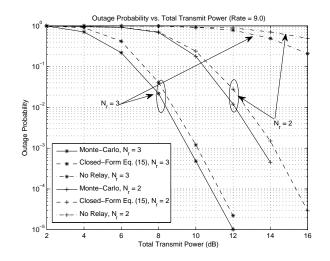


Fig. 5. Outage probability vs. SNR of the DF-MSC-opt scheme for different  $N_r$  under  $\sigma_{SR}^2 = 10 dB$ , K = 6 and M = 15.

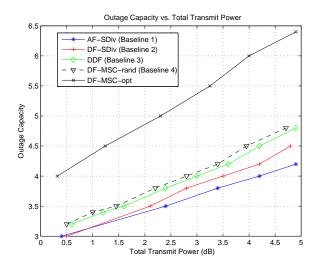


Fig. 6. Outage capacity comparison of different schemes for  $N_r = 3$ , K = 3 and M = 15. The channel variances of the S-R, R-D, S-D links are normalized to unity.

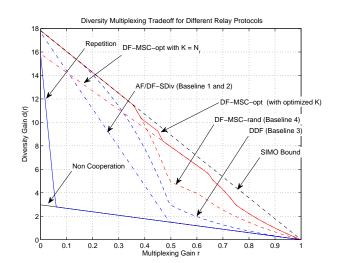


Fig. 7. Diversity-multiplexing tradeoff comparison of different relay protocols.

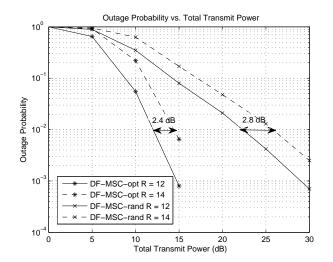


Fig. 8. Monte-Carlo simulation results for outage curves corresponding to  $\Delta R = 2$  bits/channel use for  $N_r = 3$ , K = 3 and M = 15 case with normalized S-R, R-D, S-D channel variances.

