Coordinated Port Selection and Beam Steering Optimization in a Multi-Cell Distributed Antenna System using Semidefinite Relaxation

Talha Ahmad, Ramy H. Gohary, *Member, IEEE*, Halim Yanikomeroglu, *Member, IEEE*, Saad Al-Ahmadi, and Gary Boudreau, *Senior Member, IEEE*

***Abstract*—In this paper, we consider coordinated downlink transmission in a cellular system wherein each base station (BS) has multiple geographically dispersed antenna ports. Each port uses a fixed transmit power and the goal of the BSs is to collectively determine the subset of ports and the corresponding beam steering coefficients that maximize the minimum signal-to-interference-plus-noise ratio observed by the user terminals. This problem is NP-hard. To circumvent this difficulty, a two-stage polynomial-complexity technique that relies on semidefinite relaxation and Gaussian randomization is developed. It is shown that, for the considered scenarios, the port state vectors and beam steering coefficients generated by the proposed technique yield a performance comparable to that yielded by exhaustive search, but with a significantly less computational complexity. It is also shown that the proposed technique results in significant power savings when compared with other transmission strategies proposed in the literature.**

***Index Terms*—Distributed antenna systems, remote radio heads, multi-cell coordination, port selection, beam steering optimization, semidefinite relaxation, Gaussian randomization.**

I. INTRODUCTION

WIRELESS communication systems employing multiple antennas can achieve higher data rates and better coverage than their single antenna counterparts. However, the potential coverage gains of using multiple antennas are not realized for systems in which the antennas are co-located. For instance, in such systems, user terminals (UTs) that are far from the base station (BS) are likely to receive highly attenuated signals. This drawback can be overcome by dispersing the BS antennas over the coverage area [1] and using a coordinated multi-point (CoMP) transmission strategy to maximize the signal-to-interference-plus-noise ratios (SINRs) of the UTs.

In this paper, we consider the downlink of a multi-cell distributed antenna system. A set of antenna ports (or remote radio heads (RRHs) [2]) is available in each cell. As is customary in orthogonal transmission schemes, e.g., orthogonal frequency division multiple access (OFDMA), inter-user interference is mitigated by assigning each frequency-time resource block (RB) to at most one UT. To improve design efficiency and facilitate system-wide implementation of the distributed antenna system, the transmit power used by the ports is assumed to be fixed. Because the wireless medium is shared, the transmissions of the ports in each cell interfere with those of the ports using the same RB in other cells.

Despite the envisioned benefits of using multiple antenna ports [3], [4], poor selection of these ports and their antenna weights can yield low SINRs, resulting in undesirable performance. This has been demonstrated in [5] for cellular distributed antenna systems with no coordination among the BSs. In [6], a system with no BS coordination, similar to the one in [5], is considered. For this system the weights of the antenna ports are chosen to match the phases of the channel coefficients and either a single-port or an all-port transmission strategy is selected.

In contrast with both [5] and [6], in this work we consider a cluster of cells in which the BSs organize their transmissions in a coordinated manner. Since coordination among multiple cells is a generalization of coordination within a single cell, the CoMP system considered herein subsumes those considered in [5] and [6], and offers a greater number of degrees of design freedom. Unlike [6], in which the transmission strategies are selected and fixed prior to choosing the antenna weights, herein we consider the joint optimization of the ports to be used for transmission and their corresponding weights, which we refer to as beam steering coefficients. The joint optimization of these parameters allows further exploitation of the coordination among the BSs to enable the UTs to achieve higher SINRs. However, this problem can be shown to be NP-hard [7], [8], which implies that finding the global optimal solution is computationally prohibitive for many practical systems. To circumvent this difficulty, we propose a novel polynomial-complexity two-stage approach that will be shown to yield close-to-optimal solutions efficiently.

In the first stage of this approach, the beam steering

Manuscript received June 30, 2011; revised November 11, 2011; accepted December 23, 2011. The associate editor coordinating the review of this paper and approving it for publication was Y. J. Zhang.

T. Ahmad, R.H. Gohary, and H. Yanikomeroglu are with the Department of Systems and Computer Engineering, Carleton University, Ottawa, Ontario, Canada (e-mail: {tahmad,gohary,halim}@sce.carleton.ca).

S. Al-Ahmadi is with the Electrical Engineering Department, King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia (e-mail: saadbd@kfupm.edu.sa).

G. Boudreau is with Ericsson Canada, Ottawa, Ontario, Canada (e-mail: gary.boudreau@ericsson.com).

This work was supported in part by Research In Motion Limited, in part by the Natural Sciences and Engineering Research Council (NSERC) of Canada, and the NSERC CGS M award.

A preliminary version of this work has been accepted to the 2012 IEEE International Conference on Communications.

Digital Object Identifier 10.1109/TWC.2012.030512.111256

coefficients are chosen to match the phases of the complex channel gains between the ports and their intended UTs. This choice was shown in [6] to maximize the SINR under a constant interference power assumption. Although interference power is generally not constant, herein we select the beam steering coefficients as in [6] to facilitate developing a strategy for port selection. Unfortunately, for any fixed beam steering coefficients, including the ones proposed in [6], the problem of determining the set of ports that maximizes the minimum SINR of the UTs is an NP-hard binary-constrained optimization problem. To tackle this problem, we use the semidefinite relaxation (SDR) technique [9] to relax the binary constraints. Noting that the relaxed problem possesses a quasi-linear structure, we solve it using a series of convex feasibility problems with polynomial complexity. Gaussian randomization is then used to efficiently obtain close-to-optimal sets of port states [10]. Our simulations show that the performance of the proposed technique approaches that of the optimal set of port states for the given set of beam steering coefficients with a relatively small number of random Gaussian vectors. It is also shown that this technique outperforms the single-port and all-port transmission strategies considered in [6], and provides significant power savings.

In the second stage, candidate port state vectors generated in the first stage are considered. For each such vector, we consider the problem of determining the beam steering coefficients that maximize the minimum SINR. Unfortunately, this problem is non-convex, and in fact, a variant of it was shown in [8] to be NP-hard. Similar to the first stage, SDR-based Gaussian randomization provides a candidate technique for efficiently generating close-to-optimal solutions for the optimization problem in the second stage. A technique similar to that used in the second stage was used in [8], [11], [12] for solving beamforming problems with total power constraints. Using this technique for each candidate port state vector, we obtain beam steering coefficients that yield significantly better performance than those used in the first stage.

The paper is organized as follows. The system model and problem formulation are described in Section II. In Section III, the problem of finding close-to-optimal candidate port state vectors is formulated for given beam steering coefficients. For each candidate vector, the problem of optimizing the beam steering coefficients is considered in Section IV. In Section V, the techniques developed in Sections III and IV are used to develop the two-stage approach, which generates an approximate solution to the original problem in Section II. The computational complexity of the proposed techniques is analyzed in Section VI. In Section VII, simulation results are provided, and Section VIII concludes the paper.

Notation: Scalars are denoted by regular-face lower-case letters, and column vectors and matrices are denoted by lower-case and upper-case bold-face letters, respectively. The superscripts $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^*$ denote the transpose, Hermitian transpose, and complex conjugate operators, respectively, and $\mathrm{Tr}(\cdot)$, $|\cdot|$, and $\|\cdot\|_2$ denote the trace, the absolute value, and the 2-norm operators, respectively. The operator $\mathrm{diag}(\cdot)$ denotes a vector containing the diagonal entries of the matrix argument. The notation $\boldsymbol{W}_1 \succeq \boldsymbol{W}_2$ is used to indicate that the matrix $\boldsymbol{W}_1 - \boldsymbol{W}_2$ is positive semidefinite (PSD). The operator $E\{\cdot\}$ denotes expectation, and $\mathcal{N}(\cdot)$ and $\mathcal{CN}(\cdot)$ denote the real and complex Gaussian distributions, respectively. The real part of a complex argument is denoted by $\Re\{\cdot\}$. The q-th entry of any vector $\boldsymbol{x} \in \mathbb{C}^N$ is denoted by $[\boldsymbol{x}]_q$. For two square matrices $\boldsymbol{A} \in \mathbb{C}^{M\times M}$ and $\boldsymbol{B} \in \mathbb{C}^{N\times N}$, we use $\boldsymbol{A} \oplus \boldsymbol{B}$ to denote the block diagonal matrix $\begin{bmatrix} \boldsymbol{A} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{B} \end{bmatrix}$, where $\mathbf{0}$ is the all-zero matrix with conforming dimensions.

II. System Model and Problem Statement

Consider a multi-cell distributed antenna system consisting of a cluster of M cells which use the same set of RBs. The BS in each cell is connected to L distributed single-antenna ports with high-speed communication links; e.g., optical fibre. The transmissions of the BSs in the cluster are coordinated by a central network entity, which is assumed to have reliable knowledge of the channel gains between the LM ports in the cluster and each UT in the M cells. Although having accurate channel knowledge increases the computational and communication burden on the network, the extent of this burden depends on the value of M, which is a design parameter that trades off implementation complexity for performance.

In the system considered herein, each UT is assumed to have one antenna and, as in OFDMA-based systems, at most one UT is assigned the same RB in each cell. The UTs are assumed to be served by the ports in their respective cells. Although it is possible to relax this assumption and allow the UTs close to the cell periphery to be served by ports in neighbouring cells, such a relaxation complicates the design and will not be considered herein.

Consider a specific RB, wherein a single UT is served in each cell. Let $\alpha_{\ell m} \in \{0, 1\}$ be the binary coefficient representing the on-off state of the ℓ-th port in the m-th cell on this RB, and let $w_{\ell m}$ and $P_{\ell m}$ denote the corresponding complex beam steering coefficient and the fixed transmit power of this port, respectively, for $\ell = 1, \ldots, L$ and $m = 1, \ldots, M$.

The received signal of the UT in the m-th cell is given by

$$y_m = \sum_{\ell=1}^{L} \alpha_{\ell m} \sqrt{P_{\ell m}} h_{\ell mm} w_{\ell m} x_m + \sum_{n=1, n\neq m}^{M} \sum_{\ell=1}^{L} \alpha_{\ell n} \sqrt{P_{\ell n}} h_{\ell nm} w_{\ell n} x_n + n_m, \quad \forall m, \quad (1)$$

where $h_{\ell nm}$ is the complex-valued channel gain between the ℓ-th port of the n-th cell and the UT in the m-th cell, and x_m is the normalized data symbol of the UT satisfying $\mathrm{E}\{x_m x_n\} = \delta_{mn}$, where δ_{mn} is equal to 1 when $m = n$, and zero otherwise. The additive white noise of the UT in the m-th cell is represented by $n_m \sim \mathcal{CN}(0, \sigma^2)$.

Let $\boldsymbol{\alpha}$ and $\boldsymbol{w}$ be the vectors containing the LM port states and beam steering coefficients, $\{\alpha_{\ell m}\}$ and $\{w_{\ell m}\}$, respectively. The SINR of the UT in the m-th cell can be expressed as

$$\mathrm{SINR}_m(\boldsymbol{\alpha}, \boldsymbol{w}) = \frac{\left|\sum_{\ell=1}^{L} \alpha_{\ell m} \sqrt{P_{\ell m}} h_{\ell mm} w_{\ell m}\right|^2}{\sigma^2 + \sum_{n=1, n\neq m}^{M} \left|\sum_{\ell=1}^{L} \alpha_{\ell n} \sqrt{P_{\ell n}} h_{\ell nm} w_{\ell n}\right|^2}, \quad \forall m. \quad (2)$$

In current cellular systems, a significant fraction of UTs, including those near the cell periphery, suffer from poor network coverage. This drawback can be effectively mitigated by proper coordination of geographically dispersed antenna ports. To facilitate system-wide implementation, it is desirable for the antenna ports to operate at fixed power levels. This helps to reduce the cost of building the antenna ports and to improve the operational efficiency of their power amplifiers. Motivated by these considerations, in the forthcoming analysis, we will restrict our attention to the practical case of fixed antenna port powers. In this case, coordinated transmission can be achieved by selecting the active antenna ports and their beam steering coefficients that maximize the SINR of the UT with the least favourable channel conditions. Although it is possible to consider other objectives, including those that yield Pareto optimal port state and beam steering coefficient vectors, finding a good approximation of these vectors appears to be complicated.

The port state and beam steering coefficient vectors that maximize the minimum SINR in (2) can be obtained by solving the following optimization problem:

$$\max_{\boldsymbol{\alpha},\boldsymbol{w}} \quad \min_{m} \ \mathrm{SINR}_m(\boldsymbol{\alpha},\boldsymbol{w}), \tag{3a}$$

$$\text{subject to} \quad \boldsymbol{\alpha} \in \{0,1\}^{LM}, \tag{3b}$$

$$\left|[\boldsymbol{w}]_q\right| = 1, \quad q = 1,\ldots,LM. \tag{3c}$$

The constraint in (3b) ensures that the state of each port is either off, implying that no power is allocated to this port, or on, implying that the port operates at full power. The constraints in (3c) ensure that the entries of $\boldsymbol{w}$ lie on the unit circle. Hence, varying these entries will steer the direction of the beams radiated by the antenna ports without changing their power. Neither the objective nor the constraints of this problem are convex and hence, this problem is difficult to solve jointly for $\boldsymbol{\alpha}$ and $\boldsymbol{w}$. For the objective, it can be shown that the SINR of the m-th UT is a rational function of biquadratic terms in $\boldsymbol{\alpha}$ and $\boldsymbol{w}$. For the constraints, it can be shown that, for any given $\boldsymbol{\alpha}$, finding the optimal beam steering coefficient vector, $\boldsymbol{w}$, satisfying (3c) is NP-hard [8]. In a complementary fashion, for any given $\boldsymbol{w}$, finding the optimal port state vector, $\boldsymbol{\alpha}$, satisfying (3b) is also NP-hard [7]. Despite their inherent difficulty, we will show that each of these problems can be cast in a form amenable to techniques that efficiently yield close-to-optimal solutions. To exploit this observation, we will seek an approximate two-stage algorithm for the problem in (3). The stages of this algorithm are described in Sections III and IV below.

It is worth noting that the formulation in (3) assumes that accurate channel gains are available at the coordinating network entity. As such, this formulation yields an upper bound on the performance that can be achieved when these gains are not accurately known.

III. Coordinated Multi-Cell Port Selection

In this section, we will seek to select the set of ports that maximizes the minimum SINR observed by all UTs when the beam steering coefficients are given.

Let $\boldsymbol{w}_0$ be the vector of the beam steering coefficients. In [6] it was shown that, when the interference power is fixed, the $\boldsymbol{w}_0$ that maximizes the SINR is the one in which the entries are chosen to match the phases of the channel between the ports and their intended UTs; that is, using q to denote $(m-1)L+\ell$, the q-th entry of $\boldsymbol{w}_0$ can be expressed as

$$[\boldsymbol{w}_0]_q \triangleq e^{-\jmath\angle h_{\ell mm}}. \tag{4}$$

From (2) it can be seen that the interference depends on the choice of $\boldsymbol{w}_0$, and hence the aforementioned assumption of fixed interference does not necessarily hold. In other words, the choice of $\boldsymbol{w}_0$ in (4) is not necessarily optimal. In fact, the optimal beam steering coefficient of any given port can be shown to depend on the channel gains between all the active ports and the UTs. However, this dependence complicates the search for the optimal port state and beam steering coefficient vectors. One way to facilitate this search, is to select the initial beam steering coefficient vector as in (4); that is, independently of the port state vector.

To make the SINR expression in (2) amenable to the optimization technique employed hereinafter, we will cast this expression using vector notation. In particular, using $\boldsymbol{w} = \boldsymbol{w}_0$ in (2),

$$\mathrm{SINR}_m(\boldsymbol{\alpha},\boldsymbol{w}_0) = \frac{\boldsymbol{\alpha}^T\boldsymbol{C}_m\boldsymbol{\alpha}}{\sigma^2 + \boldsymbol{\alpha}^T\boldsymbol{D}_m\boldsymbol{\alpha}} \quad \forall m, \tag{5}$$

where $\boldsymbol{C}_m \in \mathbb{R}^{LM\times LM}$ and $\boldsymbol{D}_m \in \mathbb{R}^{LM\times LM}$ are block-diagonal matrices defined as

$$\boldsymbol{C}_m = \oplus_{n=1}^{m-1}\boldsymbol{0}_L \oplus \boldsymbol{B}_{m,m} \oplus_{n=m+1}^{M} \boldsymbol{0}_L, \quad \text{and} \tag{6}$$

$$\boldsymbol{D}_m = \oplus_{n=1}^{m-1}\boldsymbol{B}_{n,m} \oplus \boldsymbol{0}_L \oplus_{n=m+1}^{M} \boldsymbol{B}_{n,m}, \tag{7}$$

where $\oplus$ is the direct sum operation defined in the Notation paragraph of the Introduction (see also [13, Sec. 0.9.2]), $\boldsymbol{0}_L$ is an $L\times L$ all-zero matrix, and the $\ell_1\ell_2$-th entry of $\boldsymbol{B}_{n,m} \in \mathbb{R}^{L\times L}$ is

$$[\boldsymbol{B}_{n,m}]_{\ell_1\ell_2} = \sqrt{P_{\ell_1 n}P_{\ell_2 n}}\,\Re\{h_{\ell_1 nm}h^*_{\ell_2 nm}w_{\ell_1 n}w^*_{\ell_2 n}\}, \quad \ell_1,\ell_2 = 1,\ldots,L, \quad n,m = 1,\ldots,M. \tag{8}$$

Defining the length-L vector $\boldsymbol{b}_{n,m} \triangleq \left[h_{1nm}w_{1n}\sqrt{P_{1n}} \ \cdots \ h_{Lnm}w_{Ln}\sqrt{P_{Ln}}\right]$, it can be verified that $\boldsymbol{B}_{n,m} = \Re\{\boldsymbol{b}_{n,m}\boldsymbol{b}^H_{n,m}\}$. Hence, it can be seen that $\boldsymbol{B}_{n,m}$ is PSD, and $\mathrm{rank}(\boldsymbol{B}_{m,m}) = 1$, and $\mathrm{rank}(\boldsymbol{B}_{n,m}) = 2$ for $n \neq m$. Subsequently, $\boldsymbol{C}_m$ and $\boldsymbol{D}_m$ are PSD and their ranks are 1 and $2(M-1)$, respectively for all m. Using this notation, the problem corresponding to (3) for selecting the SINR maximizing set of ports with the given $\boldsymbol{w}_0$ can be cast as

$$\max_{\boldsymbol{\alpha}} \quad \min_{m} \ \frac{\boldsymbol{\alpha}^T\boldsymbol{C}_m\boldsymbol{\alpha}}{\sigma^2 + \boldsymbol{\alpha}^T\boldsymbol{D}_m\boldsymbol{\alpha}}, \tag{9a}$$

$$\text{subject to} \quad \boldsymbol{\alpha} \in \{0,1\}^{LM}. \tag{9b}$$

This is a binary-constrained problem, which can be shown to be NP-hard [7]. To find a close-to-optimal solution for this problem, we introduce the vector $\boldsymbol{\beta} = 2\boldsymbol{\alpha} - \boldsymbol{1}$ and use the SDR technique; see e.g., [9]. Using the definition of $\boldsymbol{\beta}$, it can be seen that $\boldsymbol{\beta} \in \{-1,1\}^{LM}$, and

$$\boldsymbol{\alpha} = (\boldsymbol{\beta} + \boldsymbol{1})/2. \tag{10}$$

Substituting from (10) in (9), the non-homogeneous quadratic terms in the numerator and denominator of the resulting

SINR expression in the objective function can be expressed as $[\boldsymbol{\beta}^T \quad 1]\boldsymbol{E}_m[\boldsymbol{\beta}^T \quad 1]^T$, and $[\boldsymbol{\beta}^T \quad 1]\boldsymbol{F}_m[\boldsymbol{\beta}^T \quad 1]^T$, respectively, where

$$\boldsymbol{E}_m \triangleq \begin{bmatrix} \boldsymbol{C}_m & \boldsymbol{C}_m\mathbf{1} \\ \mathbf{1}^T\boldsymbol{C}_m & \mathbf{1}^T\boldsymbol{C}_m\mathbf{1} \end{bmatrix}, \quad \text{and} \tag{11}$$

$$\boldsymbol{F}_m \triangleq \begin{bmatrix} \boldsymbol{D}_m & \boldsymbol{D}_m\mathbf{1} \\ \mathbf{1}^T\boldsymbol{D}_m & \mathbf{1}^T\boldsymbol{D}_m\mathbf{1} + 4\sigma^2 \end{bmatrix}. \tag{12}$$

Using this notation, and letting $\boldsymbol{\Phi} = \boldsymbol{\beta}\boldsymbol{\beta}^T$, the transformed problem can be cast as

$$\max_{\boldsymbol{\Phi},\boldsymbol{\beta}} \quad \min_m \frac{\mathrm{Tr}\left(\boldsymbol{E}_m \begin{bmatrix} \boldsymbol{\Phi} & \boldsymbol{\beta} \\ \boldsymbol{\beta}^T & 1 \end{bmatrix}\right)}{\mathrm{Tr}\left(\boldsymbol{F}_m \begin{bmatrix} \boldsymbol{\Phi} & \boldsymbol{\beta} \\ \boldsymbol{\beta}^T & 1 \end{bmatrix}\right)}, \tag{13a}$$

$$\text{subject to} \quad \boldsymbol{\Phi} - \boldsymbol{\beta}\boldsymbol{\beta}^T = \mathbf{0}, \quad \mathrm{diag}(\boldsymbol{\Phi}) = \mathbf{1}. \tag{13b}$$

In this formulation, the binary constraint $\boldsymbol{\beta} \in \{-1, 1\}^{LM}$ is replaced with the equivalent constraint $\mathrm{diag}(\boldsymbol{\Phi}) = \mathbf{1}$. To see the equivalence, note that when $\boldsymbol{\Phi} = \boldsymbol{\beta}\boldsymbol{\beta}^T$, the diagonal entries of $\boldsymbol{\Phi}$ will be equal to $\beta_{\ell m}^2 = 1, \forall\ \ell, m$, which is satisfied if and only if $\boldsymbol{\beta} \in \{-1, 1\}^{LM}$.

A. Positive semidefinite relaxation

The first constraint in (13b) imposes a non-convex rank-1 constraint and results in the NP-hardness of (13). To obtain a close-to-optimal solution, we consider a relaxed version of (13) in which the first equality constraint in (13b) is replaced with the generalized matrix inequality

$$\boldsymbol{X} - \boldsymbol{x}\boldsymbol{x}^T \succeq \mathbf{0}, \tag{14}$$

where $\boldsymbol{X}$ and $\boldsymbol{x}$ are the optimization variables corresponding to $\boldsymbol{\Phi}$ and $\boldsymbol{\beta}$ in the original problem, respectively. Neither the constraint in (14) nor the objective of the relaxed problem is convex. To cast it in a more convenient form, we introduce an auxiliary variable, t, that lower-bounds the objective. Note that $\boldsymbol{X} - \boldsymbol{x}\boldsymbol{x}^T$ is the Schur complement [14] of the matrix

$$\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{x} \\ \boldsymbol{x}^T & 1 \end{bmatrix}, \tag{15}$$

and is PSD if and only if $\boldsymbol{\Psi}$ is PSD. Using this observation, the constraint in (14) will be cast as a convex linear matrix inequality. Now, the relaxed problem can be rewritten as

$$\max_{t,\boldsymbol{\Psi}} \quad t, \tag{16a}$$

$$\text{subject to} \quad t \le \frac{\mathrm{Tr}(\boldsymbol{E}_m\boldsymbol{\Psi})}{\mathrm{Tr}(\boldsymbol{F}_m\boldsymbol{\Psi})}, \quad \forall m, \tag{16b}$$

$$\boldsymbol{\Psi} \succeq \mathbf{0}, \tag{16c}$$

$$\mathrm{diag}(\boldsymbol{\Psi}) = \mathbf{1}. \tag{16d}$$

The problem in (16) is still non-convex in $(t, \boldsymbol{\Psi})$ because each of the inequality constraints in (16b) involves products of the form $t\boldsymbol{\Psi}$. Fortunately, however, it can be seen that these constraints are quasi-linear in $(t, \boldsymbol{\Psi})$. This is because both the superlevel and the sublevel sets corresponding to a fixed t are convex. Despite this quasi-linearity, solving (16) may still be difficult because both $\boldsymbol{E}_m$ and $\boldsymbol{F}_m$ are rank-deficient, for every m. Hence, it may happen that both $\mathrm{Tr}\,(\boldsymbol{E}_m\boldsymbol{\Psi})$ and $\mathrm{Tr}\,(\boldsymbol{F}_m\boldsymbol{\Psi})$ vanish simultaneously, resulting in the right hand side of (16b) assuming an indeterminate quantity. To show that such an occurrence is not possible, it suffices to show that $\mathrm{Tr}(\boldsymbol{F}_m\boldsymbol{\Psi})$ is strictly greater than zero. In particular, we have the following result:

Lemma 1: For the matrix $\boldsymbol{F}_m$ defined in (12) and the matrix $\boldsymbol{\Psi}$ defined in (15),

$$\mathrm{Tr}\big(\boldsymbol{F}_m\boldsymbol{\Psi}\big) > 0, \quad \forall m.$$

Proof: See Appendix A. ■

Expressing the constraints in (16b) in the form $\mathrm{Tr}\big((t\boldsymbol{F}_m - \boldsymbol{E}_m)\boldsymbol{\Psi}\big) \le 0$, $\forall\, m$, Lemma 1 can now be invoked to show that for any $\boldsymbol{\Psi}$ with the structure in (15) the left hand side of this inequality is monotonically increasing in t. Using this observation, the optimal value of t, $t^\star$, can be obtained by solving a sequence of convex feasibility problems, each of the form

$$\text{find} \quad \boldsymbol{\Psi} \text{ with the structure in (15)}, \tag{17a}$$

$$\text{subject to} \quad \mathrm{Tr}\big((t_0\boldsymbol{F}_m - \boldsymbol{E}_m)\boldsymbol{\Psi}\big) \le 0, \quad \forall m, \tag{17b}$$

$$\boldsymbol{\Psi} \succeq \mathbf{0}, \tag{17c}$$

$$\mathrm{diag}(\boldsymbol{\Psi}) = \mathbf{1}. \tag{17d}$$

For each instance of this problem, the value of t_0 is fixed and represents the minimum SINR observed by the UTs. This SINR is upper bounded by

$$t_{\max} = \min_{m=1,\ldots,M} (\mathbf{1}^T\boldsymbol{C}_m\mathbf{1})/\sigma^2, \tag{18}$$

which corresponds to a situation in which there is no interference and all ports are active. Hence, it can be seen that the optimal t_0 must lie in $[0, t_{\max}]$.

Using an argument analogous to the one in [15, Section 4.2.5], it can be seen that if, for a particular value of t_0, (17) is feasible then $t_0 \le t^\star$. Conversely, if for this value of t_0, (17) is infeasible then $t_0 > t^\star$. Hence, the optimal value of t_0 must lie on the boundary of the feasible set of (16) and can be found using a bisection search.

Note that, being bilinear in t and $\boldsymbol{\Psi}$, the problem in (16) is amenable to generic optimization techniques, including the Alternate Convex Search [16]. However, guarantees for convergence and for reaching the global optimal solution using this method are generally not available. In contrast, exploiting the quasi-linearity of (16), the approach proposed herein is guaranteed to converge to the global optimal solution with polynomial complexity as shown in Section VI.

B. Randomization for coordinated port selection

Let $\boldsymbol{X}^\star$ and $\boldsymbol{x}^\star$ denote the optimal solution of (16) corresponding to $t = t^\star$ obtained by the bisection search. To obtain a candidate solution of the optimization problem in (13), we will use the Gaussian randomization technique, which is known to yield a close-to-optimal solution for NP-hard optimization problems with a similar underlying structure; see e.g., [10].

To apply this technique to our current problem, a set of J random vectors $\mathcal{V} = \{\boldsymbol{v}^{(j)}\}_{j=1}^J$, where $\boldsymbol{v}^{(j)} \in \mathbb{R}^{LM\times 1}$ for all j, is generated from the Gaussian distribution $\mathcal{N}(\boldsymbol{x}^\star, \boldsymbol{X}^\star - \boldsymbol{x}^\star\boldsymbol{x}^{\star T})$. For sufficiently large J, the vectors in $\mathcal{V}$ provide an approximate solution to the following stochastic optimization problem:

$$\max_{\substack{\boldsymbol{X}^\star=\mathrm{E}\{\boldsymbol{v}\boldsymbol{v}^T\}\\ \boldsymbol{x}^\star=\mathrm{E}\{\boldsymbol{v}\}}} \quad t, \tag{19a}$$

$$\text{subject to} \quad \mathrm{E}\Big\{[\boldsymbol{v}^T \quad 1]\big(t\boldsymbol{F}_m - \boldsymbol{E}_m\big)[\boldsymbol{v}^T \quad 1]^T\Big\} \le 0,\ \forall m, \tag{19b}$$

$$\mathrm{E}\{[\boldsymbol{v}]_r^2\} = 1, \quad r = 1, \ldots, LM. \tag{19c}$$

Since $\boldsymbol{X}^\star - \boldsymbol{x}^\star\boldsymbol{x}^{\star T} \succeq \boldsymbol{0}$, it can be seen that this optimization problem is equivalent to the one in (16). Hence, the set of vectors in $\mathcal{V}$ solves the problem in (16) on average [9].

Our goal now is to use the vectors in $\mathcal{V}$ to extract candidate solutions to the problem in (13). To do so, each realization of $\boldsymbol{v}^{(j)} \in \mathcal{V}$ is quantized. In particular, for each $\boldsymbol{v}^{(j)} \in \mathcal{V}$, a candidate binary solution $\tilde{\boldsymbol{\beta}}^{(j)}$ is obtained as follows:

$$\tilde{\boldsymbol{\beta}}^{(j)} = \mathrm{sgn}(\boldsymbol{v}^{(j)} - \boldsymbol{x}^\star), \qquad j = 1, \ldots, J, \tag{20}$$

where $\mathrm{sgn}(\cdot)$ is the element-wise signum function. Using (10), the corresponding candidate solutions of (9) are obtained and the one yielding the largest objective is chosen; i.e.,

$$\boldsymbol{\alpha}^\star = \arg \max_{j=1,\ldots,J} \min_{m=1,\ldots,M} \mathrm{SINR}_m(\tilde{\boldsymbol{\alpha}}^{(j)}, \boldsymbol{w}_0). \tag{21}$$

In Section VII, it will be shown that when the beam steering coefficients are fixed, the above SDR technique with Gaussian randomization provides a close-to-optimal solution of the port selection problem with a relatively small J.

IV. Coordinated Beam Steering Optimization

In the previous section, we considered the problem of selecting the antenna ports that maximize the minimum SINR when the beam steering coefficients are fixed. We now consider the complementary problem in which the port state vectors are fixed and the beam steering coefficients are to be optimized.

Let $\boldsymbol{\alpha}_0$ be a given port state vector. Analogous to the approach used in Section III, we will use vector notation to express the SINR in a convenient form that facilitates the optimization of the beam steering coefficients.

Define M^2 matrices $\{\boldsymbol{Q}_{n,m}\}_{n,m=1}^M$ such that the $\ell_1\ell_2$-th entry of the nm-th matrix is given by

$$[\boldsymbol{Q}_{n,m}]_{\ell_1\ell_2} = \alpha_{\ell_1 n} h_{\ell_1 nm} h^*_{\ell_2 nm} \alpha_{\ell_2 n}\sqrt{P_{\ell_1 n}P_{\ell_2 n}}, \quad \ell_1, \ell_2 = 1, \ldots, L, \quad n, m = 1, \ldots, M. \tag{22}$$

Furthermore, define block-diagonal matrices $\boldsymbol{S}_m \in \mathbb{C}^{LM\times LM}$ and $\boldsymbol{T}_m \in \mathbb{C}^{LM\times LM}$ as

$$\boldsymbol{S}_m = \oplus_{n=1}^{m-1}\boldsymbol{0} \oplus \boldsymbol{Q}_{m,m} \oplus_{n=m+1}^{M} \boldsymbol{0}, \quad \text{and} \tag{23}$$

$$\boldsymbol{T}_m = \oplus_{n=1}^{m-1}\boldsymbol{Q}_{n,m} \oplus \boldsymbol{0} \oplus_{n=m+1}^{M} \boldsymbol{Q}_{n,m}. \tag{24}$$

Using an argument analogous to the one used in Section III, it can be shown that the matrices $\{\boldsymbol{Q}_{n,m}\}_{n,m=1}^M$ are PSD and rank-1, and that the matrices $\{\boldsymbol{S}_m\}_{m=1}^M$ and $\{\boldsymbol{T}_m\}_{m=1}^M$ are PSD with ranks 1 and $M-1$, respectively.

Using this notation, the SINR of the UT in the m-th cell can be expressed as

$$\mathrm{SINR}_m(\boldsymbol{\alpha}_0, \boldsymbol{w}) = \frac{\boldsymbol{w}^H\boldsymbol{S}_m\boldsymbol{w}}{\sigma^2 + \boldsymbol{w}^H\boldsymbol{T}_m\boldsymbol{w}}, \quad \forall m. \tag{25}$$

With the port state vector being fixed as $\boldsymbol{\alpha}_0$, the optimization in (3) reduces to

$$\max_{\boldsymbol{w}} \quad \min_{m} \frac{\boldsymbol{w}^H\boldsymbol{S}_m\boldsymbol{w}}{\sigma^2 + \boldsymbol{w}^H\boldsymbol{T}_m\boldsymbol{w}}, \tag{26a}$$

$$\text{subject to} \quad \big|[\boldsymbol{w}]_q\big| = 1, \quad q = 1, \ldots, LM. \tag{26b}$$

This problem is non-convex, and is known to be NP-hard [8]. To find a close-to-optimal solution, we utilize a variation of the SDR-based Gaussian randomization technique used in the first stage.

Similar to the approach used in Section III, in applying the SDR technique, the optimization problem in (26) is relaxed by letting $\boldsymbol{\Upsilon} = \boldsymbol{w}\boldsymbol{w}^H$ and subsequently dropping the rank-1 constraint on $\boldsymbol{\Upsilon}$. Let $\boldsymbol{W} \in \mathbb{C}^{LM\times LM}$ be the counterpart of $\boldsymbol{\Upsilon}$ in the relaxed problem. Using a new variable s to lower bound the objective in (26a), the relaxed optimization problem can be expressed as in (16), where t, $\boldsymbol{E}_m$, $\boldsymbol{F}_m$, and $\boldsymbol{\Psi}$ are replaced with s, $\boldsymbol{S}_m$, $\boldsymbol{T}_m$, and $\boldsymbol{W}$, respectively. Analogous to the discussion following (16), this relaxed problem is non-convex, but can be shown to be quasi-linear. Let s_0 denote the value of s at any iteration of the corresponding bisection search. We seek the maximum value of s_0 for which the following feasibility problem has a solution.

$$\text{find} \quad \boldsymbol{W}, \tag{27a}$$

$$\text{subject to} \quad s_0\sigma^2 + \mathrm{Tr}\big((s_0\boldsymbol{T}_m - \boldsymbol{S}_m)\boldsymbol{W}\big) \le 0, \quad \forall m, \tag{27b}$$

$$\mathrm{diag}(\boldsymbol{W}) = \boldsymbol{1}, \tag{27c}$$

$$\boldsymbol{W} \succeq \boldsymbol{0}. \tag{27d}$$

It is straightforward to see that this problem is convex and can be solved efficiently for a given s_0. However, to facilitate the bisection search, it is desirable to upper-bound s_0.

Let $s_{\max}$ denote the upper bound on the value of s_0. A candidate $s_{\max}$ can be obtained by upper bounding the SINR expression in (25). In particular, since $\boldsymbol{T}_m \succeq \boldsymbol{0}$,

$$\mathrm{SINR}_m(\boldsymbol{\alpha}_0, \boldsymbol{w}) \le \frac{\boldsymbol{w}^H\boldsymbol{S}_m\boldsymbol{w}}{\sigma^2}$$

$$\le \frac{\|\boldsymbol{w}\|_2^2}{\sigma^2}\|\boldsymbol{S}_m\|_2 \tag{28}$$

$$\le \frac{LM}{\sigma^2}\|\boldsymbol{S}_m\|_2, \tag{29}$$

where (28) follows from the submultiplicative property of the 2-norm [14], and (29) follows from the fact that $\|\boldsymbol{w}\|_2^2 \le LM$. Hence, a candidate value of $s_{\max}$ is $\min_{m=1,\ldots,M}(LM\|\boldsymbol{S}_m\|_2)/\sigma^2$.

Let $\boldsymbol{W}^\star$ denote the solution of the relaxed problem corresponding to the optimal s_0 obtained from the bisection search. The Gaussian randomization technique is then used to generate an approximate solution of the original problem in (26). To apply this technique, a set of K random vectors, $\mathcal{Z} = \{\boldsymbol{z}^{(k)}\}_{k=1}^K$, is drawn from the Gaussian distribution $\mathcal{CN}(\boldsymbol{0}, \boldsymbol{W}^\star)$. Candidate beam steering coefficients, $\{[\tilde{\boldsymbol{w}}^{(k)}]_q\}_{q=1}^{LM}$, that lie on the unit circle are obtained by normalizing the entries of each realization of $\boldsymbol{z}^{(k)} \in \mathcal{Z}$; i.e.,

$$[\tilde{\boldsymbol{w}}^{(k)}]_q = \frac{[\boldsymbol{z}^{(k)}]_q}{\big|[\boldsymbol{z}^{(k)}]_q\big|}, \quad q = 1, \ldots, LM. \tag{30}$$

The close-to-optimal solution generated by the Gaussian randomization is the one that yields the largest minimum SINR; i.e.,

$$\boldsymbol{w}^{\star} = \arg \max_{k=1,\ldots,K} \min_{m=1,\ldots,M} \mathrm{SINR}_m(\boldsymbol{\alpha}_0, \tilde{\boldsymbol{w}}^{(k)}). \qquad (31)$$

Using an analogous approach to the one in [12], it can be shown that, for a given $\boldsymbol{\alpha}_0$, the above technique yields a close-to-optimal solution to the problem in (26).

V. An approximate solution to the joint optimization problem

In this section, we use the techniques developed in Sections III and IV to develop a two-stage approach for generating an efficiently-computable approximate solution to the problem in (3).

In the first stage, the weights are chosen as in (4), the matrices $\{\boldsymbol{E}_m, \boldsymbol{F}_m\}_{m=1}^{M}$ are constructed, and a set of J port state vectors are generated using (20) and (10). Out of those J vectors, the $\hat{J} \leq J$ candidates that yield the largest minimum SINR are selected. Notice that the close-to-optimal solution of (9) generated by (21) corresponds to setting $\hat{J} = 1$ and does not necessarily yield a close-to-optimal solution of the problem in (3), as we will show later.

In the second stage, each of the $\hat{J}$ candidate vectors generated in the first stage is used to construct the matrices $\{\boldsymbol{S}_m, \boldsymbol{T}_m\}_{m=1}^{M}$ and the corresponding close-to-optimal beam steering coefficients are generated using (31). The approximate solution of (3) is chosen to be the pair of port state vector and beam steering coefficient vector that yield the largest objective in (3a).

It will be shown in Section VII that this two-stage approach with relatively small J and $\hat{J}$ yields close-to-optimal solutions that perform significantly better than the port states selected in the first stage with the initial beam steering coefficients.

Although it is possible to iterate between the two stages, our numerical results suggest that this technique provides negligible difference and does not guarantee a better performance. This is due to the fact that the original problems for optimizing $\boldsymbol{\alpha}$ and $\boldsymbol{w}$ are non-convex. Hence, using different initial vectors, including close-to-optimal ones, does not necessarily yield final vectors that perform better than those obtained using other sub-optimal initial vectors. Furthermore, the numerical results in Section VII below show a relatively small gap between the jointly optimal port state and beam steering coefficient vectors obtained by exhaustive search and those obtained by the proposed two-stage approach. This suggests that iterating between the two stages would yield a marginal performance gain, but would incur a significant additional computational cost.

In the next section, we will provide bounds on the computational complexity of the proposed techniques. In particular, we will show that each of the techniques that yield close-to-optimal solutions in Sections III and IV has a polynomial complexity. Hence, the technique proposed in this section for obtaining an approximate solution to the joint optimization problem in (3) also has polynomial complexity.

VI. Complexity Analysis

A. Computational complexity of the first stage

In the first stage of the approach proposed in the previous section, the beam steering coefficients are fixed. In this case, the optimal port state vector could be found by exhaustive search over all possible vectors. The computational complexity of this approach is $\mathcal{O}\big(2^{LM}\big)$, and hence it is inefficient for large L and M.

In contrast, the SDR-based Gaussian randomization technique proposed in Section III involves solving a sequence of convex problems, each with a PSD constraint; cf. (17). Using interior point based solvers, e.g., `SeDuMi` [17], the complexity of solving problems of this form is $\mathcal{O}\big((LM)^{6.5}\log(1/\epsilon_0)\big)$, where $\epsilon_0 > 0$ is the solution accuracy [18]. However, the problem in (17) has a particular structure that can exploited to develop more efficient solving techniques. For instance, the primal-dual path-following interior-point method developed in [19] has been particularized in [9] to solve a PSD-constrained convex optimization problem similar to the one in (17) with complexity $\mathcal{O}\big((LM)^{4.5}\log(1/\epsilon_0)\big)$.

Let $\epsilon_1 > 0$ be the solution accuracy of the bisection search used in Section III. Since this search is over the interval $[0, t_{\max}]$ and its convergence rate is exponential, the number of bisection search iterations is given by $\log(t_{\max}/\epsilon_1)$, where $t_{\max}$ is defined in (18). For the Gaussian randomization procedure described in Section III-B, the computational complexity of generating and evaluating the objective corresponding to the J random samples is $\mathcal{O}\big((LM)^2 J\big)$. Now, combining these observations, it can be seen that the complexity of the proposed port selection technique is $\mathcal{O}\big((LM)^{4.5}\log(1/\epsilon_0)\log(t_{\max}/\epsilon_1) + (LM)^2 J\big)$.

B. Computational complexity of the second stage

In the second stage, close-to-optimal beam steering coefficients for a given port state vector could be obtained by discretizing each coefficient using N_b bins. Then, the coefficients that yield the largest minimum SINR could be determined by exhaustive search over all possible bin combinations. The complexity of such an approach would be $\mathcal{O}\big(N_b^{LM}\big)$, which is computationally inefficient, similar to the exhaustive search in the port selection problem. In Section IV, the SDR-based Gaussian randomization technique is used to generate close-to-optimal beam steering coefficients. Using a discussion analogous to the one in Section VI-A, it can be shown that the complexity of this technique is $\mathcal{O}\big((LM)^{4.5}\log(1/\epsilon_0)\log(s_{\max}/\epsilon_1) + (LM)^2 K\big)$.

C. Computational complexity of the two-stage approach

Using exhaustive search to solve the joint optimization problem in (3) with discretized beam steering coefficients involves a complexity of $\mathcal{O}\big((2N_b)^{LM}\big)$, which is computationally prohibitive, even for a relatively small system. For example, for a two-cell cluster with seven ports in each cell, and $N_b = 100$, the exhaustive search involves about $200^{14} \approx 1.64 \times 10^{32}$ combinations.

Using the two-stage approach presented in Section V and the complexity discussions in Sections VI-A and VI-B, it can

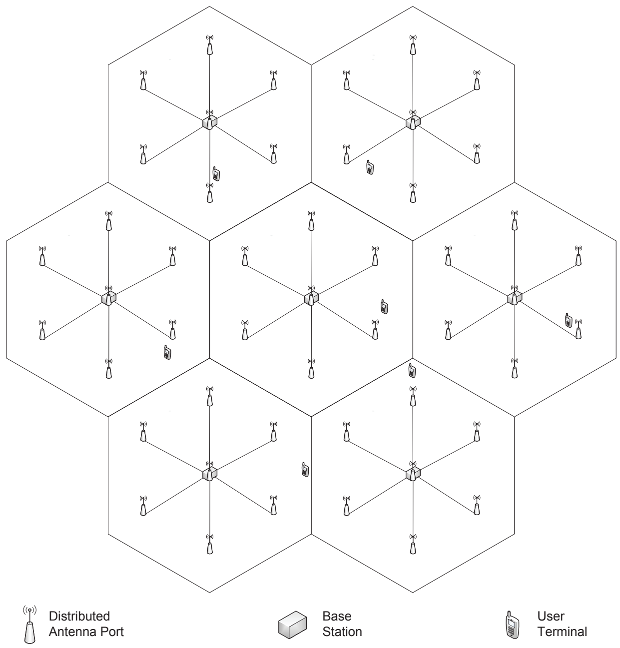


Fig. 1. A seven-cell distributed antenna system with seven ports per cell.

be seen that the complexity of this technique is bounded by

$$\mathcal{O}\Big((LM)^{4.5}\log(1/\epsilon_0)\big(\hat{J}\log(s_{\max}/\epsilon_1) + \log(t_{\max}/\epsilon_1)\big) + (LM)^2(K\hat{J}+J)\Big).$$

The results of this section are summarized in Table I.

VII. Performance Evaluation

In this section, we use Monte Carlo simulation to assess the performance of the port selection and the beam steering optimization techniques presented in Sections III and IV, respectively, and the performance of the two-stage approach presented in Section V. In the numerical results reported herein, solutions to the sequence of feasibility problems in (17) and (27) are obtained using the software package `CVX` [20] with an underlying `SeDuMi` solver [17].

The cellular system used for the simulation consists of M hexagonal cells, each with circumradius r_c. The BS in each cell is connected to seven ports; i.e., $L = 7$ [5], [6]. To evenly cover the geographic area of the cell, six of these ports are located uniformly at a distance of $\frac{2}{3}r_c$ from the BS at the center of the cell, while the seventh port is co-located with the BS. Such a system is illustrated in Fig. 1, where $M = 7$. All ports transmit at a fixed power P, i.e., $P_{\ell n} = P, \forall \ell, n$ [5]. In each iteration, the UTs are dropped randomly in each cell.

We consider a standard communication channel model with quasi-static frequency-flat Rayleigh fading, log-normal shadowing, and path loss components. As such, each complex channel gain can be expressed as $h_{\ell nm} = \sqrt{\rho(d_{\ell nm})s_{\ell nm}}h'_{\ell nm}$, where $\rho(\cdot)$ is a path loss function, which depends on the propagation environment [21], and $d_{\ell nm}$ is the distance between the ℓ-th port of the n-th cell and the UT in the m-th cell. Shadowing is represented by $s_{\ell nm}$, which is log-normal distributed with 0 dB mean and standard deviation σ_s in dB, and fading is represented by $h'_{\ell nm}$, which is complex Gaussian distributed with zero mean and unit variance.

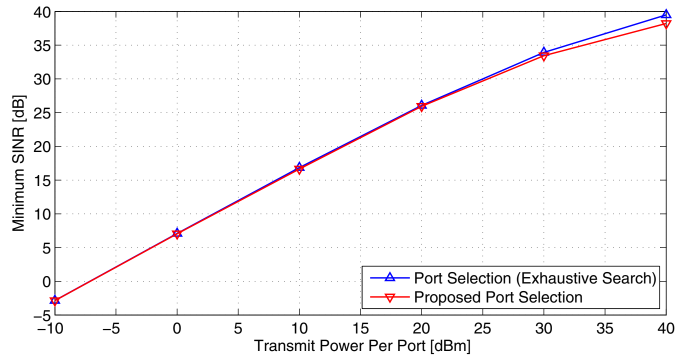


Fig. 2. A comparison between the largest minimum SINR achieved by port selection with exhaustive search and with the technique proposed in the first stage for a two-cell cluster in the SMa scenario.

To simulate practical communication scenarios, we have selected the distance values and the log-normal shadowing and path loss parameters corresponding to the suburban macro-cell (SMa) and urban macro-cell (UMa) IMT-Advanced scenarios [21, Sections 8.4.2, A-1.3.1]. For the SMa scenario, the distance between the BSs is 1299 m, and for the UMa scenario, this distance is 500 m. The corresponding shadowing standard deviation, σ_s, is 8 dB and 6 dB, respectively. For both scenarios, the noise power, σ^2, is chosen to be -114 dBm [21], [22] and the path loss channel model is chosen to be the non-line-of-sight one in [21]. In this model, setting the carrier frequency to 2 GHz, the elevation of each antenna port to 15 m, and the elevation of each UT to 1.5 m yields the following path loss function:

$$\rho(d_{\ell nm}) = 10^{1.866+4.032\log_{10}(d_{\ell nm})}.$$

Example 1: In this example, the SMa scenario is considered. The largest minimum SINR that is achieved by the port selection technique in Section III is depicted in Fig. 2 when the beam steering coefficients are fixed as in (4). This SINR is compared with the corresponding SINR achieved using the exhaustive search in Section VI. To facilitate exhaustive search, in this example we consider the case of a two-cell cluster (i.e., $M = 2$), which requires searching over $2^{14} \approx 1.64 \times 10^4$ port state vectors. The resulting SINRs are averaged over 500 independent channel realizations. For each iteration, the number of the Gaussian samples, J, is chosen to be 100. From Fig. 2, it can be seen that the performance of the proposed port selection technique approaches that of the optimal solution for the entire range of P.

In Fig. 3, the sum rate achieved by the proposed port selection technique is compared with the maximum achievable sum rate obtained using exhaustive search over all possible port state vectors. It can be seen from the figure that the sum rate achieved by the proposed technique is comparable with the maximum sum rate, but achieving it requires significantly less computation; polynomial complexity for the proposed technique instead of exponential complexity for the exhaustive search. □

Example 2: In this example, we consider the SMa scenario in a cluster of $M = 7$ cells. In Fig. 4, the largest

TABLE I
COMPLEXITY OF THE PROPOSED TECHNIQUES AND THE CORRESPONDING EXHAUSTIVE SEARCH

Technique		Complexity
Port selection	Exhaustive	$\mathcal{O}(2^{LM})$
	Proposed	$\mathcal{O}\big((LM)^{4.5}\log(\frac{1}{\epsilon_0})\log(\frac{t_{\max}}{\epsilon_1})+(LM)^2J\big)$
Beam steering optimization	Exhaustive	$\mathcal{O}(N_b^{LM})$
	Proposed	$\mathcal{O}\big((LM)^{4.5}\log(\frac{1}{\epsilon_0})\log(\frac{s_{\max}}{\epsilon_1})+(LM)^2K\big)$
Port selection and beam steering optimization	Exhaustive	$\mathcal{O}\big((2N_b)^{LM}\big)$
	Proposed	$\mathcal{O}\Big((LM)^{4.5}\log(\frac{1}{\epsilon_0})\big(\hat{J}\log(\frac{s_{\max}}{\epsilon_1})+\log(\frac{t_{\max}}{\epsilon_1})\big)+(LM)^2(K\hat{J}+J)\Big)$

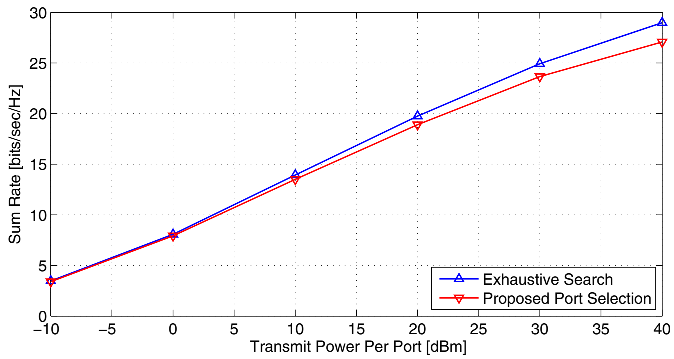


Fig. 3. A comparison between the largest sum rate achieved by port selection with exhaustive search and with the technique proposed in the first stage for a two-cell cluster in the SMa scenario.

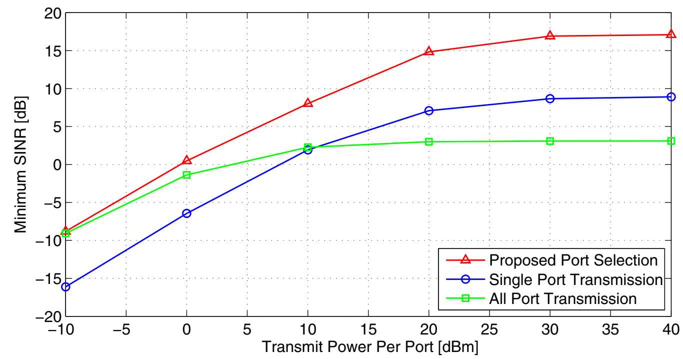


Fig. 4. A comparison between the largest minimum SINR achieved by the port selection technique proposed in the first stage, and that achieved by the one-port (without coordination) and all-port strategies for a seven-cell cluster in the SMa scenario.

minimum SINR achieved by the port selection technique in Section III is compared with that achieved by other candidate transmission strategies, namely transmission from a single port and from all ports in each cell, which were investigated in [6]. The SINRs are averaged over 5000 independent channel realizations. For such a system, obtaining the optimal solution using exhaustive search is computationally prohibitive since this search involves $2^{49} \approx 5.63 \times 10^{14}$ port state vectors. However, the proposed technique can provide close-to-optimal solutions in polynomial time as discussed in Section VI. For each iteration, the port that is chosen in each cell in the case of single-port transmission is the one with the largest channel gain to the UT in that cell. For the SDR-based Gaussian randomization technique used for port selection, the number of the Gaussian samples, J, is chosen to be 300. It can be seen from Fig. 4 that the proposed technique achieves significantly larger minimum SINR than the two baseline transmission strategies, particularly at higher values of P.

Fig. 5 shows the average number of ports per cell that are activated by the proposed port selection technique. It can be observed from the figure that the number of active ports is relatively high at low values of P and decreases as P increases. This observation is consistent with the performance results depicted in Fig. 4, wherein the all-port transmission strategy outperforms the single-port one when P is low, and vice versa when P is high. The results in Fig. 5 also indicate that the proposed port selection technique leads to significant power savings by deactivating a considerable percentage of the available ports when P is relatively high. □

Example 3: In Fig. 6, we consider a comparison similar to the one in Example 2, but for the UMa scenario. Comparing the results in this figure with the corresponding results in

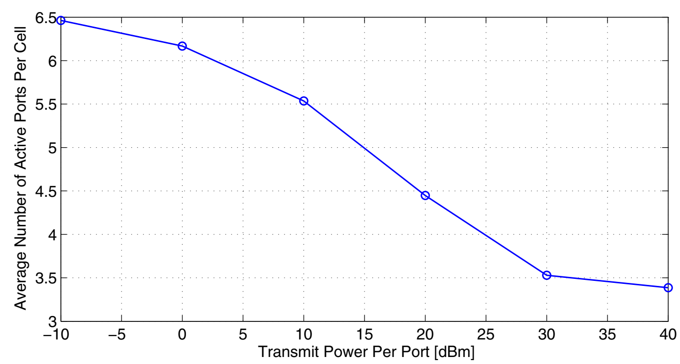


Fig. 5. The average number of ports activated per cell by the selection technique proposed in the first stage of an SMa seven-cell cluster.

Fig. 4, it can be seen that, at lower values of P, the achieved minimum SINRs are higher for the UMa scenario than the SMa one, and vice versa at higher values of P. This behaviour can be attributed to the fact that at low values of P, the system is noise-limited, whereas at high values of P, it is interference-limited. From the figure, it can also be noticed that when P is low, the performance of the proposed technique is close to that of the all-port transmission strategy. This observation suggests that when the power per port is low, the SINR maximizing strategy is to use a greater number of ports for transmission in each cell. This effect is further demonstrated in Fig. 7, where it can be observed that the number of active ports per cell approaches L as P decreases. □

Example 4: In this example, the performance of the two-stage approach proposed in Section V is evaluated for a system with $M = 2$ in the SMa scenario, and compared with a computationally-expensive close-to-optimal joint solution.

TABLE II
CHANNEL GAINS BETWEEN THE PORTS AND THE UTS IN EACH CELL $(\times 10^{-6})$

Port #	UT in the 1st cell		UT in the 2nd cell	
	1st Cell	2nd Cell	1st Cell	2nd Cell
1	$-0.0565 + j0.0801$	$0.1369 - j0.1623$	$-0.0613 + j0.0329$	$0.1892 + j0.7350$
2	$-0.3688 + j0.0382$	$-0.1055 + j0.0519$	$0.0379 - j0.0114$	$-1.2478 - j13.248$
3	$1.8197 - j0.7746$	$0.0181 + j0.0093$	$0.2241 + j0.0023$	$-0.5654 + j0.4546$
4	$-0.2693 - j4.1101$	$-0.1455 - j0.0391$	$-0.1529 - j0.1268$	$-0.0598 + j0.0116$
5	$0.0076 - j0.1963$	$-0.1746 - j0.1039$	$0.0001 - j0.0009$	$0.1900 + j0.0674$
6	$0.3743 + j0.0490$	$-1.3740 - j0.5114$	$-0.0079 + j0.0075$	$0.4124 - j0.5146$
7	$-0.0459 - j0.0178$	$0.0446 - j0.0564$	$0.0056 + j0.0068$	$0.3308 + j0.5633$

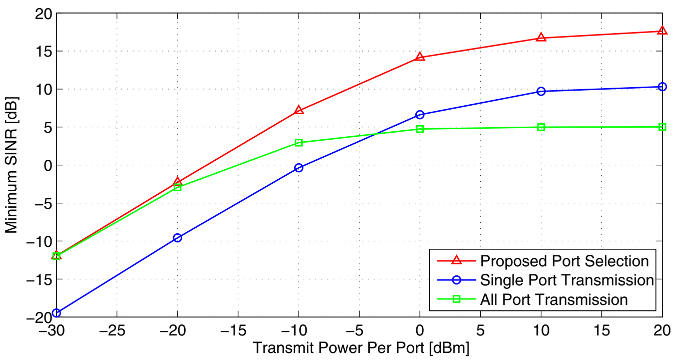


Fig. 6. A comparison between the largest minimum SINR achieved by the port selection technique proposed in the first stage, and that achieved by the one-port (without coordination) and all-port strategies for a seven-cell cluster in the UMa scenario.

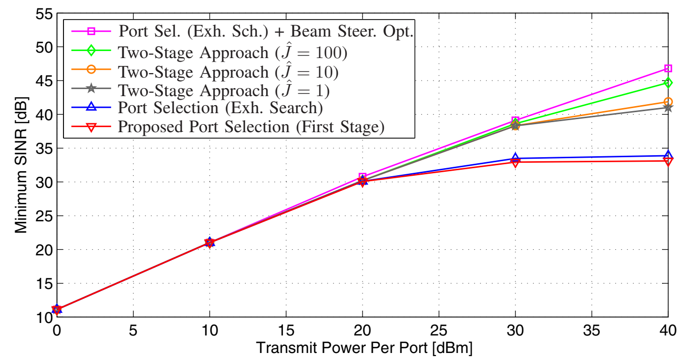


Fig. 8. A comparison between the largest minimum SINR achieved by the proposed two-stage approach, exhaustive-search, and the port selection technique in the first stage. A two-cell cluster in the SMa scenario.

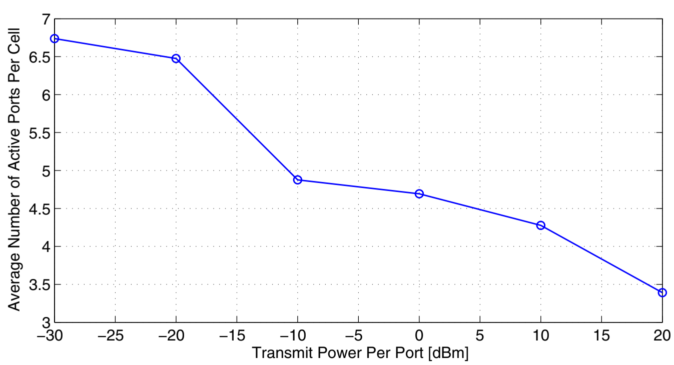


Fig. 7. The average number of ports activated per cell by the selection technique proposed in the first stage of a UMa seven-cell cluster.

Due to the complexity of the latter approach, a single channel realization is simulated. The channel gains for this realization are provided in Table II.

The close-to-optimal solution is obtained through exhaustive search over all possible port state vectors, and by using the technique proposed in Section IV to generate close-to-optimal beam steering coefficients for each vector. For the proposed two-stage approach, 100 Gaussian samples are generated in the first and second stages; i.e., $J = K = 100$.

In Fig. 8, we show the largest minimum SINRs achieved by the two-stage approach described in Section V with $\hat{J} = 1$, 10, and 100, and by the exhaustive search described above. For comparison, we also show the largest minimum SINRs achieved by the techniques in Sections III and IV. From the figure, it can be seen that the performance of the two-stage technique approaches that of the close-to-optimal joint solution as $\hat{J}$ increases. Both techniques outperform those presented in Sections III and IV for the given initial beam steering coefficients and the port state vector generated in the first stage, respectively, particularly at higher transmit powers.

The simulation time taken to generate the results reported in Fig. 8 is approximately 3.3 seconds, whereas the corresponding time for an SMa seven-cell system, not shown herein, is approximately 16.2 seconds. These values have been obtained using Matlab-based simulations on a computer with a 3.6 GHz Intel Core i7 processor. □

VIII. CONCLUSION

In this paper, we considered coordinated downlink transmission in a multi-cell distributed antenna system. A novel two-stage SDR-based polynomial-complexity technique for jointly optimizing the port states and the beam steering coefficients was developed. It was shown that the port state vector generated by the technique proposed in the first stage yields comparable performance to the optimal vector, which is obtained using exponential-complexity exhaustive search. Additionally, the performance of the two-stage technique was shown to be comparable with that of a close-to-optimal one, but with a significantly less computational cost. Finally, it was shown that, by deactivating selected ports, the two-stage technique can provide significant power savings without compromising performance.

ACKNOWLEDGEMENT

The authors would like to thank Dr. Chandra (Sekhar) Bontu and Dr. Jim Womack of Research In Motion, and Dr. Cenk Toker of Hacettepe University, Ankara, Turkey, for their comments and support.

Appendix A
Proof of Lemma 1

Using the definitions of $\boldsymbol{F}_m$ and $\boldsymbol{\Psi}$ in (12) and (15), respectively, we have

$$\begin{aligned}\mathrm{Tr}\big(\boldsymbol{F}_m\boldsymbol{\Psi}\big) &= \mathrm{Tr}\left(\begin{bmatrix}\boldsymbol{D}_m & \boldsymbol{D}_m\mathbf{1}\\ \mathbf{1}^T\boldsymbol{D}_m & \mathbf{1}^T\boldsymbol{D}_m\mathbf{1}+4\sigma^2\end{bmatrix}\begin{bmatrix}\boldsymbol{X} & \boldsymbol{x}\\ \boldsymbol{x}^T & 1\end{bmatrix}\right)\\ &= 4\sigma^2 + \mathrm{Tr}\left(\boldsymbol{D}_m\boldsymbol{X} + 2\boldsymbol{x}^T\boldsymbol{D}_m\mathbf{1} + \mathbf{1}^T\boldsymbol{D}_m\mathbf{1}\right).\end{aligned} \tag{32}$$

Using (14), the matrix $\boldsymbol{X}$ can be expressed as

$$\boldsymbol{X} = \boldsymbol{X}_0 + \boldsymbol{x}\boldsymbol{x}^T, \tag{33}$$

where $\boldsymbol{X}_0 \succeq \mathbf{0}$. Using this observation in (32), we have

$$\begin{aligned}\mathrm{Tr}\big(\boldsymbol{F}_m\boldsymbol{\Psi}\big) &= 4\sigma^2 + \mathrm{Tr}(\boldsymbol{D}_m\boldsymbol{X}_0 + \boldsymbol{x}^T\boldsymbol{D}_m\boldsymbol{x}\\ &\qquad + 2\boldsymbol{x}^T\boldsymbol{D}_m\mathbf{1} + \mathbf{1}^T\boldsymbol{D}_m\mathbf{1})\\ &= 4\sigma^2 + \mathrm{Tr}\left(\boldsymbol{D}_m\boldsymbol{X}_0\right) + (\boldsymbol{x}+\mathbf{1})^T\boldsymbol{D}_m(\boldsymbol{x}+\mathbf{1}).\end{aligned}$$

The statement of the lemma follows from noting that $\boldsymbol{X}_0$ and $\boldsymbol{D}_m$ are PSD for all m, and $\sigma^2 > 0$.

References

[1] A. Saleh, A. Rustako, and R. Roman, "Distributed antennas for indoor radio communications," *IEEE Trans. Commun.*, vol. 35, pp. 1245–1251, Dec. 1987.

[2] A. Ghosh, R. Ratasuk, B. Mondal, N. Mangalvedhe, and T. Thomas, "LTE-Advanced: next-generation wireless broadband technology," *IEEE Wireless Commun. Mag.*, vol. 17, pp. 10–22, June 2010.

[3] H. Yanikomeroglu and E. Sousa, "CDMA distributed antenna system for indoor wireless communications," in *Proc. 1993 IEEE Int. Conf. on Universal Personal Commun.*, vol. 2, pp. 990–994.

[4] L. Dai, S. Zhou, and Y. Yao, "Capacity analysis in CDMA distributed antenna systems," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 2613–2620, Nov. 2005.

[5] J. Park, E. Song, and W. Sung, "Capacity analysis for distributed antenna systems using cooperative transmission schemes in fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 586–592, Feb. 2009.

[6] W. Choi and J. Andrews, "Downlink performance and capacity of distributed antenna systems in a multicell environment," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 69–73, Jan. 2007.

[7] L. Vandenberghe and S. Boyd, "Semidefinite programming," *SIAM Rev.*, vol. 38, pp. 49–95, Mar. 1996.

[8] N. Sidiropoulos, T. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Trans. Signal Process.*, vol. 54, pp. 2239–2251, June 2006.

[9] Z.-Q. Luo, W.-K. Ma, A.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Process. Mag.*, vol. 27, pp. 20–34, May 2010.

[10] M. Goemans and D. Williamson, "Improved approximation algorithms for maximum cut and satisfiability problem using semi-definite programming," *J. ACM*, vol. 42, pp. 1115–1145, Nov. 1995.

[11] E. Karipidis, N. Sidiropoulos, and Z.-Q. Luo, "Quality of service and max-min fair transmit beamforming to multiple cochannel multicast groups," *IEEE Trans. Signal Process.*, vol. 56, pp. 1268–1279, Mar. 2008.

[12] T.-H. Chang, Z.-Q. Luo, and C.-Y. Chi, "Approximation bounds for semidefinite relaxation of max-min-fair multicast transmit beamforming problem," *IEEE Trans. Signal Process.*, vol. 56, pp. 3932–3943, Aug. 2008.

[13] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.

[14] G. H. Golub and C. F. van Loan, *Matrix Computations*, 3rd edition. The Johns Hopkins University Press, 1996.

[15] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[16] J. Gorski, F. Pfeuffer, and K. Klamroth, "Biconvex sets and optimization with biconvex functions: a survey and extensions," *Math. Meth. of Oper. Res.*, vol. 66, pp. 373–407, Dec. 2007.

[17] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optim. Methods Softw.*, vol. 11–12, pp. 625–653, 1998.

[18] Z.-Q. Luo, T. Davidson, G. Giannakis, and K. M. Wong, "Transceiver optimization for block-based multiple access through ISI channels," *IEEE Trans. Signal Process.*, vol. 52, pp. 1037–1052, Apr. 2004.

[19] C. Helmberg, F. Rendl, R. J. Vanderbei, and H. Wolkowicz, "An interior-point method for semidefinite programming," *SIAM J. Optim.*, vol. 6, no. 2, pp. 342–361, 1996.

[20] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 1.21," Jan. 2011. Available: http://cvxr.com/cvx.

[21] International Telecommunication Union (ITU), "Guidelines for evaluation of radio interface technologies for IMT-Advanced," TR M.2135-1, ITU-R, Dec. 2009. Available: http://www.itu.int/pub/R-REP-M.2135-1-2009.

[22] 3rd Generation Partnership Project (3GPP), "Evolved Universal Terrestrial Radio Access (E-UTRA); Radio Frequency (RF) system scenarios," TR 36.942, 3GPP, Dec. 2010. Available: http://www.3gpp.org/ftp/Specs/html-info/36942.htm.

Talha Ahmad received the B.Eng. degree in Electrical Engineering in 2009, and the M.A.Sc. degree in Electrical and Computer Engineering in 2011 from Carleton University, Ottawa, Ontario, Canada. In 2010, he served as the Registration Chair for the IEEE Vehicular Technology Conference Fall 2010. Since 2011, he has been with Ericsson Canada, where he is currently a member of the LTE software development team. His research interests include wireless communications and digital signal processing with a focus on coordinated multipoint transmission and reception technologies, and distributed antenna systems.

Ramy Gohary (S'02–M'06) received the B.Eng. (Hons.) degree from Assiut University, Egypt, in 1996, the M.Sc. degree from Cairo University, Egypt, in 2000, and the Ph.D. degree from McMaster University, Ontario, Canada, in 2006, all in electronics and communications engineering. He received the Natural Sciences and Engineering Research Council visiting fellowship award in 2007.

Dr. Gohary was a visiting fellow with the Terrestrial Wireless Systems Branch, Communications Research Centre, Canada. He is currently the project manager of the Carleton-RIM (Research In Motion) research project.

His research interests include analysis and design of MIMO wireless communication systems, applications of optimization and geometry in signal processing and communications, information theoretic aspects of multiuser communication systems, and applications of iterative detection and decoding techniques in multiple antenna and multiuser systems.

Halim Yanikomeroglu was born in Giresun, Turkey, in 1968. He received the B.Sc. degree in Electrical and Electronics Engineering from the Middle East Technical University, Ankara, Turkey, in 1990, and the M.A.Sc. degree in Electrical Engineering (now ECE) and the Ph.D. degree in Electrical and Computer Engineering from the University of Toronto, Canada, in 1992 and 1998, respectively. He was with the R&D Group of Marconi Kominikasyon A.S., Ankara, Turkey, during 1993-1994.

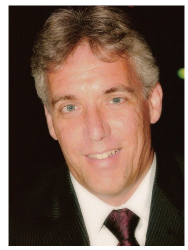
Since 1998 Dr. Yanikomeroglu has been with the Department of Systems and Computer Engineering at Carleton University, Ottawa, where he is now a Full Professor. Dr. Yanikomeroglu's research interests cover many aspects of the physical, medium access, and networking layers of wireless communications with a special emphasis on next-generation cellular system. In recent years, Dr. Yanikomeroglu's research has been funded by Huawei (Canada and China), Research In Motion (Canada), Samsung Advanced Institute of Technology (SAIT, Korea), Communications Research Centre of Canada (CRC), Nortel, NSERC (Natural Sciences and Engineering Research Council of Canada), and Carleton University. Dr. Yanikomeroglu is a recipient of the Carleton University Faculty Graduate Mentoring Award in 2010, the Carleton University Graduate Students Association Excellence Award in Graduate Teaching in 2010, and the Carleton University Research Achievement Award in 2009. He is a Distinguished Lecturer for the IEEE Vehicular Technology Society

Dr. Yanikomeroglu has been involved in the steering committees and technical program committees of numerous international conferences in communications; he has also given about 20 tutorials in such conferences. Dr. Yanikomeroglu is a member of the Steering Committee of the IEEE Wireless Communications and Networking Conference (WCNC), and has been involved in the organization of this conference over the years, including serving as the Technical Program Co-Chair of WCNC 2004 and the Technical Program Chair of WCNC 2008; he will also serve as the Technical Program Co-Chair of WCNC 2014 to be held in Istanbul. Dr. Yanikomeroglu was the General Co-Chair of the IEEE Vehicular Technology Conference Fall 2010 held in Ottawa.

Dr. Yanikomeroglu is an editor for IEEE TRANSACTIONS ON COMMUNICATIONS. He was an editor for IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and IEEE COMMUNICATIONS SURVEYS & TUTORIALS. He was the Chair of the IEEE's Technical Committee on Personal Communications (now called, Wireless Technical Committee).

Dr. Yanikomeroglu is spending the 2011-2012 academic year at TOBB University of Economics and Technology, Ankara, Turkey, as a Visiting Professor. He is also an Adjunct Professor at King Saud University's Advanced Technology Research Institute (Riyadh, Saudi Arabia). Dr. Yanikomeroglu is a registered Professional Engineer in the province of Ontario, Canada.

Saad Al-Ahmadi received the M. Sc. in Electrical Engineering from King Fahd University of Petroleum and Minerals (KFUPM), Dhahran, Saudi Arabia, in 2002 and the Ph. D. in Electrical and Computer Engineering from Carleton University, Ottawa, Canada, in 2010. In 2010, he joined the Department of Electrical Engineering at KFUPM as an assistant professor. His current research interests include channel characterization, design, and performance analysis of distributed MIMO systems.



Gary Boudreau (M'84–SM'11) received the B.A.Sc. in Electrical Engineering from the University of Ottawa in 1983, the M.A.Sc. in Electrical Engineering from Queens University in 1984 and the Ph.D. in Electrical Engineering from Carleton University in 1989. From 1984 to 1989 he was employed as a communications systems engineer with Canadian Astronautics Limited after which from 1990 to 1993 he worked as a satellite systems engineer for MPR Teltech Ltd. For the period spanning 1993 to 2009 he was employed by Nortel Networks in a variety of communication systems and management roles within the CDMA and LTE basestation product groups. In 2010 he joined Ericsson Canada where he is currently working in the LTE systems architecture group. His interests include digital and wireless communications as well as digital signal processing.