# Sequential Likelihood Ratio Test under Incomplete Signal Model for Spectrum Sensing

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***Abstract*—Detecting the existence of the transmitter emitting signals is an important mechanism in many applications, e.g., the spectrum sensing in the cognitive radio. In conventional detection schemes, the predefined number of samples is taken for detection and the statistics of the signals are assumed to be available in the signal model. However, under the ubiquitous fading effects and the non-cooperation of the targets, the signal statistics are not accurately obtainable at the detector. In this paper, we propose a sequential detector operating on the signal model described by the autoregressive moving average (ARMA) process without assuming known coefficients. The sequential detector for the ARMA model is derived by using the likelihood ratio test framework and the predictive distributions of the ARMA process. The novelties the proposed sequential detector include: 1) performing detection without requiring complete knowledge of the signal; 2) using smaller number of samples to reach the decision on average; and 3) allowing user-specified probabilities of detection and false alarm. We derive the approximate average number of samples required to reach the decision. The energy detector and sequential energy detector are compared with the proposed sequential detector by simulations. The results show the sequential detector uses the smaller average number of samples than the energy detector and sequential energy detector to termination.**

***Index Terms*—Sequential detector, incomplete signal model, ARMA, cognitive radio, spectrum sensing, target detection.**

## I. INTRODUCTION

DETECTING the existence or absence of non-cooperative transmitters is an important mechanism in many engineering applications. One of the important applications arises in the spectrum sensing for the cognitive radios [1-12]. With the increasing number of wireless applications, the demand for the spectra increases and the efficient utilization of spectrum is crucial in supporting the applications. The conventional spectrum allocation scheme, which restricts the spectrum usage to be available only to the licensed users, results in low spectrum utilization [1-4]. The low spectrum utilization in current usage model leads to the idea of increasing spectrum utilizations by allowing the unlicensed users to dynamically access the spectra. The concept of the cognitive radio is to allow the unlicensed users to access the channels under the condition that the intended channel is not occupied by the licensed users. Therefore, the unlicensed user must detect the existence of licensed users on the intended channel before accessing the channel. The undesired channel conflicts occur if the unlicensed user accesses the channel occupied by the licensed user. In other words, the missed detection at the unlicensed user causes channel conflicts, and the false alarm in the detection causes the loss of channel opportunity for the unlicensed users. Therefore, the accurate spectrum sensing is a crucial mechanism to enable the cognitive radio. Besides the cognitive radio, the spectrum sensing is applicable to various applications, including the opportunistic spectrum access for ad hoc networks [13] and the non-cooperative target detection and tracking [14-15].

In the spectrum sensing, the energy detector [5] is the most widely adopted approach due to its simplicity. The energy detector collects the predefined number of samples and computes the energy of the collected samples. The decision is made by comparing the sample energy with a threshold. The design of the energy detector allows the implementation of CFAR criterion [16]. Besides the energy detector, the likelihood ratio test approach is investigated in [17], where the statistics or the waveforms of the signals are required in the decision making. The cyclostationary feature detector was discussed in [4]-[6]. In [15], the AR process is used to model the non-cooperative radar or sonar emitters, and the fixed amount of samples are collected to detect the AR process with unknown Doppler frequency shift and phase. The detection problem is formulated as a spectral estimation problem. As opposed to conventional detector using fixed number of samples, the sequential detector [18-20] has the probabilistic termination time. Upon the collection of a sample, the sequential detector makes a decision to terminate or continue to the next sample. The sequential detector [18-20] is designed using the iterative formulation of likelihood ratio, which requires the complete statistics of the signal of interest. Variations of the sequential detector include the sequential energy detector [11] and the sequential change detector [21]. The sequential energy detector [11] uses sequential likelihood ratio of the sample energy for decision. The sequential change detector [21] continuously monitors the signal of interest and detects abrupt spectral changes. In [3][12], the spectrum sensing using distributed decision fusion is investigated. In existing approaches, all or parts of the following are assumed available: 1) the fixed number of samples capable of supporting the desired detection accuracies; 2) the a priori information of the signal of interest. Nevertheless, in the cognitive radio, the spectrum sensing is intended to detect non-cooperative primary transmitters in a highly dynamic environment, where the previous assumptions on the primary transmitters encounter difficulties. The cognitive radio requires spectrum sensing without assuming strong information availability at the detector. Besides, in

Manuscript received April 21, 2010; revised November 19, 2010, May 5, 2011, and July 26, 2012; accepted December 4, 2012. The associate editor coordinating the review of this paper and approving it for publication was M. Torlak.

This work was supported by the National Science Council of Taiwan, under grant number NSC101-2221-E-001-002, NSC101-2221-E-001-008, NSC101-2622-E-001-001-CC3, and 32T-1010721-1C.

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Digital Object Identifier 10.1109/TWC.2012.12.100663

wireless communications, the signals traverse various physical structures of the environments, which cause fading effects. In many research works, e.g., [22]-[25], the Rayleigh, Rician, and Nakagami channels are studied and considered applicable to environments which wireless users often encounter. Since the fading effects are ubiquitous, the accuracies of the detections are influenced by the fading. Besides, the transmitters are often non-cooperative in the detections. The fading effects and the non-cooperation of the transmitters render the signal model unavailable to the detectors. The amounts of knowledge on the signal model, required by the conventional detectors, may not be accurately available in such scenarios. To facilitate the detections in these scenarios, the detection scheme needs to be designed without assuming the complete knowledge of the signal model, which we call incomplete signal model in this paper. Besides, the availability of statistics in the primary signal often requires certain degree of cooperation from the primary users and the fading estimation, which poses great challenges from the primary community [59][63] and difficulties in the legacy compatibility. This observation motivates our investigation of detection under incomplete signal model, which demands only little amount of primary knowledge and primary cooperation.

The autoregressive (AR) process has been known to be capable of modeling various signals. In [26-27], the AR processes are used to model the fading channels. In [28], the non-cooperative narrowband interference is modeled by AR process. The non-cooperative narrowband interference signal is identical to the non-cooperative primary signals in the cognitive radios. In [29], the AR modeling of channel occupations is verified through experimental data. In [30-31], the AR model is used to describe the data collected by the synthetic aperture radar. In [32], the AR is used to model the signals emitted by the non-cooperative target, where the AR modeling is verified through experimental data. Besides, the AR process is used to model the unknown digitally modulated signals emitted by non-cooperative targets [33], spectrum holes in the cognitive radio [34], and the unknown color noises [35]. The primary transmitter of interest in the cognitive radio is identical to the non-cooperative transmitters in [26]-[34]. In other words, the primary transmitter in this paper is in the identical situation as the transmitters in [26]-[34], which may perform power control as in the communication system and experience shadowing effects. The detector at the secondary user in the cognitive radio is in the identical situation as the detectors in [26]-[34], which operate without cooperation of the signal emitters. With the experimental results and commonly practiced adoption in [26]-[34], we adopt the AR model to approximate the signals emitted from the primary transmitters in the cognitive radio. The received signals, consisting of the AR processes and noises, can be represented by the autoregressive moving average (ARMA) processes [36].

In the proposed sequential detector under the incomplete ARMA signal model, although the received signals are assumed to be in the category of the ARMA processes, the coefficients are not assumed to be known at the detector. Therefore, the flexibilities in the AR model can be fully utilized to accommodate various primary signals in different scenarios. One of the major challenges of opportunistic spectrum usage is the concern from the primary-user community [59,63]; our proposed design in a rather conservative perspective is aimed to hopefully reduce the concerns and facilitate smooth transition from the fixed frequency allocation to the opportunistic usage model. The sequential detector sequentially and adaptively exploits the information from the receiving signals. We derive the sequential test statistic and the decision thresholds.

The major contribution of this paper is to propose a sequential detection algorithm for the incomplete signal model. The novelty of the proposed detector includes: 1) The proposed detector is capable of performing detection without requiring complete knowledge of the primary signal. 2) The proposed detector is able to use smaller number of samples to reach the decision on average, as shown in the simulation section. 3) The proposed detector allows user-specified probabilities of detection and false alarm.

This paper is organized as follows. In Section II, the incomplete signal model is discussed. In Section III, the proposed sequential detector is described in detail. The expected number of samples at termination is analyzed in Section IV. The comparison with energy detector and sequential energy detector is discussed in Section V. The performances are verified through examples in Section VI. The concluding remarks are given in Section VII.

## II. Incomplete Signal Model

### A. Scenarios Causing Incomplete Knowledge

In detecting the non-cooperative transmitter that emits wireless narrowband signals, several factors are often encountered in the physical environments. First, since the transmitter is non-cooperative, the transmitter-specific mechanisms, e.g., the power control mechanisms [37] in the cellular systems or the information contents of the signals, are unknown to the detector. Secondly, the physical environments, including the scatters, deflection, and reflections, cause fading effects on the signals. These transmitter-specific mechanisms and fading effects make it difficult for the detectors to obtain the complete properties of the signals. For example in detecting a mobile terminal emitting radio frequency signals, the mobile terminals move and perform the power control mechanisms. The speeds and path losses are unknown at the detector. As the results of the unknown speeds and path losses, the channel autocorrelations and signal powers are unknown. In these types of applications, the model without assuming complete knowledge of the signals is needed in the detections.

### B. Model Constraints of Detecting Unknown Signal

In formulating the model for detections, we denote $H_0$ as the hypothesis of the signal absence and $H_1$ as the hypothesis of the signal presence. In the unknown signal model without assuming any constraints on the signals, the $H_0$ could be considered as equivalent to $H_1$ with the signal equal to 0. In other words, if the signal is allowed to be arbitrary, the $H_0$ is a special case of $H_1$. Therefore, the detection on the model, which imposes no constraints on the signals, can be reduced to a trivial decision, where $H_1$ is always the correct decision. Such trivial decisions provide no useful information to the applications. To resolve this problem, we adopt the constraint on the signal which assumes the lower bound on the signal power. This constraint of the lower bound on the signal power makes $H_1$ and $H_0$ mutually exclusive, such that the decisions provide the meaningful information. The usage of detection lower bound also follows general practices in IEEE 802.22 requirements. The practical lower bound of the signal power can be obtained through the physical parameters of the detection scenarios, e.g., the cellular systems [38], TV bands [4, 59], and wireless microphones [59]. In this paper, the examples focus on the cognitive radios, where the signal-noise-ratio (SNR) of interest is at the range of -20 dB [4]. The transmitter with its SNR less than -20 dB at the detector is far away and the probability of channel collision is negligible.

### C. Formulation of Incomplete Signal Model

The AR processes have been applied to model a wide category of signals, e.g., fading channels [26-27], non-cooperative narrowband interferences [28], non-cooperative radar signals [30-31], and digitally modulated signals [33]. In the cognitive radio, the primary signals are similar to the non-cooperative signals described by AR process in [26-28][30-31][33]. The major feature in the scenario of the cognitive radio is the unavailability of primary information at the detector. The unavailability of primary information at the secondary detector renders it impossible to properly design sampling rate for uncorrelated samples. Furthermore, a properly designed sampling rate with uncorrected samples for a specific primary transmitter is unable to induce uncorrelated samples for other transmitters with different physical parameters. Therefore, the non-zero sample correlation generally exists in the samples in the scenarios of cognitive radios. Because of the flexibilities and wide applicability of AR processes, we adopt the complex AR Gaussian processes to model the primary signal. The signal model is formulated as

$$x_t = \begin{cases} n_t, & H_0 \\ \alpha_t + n_t, & H_1 \end{cases}, \quad (1)$$

where $n_t$ is the complex white Gaussian noise with mean 0 and variance $\sigma_n^2$, and $\alpha_t$ is the complex signal. The $t$ is the discrete integer time index. In this model, the mean 0 and variance $\sigma_n^2$ of the $n_t$ are assumed to be known a priori while the complete knowledge of the $\alpha_t$ is not assumed. The only assumed knowledge of $\alpha_t$ is the lower bound of its signal power and that $\alpha_t$ belongs to the category of complex Gaussian AR processes. We do not assume the detector knows the parameters of the complex Gaussian AR process of $\alpha_t$. Denoting * as the complex conjugate, the variance is expressed as $Var[\alpha_t] = E[\alpha_t \alpha_t^*] = \sigma_\alpha^2$. In our model, the $\sigma_\alpha^2$ is lower bounded. Knowing the $\sigma_n^2$ a priori, specifying the lower bound on $\sigma_\alpha^2$ is equivalent to specifying the lower bound on the SNR, denoted by $S$, i.e., $\frac{\sigma_\alpha^2}{\sigma_n^2} > S$. The $\alpha_t$ can be described by the AR expression with order $q$ as

$$\alpha_t = a_1\alpha_{t-1} + a_2\alpha_{t-2} + a_3\alpha_{t-3} + ... + a_q\alpha_{t-q} + w_t, \quad (2)$$

where $w_t$ is the complex white Gaussian driving process. The received signal under $H_1$ is $x_t = \alpha_t + n_t$, which can be reformulated as $\alpha_t = x_t - n_t$. Substituting $\alpha_t = x_t - n_t$ into (2) for all relevant values of $t$, we obtain

$$\begin{aligned} x_t - n_t =& a_1(x_{t-1} - n_{t-1}) + a_2(x_{t-2} - n_{t-2}) \\ &+ a_3(x_{t-3} - n_{t-3}) + ... + a_q(x_{t-q} - n_{t-q}) + w_t \end{aligned} \quad (3)$$

which can be reformulated as

$$\begin{aligned} x_t =& a_1x_{t-1} + a_2x_{t-2} + a_3x_{t-3} + ... + a_qx_{t-q} + w_t \\ &+ n_t - a_1n_{t-1} - a_2n_{t-2} - ... - a_qn_{t-q}. \end{aligned} \quad (4)$$

The derivations in (3)(4) show that the received signal, $x_t$, is a complex ARMA process.

## III. Sequential Detection under Incomplete Model

At each time instant $t$, the sequential detector collects $x_t$, updates the decision statistic $\Lambda_t$, and makes the decision $D_t$. The decision $D_t$ is made by choosing one of the $H_1$, $H_0$, or $continue$, expressed as

$$D_t = \begin{cases} H_1 \\ H_0 \\ continue \end{cases}, \quad (5)$$

where the decision criteria of choosing $H_1$, $H_0$, or $continue$ are discussed in the following sections. If $D_t$ is chosen to be $H_1$ or $H_0$, the sequential detection scheme terminates at the time instant $t$, which is called the termination time. If $D_t$ is chosen as $continue$, the sequential detector continues to time $t+1$, collects $x_{t+1}$, and perform the same decision process as (5). As the design goal to reduce the complexity of the sequential detection scheme, the $\Lambda_t$ must be sequentially updated without preserving all the past observations. In other words, the statistic $\Lambda_t$ must be updated, based on finite length of the past observations. The procedures of sequentially updating $\Lambda_t$ are discussed in the following sections.

### A. Deriving the Sequential Likelihood Ratio Test of ARMA

The sequential detector is assumed to start observing the first sample at time 1. At time $t$, the observed samples are $x_1, x_2, ..., x_t$, which generate the logarithmic likelihood ratio as

$$\Lambda_t(x_1, x_2, ..., x_t) = \log \frac{P(x_1, x_2, ..., x_t|H_1)}{P(x_1, x_2, ..., x_t|H_0)}. \quad (6)$$

The logarithmic likelihood ratio $\Lambda_t(x_1, x_2, ..., x_t)$ is the function of all samples accumulated from time 1 to time $t$. We express $\Lambda_t(x_1, x_2, ..., x_t)$ as $\Lambda_t$ to simplify the notations when there are no risks of confusions. We denote the probability ratio of the sample at time $t$ conditioned on the past $p+1$ samples as $\lambda_t$, i.e.,

$$\lambda_t(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}) = \log \frac{P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1)}{P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_0)}. \quad (7)$$

When there are no risks of confusions, we simplify $\lambda_t(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1})$ as $\lambda_t$ to simplify the notations. Using derivations in Appendix, we obtain

$$\Lambda_t = \sum_{h=p+2}^{t} \lambda_h. \quad (8)$$

The sequential update of the test statistic can be expressed by

$$\Lambda_t = \Lambda_{t-1} + \lambda_t. \tag{9}$$

### *B. Predictive probability density function and decision rule*

To compute the test statistic, the sequential detector requires the explicit expression of $\lambda_t$. We derive explicit expressions for $P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1)$ and $P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_0)$ to obtain $\lambda_t$ by (7). We denote the autocovariance as $r_{x,k} = \text{cov}(x_t, x_{t+k}) = E[x_t x_{t+k}^*]$ and denote the column vectors of the observed past $p+1$ samples at time $t$ as

$$X_{t,p} = \begin{bmatrix} x_{t-p-1} \\ \vdots \\ x_{t-2} \\ x_{t-1} \end{bmatrix}, X_{t,p,\#} = \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-p-1} \end{bmatrix}. \tag{10}$$

The covariance matrix and covariance column vector are denoted by

$$R_x = E[XX^H] = \begin{bmatrix} r_{x,0} & r_{x,1} & r_{x,2} & \cdots & r_{x,p} \\ r_{x,-1} & r_{x,0} & & & \\ r_{x,-2} & & r_{x,0} & & \vdots \\ \vdots & & & \ddots & \\ r_{x,-p} & & \cdots & & r_{x,0} \end{bmatrix},$$

$$\Upsilon_x = \begin{bmatrix} r_{x,1} \\ r_{x,2} \\ \vdots \\ r_{x,p+1} \end{bmatrix}. \tag{11}$$

The $N(x; \mu, \Omega)$ is used to denote the two-dimensional Gaussian pdf with the real component and imaginary component of $x$ as the two random variables. In $N(x; \mu, \Omega)$, the $\mu$ is the mean vector and $\Omega$ is the covariance matrix. Under $H_0$, the $x_t$ is complex white Gaussian sequence with mean $\mu_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and covariance $\Omega_0 = \begin{bmatrix} \sigma_n^2/2 & 0 \\ 0 & \sigma_n^2/2 \end{bmatrix}$. Therefore, we obtain

$$P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_0) = N(x_t; \mu_0, \Omega_0). \tag{12}$$

The estimated $r_{x,k}$ at time $t$ is denoted as $r_{x,k,t}$. Using sample average as the estimation of $r_{x,k}$, the sequential updating relationship of the of the $r_{x,k,t}$ and $r_{x,k,t-1}$ is expressed as

$$r_{x,k,t} = \frac{r_{x,k,t-1} \times (t-1) + x_{t-k} x_t^*}{t}, \tag{13}$$

for valid values of $k$. Using (13) to update $r_{x,k,t}$ sequentially, and using the estimated $r_{x,k,t}$ as the $r_{x,k}$ in (11) at each time $t$, the elements in $R_x$ and $\Upsilon_x$ are sequentially updated.

For the $x_t$ modeled by the ARMA process, we have the one-step predictive mean and variance [41] expressed by

$$E[x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}] = \Upsilon_x{}' R_x^{-1} X_{t,p,\#}, \tag{14}$$

$$Var[x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}] = r_{x,0} - \Upsilon_x{}' R_x^{-1} \Upsilon_x, \tag{15}$$

where the $'$ represents the $Hermitian$ operator, i.e., $\Upsilon_x{}'$ represents the transpose and complex conjugate of $\Upsilon_x$. It is noted that one-step predictive mean (14) and variance (15) can be sequentially updated by (13). By denoting $\mu_1 = \begin{bmatrix} \text{Re}(\Upsilon_x{}' R_x^{-1} X_{t,p,\#}) \\ \text{Im}(\Upsilon_x{}' R_x^{-1} X_{t,p,\#}) \end{bmatrix}$ and $\Omega_1 = \begin{bmatrix} (r_{x,0} - \Upsilon_x{}' R_x^{-1} \Upsilon_x)/2 & 0 \\ 0 & (r_{x,0} - \Upsilon_x{}' R_x^{-1} \Upsilon_x)/2 \end{bmatrix}$, the predictive pdf can be expressed as

$$P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1) = N(x_t; \mu_1, \Omega_1). \tag{16}$$

By combining (7)(12)(16), we obtain

$$\lambda_t = \frac{N(x_t; \mu_1, \Omega_1)}{N(x_t; \mu_0, \Omega_0)}. \tag{17}$$

The decision at time $t$ is described as

$$D_t = \begin{cases} H_1 & \text{if } \Lambda_t > \log A \\ H_0 & \text{if } \Lambda_t < \log B \\ \textit{continue} & \text{if } \log A \leq \Lambda_t \leq \log B \end{cases}, \tag{18}$$

where the decision boundaries A and B can be set [20] by

$$A = \frac{1 - P_M}{P_{FA}}, \; B = \frac{P_M}{1 - P_{FA}}. \tag{19}$$

If $D_t$ is $H_1$ or $H_0$, the sequential detection is completed and terminated. If $D_t$ is $continue$, the detector takes $x_{t+1}$ and repeat the same process at time $t+1$. The processes repeat till a decision, $H_1$ or $H_0$, is made.

### *C. Initialization*

It is noticed that the white Gaussian noise is the special case of the ARMA process. Therefore, under $H_0$, the $N(x_t; \mu_1, \Omega_1)$ estimated from the noises of $H_0$ approaches $N(x_t; \mu_0, \Omega_0)$, which renders $\lambda_t$ in (7) close to zero and stagnates the progress of $\Lambda_t$. To resolve this, the constraint of the SNR lower bound, i.e., the $S$, must be imposed on the $P(x_t \,|\, x_{t-1}, x_{t-2}, \cdots, x_{t-p-1}, H_1)$ in the computation of $\lambda_t$. Given the $S$ as the lower bound on the SNR, i.e., $\frac{\sigma_\alpha^2}{\sigma_n^2} > S$, we obtain $r_{x,0} = \sigma_\alpha^2 + \sigma_n^2 > (1+S)\sigma_n^2$. We denote $\mu_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Omega_2 = \begin{bmatrix} (S+1)\sigma_n^2/2 & 0 \\ 0 & (S+1)\sigma_n^2/2 \end{bmatrix}$. By using the maximum entropy criterion under the constraint $r_{x,0} > (1+S)\sigma_n^2$, the maximum-entropy pdf for $P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1)$ is $N(x_t; \mu_2, \Omega_2)$. Thus, the $N(x_t; \mu_2, \Omega_2)$ is used as the initial pdf of $P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1)$. Using $N(x_t; \mu_2, \Omega_2)$ as the $P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1)$ in (7) and combining (12), we obtain

$$\lambda_t = \frac{N(x_t; \mu_2, \Omega_2)}{N(x_t; \mu_0, \Omega_0)}. \tag{20}$$

The threshold on the power of the received signal is used to trigger the update of $P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1)$ by the $N(x_t; \mu_1, \Omega_1)$. The power of the received signal is $r_{x,0}$. At time $t$, the mean of the sample variance under $H_0$ is $\sigma_n^2$, the variance of the sample variance under $H_0$ is $\frac{2\sigma_n^4}{t}$, which gives the standard deviation as $\sigma_n^2\sqrt{\frac{2}{t}}$. The criterion is to update the $P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1)$ by the $N(x_t; \mu_1, \Omega_1)$ if the estimated signal power is larger than the mean of the noise variance plus $k$ deviations. The expression of the threshold to trigger the update $P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1)$ by $N(x_t; \mu_1, \Omega_1)$ is $r_{x,0} > \sigma_n^2(1 + k\sqrt{\frac{2}{t}})$.

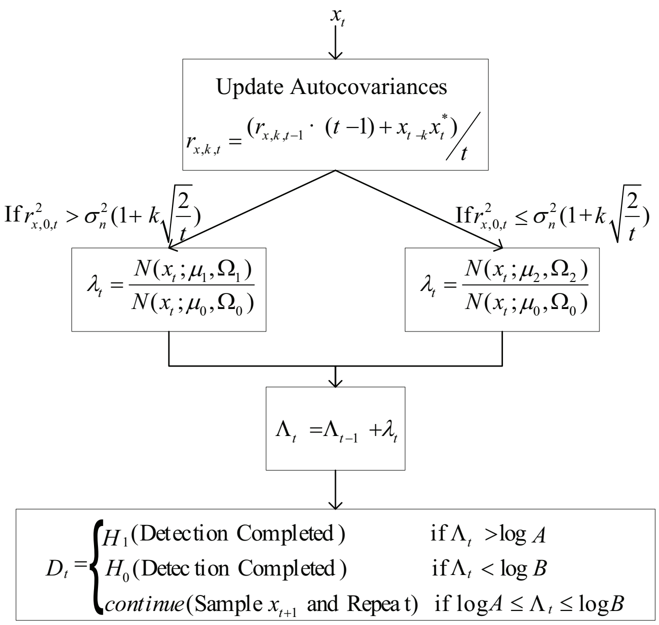


Fig. 1. The diagram of the proposed sequential detector.

### D. Summary and Discussions

The sequential scheme is illustrated in Fig. 1 and summarized in the following.

1) Take the sample $x_t$.
2) Update the autocovariances by $r_{x,k,t} = (r_{x,k,t-1} \times (t-1) + x_{t-k} x_t^*)/t$.
3) Update $\mu_1$, $\Omega_1$, $\mu_2$, and $\Omega_2$.
4) If $r_{x,0,t} > \sigma_n^2(1+k\sqrt{\frac{2}{t}})$, $\lambda_t = \frac{N(x_t;\mu_1,\Omega_1)}{N(x_t;\mu_0,\Omega_0)}$, else $\lambda_t = \frac{N(x_t;\mu_2,\Omega_2)}{N(x_t;\mu_0,\Omega_0)}$.
5) Update the test statistic by $\Lambda_t = \Lambda_{t-1} + \lambda_t$.
6) Make the decision by

$$D_t = \begin{cases} H_1 & \text{if } \Lambda_t > \log A \\ H_0 & \text{if } \Lambda_t < \log B \\ continue & \text{if } \log A \leqslant \Lambda_t \leqslant \log B \end{cases}.$$

In certain scenarios, more primary knowledge may be obtainable through white space database or other auxiliary means, e.g., the TV white space database [63], Geo-location [62,63], or statistical modeling of white space [61]. The design of a sequential detector with partial knowledge of the primary signal can be facilitated through conditioning the above formulations on the available information or through Bayes rule. The detection accuracies of the small-scale primary users [64] can also be improved, since the proposed algorithm allows the user-specified detection accuracies. The sequential detector can be further extended to the cooperative operation for better detecting the small-scale devices.

## IV. Expected Number of Samples at Termination

The sequential detection scheme terminates at the time when a decision $H_0$ or $H_1$ is made. The number of samples at the termination, denoted as $\Im$, is a random variable. With probability 1, the sequential process terminates within finite number of samples [18]. One of the important properties in the sequential detection scheme is the expected number of termination samples, expressed as $E[\Im]$. The $E[\Im]$ indicates, on average, the number of samples required for the sequential scheme to reach a decision. In many applications, the smaller $E[\Im]$ is meritorious, since the timely decisions usually facilitate quick responses of the applications and lower the computation complexities of the detector. The approximate expression of $E[\Im]$ is analyzed in this section.

In [42], the $g(P_{FA}, P_M)$ is defined as

$$g(P_{FA}, P_M) = (1-P_{FA})\log B + P_M \log A. \tag{21}$$

By the above $g(P_{FA}, P_M)$, the expected termination time of the sequential probability ratio test is expressed [20] as

$$E[\Im|H_0] = \frac{g(P_{FA}, P_M)}{E[\lambda_t|H_0]}, \tag{22}$$

$$E[\Im|H_1] = \frac{-g(P_M, P_{FA})}{E[\lambda_t|H_1]}, \tag{23}$$

where $E[\lambda_t \mid H_0]$ is the expected step size conditioned on $H_0$, and $E[\lambda_t \mid H_1]$ is the expected step size conditioned on $H_1$. The $E[\lambda_t \mid H_0]$ and $E[\lambda_t \mid H_1]$ are expressed as

$$\begin{aligned} E[\lambda_t|H_0] = \int_{-\infty}^{\infty} & \lambda_t(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_0) \\ & \times P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_0)dx_t, \end{aligned} \tag{24}$$

$$\begin{aligned} E[\lambda_t|H_1] = \int_{-\infty}^{\infty} & \lambda_t(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1) \\ & \times P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1)dx_t. \end{aligned} \tag{25}$$

In (24), the estimated $\lambda_t(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_0)$ under $H_0$ is approximately equal to (20). Therefore, by using (12)(20), the (24) can be reformulated by

$$E[\lambda_t|H_0] = \int_{-\infty}^{\infty} \frac{N(x_t;\mu_2,\Omega_2)}{N(x_t;\mu_0,\Omega_0)} N(x_t;\mu_0,\Omega_0)dx_t. \tag{26}$$

By (22)(26), the $E[\Im \mid H_0]$ is expressed as

$$\begin{aligned} E[\Im|H_0] =& \frac{g(P_{FA}, P_M)}{E[\lambda_t|H_0]} \\ =& \frac{g(P_{FA}, P_M)}{\int_{-\infty}^{\infty} \frac{N(x_t;\mu_2,\Omega_2)}{N(x_t;\mu_0,\Omega_0)} N(x_t;\mu_0,\Omega_0)dx_t}. \end{aligned} \tag{27}$$

We denote the real SNR as $\zeta$, the mean as $\mu_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and the covariance matrix as $\Omega_3 = \begin{bmatrix} (\zeta+1)\sigma_n^2/2 & 0 \\ 0 & (\zeta+1)\sigma_n^2/2 \end{bmatrix}$. Under $H_1$, the $P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1)$ in (25) is approximately equal to $N(x_t;\mu_3,\Omega_3)$. It is noted that the $N(x_t;\mu_3,\Omega_3)$ is to denote the sample distribution under the SNR=$\zeta$ for the purpose of analysis. The $N(x_t;\mu_3,\Omega_3)$ is inherently different from the sequentially updated $N(x_t;\mu_1,\Omega_1)$ in Section III. There are two stages for $\lambda_t$ expressed by (17) and (20) separately. At the first stage, the $\lambda_t(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_0)$ is expressed by (20). Using $N(x_t;\mu_3,\Omega_3)$ as $P(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1)$ and (20) as $\lambda_t(x_t|x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_0)$ in (25), we obtain

$$\begin{aligned} & E[\lambda_t|H_1, \zeta, 1st\ stage] \\ & = \int_{-\infty}^{\infty} \frac{N(x_t;\mu_2,\Omega_2)}{N(x_t;\mu_0,\Omega_0)} N(x_t;\mu_3,\Omega_3)dx_t. \end{aligned} \tag{28}$$

At the second stage, the $\lambda_t(x_t|x_{t-1}, x_{t-2}, \cdots, x_{t-p-1}, H_0)$ is expressed by $\frac{N(x_t;\mu_3,\Omega_3)}{N(x_t;\mu_0,\Omega_0)}$. Using $N(x_t; \mu_3, \Omega_3)$ as $P(x_t|x_{t-1}, x_{t-2}, \cdots, x_{t-p-1}, H_1)$ and $\frac{N(x_t;\mu_3,\Omega_3)}{N(x_t;\mu_0,\Omega_0)}$ as $\lambda_t(x_t|x_{t-1}, x_{t-2}, \cdots, x_{t-p-1}, H_0)$ in (25), we obtain

$$E[\lambda_t|H_1, \zeta, 2nd\ stage] = \int_{-\infty}^{\infty} \frac{N(x_t; \mu_3, \Omega_3)}{N(x_t; \mu_0, \Omega_0)} N(x_t; \mu_3, \Omega_3) dx_t. \quad (29)$$

Under $H_1$, the approximated time, in units of samples, of the detector being in the first stage is denoted as $E_{th}$ and expressed as $E_{th} = 2\left(\frac{k}{\zeta}\right)^2$. Therefore, under $H_1$ in the first stage, the expected step size is $E[\lambda_t|H_1, \zeta, 1st\ stage]$ when the time is smaller than $E_{th} = 2\left(\frac{k}{\zeta}\right)^2$. Under $H_1$ in the second stage, the expected step size is $E[\lambda_t|H_1, \zeta, 2nd\ stage]$ when the time is larger than or equal to $E_{th} = 2\left(\frac{k}{\zeta}\right)^2$. Summarizing the step sizes and conditions under $H_1$, we obtain

$$\begin{aligned} E[\Im|H_1, \zeta] &= E[\lambda_t|H_1, \zeta, 1st\ stage] \\ &\quad + E[\lambda_t|H_1, \zeta, 2nd\ stage] \\ &= \min\left[\frac{-g(P_M, P_{FA})}{E[\lambda_t|H_1, \zeta, 1st\ stage]}, E_{th}\right] + \\ &\max\left[\frac{-g(P_M, P_{FA}) - E[\lambda_t|H_1, \zeta, 1st\ stage]E_{th}}{E[\lambda_t|H_1, \zeta, 2nd\ stage]}, 0\right]. \end{aligned} \quad (30)$$

For the SNR $\zeta$ distributed by pdf $f(\zeta)$, we have

$$E[\Im|H_1] = \int E[\Im|H_1, \zeta] f(\zeta) d\zeta. \quad (31)$$

Summarizing the derivation, the expected termination time can be expressed by

$$E[\Im] = P_{H_0} E[\Im|H_0] + P_{H_1} E[\Im|H_1]. \quad (32)$$

Therefore, to evaluate $E[\Im]$, the $E[\lambda_t \mid H_0]$ and $E[\lambda_t \mid H_1]$ are first evaluated by (27)(31). Then, $E[\Im]$ can be evaluated through (32).

## V. Comparisons with Energy Detector and Sequential Energy Detector

The energy detector collects a fixed number of samples and makes the decision based on the collected samples. In this paper, we use $T$ to denote the pre-determined fixed number of samples for conventional energy detector. The test statistic of the energy detector is denoted and expressed by

$$\lambda_{eng,T} = \sum_{t=1}^{T} x_t x_t^*. \quad (33)$$

The decision of the energy detector, denoted as $D_{eng}$, is expressed [5] as

$$D_{eng} = \begin{cases} H_1, & \text{if } \lambda_{eng,T} > \gamma \\ H_0, & \text{if } \lambda_{eng,T} \le \gamma \end{cases}. \quad (34)$$

The energy detector uses central limit theorem to approximate the test statistics by the Gaussian distributions, i.e.,

$$D_{eng} = \begin{cases} \lambda_{eng,T} \sim N(\lambda_{eng,T}; T\sigma_n^2, 2T\sigma_n^4), & H_0 \\ \lambda_{eng,T} \sim N(\lambda_{eng,T}; T(\sigma_n^2 + \sigma_\alpha^2), 2T(\sigma_n^2 + \sigma_\alpha^2)^2), & H_1 \end{cases}. \quad (35)$$

TABLE I
The Targeted $(P_M, P_{FA})$ and the Simulation Results Under the Settings of Example 1

| Targeted $(P_M, P_{FA})$ | Sequential Detector | Sequential Energy Detector | Energy Detector |
|---|---|---|---|
| $(0.1, 0.1)$ | $(0.0697, 0.0904)$ | $(0.0722, 0.1031)$ | $(**, 0.0382)$ |
| $(0.05, 0.05)$ | $(0.0357, 0.0517)$ | $(0.0332, 0.0498)$ | $(**, 0.0189)$ |
| $(0.03, 0.03)$ | $(0.0203, 0.0314)$ | $(0.0217, 0.0302)$ | $(**, 0.0113)$ |
| $(0.01, 0.01)$ | $(0.0067, 0.0096)$ | $(0.0058, 0.0093)$ | $(**, 0.0035)$ |

**Smaller than $10^{-3}$

By (34)(35), the $P_{FA}$ and $P_D$ are solved and expressed as

$$P_{FA} = Q\left(\frac{\gamma - T\sigma_n^2}{\sqrt{2T\sigma_n^4}}\right),\ P_D = Q\left(\frac{\gamma - T(\sigma_\alpha^2 + \sigma_n^2)}{\sqrt{2T(\sigma_\alpha^2 + \sigma_n^2)^2}}\right). \quad (36)$$

Solving for $T$ in (36), the $T$ is expressed by $T = 2[(Q^{-1}(P_{FA}) - Q^{-1}(P_D))\frac{\sigma_n^2}{\sigma_\alpha^2} - Q^{-1}(P_D)]^2$.

For the given $P_{FA}$, $P_D$, and the SNR lower bound $S$, the energy detector calculates the fixed number of samples by $T = 2[(Q^{-1}(P_{FA}) - Q^{-1}(P_D))S^{-1} - Q^{-1}(P_D)]^2$. By (36) for the given $P_{FA}$, the detection threshold of the statistic is $\gamma = \sqrt{2T\sigma_n^4}Q^{-1}(P_{FA}) + T\sigma_n^2 = \left(\sqrt{2T}Q^{-1}(P_{FA}) + T\right)\sigma_n^2$. The energy detector performs the detections based on the fixed number of samples, i.e., the calculated $T$.

It is noted that, in many applications, the samples are correlated due to the narrowband property of the signals. The energy detector does not incorporate the properties of the autocorrelations in the formulations of the detection scheme.

Besides the energy detector, the sequential energy detector in [11] is briefly described for comparison. We denote the logarithmic likelihood ratio of the sequential energy detector as $\Lambda_{SeqEng,t}$. The sequential update of the logarithmic likelihood ratio can be expressed [11] as

$$\begin{aligned} \Lambda_{SeqEng,t} =& \Lambda_{SeqEng,t-1} + \frac{1}{2}\log\frac{\sigma_n^2}{\sigma_\alpha^2 + \sigma_n^2} \\ &+ \frac{\sigma_\alpha^2 \log e}{2\sigma_n^2(\sigma_\alpha^2 + \sigma_n^2)} x_t x_t^*. \end{aligned} \quad (37)$$

The decision rule is described by

$$D_t = \begin{cases} H_1 & \text{if } \Lambda_{SeqEng,t} > \log A \\ H_0 & \text{if } \Lambda_{SeqEng,t} < \log B \\ continue & \text{if } \log A \le \Lambda_{SeqEng,t} \le \log B \end{cases}. \quad (38)$$

The details of implementation and results are described in the following section.

## VI. Numerical Examples

### A. Example 1-Random SNR with Random Mobility

In this example, we simulate the detection of mobile transmitters emitting primary signals traversing fading channels. In the simulation scenario, the received SNR at detector is uniformly distributed between $-20$ dB to 10 dB. The prior probabilities of $H_1$ and $H_0$ are 0.5 separately. The primary signal under $H_1$ is the BPSK modulated signal traversing fading channels generated by the Jake's model [23]. In the Jake's model, the autocorrelation of the channel is expressed as $J_0(2\pi f_D \Delta|\tau|)$, where the $\tau$ represents the index of the time lag, and the $\Delta$ represents the sampling interval. The sampling interval $\Delta$ is set as $1/10000$ second,

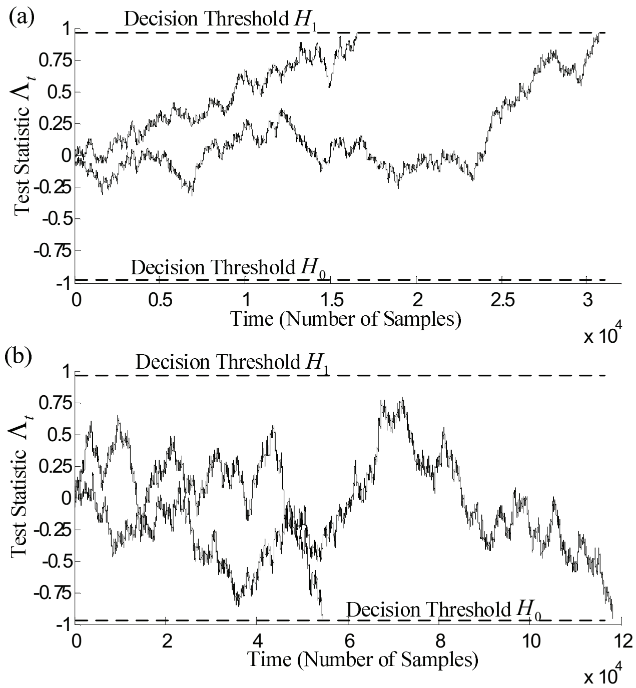


Fig. 2. Realizations of the test statistic $\Lambda_t$ in example 1 at $P_{FA} = 0.1$ and $P_M = 0.1$. (a) Two realizations under the truth of $H_1$. (b) Two realizations under the truth of $H_0$.

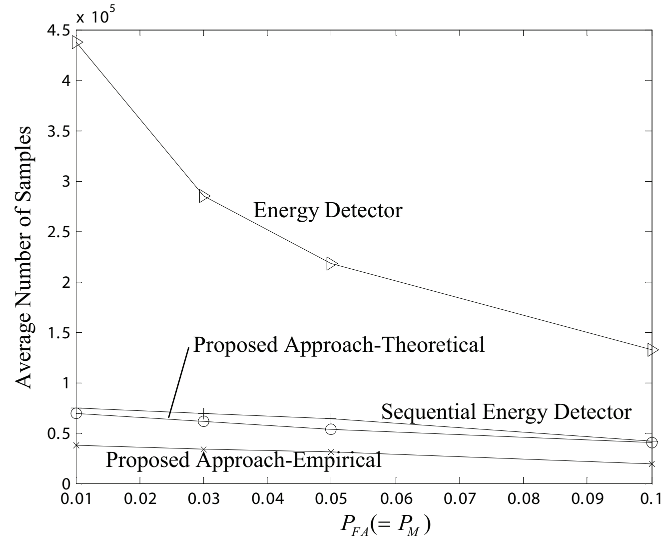


Fig. 3. The average number of samples at $P_M = P_{FA} = 0.01, 0.03, 0.05$, and $0.1$, under the settings of example 1.

TABLE II
THE TARGETED $(P_M, P_{FA})$ AND THE SIMULATION RESULTS IN EXAMPLE 2 AT SNR=-20 DB

| Targeted $(P_M, P_{FA})$ | Proposed Sequential Detector($-20$ dB) | Sequential Energy Detector($-20$ dB) | Energy Detector($-20$ dB) |
|---|---|---|---|
| $(0.1, 0.1)$ | $(0.0741, 0.1037)$ | $(0.0634, 0.1023)$ | $(0.1034, 0.0975)$ |
| $(0.05, 0.05)$ | $(0.0328, 0.0469)$ | $(0.0312, 0.0465)$ | $(0.0453, 0.0531)$ |
| $(0.03, 0.03)$ | $(0.0189, 0.0312)$ | $(0.0159, 0.0301)$ | $(0.0298, 0.0319)$ |
| $(0.01, 0.01)$ | $(0.0067, 0.0095)$ | $(0.0074, 0.0103)$ | $(0.0103, 0.0983)$ |

i.e., the sampling frequency is 10000 samples per second. The Doppler shift, $f_D$, is described by $f_D = \frac{v f_c}{3\times10^8}$, where the carrier frequency $f_c$ is chosen as the GSM band at 900 MHz and the transmitter speed, $v$ , is uniformly distributed between 3 meters/second (m/s) and 30 m/s. The detection schemes, including the sequential detector, sequential energy detector, and the energy detector, are performed. The simulation are performed with the targeted $(P_{FA}, P_M) = (0.01, 0.01)$, $(0.03, 0.03)$, $(0.05, 0.05)$, $(0.1, 0.1)$, respectively.

The examples of realizations of $\Lambda_t$ are shown in Fig. 2. The simulation and theoretical results of average number of samples to reach decision are shown in Fig. 3. The detection accuracies are shown in Table I. In Fig. 3, the energy detector and the sequential energy detector require more number of samples to reach the decision than the proposed sequential detector. In Table I, the resulting $(P_{FA}, P_M)$ for the sequential detector, sequential energy detector, and the energy detector satisfies the targeted $(P_{FA}, P_M)$. Observing Fig. 3 and Table I, the proposed sequential detector achieves smaller number of samples while fulfilling the targeted $(P_{FA}, P_M)$.

### *B. Example 2-Fixed SNR with Random Mobility*

In this example, the scenarios of the fixed SNR at the receiver are investigated. The SNRs are $-20$ dB and 0 dB separately. Other parameters are the same as those in the example 1.

The examples of realizations of $\Lambda_t$ are shown in Fig. 4. The average number of samples is shown in Fig. 5. The $(P_{FA}, P_M)$ is shown in Table II and III. In Fig. 5, at the same SNR, the energy detector and sequential energy detector require more number of samples to reach the decision than the proposed sequential detector. In Table II and III, the resulting $(P_{FA}, P_M)$ for the proposed sequential detector, sequential energy detector, and the energy detector satisfy the targeted $(P_{FA}, P_M)$. Observing Fig. 5, Table II and III, the proposed sequential detector achieves smaller number of samples while fulfilling the targeted $(P_{FA}, P_M)$. Observing the $(P_{FA}, P_M)$ of the proposed sequential detector in Table II and III, the detection performances on the lower SNR ($-20$ dB) scenarios are comparable with the performances on higher SNR (0 dB) scenarios. Thus, the detections on the transmitters with lower SNRs are not unfairly disadvantaged by our sequential detection scheme.

Because of the unavailability of the signal SNR for the primary signal, the energy detector uses the conservative detection strategy and performs much better than the targeted performance by using a much more samples than necessary. The performances of our proposed approach are closer to the targeted detection accuracy. Comparing the average number of samples for the $-20$ dB and 0 dB scenarios, our proposed approach can adaptively terminate its sampling and make its decision based on its accumulated likelihood ratio, which enable our proposed approach to achieve smaller termination time.

### *C. Example 3-Unequal Probability of False Alarm and Probability of Missed Detection*

In cognitive radios, the event of missed detection, measured by $P_M$, causes channel conflicts which are often considered more harmful than the event of missed channel opportunity,

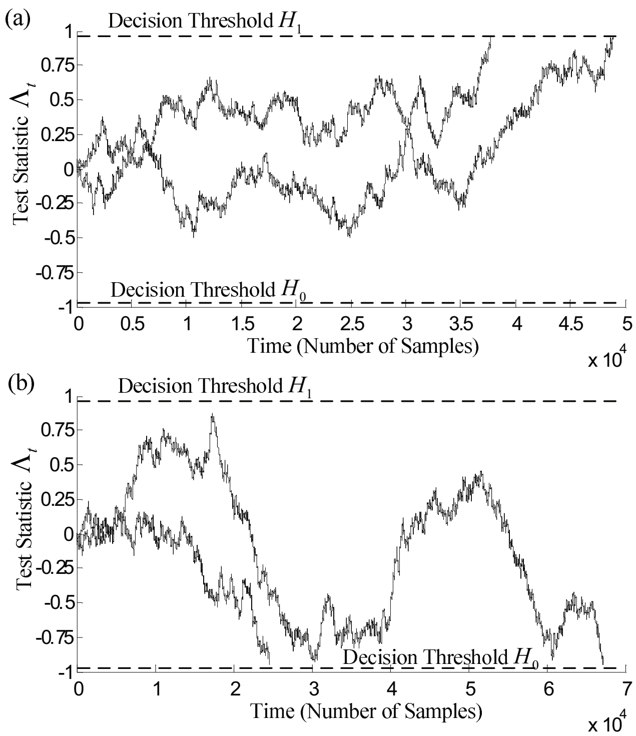


Fig. 4. Realizations of the test statistic $\Lambda_t$ in example 2 at $P_{FA} = 0.1$ and $P_M = 0.1$. (a) Two realizations under the truth of $H_1$. (b) Two realizations under the truth of $H_0$.

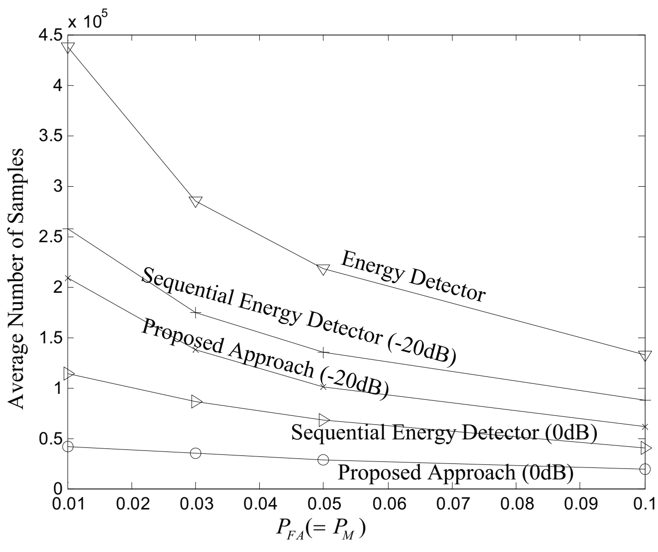


Fig. 5. The average number of samples at $P_M = P_{FA} = 0.01$, 0.03, 0.05, and 0.1, under the settings of example 2.

i.e., the false alarm measured by $P_{FA}$. Therefore, it is often desired to design the detector with $P_M$ lower than $P_{FA}$. In this example, we simulate the design targeting performances of $(P_{FA}, P_M)$=$(0.1, 0.08)$, $(0.1, 0.05)$, $(0.05, 0.03)$, and $(0.05, 0.01)$, respectively. The simulation settings are the same as example 1, except the $P_M$ and $P_{FA}$ described in this section.

The results are shown in Table IV. We observe that the proposed approach achieves the lower average number of samples than the sequential energy detector, while their detection accuracies are comparable. The proposed approach and

TABLE III
THE TARGETED $(P_M, P_{FA})$ AND THE SIMULATION RESULTS IN EXAMPLE 2 AT SNR=0 DB

| Targeted $(P_M, P_{FA})$ | Proposed Sequential Detector(0 dB) | Sequential Energy Detector(0 dB) | Energy Detector(0 dB) |
|---|---|---|---|
| $(0.1, 0.1)$ | $(0.0530, 0.1019)$ | $(0.0520, 0.0937)$ | $(**, 0.0305)$ |
| $(0.05, 0.05)$ | $(0.0231, 0.0501)$ | $(0.0217, 0.0491)$ | $(**, 0.0167)$ |
| $(0.03, 0.03)$ | $(0.0125, 0.0297)$ | $(0.0131, 0.0310)$ | $(**, 0.0102)$ |
| $(0.01, 0.01)$ | $(0.0051, 0.0105)$ | $(0.0045, 0.0101)$ | $(**, 0.0024)$ |

**Smaller than $10^{-3}$

the sequential energy detector achieve much lower average number of samples than the energy detector. The results in this example show the effectiveness of the proposed approach in the scenario of unequal $P_M$ and $P_{FA}$ in the cognitive radio.

Besides the detection accuracies, the comparisons of complexities are discussed as follows. At each instant of taking a sample, the proposed sequential detector performs Step.1)-6) in Section III-D, whose complexity is fixed per sample. Therefore, the computational complexity per detection of the proposed detector grows linearly with the number of samples to termination. With the expected termination time denoted as $E[\Im]$, the overall complexity per detection is $O(E[\Im])$. At each instant of taking a sample, the sequential energy detector performs (37)(38), whose complexity is fixed per sample. Therefore, the computational complexity per detection of the sequential energy detector grows linearly with the number of samples to termination, and the overall complexity per detection is $O(E[\Im])$. The complexities of our proposed sequential detector and the energy sequential detector grow at the same order with the number of samples to termination. As observed in the numerical results, the expected termination times of the sequential energy detector are roughly 1.5 to 2 times higher than that of the proposed approach, depending on the scenarios. Therefore, the complexity of the sequential energy detector is slightly higher than the complexity of the proposed sequential detector.

## VII. CONCLUSION

We investigate the sequential detector based on AR model for the incomplete signal model suitable for non-cooperative signal sources. With the goals to accommodate the uncertainties and unavailability of the signal characteristics, the incomplete signal model assumes only the knowledge of AR process on the primary signal. The incomplete model provides the detection framework with looser assumptions than the previously studied models. We derive the sequential detector for the incomplete model. The sequential detector utilizes likelihood ratio test framework and uses the predictive distributions of the ARMA process. The predictive distributions are sequentially updated by the sample covariances. To measure the efficiency of the sequential detector, the approximate expected number of samples to reach the decision is derived.

The energy detector and the sequential energy detector are compared with the sequential detector. The numerical examples simulating the detections of the mobile transmitters are demonstrated. The sequential detector is shown to reach the decision with smaller average number of samples than the energy detector and sequential energy detector while satisfying the desired probabilities of miss and false alarm.

The incorporation of more information, e.g., the cyclostationarity, the fading statistics, and SNR, may have potentials

TABLE IV
THE TARGETED $(P_M, P_{FA})$ AND THE SIMULATION RESULTS UNDER THE SETTING IN EXAMPLE 3

| | Sequential Detector | | Sequential Energy Detector | | Energy Detector | |
|---|---|---|---|---|---|---|
| Targeted $(P_M, P_{FA})$ | $(P_M, P_{FA})$ | ANS | $(P_M, P_{FA})$ | ANS | $(P_M, P_{FA})$ | ANS |
| $(0.08, 0.1)$ | $(0.0496, 0.0876)$ | $2.18 \times 10^4$ | $(0.0506, 0.0905)$ | $4.46 \times 10^4$ | $(**, 0.0393)$ | $1.45 \times 10^4$ |
| $(0.05, 0.1)$ | $(0.0301, 0.0902)$ | $2.64 \times 10^4$ | $(0.0320, 0.0921)$ | $5.54 \times 10^4$ | $(**, 0.0326)$ | $1.73 \times 10^4$ |
| $(0.03, 0.05)$ | $(0.0165, 0.0438)$ | $3.12 \times 10^4$ | $(0.0153, 0.0480)$ | $6.33 \times 10^4$ | $(**, 0.0203)$ | $2.51 \times 10^4$ |
| $(0.01, 0.05)$ | $(0.0062, 0.0463)$ | $3.45 \times 10^4$ | $(0.0065, 0.0442)$ | $7.02 \times 10^4$ | $(**, 0.0161)$ | $3.19 \times 10^4$ |

** Smaller than $10^{-3}$

to improve the detection accuracies, and may be the direction of further investigations.

## APPENDIX
## PROOF OF PROPOSITION 3

Applying chain rules on (6), we obtain (39) in the next page. Because the autocovariances tend to zero for large time lags, we denote $p+1$ as the time lag by which the past $p+1$ samples capture most of the information of the probabilities in (39). The $\frac{P(x_t|x_{t-1},x_{t-2},...,x_1,H_1)}{P(x_t|x_{t-1},x_{t-2},...,x_1,H_0)}$ in (39) can be truncated to depend only on the past $p+1$ samples, i.e,

$$\frac{P(x_t|x_{t-1},x_{t-2},...,x_1,H_1)}{P(x_t|x_{t-1},x_{t-2},...,x_1,H_0)} := \frac{P(x_t|x_{t-1},x_{t-2},...,x_{t-p-1},H_1)}{P(x_t|x_{t-1},x_{t-2},...,x_{t-p-1},H_0)}, \quad (40)$$

where ":=" denotes "approximately equal to". The $p$ is selected to be approximately equal to or slightly larger than the underlying order of AR process in (2), and the $p$ can be determined using the commonly practiced information-theoretical criterion [39][40]. By using (40) in (39), we obtain (41) in the next page. Therefore, the (41) leads to the expression in (8).

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$$\begin{aligned}\Lambda_t =& \log\frac{P(x_1,x_2,\cdots,x_t|H_1)}{P(x_1,x_2,\cdots,x_t|H_0)}\\ =& \log\left\{\frac{P(x_t|x_{t-1},x_{t-2},\cdots,x_1,H_1)P(x_{t-1}|x_{t-2},x_{t-3},\cdots,x_1,H_1)\cdots P(x_{p+2}|x_{p+1},x_p,\cdots,x_1,H_1)}{P(x_t|x_{t-1},x_{t-2},\cdots,x_1,H_0)P(x_{t-1}|x_{t-2},x_{t-3},\cdots,x_1,H_0)\cdots P(x_{p+2}|x_{p+1},x_p,\cdots,x_1,H_0)}\right.\\ &\times\log\left.\frac{P(x_{p+1}|x_p,x_{p-1},\cdots,x_1,H_1)P(x_p|x_{p-1},x_{p-2},\cdots,x_1,H_1)\cdots P(x_1|H_1)}{P(x_{p+1}|x_p,x_{p-1},\cdots,x_1,H_0)P(x_p|x_{p-1},x_{p-2},\cdots,x_1,H_0)\cdots P(x_1|H_0)}\right\}. \qquad (39)\end{aligned}$$

$$\begin{aligned}\Lambda_t =& \log\left\{\frac{P(x_t|x_{t-1},x_{t-2},\cdots,x_{t-p-1},H_1)P(x_{t-1}|x_{t-2},x_{t-3},\cdots,x_{t-p-2},H_1)\cdots P(x_{p+2}|x_{p+1},x_p,\cdots,x_1,H_1)}{P(x_t|x_{t-1},x_{t-2},\cdots,x_{t-p-1},H_0)P(x_{t-1}|x_{t-2},x_{t-3},\cdots,x_{t-p-2},H_0)\cdots P(x_{p+2}|x_{p+1},x_p,\cdots,x_1,H_0)}\right.\\ =& \log\frac{P(x_t|x_{t-1},x_{t-2},\cdots,x_{t-p-1},H_1)}{P(x_t|x_{t-1},x_{t-2},\cdots,x_{t-p-1},H_0)}+\log\frac{P(x_{t-1}|x_{t-2},x_{t-3},\cdots,x_{t-p-2},H_1)}{P(x_{t-1}|x_{t-2},x_{t-3},\cdots,x_{t-p-2},H_0)}+\cdots+\log\frac{P(x_1|H_1)}{P(x_1|H_0)}. \qquad (41)\end{aligned}$$


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