

Two-Way Transmission Capacity of Wireless Ad-hoc Networks

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Abstract

The transmission capacity of an ad-hoc network is the maximum density of active transmitters per unit area, given an outage constraint at each receiver for a fixed rate of transmission. Most prior work on finding the transmission capacity of ad-hoc networks has focused only on one-way communication where a source communicates with a destination and no data is sent from the destination to the source. In practice, however, two-way or bidirectional data transmission is required to support control functions like packet acknowledgements and channel feedback. This paper extends the concept of transmission capacity to two-way wireless ad-hoc networks by incorporating the concept of a two-way outage with different rate requirements in both directions. Tight upper and lower bounds on the two-way transmission capacity are derived for frequency division duplexing. The derived bounds are used to derive the optimal solution for bidirectional bandwidth allocation that maximizes the two-way transmission capacity, which is shown to perform better than allocating bandwidth proportional to the desired rate in both directions. Using the proposed two-way transmission capacity framework, a lower bound on the two-way transmission

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capacity with transmit beamforming using limited feedback is derived as a function of bandwidth, and bits allocated for feedback.

I. INTRODUCTION

The transmission capacity of an ad-hoc wireless network is the maximum allowable spatial density of transmitting nodes, satisfying a per transmitter receiver rate, and outage probability constraint [1]–[4]. Essentially, the transmission capacity characterizes the maximum number of transmissions per unit area that can be simultaneously supported in an ad-hoc network under a quality of service constraint. The transmission capacity framework allows the application of the rich tool set of stochastic geometry to derive closed-form bounds for the interference distribution in a spatial network when the locations of nodes form a Poisson point process (PPP) [5].

In prior work, the transmission capacity has been used successfully to characterize the effect of various physical and medium access (MAC) layer techniques on the ad-hoc network capacity, such as successive interference cancelation [6], multiple antennas [7]–[10], and guard-zone based scheduling [11]. Most of the prior work on finding the transmission capacity has been limited to one-way communication (no data communication from the destination to the source), and precludes the possibility of two-way communication. In two-way (bidirectional) communication the destination also has data to send to its source, e.g. channel state feedback [12], packet acknowledgement [13], or route initiation and update requests [14].

In this paper we define the two-way transmission capacity, and derive tight upper and lower bounds on it when the transmitter location are distributed as a Poisson point process (PPP) distributed. The bounds are used to characterize the dependence of the two-way transmission capacity on the key system parameters, e.g. bandwidth allocation in two directions given a data rate requirement. We consider an ad-hoc network with two-way communication, where each source destination pair has data to exchange in both directions. We consider a general system where the data requirement in both directions can be different, and a frequency division duplex

(FDD) communication model, where two separate frequency carriers are used for two directions, thereby forming a full-duplex link.

In a two-way communication model, where the transmitter locations are modeled as a PPP, the interference received in both directions is correlated, and hence the joint success probability in two directions is not equal to the product of the success probabilities in each direction. Therefore finding the exact expression for the joint success probability is complicated. To obtain meaningful insights on the two-way transmission capacity, we derive tight upper and lower bounds on the two-way transmission capacity with FDD, assuming that the channel coefficients on separate frequencies are independent and all the channel coefficients are Rayleigh distributed. The upper and lower bound only differ by a constant, i.e. the bounds have identical dependence on the parameters of interest (rate requirements, and bandwidths allocated in each direction). Thus, the derived bounds establish the two-way transmission capacity up to a constant.

The results of this paper in part have been presented in [15], [16]. The differences between [15], [16] and the present paper are as follows. For simplification of analysis, [15] assumed that the interference received in both directions is independent. The independence assumption was removed in [16], and upper and lower bounds on the two-way transmission capacity that derived which were shown to be tight. Compared to [16], the present paper extends the two-way transmission capacity framework to quantify the loss in transmission capacity with practical limited feedback [17] in comparison to genie-aided feedback (channel coefficients are known exactly, and without any cost at the transmitter), when the transmitter is equipped with multiple antenna and uses beamforming to transmit its signal to the receiver. In addition to this, the present paper offers more clarity of exposition, complete proof of Theorem 2, and added simulation results for more insights into the effects of two-way communication.

Using the derived bounds on the two-way transmission capacity, we find the optimal bandwidth allocation in two directions that maximizes the transmission capacity. The optimal bandwidth allocation problem is shown to be a convex program in a single variable which can be solved

easily by finding the value where the function derivative is zero. Using the optimal bandwidth allocation solution, we show that an intuitive strategy that allocates the bandwidth in proportion to the desired rate in each direction is optimal only for symmetric traffic (same rate requirement in both directions) and performs poorly for asymmetric traffic in comparison to the optimal strategy. Examples of asymmetric traffic are channel feedback, and ack/nack messages, where there is huge disparity between the data rates required in two directions.

There is extensive related work on resource allocation in wireless ad hoc networks, but almost all of it focused on one-way communication. For instance, prior work studied the spectrum sharing between two one-way spatial networks in [18], between a spatial network and a cellular uplink network in [19], and one-way spatial networks where the total bandwidth is optimally split into sub-bands to maximize the transmission capacity [20]. Our bandwidth allocation, however, studies the bandwidth sharing between two directions within a single two-way spatial network.

As an application of the proposed two-way transmission capacity framework, we evaluate the performance degradation with practical limited channel feedback in comparison to genie aided channel feedback, when the transmitter has multiple antennas and uses beamforming for transmitting its signal to the receiver. We account for both the bandwidth used, and the bits required for feedback, to derive a lower bound on the two-way transmission capacity with transmit beamforming using limited feedback. We show that with practical limited channel feedback, the two-way transmission capacity is substantially reduction compared to the genie-aided case. The severe degradation results because with increasing the number of feedback bits, the transmission capacity increases sub-linearly due to improvement in signal strength, however, decreases exponentially because of the stringent requirement of feedback bits to be correctly decoded.

Notation: The expectation of function $f(x)$ with respect to x is denoted by $\mathbb{E}(f(x))$. A circularly symmetric complex Gaussian random variable x with zero mean and variance σ^2 is denoted as $x \sim \mathcal{CN}(0, \sigma^2)$. Let S_1 be a set and S_2 be a subset of S_1 . Then $S_2 \setminus S_1$ denotes the

set of elements of S_1 that do not belong to S_2 . The integral $\int_0^\infty x^{k-1}e^{-x}dx$ is denoted by $\Gamma(x)$. We use the symbol $:=$ to define a variable.

II. SYSTEM MODEL

Consider an ad-hoc network with two sets of nodes $\mathcal{T} := \{Tx_n, n \in \mathbb{N}\}$, and $\mathcal{R} = \{Rx_n, n \in \mathbb{N}\}$, where Tx_n and Rx_n want to exchange data between each other for each n . We assume that each Tx_n and Rx_n have a single antenna. We consider a slotted Aloha random access protocol, where at any given time, the pair (Tx_n, Rx_n) transmits data to each other with an access probability P_a for each n , independently of all other nodes. We assume that the distance between each Tx_n and Rx_n is d . Let the location of Tx_n be T_n , and Rx_n be R_n . The set $\Phi_T = \{T_n\}$ is modeled as a homogenous PPP on a two-dimensional plane with intensity λ_0 , similar to [1], [2], [9]. Since R_n is at a fixed distance d in a random direction from the T_n , the set $\Phi_R := \{R_n\}$ is also a homogenous PPP on a two-dimensional plane with intensity λ_0 , because of the random translation invariance property of PPP [21]. Because of the assumed Aloha random access protocol, at any given time, the active transmitter receiver location processes $\Phi_T^a := \{T_n | Tx_n \text{ is active}\}$, and $\Phi_R^a := \{R_n | Rx_n \text{ is active}\}$ are homogenous PPPs on a two-dimensional plane with intensity $\lambda = P_a \lambda_0$. We consider a frequency division duplex system, where the total available bandwidth is F_{total} , out of which F_{TR} is dedicated for $Tx_n \rightarrow Rx_n$ communication to support a rate demand B_{TR} bits for all n , and the rest $F_{RT} := F_{total} - F_{TR}$ for the $Rx_n \rightarrow Tx_n$ communication to support a rate demand of B_{RT} bits for all n .

In a time slot when the pair (Tx_0, Rx_0) is active, the received signal at receiver Rx_0 is

$$y_0 = \sqrt{P_t}d^{-\alpha/2}h_0x_0 + \sum_{T_n \in \Phi_T^a \setminus \{T_0\}} \sqrt{P_t}d_{T_n}^{-\alpha/2}h_{0n}x_n + z_0, \quad (1)$$

and the received signal at receiver Tx_0 is

$$w_0 = \sqrt{P_t}d^{-\alpha/2}g_0u_0 + \sum_{R_n \in \Phi_R^a \setminus \{R_0\}} \sqrt{P_t}d_{Rn}^{-\alpha/2}g_{0n}u_n + v_0, \quad (2)$$

where P_t is the transmit power, h_0 is the channel between Tx_0 and Rx_0 , and g_0 is the channel from Rx_0 and Tx_0 , h_{0n} and g_{0n} is the channel between Tx_n and Rx_0 , and Rx_n and Tx_0 , respectively, d_{Tn} and d_{Rn} are the distances between Tx_n and Rx_0 , and Rx_n and Tx_0 , respectively, $\alpha > 2$ is the path loss exponent, $x_n, u_n \in \mathcal{CN}(0, 1)$ are signals transmitted from Tx_n and Rx_n , respectively, and z_0, v_0 is the additive white Gaussian noise. The ad-hoc network is assumed to be interference limited [1], thus we drop the noise contribution from the received signal. We assume that h_0, g_0, h_{0n} , and g_{0n} are independent and identically distributed with $\mathcal{CN}(0, 1)$ to model a Rayleigh fading channel.

With the received signal model (1) and (2), the signal to interference ratio (SIR) for the transmission from $Tx_0 \rightarrow Rx_0$ and from $Rx_0 \rightarrow Tx_0$ are

$$SIR_{TR} := \frac{d^{-\alpha}|h_0|^2}{\sum_{T_n \in \Phi_T^a \setminus \{T_0\}} d_{Tn}^{-\alpha}|h_{0n}|^2}, \quad SIR_{RT} := \frac{d^{-\alpha}|g_0|^2}{\sum_{R_n \in \Phi_R^a \setminus \{R_0\}} d_{Rn}^{-\alpha}|g_{0n}|^2}.$$

Assuming interference as noise, the mutual information [22] for the Tx_0 to Rx_0 communication using bandwidth F_{TR} , and for the Rx_0 to Tx_0 communication using bandwidth $F_{total} - F_{TR}$ are

$$MI_{TR} := F_{TR} \log(1 + SIR_{TR}) \text{ bits/sec}, \quad MI_{RT} := (F_{total} - F_{TR}) \log(1 + SIR_{RT}) \text{ bits/sec}.$$

Recall that the rate requirement for the $Tx_0 \rightarrow Rx_0$ transmission is B_{TR} bits, and for the $Rx_0 \rightarrow Tx_0$ communication is B_{RT} bits. Thus, to account for the two-way or bidirectional nature of communication, we define the success probability (complement of the outage probability ϵ) as the probability that communication in both directions is successful simultaneously, i.e.

$$P_{success} = P(MI_{TR} > B_{TR}, MI_{RT} > B_{RT}). \quad (3)$$

Let λ be maximum density of nodes per unit area that can support rate B_{TR} from $Tx_0 \rightarrow Rx_0$, and B_{RT} bits from $Rx_0 \rightarrow Tx_0$ with success probability $P_{success} = 1 - \epsilon$, using bandwidth F_{total} .

Definition 1: The two-way transmission capacity C_ϵ is defined as

$$C_\epsilon := (1 - \epsilon)\lambda \left(\frac{B_{TR} + B_{RT}}{F_{Total}} \right) \text{ bits/sec/Hz/m}^2.$$

The problem to solve is to find the λ and consequently C_ϵ for a given rate B_{TR} , B_{RT} , outage probability ϵ and bandwidth F_{total} .

To compute the success probability we consider a typical transmitter receiver pair (Tx_0, Rx_0) . Using the stationarity of the homogenous PPP and Slivnyak's Theorem [19] (Page 121), it follows that the statistics of the signal received at the typical receiver are identical to that of any other receiver. Hence the outage probability is invariant with the choice of the receiver. Slivnyak's Theorem also states that the locations of the interferers for the typical transmitter and receiver (Tx_0, Rx_0) , i.e. $\Phi_T^a \setminus \{T_0\}$ and $\Phi_R^a \setminus \{R_0\}$ are also homogenous PPPs, each with intensity λ .

III. COMPUTING THE TWO-WAY TRANSMISSION CAPACITY

In this section we derive an upper and lower bound on the two-way transmission capacity. To derive a lower bound we use the Fortuin, Kastelyn, Ginibre (FKG) inequality [23], while for deriving an upper bound we make use of the Cauchy-Schwartz inequality. Before stating the FKG inequality, we need the following definitions.

Definition 2: A random variable X defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ is called increasing if $X(\omega) \leq X(\omega')$ whenever $\omega \leq \omega'$, for some partial ordering on $\omega, \omega' \in \Omega$. X is called decreasing if $-X$ is increasing.

Example 1: SIR_{TR} and SIR_{RT} are decreasing random variables.

For the PPP under consideration, let $\omega = (a_1, a_2, \dots)$ where for $n \in \mathbb{N}$,

$$a_n = \begin{cases} 1 & \text{if } Tx_n \text{ is active,} \\ 0 & \text{otherwise.} \end{cases}$$

Then $\omega' \geq \omega$, if $a'_n \geq a_n, \forall n$, i.e. configuration ω' contains at least those interferers which are present in configuration ω . Recall our definition of $SIR_{TR} = \frac{d^{-\alpha}|h_0|^2}{\sum_{T_n \in \Phi_T^a \setminus \{T_0\}} d_{T_n}^{-\alpha}|h_{0n}|^2}$. Clearly,

if there are more interferers present, SIR_{TR} decreases, i.e. considering SIR_{TR} as a random variable $SIR_{TR}(\omega) \geq SIR_{TR}(\omega')$, if $\omega \leq \omega'$. Thus SIR_{TR} is a decreasing random variable and so is SIR_{RT} .

Definition 3: Let A be an event in \mathcal{F} , and \mathcal{I}_A be the indicator function of A . Then the event $A \in \mathcal{F}$ is called increasing if $\mathcal{I}_A(\omega) \leq \mathcal{I}_A(\omega')$, whenever $\omega \leq \omega'$. The event A is called decreasing if its complement A^c is increasing.

Example 2: The success event $\{SIR > \beta\}$ is a decreasing event, since if $\omega' \in \{SIR > \beta\}$ and $\omega' \geq \omega$, then $\omega \in \{SIR > \beta\}$.

Lemma 1: (FKG Inequality [23])

(a) If both X and Y are increasing or decreasing random variables with $\mathbb{E}\{X^2\} < \infty$, and $\mathbb{E}\{Y^2\} < \infty$, then $\mathbb{E}\{XY\} \geq \mathbb{E}\{X\}\mathbb{E}\{Y\}$.

(b) If both $A, B \in \mathcal{F}$ are increasing or decreasing events then $P(AB) \geq P(A)P(B)$.

Now we are ready to derive bounds on the two-way transmission capacity. From (3), the success probability is

$$P_{success} = P\left(SIR_{TR} > 2^{\frac{B_{TR}}{F_{TR}}} - 1, SIR_{RT} > 2^{\frac{B_{RT}}{F_{RT}}} - 1\right).$$

Let $\beta_1 := d^\alpha \left(2^{\frac{B_{TR}}{F_{TR}}} - 1\right)$, $\beta_2 := d^\alpha \left(2^{\frac{B_{RT}}{F_{RT}}} - 1\right)$, $I_{TR} := \sum_{T_n \in \Phi_T^a \setminus \{T_0\}} d_{T_n}^{-\alpha} |h_{0n}|^2$, and $I_{RT} := \sum_{R_n \in \Phi_R^a \setminus \{R_0\}} d_{R_n}^{-\alpha} |g_{0n}|^2$. Then,

$$\begin{aligned} P_{success} &= P\left(\frac{|h_0|^2}{I_{TR}} > \beta_1, \frac{|g_0|^2}{I_{RT}} > \beta_2\right), \\ &\stackrel{(a)}{=} \mathbb{E}\left\{e^{-\beta_1 I_{TR}} e^{-\beta_2 I_{RT}}\right\}, \\ &= \mathbb{E}\left\{e^{-\beta_1 \left(\sum_{T_n \in \Phi_T^a \setminus \{T_0\}} d_{T_n}^{-\alpha} |h_{0n}|^2\right)} e^{-\beta_2 \left(\sum_{R_n \in \Phi_R^a \setminus \{R_0\}} d_{R_n}^{-\alpha} |g_{0n}|^2\right)}\right\}, \\ &\stackrel{(b)}{=} \mathbb{E}\left\{\prod_{T_n \in \Phi_T^a \setminus \{T_0\}} \left(\frac{1}{1 + \beta_1 d_{T_n}^{-\alpha}}\right) \prod_{R_n \in \Phi_R^a \setminus \{R_0\}} \left(\frac{1}{1 + \beta_2 d_{R_n}^{-\alpha}}\right)\right\}, \end{aligned} \quad (4)$$

where (a) follows since $P(|h_0|^2 > x) = P(|g_0|^2 > x) = e^{-x}$, and h_0 and g_0 are independent, and (b) follows by taking the expectation with respect to h_{0n} , and g_{0n} , and noting that h_{0n} , and g_{0n} are independent and exponentially distributed.

The difficulty in evaluating the expectation with respect to $\{d_{Tn}\}$ and $\{d_{Rn}\}$ in the success probability (4) lies in the fact that d_{Tn} and d_{Rn} are not independent. To visualize this, consider a network where there are only two active pairs of nodes, (Tx_0, Rx_0) , and (Tx_1, Rx_1) as depicted in Figure 1. For the receiver Rx_0 receiving over bandwidth F_{TR} , the transmission from Tx_1 is interference. As defined before, the distance between Rx_0 and Tx_1 be d_{T1} . Thus, the interference power at Rx_0 is $d_{T1}^{-\alpha}|h_{01}|^2$. Similarly, for Tx_0 receiving over bandwidth F_{RT} , the transmission from Rx_1 is interference. The distance between Rx_1 and Tx_0 be d_{R1} . Thus, the interference power at Tx_0 is $d_{R1}^{-\alpha}|g_{01}|^2$. For the case when d is very small $d \rightarrow 0$, $d_{R1} \approx d_{T1}$, and thus distances d_{R1} and d_{T1} are not independent. Moreover, explicitly computing the correlation between d_{Tn} and d_{Rn} is also a hard problem. Thus, to get a meaningful insight into the two-way transmission capacity we derive a lower and upper bound.

Lower Bound: Similar to Example 1, $\prod_{Tn \in \Phi_T^a \setminus \{T_0\}} \left(\frac{1}{1 + \beta_1 d_{Tn}^{-\alpha}} \right)$ and $\prod_{Rn \in \Phi_R^a \setminus \{R_0\}} \left(\frac{1}{1 + \beta_1 d_{Rn}^{-\alpha}} \right)$ are decreasing random variables, since each term in the product is less than 1, and with the increasing the number of terms (number of interferers) in the product the total value of each expression decreases. Thus, using Lemma 1, from (4)

$$\begin{aligned}
P_{success} &\geq \mathbb{E} \left\{ \prod_{Tn \in \Phi_T^a \setminus \{T_0\}} \left(\frac{1}{1 + \beta_1 d_{Tn}^{-\alpha}} \right) \right\} \mathbb{E} \left\{ \prod_{Rn \in \Phi_R^a \setminus \{R_0\}} \left(\frac{1}{1 + \beta_2 d_{Rn}^{-\alpha}} \right) \right\}, \quad (5) \\
&\stackrel{(c)}{=} e^{\left(-\lambda \int_{\mathbb{R}^2} 1 - \left(\frac{1}{1 + \beta_1 x^{-\alpha}} \right) dx \right)} e^{\left(-\lambda \int_{\mathbb{R}^2} 1 - \left(\frac{1}{1 + \beta_2 x^{-\alpha}} \right) dx \right)}, \\
&= e^{\left(-2\pi\lambda \int_0^\infty \left(\frac{\beta_1 x^{-\alpha+1}}{1 + \beta_1 x^{-\alpha}} \right) dx \right)} e^{\left(-2\pi\lambda \int_0^\infty \left(\frac{\beta_2 x^{-\alpha+1}}{1 + \beta_2 x^{-\alpha}} \right) dx \right)}, \\
&= e^{-\lambda c_1 \beta_1^{\frac{2}{\alpha}}} e^{-\lambda c_1 \beta_2^{\frac{2}{\alpha}}}, \\
&= e^{-\lambda c_1 \left(\beta_1^{\frac{2}{\alpha}} + \beta_2^{\frac{2}{\alpha}} \right)}, \quad (6)
\end{aligned}$$

where (c) follows from the probability generating functional of the Poisson point process [24,

Example 4.2], and $c_1 = \frac{2\pi^2 C_{sc}(\frac{2\pi}{\alpha})}{\alpha}$ is a constant, where C_{sc} is co-secant.

Upper Bound: Using the Cauchy-Schwartz inequality, from (4)

$$\begin{aligned}
P_{success} &\leq \left[\mathbb{E} \left\{ \prod_{T_n \in \Phi_T^a \setminus \{T_0\}} \left(\frac{1}{1 + \beta_1 d_{T_n}^{-\alpha}} \right)^2 \right\} \mathbb{E} \left\{ \prod_{R_n \in \Phi_R^a \setminus \{R_0\}} \left(\frac{1}{1 + \beta_2 d_{R_n}^{-\alpha}} \right)^2 \right\} \right]^{\frac{1}{2}}, \\
&\stackrel{(d)}{=} \left[e^{-\lambda \left(\int_{\mathbb{R}^2} 1 - \left(\frac{1}{1 + \beta_1 x^{-\alpha}} \right)^2 dx \right)} e^{-\lambda \int_{\mathbb{R}^2} 1 - \left(\frac{1}{1 + \beta_2 x^{-\alpha}} \right)^2 dx} \right]^{\frac{1}{2}}, \\
&= \left[e^{-2\pi\lambda \left(\int_{\mathbb{R}^2} \left(\frac{\beta_1^2 x^{-2\alpha+1} + 2\beta_1 x^{-\alpha+1}}{(1 + \beta_1 x^{-\alpha})^2} \right) dx \right)} e^{-2\pi\lambda \left(\int_{\mathbb{R}^2} \left(\frac{\beta_2^2 x^{-2\alpha+1} + 2\beta_2 x^{-\alpha+1}}{(1 + \beta_2 x^{-\alpha})^2} \right) dx \right)} \right]^{\frac{1}{2}}, \\
&= e^{-\lambda c_2 \beta_1^{\frac{2}{\alpha}}} e^{-\lambda c_2 \beta_2^{\frac{2}{\alpha}}}, \\
&= e^{-\lambda c_2 \left(\beta_1^{\frac{2}{\alpha}} + \beta_2^{\frac{2}{\alpha}} \right)}, \tag{7}
\end{aligned}$$

where (d) follows from the probability generating functional of the Poisson point process [24, Example 4.2], and $c_2 = \frac{\pi^2 C_{sc} \left(\frac{2\pi}{\alpha} \right) (\alpha+2)}{\alpha^2}$ is a constant, different from the constant c_1 of the lower bound.

Theorem 1: The two-way transmission capacity is upper and lower bounded by

$$\frac{(1 - \epsilon) \ln(1 - \epsilon) \frac{B_{TR} + B_{RT}}{F_{Total}}}{c_1 \left(\beta_1^{\frac{2}{\alpha}} + \beta_2^{\frac{2}{\alpha}} \right)} \leq C_\epsilon \leq \frac{(1 - \epsilon) \ln(1 - \epsilon) \frac{B_{TR} + B_{RT}}{F_{Total}}}{c_2 \left(\beta_1^{\frac{2}{\alpha}} + \beta_2^{\frac{2}{\alpha}} \right)} \text{ bits/sec/Hz/m}^2,$$

where c_1 and c_2 are constants, and $c_2/c_1 = \frac{1}{2} + \frac{1}{\alpha}$.

Proof: With $P_{success} = 1 - \epsilon$, and using the definition of C_ϵ (1), the result follows from (6) and (7). ■

Discussion: In this section we derived an upper and lower bound on the two-way transmission capacity. The upper and lower bound only differ by a constant, and, most importantly, both have identical dependence on the parameters of interest in the two-way communication, β_1 and β_2 . Thus, the derived bounds establish the two-way transmission capacity up to a constant. The derived upper and lower bounds for the two-way transmission capacity are in a fairly simple form and can be used to calculate the two-way transmission capacity for given rates B_{TR} , B_{RT} , success probability ϵ , F_{TR} and F_{total} . Since the upper and lower bound are identical functions of β_1 and β_2 , an added advantage of our bounds on the two-way transmission capacity expression

is that they can be used to find the optimal value of F_{TR} for given rates B_{TR} , B_{RT} , success probability $1 = \epsilon$, and F_{total} . The optimal bandwidth allocation that maximizes the two-way transmission capacity is derived next in the Section IV.

IV. TWO-WAY BANDWIDTH ALLOCATION

In Section III, we derived the two-way transmission capacity of ad-hoc networks within a constant as a function of bandwidth allocated to the $Tx_0 \rightarrow Rx_0$ and $Rx_0 \rightarrow Tx_0$ connections. Since the total bandwidth F_{total} is finite, an important question to answer is: what is the optimal bandwidth allocation between that maximizes the transmission capacity? For the special case of equal rate requirement in both directions, i.e. $B_{TR} = B_{RT}$, equal bandwidth allocation is optimal. For the non-symmetric case, however, the answer is not that obvious and is derived in the following theorem.

Theorem 2: The optimum bidirectional bandwidth allocation that maximizes the transmission capacity with two-way communication is $F_{TR}^* = x^*$ and $F_{RT}^* = F_{TR}^*$ where x^* is the unique positive solution to the following equation:

$$\frac{1}{B_{TR}} h\left(\frac{B_{TR}}{x}\right) - \frac{1}{B_{RT}} h\left(\frac{B_{RT}}{F_{total} - x}\right) = 0 \quad (8)$$

where $h(t) = t^2 2^t (2^t - 1)^{(\delta-1)}$ for $0 < t < F_{total}$.

Proof: Neglecting the constant, the two-way transmission capacity is

$$C = (1 - \epsilon) \lambda \left(\frac{B_{TR} + B_{RT}}{F_{total}} \right) = (1 - \epsilon) \frac{-\ln(1 - \epsilon)}{d^2 \left(\left(2^{\frac{B_{TR}}{F_{TR}}} - 1 \right)^{\frac{2}{\alpha}} + \left(2^{\frac{B_{RT}}{F_{RT}}} - 1 \right)^{\frac{2}{\alpha}} \right)} \left(\frac{B_{TR} + B_{RT}}{F_{total}} \right).$$

To derive the optimal bandwidth partitioning, i.e. the optimal F_{TR} that maximizes C , we need to minimize $\left(\left(2^{\frac{B_{TR}}{F_{TR}}} - 1 \right)^{\frac{2}{\alpha}} + \left(2^{\frac{B_{RT}}{F_{RT}}} - 1 \right)^{\frac{2}{\alpha}} \right)$.

Let $\delta := \frac{2}{\alpha}$. Let $f(x) := \left(\left(2^{\frac{B_{TR}}{x}} - 1 \right)^{\delta} + \left(2^{\frac{B_{RT}}{F_{total} - x}} - 1 \right)^{\delta} \right)$. Thus, the problem we need to solve is

$$\min_{x \in (0, F_{total})} f(x).$$

The first-order derivative of $f(x)$ is $\frac{d}{dx}f(x) = \delta \log_e 2 \left[-\frac{1}{B_{TR}} h\left(\frac{B_{TR}}{x}\right) + \frac{1}{B_{RT}} h\left(\frac{B_{RT}}{F_{total}-x}\right) \right]$, where $h(t) := t^2 2^t (2^t - 1)^{(\delta-1)}$ for $t \geq 0$. The second-order derivative of $f(x)$ is $\frac{d^2}{dx^2}f(x) = \delta \log_e 2 \left[\frac{1}{x^2} h\left(\frac{B_{TR}}{x}\right) + \frac{1}{(F_{total}-x)^2} h\left(\frac{B_{RT}}{F_{total}-x}\right) \right]$. Since $h(t)$ is monotonically increasing in t over $t \geq 0$, then we have $h(t) > h(0) = 0$ for all $t > 0$. Therefore, $\frac{d^2}{dx^2}f(x) > 0$ for all $x \in (0, F_{total})$. This means that $f(x)$ is a convex function of x over $(0, F_{total})$ and its minimum corresponds to x^* that is the unique positive solution of the following equation $\frac{d}{dx}f(x) = 0$, or equivalently, $\frac{1}{B_{TR}} h\left(\frac{B_{TR}}{x}\right) - \frac{1}{B_{RT}} h\left(\frac{B_{RT}}{F_{total}-x}\right) = 0$. ■

Discussion: In Theorem 2 we derived the optimal bandwidth allocation for two-way communication in ad-hoc networks that maximizes the transmission capacity. The result is derived by showing that the optimization problem is convex in one variable, hence the optimal solution corresponds to the value for which the function derivative is zero.

Using Theorem 2, if the traffic is symmetric, i.e., $B_{TR} = B_{RT}$, the optimal strategy is naturally allocate equal bandwidths for two directions with $F_{TR} = F_{total}/2$. This result is intuitive since the counterpart parameters in two directions are equal. For asymmetric traffic $B_{TR} \neq B_{RT}$, however, allocating bandwidths proportional to the desired rate in each direction $F_{TR} = \frac{F_{total} B_{TR}}{B_{TR} + B_{RT}}$ does not satisfy (8). Thus the proportional bandwidth allocation policy is not optimal for asymmetric traffic for maximizing the transmission capacity, and (8) must be satisfied to find the optimal policy.

V. EFFECT OF LIMITED FEEDBACK ON TWO-WAY TRANSMISSION CAPACITY WITH BEAMFORMING

In this section we consider an ad-hoc network where each transmitter Tx_n is equipped with N antennas while each receiver Rx_n has a single antenna. All other system parameters and assumptions remain the same as defined in Section II. With multiple transmit antennas, and channel state information CSI at each transmitter, transmission rate can be increased by transmitting the signal along the strongest eigenmode of the channel (called beamforming).

Beamforming, however, requires that the transmitter know the channel coefficients, which in general is a challenging problem. In a FDD system, the transmitter can learn the channel coefficients, or equivalently the optimum beamformer, through the use of a finite rate feedback channel from the receiver. Assuming a genie aided feedback (channel coefficients are exactly known at the transmitter, and without accounting for the feedback bandwidth, and SIR required for the feedback), [7] derived the transmission capacity with beamforming, and showed that the transmission capacity increases as $N^{\frac{2}{\alpha}}$ with increasing N . In reality, however, feedback requires sufficient bandwidth, and the channel coefficients can be fed back only up to a certain precision.

Limited feedback techniques [25] are commonly used in practical systems to exploit finite rate feedback channels. With limited feedback, a beamforming codebook is assumed known to both the receiver and the transmitter. The receiver computes the best beamforming vector from the beamforming codebook and sends the index of this vector back to the transmitter. The larger the codebook size, the better is the quality of feedback, and consequently better is the data rate from the transmitter to the receiver with beamforming. With a codebook size of 2^B , each codeword requires B bits of feedback. Thus, the use of a large codebook increases the required bandwidth for the feedback channel, thereby restricting the bandwidth allocated for transmitter to receiver communication. Thus, there is a three-fold tradeoff between the bandwidth allocated in forward channel, the feedback channel, and the size of the codebook. In this section, we quantify this tradeoff and evaluate its effect on the two-way transmission capacity.

The received signal at receiver R_0 over bandwidth F_{TR} is

$$y_0 = \sqrt{P_t} d^{-\alpha/2} \mathbf{h}_0^T \mathbf{b}_0 x_0 + \sum_{T_n \in \Phi \setminus \{T_0\}} \sqrt{P_t} d_{T_n}^{-\alpha/2} \mathbf{h}_{0n}^T \mathbf{b}_n x_n,$$

where P_t is the transmit power of each transmitter, \mathbf{b}_n are the beamformers used by Tx_n , $\mathbf{h}_0 \in \mathbb{C}^{N \times 1}$ is the channel between Tx_0 and Rx_0 , $\mathbf{h}_{0n} \in \mathbb{C}^{N \times 1}$ is the channel between Tx_n and Rx_0 , d_n is the distance between Tx_n and Rx_0 , x_0 and x_n are the data symbols transmitted from Tx_0 and Tx_n , respectively. For simplicity we assume that each receiver computes the

beamforming vectors \mathbf{b}_n only depending on \mathbf{h}_n , independent of the interferers' channels.

The received signal at transmitter Tx_0 corresponding to the feedback by receiver Rx_0 over bandwidth F_{RT} is

$$\mathbf{w}_0 = \sqrt{P_t} d^{-\alpha/2} \mathbf{g}_0 u_0 + \sum_{T_n \in \Phi \setminus \{T_0\}} \sqrt{P_t} d_{Rn}^{-\alpha/2} \mathbf{g}_{0n} u_n, \quad (9)$$

where $\mathbf{g}_0 \in \mathbb{C}^{N \times 1}$ is the channel between Rx_0 and Tx_0 , $\mathbf{g}_{0n} \in \mathbb{C}^{N \times 1}$ is the channel between Rx_n and Tx_0 , u_0 and u_n are the feedback signals transmitted by Rx_0 and Rx_n , respectively.

With genie-aided feedback, the optimal beamforming vector \mathbf{b}_n is known to be $\mathbf{b}_n = \mathbf{h}_n^*$. In practice, however, only a finite number of bits are available for feedback, and hence \mathbf{b}_n can be modeled as $\mathbf{b}_n = \mathbf{h}_n^* + \mathbf{e}$, where \mathbf{e} is the additive error term which represents the uncertainty due to limited feedback. The quantization error \mathbf{e} degrades the signal power compared to genie aided feedback. With B bits of feedback bits, the signal power [26] is $|\mathbf{h}_n|^2 \left(1 - c_3 \left(\frac{1}{B}\right)^{\frac{1}{N-1}}\right)$ ($c_3 < 1$ is a constant), compared to $|\mathbf{h}_n|^2$ for genie aided feedback ($B = \infty$). Thus, the SIR for Tx_0 to Rx_0 communication with B bits of feedback is

$$SIR_{TR} = \frac{d^{-\alpha} |\mathbf{h}_0^T|^2 \left(1 - c_3 \left(\frac{1}{B}\right)^{\frac{1}{N-1}}\right)}{\sum_{T_n \in \Phi \setminus \{T_0\}} d_{Tn}^{-\alpha} |\mathbf{h}_{0n}^T \mathbf{b}_0|^2}, \quad (10)$$

and the corresponding mutual information from Tx_0 to Rx_0 using bandwidth F_{TR} is

$$MI_{TR} := F_{TR} \log(1 + SIR_{TR}) \text{ bits/sec.}$$

Similarly, the SIR for the feedback link is $SIR_{RT} = \frac{d^{-\alpha} |\mathbf{g}_0(1)|^2}{\sum_{T_n \in \Phi \setminus \{T_0\}} d_{Rn}^{-\alpha} |\mathbf{g}_{0n}(1)|^2}$, and thus with bandwidth F_{RT} , the mutual information of the feedback link is

$$MI_{RT} := (F_{RT}) \log(1 + SIR_{RT}).$$

Similar to (3), we define the success probability as the probability that communication in both directions is successful simultaneously, i.e.

$$P_{success} = P(MI_{TR} > B_{TR}, MI_{RT} \geq B).$$

Consequently, with $P_{success} = (1 - \epsilon)$ the two-way transmission capacity is defined as

$$C_\epsilon = \frac{\lambda(1 - \epsilon)B_{TR}}{F_{total}} \text{ bits/sec/Hz/m}^2.$$

As stated before, in a two-way communication model, where the transmitter locations are modeled as a PPP, the interference received in both directions is correlated. Therefore, computing the success probability in closed form is a hard problem. To derive a meaningful insight into the dependence of bandwidth allocation, and feedback bits on two-way transmission capacity, we derive a lower bound on the success probability using the FKG inequality as follows.

Theorem 3: Accounting for feedback bandwidth, the two-way transmission capacity with beamforming is lower bounded by

$$C_\epsilon \geq \frac{(1 - \epsilon)\epsilon N^{\frac{2}{\alpha}}}{c_4[(\beta_1/\gamma)^{\frac{1}{\alpha}} + (\beta_3)^{\frac{1}{\alpha}}]} \frac{B_{TR}}{F_{total}} \text{ bits/sec/Hz/m}^2,$$

where $\gamma := \left(1 - c_3 \left(\frac{1}{B}\right)^{\frac{1}{N-1}}\right)$, and

$$c_4 = \left(\left(1 + \sum_{k=0}^{N-2} \frac{1}{(k+1)!} \prod_{\ell=0}^k \left(\ell - \frac{2}{\alpha} \right) \right) \left(\frac{2\pi}{\alpha} \sum_{k=0}^{N-1} \binom{N}{k} B \left(\frac{2}{\alpha} + k; N - \frac{2}{\alpha} + k \right) \right) \right)^{-1}$$

with $B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$.

Proof:

$$P_{success} = P(MI_{TR} > B_{TR}, MI_{RT} \geq B).$$

Similar to Example 3, the success events in two directions $\{MI_{TR} > B_{TR}\}$, and $\{MI_{RT} > B\}$, respectively, are decreasing events. Thus, invoking Lemma 1,

$$P_{success} \geq P(MI_{TR} > B_{TR}) P(MI_{RT} \geq B).$$

By definition,

$$\begin{aligned}
P(MI_{TR} > B_{TR}) &= P(F_{TR} \log(1 + SIR_{TR}) > B_{TR}), \\
&\stackrel{(a)}{=} P(SIR_{TR} > \beta_1), \\
&\stackrel{(b)}{=} P\left(\frac{d_0^{-\alpha} |\mathbf{h}_0^T|^2 \left(1 - |\mathbf{h}_n|^2 \left(\frac{1}{B}\right)^{\frac{1}{N-1}}\right)}{\sum_{T_n \in \Phi \setminus \{T_0\}} d_n^{-\alpha} |\mathbf{h}_{0n}^T \mathbf{b}_0|^2} > \beta_1\right), \\
&\stackrel{(c)}{\geq} P\left(\frac{d_0^{-\alpha} |\mathbf{h}_0^T|^2}{\sum_{T_n \in \Phi \setminus \{T_0\}} d_n^{-\alpha} |\mathbf{h}_{0n}^T \mathbf{b}_0|^2} > \beta_1/\gamma\right), \\
&\stackrel{(d)}{=} 1 - c_4 \lambda (\beta_1/\gamma)^{\frac{2}{\alpha}} N^{\frac{-2}{\alpha}},
\end{aligned}$$

where (a) follows from the definition of β_1 , (b) follows by substituting for SIR_{TR} (10), (c) follows by defining $\gamma := \left(1 - c_3 \left(\frac{1}{B}\right)^{\frac{1}{N-1}}\right)$, and (d) follows from Theorem 3 [7].

Directly applying Theorem 3 [7], $P(MI_{RT} \geq B) = 1 - c\lambda(\beta_3)^{\frac{2}{\alpha}} N^{\frac{-2}{\alpha}}$, where $\beta_3 = d^\alpha \left(2^{\frac{B}{F_{RT}}}\right)$

Thus, $P_{success} \geq 1 - c_4 \lambda N^{\frac{-2}{\alpha}} \left[(\beta_1/\gamma)^{\frac{2}{\alpha}} + (\beta_3)^{\frac{2}{\alpha}}\right]$. Then,

$$C_\epsilon \geq \frac{(1 - \epsilon) \epsilon N^{\frac{2}{\alpha}}}{c_4 [(\beta_1/\gamma)^{\frac{1}{\alpha}} + (\beta_3)^{\frac{1}{\alpha}}]} \frac{B_{TR}}{F_{total}} \text{ bits/sec/Hz/m}^2.$$

■

Discussion: In this section we derived a lower bound on the two-way transmission capacity when the transmitter uses beamforming with limited feedback, as a function of the bandwidth allocated in two directions, and the number of feedback bits. Note that as B (the number of feedback bits) increases, the two-way transmission capacity increases as $B^{\frac{1}{(N-1)\alpha}}$ due to the improvement in signal strength, however, decreases as $2^{\frac{-B}{\alpha}}$ because of the stringent requirement of SIR on the feedback link to be more than β_3 . Our result quantifies the degradation due to practical limited feedback in two-way transmission capacity with beamforming, compared to assuming a genie aided feedback [7]. The feedback requirement not only decreases the available bandwidth for transmitter to receiver communication, but also degrades the overall performance due to the successful reception requirement of the feedback bits.

Similar to Section IV, for a fixed value of B and B_{TR} , the optimal bandwidth allocation F_{TR} that maximizes the two-way transmission capacity upper bound can be computed using Theorem 2, since here again the optimization problem is convex. For a fixed value of F_{TR} and B_{TR} , finding the optimal B is slightly complicated since the upper bound is not a convex function of B , however, the problem is a single variable problem and can be solved easily by using techniques like bisection.

VI. NUMERICAL RESULTS

In this section we present some numerical results on the two-way transmission capacity. We adopt the simulation methodology for one-way networks presented in [27] and consider $d = 5\text{m}$, and $\alpha = 4$.

A. General Two-way Communication

Tightness of the proposed bounds: In this experiment, we consider $B_{TR} = 1.028$ kbits, $B_{RT} = 0.03$ kbits, $F_{TR} = 0.99$ MHz, and $F_{RT} = F_{\text{total}} - F_{TR} = 0.01$ MHz. Fig. 2 shows the curves for the simulated result and the bounds derived in Theorem 1 on the two-way transmission capacity as functions of the outage probability requirement. Moreover, note that the transmission capacity decreases at very high outage probability (ϵ), since the transmission capacity expressions are proportional to $-(1-\epsilon)\log(1-\epsilon)$. Intuitively, as the outage probability ϵ approaches towards 1, a high density of links is allowed in a unit area, however, most of the links fail; therefore, the amount of successfully received information actually decreases.

One-way versus two-way transmission capacity: Requiring that transmissions be successful in both directions, the two-way transmission capacity is less than the one-way transmission capacity. To quantify the loss we plot the two-way transmission capacity in comparison with the one-way transmission capacity for the same total bandwidth F_{total} and total data rates $B_{\text{total}} = B_{TR} + B_{RT}$. In particular, for the results shown in Fig. 3, we set $B_{TR} = 1.024$ kbits, $B_{RT} = 0.256$

kbits, $F_{\text{TR}} = 0.8$ MHz, and $F_{\text{RT}} = 0.2$ MHz. The simulation results show that at the outage requirement of 10%, the two-way transmission capacity is half the one-way transmission capacity.

Effect of bandwidth allocation: To highlight the effect of bandwidth allocation on the two-way transmission capacity we plot the transmission capacity as a function of F_{TR} in Fig. 4 assuming the total bandwidth is $F_{\text{total}} = 1$ MHz. For the scenario of symmetric traffic, we set the data requirements in two directions equal to 1 kbits, i.e., $B_{\text{TR}} = B_{\text{RT}} = 1.024$ kbits. In this case, we notice that the proportional allocation method is optimal. For asymmetric traffic, we consider $B_{\text{TR}} = 1.024$ Mbits and $B_{\text{RT}} = 0.056$ kbits. From Fig. 4, note that the optimal bandwidth allocation (Theorem 2) provides a gain of 36% over the proportional allocation.

B. Feedback-Based Communication

To quantify the effect of feedback on the transmission capacity we compare the transmission capacity of a feedback-based network with the corresponding one-way network with the genie-aided beamforming [7] with $N = 3$ in Fig. 5. We use $B_{\text{TR}} = 1.024$ kbits, $B_{\text{RT}} = 0.056$ kbits, feedback bits $B = 2$, $F_{\text{TR}} = 0.94$ MHz, and $F_{\text{RT}} = F_{\text{total}} - F_{\text{TR}} = 0.06$ MHz and assume that the transmitters employ Grassmannian limited feedback beamforming for transmission [25]. Moreover, of the $B_{\text{RT}} = 0.056$ kbits (or 56 bits) in the reverse direction, B bits are used for carrying the codeword index while the other bits are used for MAC header.

Tightness of the proposed lower bound: In this experiment, we set $N = 3$ antennas and $B = 2$ bits. Fig. 6 presents the simulated results for a genie-aided beamforming network and the limited-feedback beamforming network as well as the computed lower bound.

VII. CONCLUSIONS

In this paper we generalized the concept of transmission capacity to incorporate two-way communication in wireless ad-hoc networks. The two-way transmission capacity is able to capture the requirement of successful transmissions in both directions and the impact of duplexing techniques. The two-way success requirement is shown to reduce the transmission capacity

significantly compared to the corresponding one-way transmission capacity. This observation raised the question of finding the network with the maximum two-way transmission capacity among the two-way networks with the same total bandwidth given fixed desired rates in two directions. We addressed the question by providing the optimal solution for bidirectional spectrum allocation to maximize the two-way transmission capacity. The optimal solutions were determined in terms of the path-loss exponent, desired rates, and total bandwidth available.

As an application of the two-way transmission capacity framework, we also quantified the effect of practical limited channel feedback on the two-way transmission capacity with transmit beamforming. We showed that accounting for the bandwidth required for feedback, and the successful reception of the feedback bits, the transmission capacity is significantly reduced compared to the genie aided feedback.

REFERENCES

- [1] S. Weber, X. Yang, J. Andrews, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with outage constraints," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4091–4102, Dec. 2005.
- [2] S. Weber, J. Andrews, X. Yang, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with successive interference cancellation," *IEEE Trans. Inf. Theory*, vol. 53, no. 8, pp. 2799–2814, Aug. 2007.
- [3] S. Weber, J. Andrews, and N. Jindal, "An overview of the transmission capacity of wireless networks," *IEEE Trans. Wireless Commun.*, accepted in July 2010.
- [4] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "An aloha protocol for multihop mobile wireless networks," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 421–436, Feb. 2006.
- [5] J. F. C. Kingman, *Poisson Processes*. Oxford Science, 1993.
- [6] S. Weber, J. G. Andrews, X. Yang, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with successive interference cancellation," *IEEE Trans. Inform. Theory*, vol. 53, no. 8, pp. 2799–2814, 2007.
- [7] A. M. Hunter, J. G. Andrews, and S. Weber, "Capacity scaling of ad hoc networks with spatial diversity," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5058–71, Dec. 2008.
- [8] N. Jindal, J. Andrews, and S. Weber, "Rethinking MIMO for wireless networks: Linear throughput increases with multiple receive antennas," in *IEEE International Conference on Communications, 2009. ICC '09.*, June 2009, pp. 1–6.
- [9] K. Huang, J. Andrews, R. Heath Jr., D. Guo, and R. Berry, "Spatial interference cancellation for multi-antenna mobile ad hoc networks," *IEEE Trans. Inf. Theory*, submitted Jul. 2008, available on <http://arxiv.org>.

- [10] R. Vaze and R. Heath Jr., "Transmission capacity of ad-hoc networks with multiple antennas using transmit stream adaptation and interference cancelation," *IEEE Trans. Inf. Theory*, submitted Dec. 2009, available on <http://arxiv.org/abs/0912.2630>.
- [11] A. Hasan and J. G. Andrews, "The guard zone in wireless ad hoc networks," *IEEE Trans. Wireless Commun.*, vol. 6, no. 3, pp. 897–906, 2007.
- [12] C. Lo, S. Vishwanath, and R. Heath Jr., "The impact of channel feedback on opportunistic relay selection for Hybrid-ARQ in wireless networks," *to appear in IEEE Trans. Veh. Technol.*, 2008.
- [13] A. K. Singh and K. Kankipati, "TCP-ADA: TCP with adaptive delayed acknowledgement for mobile ad hoc networks," in *IEEE Wireless Commun. and Networking Conf.*, vol. 3, 2004, pp. 1685–1690 Vol.3.
- [14] E. M. Royer and C.-K. Toh, "A review of current routing protocols for ad hoc mobile wireless networks," *IEEE Personal Commun.*, vol. 6, no. 2, pp. 46–55, 1999.
- [15] K. T. Truong, S. Weber, and R. Heath Jr., "Transmission capacity of two-way communication in wireless ad hoc networks," in *IEEE International Conference on Communications (ICC), Dresden, Germany*, 14–19 June 2009, pp. 1–5.
- [16] R. Vaze, K. T. Truong, R. Heath Jr., and S. Weber, "Two-way transmission capacity of wireless ad-hoc networks," in *IEEE International Symposium on Information Theory Proceedings (ISIT)*, 13–18 June 2010, pp. 1688 –1692.
- [17] D. Love, R. Heath, V. Lau, D. Gesbert, B. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341 –1365, Oct. 2008.
- [18] L. Gropop and D. Tse, "Spectrum sharing between wireless networks," in *IEEE Infocom*, 2008, pp. 201–205.
- [19] K. Huang, V. K. N. Lau, and Y. Chen, "Spectrum sharing between cellular and mobile ad hoc networks: Transmission-capacity trade-off," *submitted to IEEE J. Select. Areas Commun.*, Aug. 2008.
- [20] N. Jindal, J. Andrews, and S. Weber, "Bandwidth partitioning in decentralized wireless networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5408–5419, Dec. 2008.
- [21] D. Daley and D. Vere-Jones, *An Introduction to the Theory of Point Processes*. Springer, 2003.
- [22] T. Cover and J. Thomas, *Elements of Information Theory*. John Wiley and Sons, 2004.
- [23] G. Grimmett, *Percolation*. Springer-Verlag, 1980.
- [24] D. Stoyan, W. Kendall, and J. Mecke, *Stochastic Gemoetry and its Applications*. John Wiley and Sons, 1995.
- [25] D. Love, R. Heath Jr., and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inf. Theory*, Oct. 2003.
- [26] B. Mondal and R. Heath Jr., "Performance analysis of quantized beamforming MIMO systems," *IEEE Trans. Signal Process.*, vol. 54, no. 12, pp. 4753–4766, Dec. 2006.
- [27] S. Weber and M. Kam, "Computational complexity of outage probability simulations in mobile ad-hoc networks," in *39th Annu. Conf. Info. Sciences and Systems (CISS)*, Baltimore, MD, Mar. 2005.

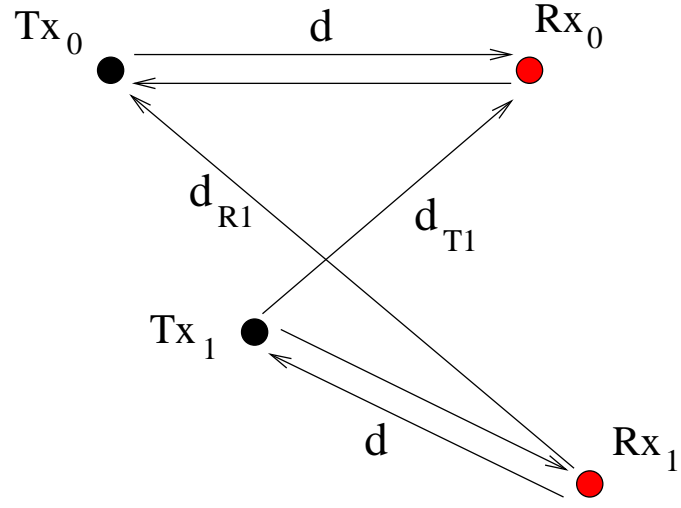


Fig. 1. Schematic of two-way communication with two pairs of nodes.

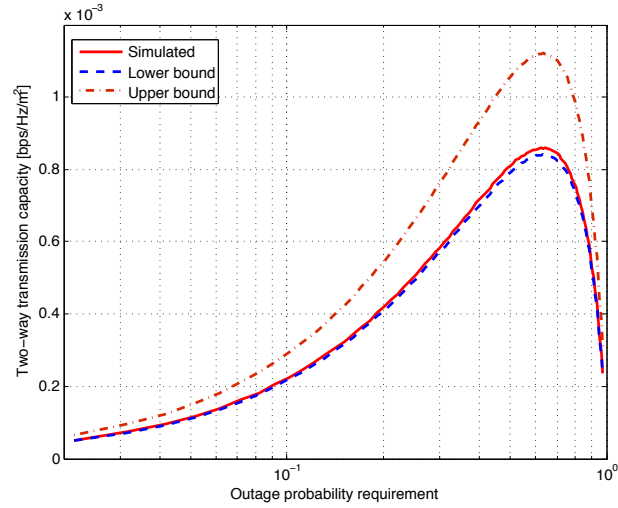


Fig. 2. Tightness of the proposed bounds on the transmission capacity of general two-way networks.

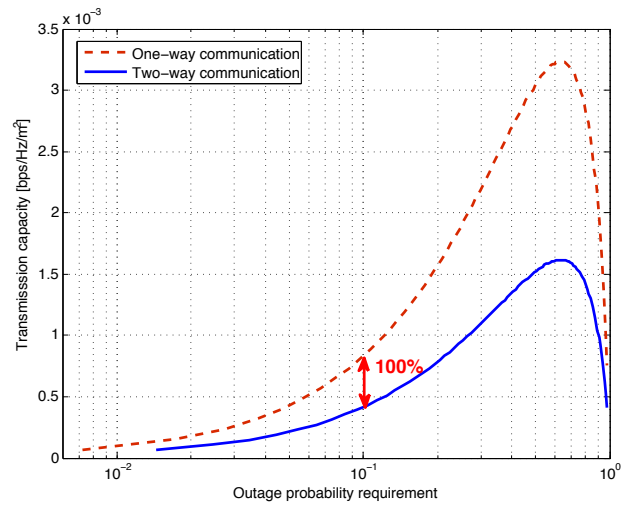


Fig. 3. Comparison of the one-way transmission capacity and the general two-way transmission capacity.

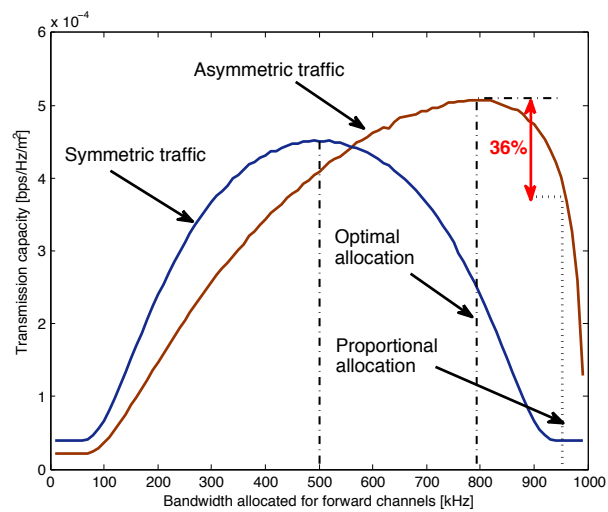


Fig. 4. Two-way transmission capacity as a function of bandwidth allocation. For symmetric traffic, the proportional allocation method is optimal, while for asymmetric traffic the optimal allocation provides a large gain over the proportional allocation.

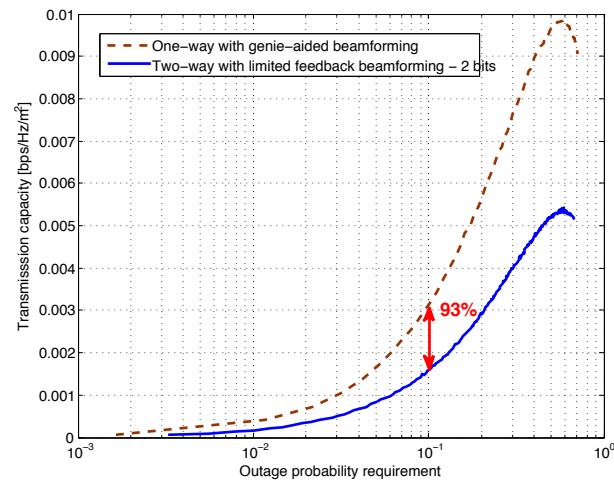


Fig. 5. Comparison of the transmission capacity of a feedback-based network with that of the corresponding one-way network.

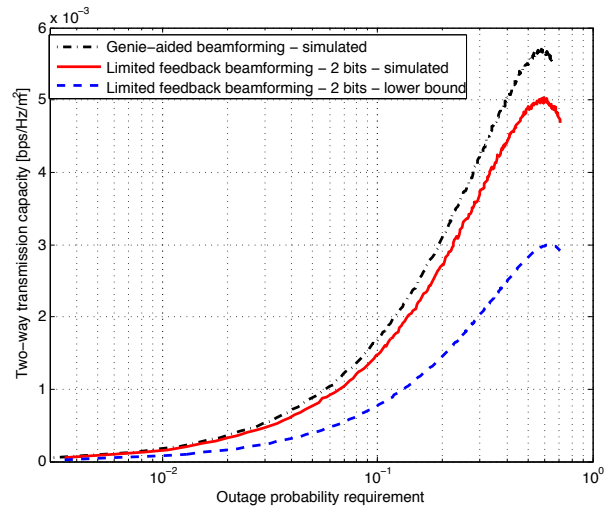


Fig. 6. Tightness of the proposed lower bound on the transmission capacity of feedback-based networks.