# Multiuser Diversity in Interfering Broadcast Channels: Achievable Degrees of Freedom and User Scaling Law

Jung Hoon Lee, Student Member, IEEE, Wan Choi, Senior Member, IEEE, and Bhaskar D. Rao, Fellow, IEEE

#### Abstract

This paper investigates how multiuser dimensions can effectively be exploited for target degrees of freedom (DoF) in interfering broadcast channels (IBC) consisting of K-transmitters and their user groups. First, each transmitter is assumed to have a single antenna and serve a singe user in its user group where each user has receive antennas less than K. In this case, a K-transmitter single-input multiple-output (SIMO) interference channel (IC) is constituted after user selection. Without help of multiuser diversity, K - 1 interfering signals cannot be perfectly removed at each user since the number of receive antennas is smaller than or equal to the number of interference. Only with proper user selection, non-zero DoF per transmitter is achievable as the number of users increases. Through geometric interpretation of interfering channels, we show that the multiuser dimensions have to be used first for reducing the DoF loss caused by the interfering signals, and then have to be used for increasing the DoF gain from its own signal. The sufficient number of users for the target DoF is derived. We also discuss how the optimal strategy of exploiting multiuser diversity can be realized by practical user selection schemes. Finally, the single transmit antenna case is extended to the multiple-input multiple-output (MIMO) IBC where each transmitter with multiple antennas serves multiple users.

#### **Index Terms**

Multiuser diversity, degrees of freedom, interference alignment measure, interfering broadcast channel

This work was supported by the Korea Research Foundation Grant funded by the Korean Government (NRF-2012-013-2012S1A2A1A01031507).

J. H. Lee and W. Choi are with Department of Electrical Engineering, Korea Advanced Institute of Science and Technology (KAIST), Daejeon 305-701, Korea (e-mail: tantheta@kaist.ac.kr, wchoi@ee.kaist.ac.kr)

B. D. Rao is with Department of Electrical and Computer Engineering, University of California, San Diego, La Jolla, CA 92093-0407 USA (e-mail: brao@ece.ucsd.edu).

#### I. INTRODUCTION

Interference is a major performance-limiting factor in modern wireless communication systems. Many interference mitigation strategies have been proposed to improve network spectral efficiency. By allowing partial or full cooperation among interfering base stations, interference can effectively be managed and spectral efficiency can be improved. Joint beamforming [1] and network MIMO (or multicell processing) [2] among base stations have been shown to be effective interference mitigation techniques. However, if cooperation among transmitters is not allowed, orthogonal multiple access has been a traditional solution to interference. In a K-user single-input single-output (SISO) interference channel (IC), for example, each user can achieve 1/K degrees of freedom (DoF) by time division multiple access.

In recent years, interference alignment (IA) techniques have received much attention [3]–[6]. The basic concept of IA is to align the interfering signals in a small dimensional subspace. In a K-user SISO IC, K/2 DoF have been shown to be achievable using IA [3]. Although IA provides a substantial asymptotic capacity gain in interference channels, there are many practical challenges for implementation. IA requires global channel state information at the transmitter (CSIT), and imperfect channel knowledge severely degrades the gain of IA. In some channel configurations, symbols should be extended in the time/frequency domain to align interfering signals. The high computational complexity is also considered as a major challenge. To ameliorate these difficulties, many IA algorithms have been proposed such as iterative IA [5] and a subspace IA [6].

For interference suppression, multiuser diversity can also be exploited by opportunistic user selection for minimizing interference. The interference reduction by multiuser diversity can be enjoyed without heavy burden on global channel knowledge because user selection in general requires only a small amount of feedback [7]–[14]. In this context, opportunistic interference alignment (OIA) has been recently proposed in [9] and has attracted much attention. In a 3-transmitter  $M \times 2M$  MIMO interfering broadcast channel (IBC), the authors of [11] proved that  $\alpha M$  (where  $\alpha \in [0, 1]$ ) DoF per transmitter is achievable when the number of users scales as  $P^{\alpha M}$ . In [12] and [13], a K-transmitter  $1 \times 3$  SIMO IBC and a K-transmitter  $1 \times (K - 1)$  SIMO IBC have been studied, respectively. For SIMO interfering multiple access channel (IMAC) constituted by K-cell uplink channels with M transmit antennas and single antenna users, the authors of [14] showed that KM DoF are achievable when the number of users scales as  $P^{(K-1)M}$ . In these schemes, user dimensions are used to align the interfering signals; each transmitter opportunistically selects a user whose interfering signals are most aligned among the users associated with the transmitter. Contrary to the conventional opportunistic user selection techniques [7], [8], [15]–[18], the OIA scheme exploits the multiuser dimensions for interference alignment.

In this paper, we investigate the optimal role of multiuser diversity for the target DoF in the IBC with K-transmitters and generalize the results of [12] [13]. For the K-transmitter SIMO IBC, each transmitter selects and serves a single user in its user group consisting of N users. Once after K-transmitters select their serving users, a K-user IC is constructed. Each user has  $N_r$  antennas less than or equal to the number of interferers, i.e.,  $N_r \leq K - 1$ . Thus, without help of multiuser diversity, interference at each user cannot be perfectly removed so that the achievable rate of each transmitter goes to zero as signal-to-noise ratio (SNR) increases. Consequently, the achievable DoF per transmitter becomes zero. However, non-zero DoF per transmitter is achievable by exploiting multiuser diversity as the number of users increases.

Since opportunistic user selection can focus on either enhancing the desired signal or decreasing interference, non-zero DoF can be obtained by properly enhancing the desired signal strength and reducing interference via user selection. That is, the non-zero DoF d comprises a DoF gain term  $d_1 \ge 0$  from the desired signal and a DoF loss term  $d_2 \ge 0$  caused by interference such that  $d_1 - d_2 = d$ , and the target DoF d can be obtained by a proper combination of  $d_1$  and  $d_2$ . However, many questions remain unsolved; what is the feasible and optimal combination of  $(d_1, d_2)$  for the target DoF d  $(= d_1 - d_2)$  and what is the sufficient number of users for the target DoF achieving strategy. We answer these fundamental questions and analytically investigate how the multiuser dimensions can be optimally exploited for the target DoF in the IBC. Specifically, from geometric interpretation of interfering channels, we define an interference alignment measure that indicates how well interference signals are aligned at each user.

Using the interference alignment measure, we first consider the K- transmitters SIMO IBC and show that the DoF gain term  $d_1$  can be achieved if the number of users scales in terms of transmit power Pas  $N \propto e^{P^{(d_1-1)}}$  and the DoF loss term can be reduced to  $d_2$  if the number of users scales as  $N \propto P^{(1-d_2)(K-N_r)}$ . From these results, we find the optimal strategy of exploiting multiuser diversity for the target DoF d in terms of the required number of users; the optimal target DoF achieving strategies  $(d_1^{\star}, d_2^{\star})$ are (1, 1-d) and (d, 0) for the target DoF  $d \in [0, 1]$  and d (> 1), respectively. We also investigate how the optimal target DoF achieving strategy  $(d_1^{\star}, d_2^{\star})$  can be realized by practical user selection schemes. Then, we extend our results to the K-transmitter MIMO IBC where each transmitter has  $N_t$  multiple antennas and serves multiple users with  $N_r$  receive antenna each. Our generalized key findings are summarized as follows:

- For the target DoF d ∈ [0, N<sub>t</sub>], (d<sup>\*</sup><sub>1</sub>, d<sup>\*</sup><sub>2</sub>) = (N<sub>t</sub>, N<sub>t</sub>-d) is the optimal target DoF achieving strategy that minimizes the required number of users. That is, the multiuser dimensions should be exploited to make the DoF loss N<sub>t</sub>-d. The sufficient number of users for this strategy scales like N ∝ P<sup>(d/N<sub>t</sub>)(KN<sub>t</sub>-N<sub>r</sub>)</sup>.
- For the target DoF d (> N<sub>t</sub>), (d<sup>\*</sup><sub>1</sub>, d<sup>\*</sup><sub>2</sub>) = (d, 0) is the optimal target DoF achieving strategy which minimizes the required number of users. That is, the multiuser dimensions should be exploited to make the DoF loss term zero as well as to make the DoF gain term d. The sufficient number of users for this strategy scales like N ∝ e<sup>P(d/Nt-1)</sup> P<sup>(KNt-Nr)</sup>.

The rest of this paper is organized as follows. In Section II, we describe the system model. In Section III, a geometric interpretation of interfering channels is provided, and the interference alignment measure is defined. Section IV derives the optimal strategies of achieving the target DoF in terms of the required number of users. In Section V, we show how various practical user selection schemes exploit multiuser diversity for the target DoF and discuss their optimality to achieve the target DoF. The system model is extended for the MIMO IBC in Section VI. Numerical results are shown in Section VII, and we conclude our paper in Section VIII.

# - Notations

Throughout the paper, we use boldface to denote vectors and matrices. The notations  $\mathbf{A}^{\dagger}$ ,  $\Lambda_i(\mathbf{A})$ , and  $V_i(\mathbf{A})$  denote the conjugate transpose, the *i*th largest eigenvalue, and the eigenvector of matrix  $\mathbf{A}$ corresponding to the *i*th largest eigenvalue. For convenience, the smallest eigenvalue, the largest eigenvalue, and the eigenvectors corresponding eigenvectors of  $\mathbf{A}$  are denoted as  $\Lambda_{\min}(\mathbf{A})$ ,  $\Lambda_{\max}(\mathbf{A})$ ,  $V_{\min}(\mathbf{A})$ , and  $V_{\max}(\mathbf{A})$ , respectively. Also,  $\mathbf{I}_n$ ,  $\mathbb{C}^n$ , and  $\mathbb{C}^{m \times n}$  indicate the  $n \times n$  identity matrix, the *n*-dimensional complex space, and the set of  $m \times n$  complex matrices, respectively.

## A. System Model

Our system model is depicted in Fig. 1. The system corresponds to the interfering broadcast channel (IBC) of which capacity is unknown. There are K transmitters with  $N_t$  transmit antennas each, and each transmitter has its own user group consisting of N users with  $N_r$  antennas each. First, each transmitter is assumed to have a single antenna, i.e.,  $N_t = 1$ , and serves a single user selected in its user group so that K-transmitter SIMO IC is opportunistically constituted. The system model with multiple transmit antennas (i.e.,  $N_t > 1$ ) becomes statistically identical with the single transmit antenna model if each transmitter uses a random precoding vector. In Section VI, we extend our system model to the K-transmitter MIMO IBC where each transmitter with multiple antennas serves the multiple users through orthonormal random beams.

In this paper, we focus on the cases that the number of receive antennas is smaller than the number of transmitters, i.e.,  $N_r < K$ . Otherwise (i.e., if  $N_r \ge K$ ), each user can suppress all interfering signals through zero-forcing like schemes so that DoF one is trivially guaranteed at each transmitter. We also assume that collaboration or information sharing among the transmitters is not allowed. Since the user selection at each transmitter is independent of the other transmitters', we only consider the achievable rate of the first transmitter without loss of generality. Note that the average achievable rate per transmitter will be same if the configurations of the transmitters are identical.

At the first transmitter, the received signal at the *n*th user denoted by  $\mathbf{y}_n \in \mathbb{C}^{N_r \times 1}$  is given by

$$\mathbf{y}_n = \mathbf{h}_{n,1} x_1 + \sum_{k=2}^{K} \mathbf{h}_{n,k} x_k + \mathbf{z}_n$$

where  $\mathbf{h}_{n,k} \in \mathbb{C}^{N_r \times 1}$  is the vector channel from the *k*th transmitter to the *n*th user whose elements are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero means and unit variance. Also,  $x_k \in \mathbb{C}^{1 \times 1}$  is the transmitted signal using random Gaussian codebook from the *k*th transmitter such that  $\mathbb{E}|x_k|^2 = P$ , where *P* is the power budget at each transmitter. Also,  $\mathbf{z}_n \in \mathbb{C}^{N_r \times 1}$  is a circularly symmetric complex Gaussian noise with zero mean and an identity covariance matrix, i.e.,  $\mathbf{z}_n \sim C\mathcal{N}(0, \mathbf{I}_{N_r})$ . Assuming perfect channel estimation at each receiver, the channel state information  $\{\mathbf{h}_{n,k}\}_{k=1}^K$  is available at the *n*th user. The received signal is postprocessed at each user using multiple receive antennas. Let  $\mathbf{v}_n \in \mathbb{C}^{N_r \times 1}$  be the postprocessing vector of the *n*th user such that  $\|\mathbf{v}_n\|^2 = 1$ . Then, the received signal after postprocessing becomes

$$\mathbf{v}_{n}^{\dagger}\mathbf{y}_{n} = \mathbf{v}_{n}^{\dagger}\mathbf{h}_{n,1}x_{1} + \sum_{k=2}^{K}\mathbf{v}_{n}^{\dagger}\mathbf{h}_{n,k}x_{k} + \mathbf{v}_{n}^{\dagger}\mathbf{z}_{n}.$$
(1)

To aid user selection at the transmitter, each user feeds one scalar value back to the transmitter. Various user selection criteria and corresponding feedback information will be discussed in the following sections. Since no information is shared among the transmitters, each transmitter independently selects a single user based on the collected information.

Let  $n^*$  be the index of the selected user at the first transmitter. Then, the average achievable rate of the first transmitter is given by

$$\mathcal{R} \triangleq \mathbb{E} \log_2 \left( 1 + \frac{P |\mathbf{v}_{n^*}^{\dagger} \mathbf{h}_{n^*,1}|^2}{1 + P \sum_{k=2}^{K} |\mathbf{v}_{n^*}^{\dagger} \mathbf{h}_{n^*,k}|^2} \right).$$
(2)

We decompose  $\mathcal{R}$  into two terms  $\mathcal{R}^+$  and  $\mathcal{R}^-$  such that  $\mathcal{R} = \mathcal{R}^+ - \mathcal{R}^-$ , which are given, respectively, by

$$\mathcal{R}^{+} = \mathbb{E}\log_2\left(1 + P\sum_{k=1}^{K} |\mathbf{v}_{n^*}^{\dagger} \mathbf{h}_{n^*,k}|^2\right),\tag{3}$$

$$\mathcal{R}^{-} = \mathbb{E}\log_2\left(1 + P\sum_{k=2}^{K} |\mathbf{v}_{n^*}^{\dagger}\mathbf{h}_{n^*,k}|^2\right).$$
(4)

Then, the achievable DoF of the first transmitter becomes

$$\lim_{P \to \infty} \frac{\mathcal{R}}{\log_2 P} = \lim_{P \to \infty} \frac{\mathcal{R}^+}{\log_2 P} - \lim_{P \to \infty} \frac{\mathcal{R}^-}{\log_2 P}.$$
(5)

We call  $\lim_{P\to\infty} \frac{\mathcal{R}^+}{\log_2 P}$  and  $\lim_{P\to\infty} \frac{\mathcal{R}^-}{\log_2 P}$  as *DoF gain term* and *DoF loss term*, respectively.

## **B.** Problem Description

The achievable rate of each transmitter depends on the number of users because multiuser dimensions are exploited for a rate increase. When there are fixed number of users, the achievable rate of each transmitter will be saturated in the high SNR region due to interferences because the number of receive antennas at each user is smaller than the number of total transmitters. Consequently, the first transmitter cannot obtain any DoF, i.e.,

$$\lim_{\substack{P \to \infty \\ \text{Fixed } N}} \frac{\mathcal{R}}{\log_2 P} = 0.$$
(6)

In this case, both the DoF gain term and the DoF loss term become one, i.e.,

$$\left(\lim_{\substack{P \to \infty \\ \text{Fixed } N}} \frac{\mathcal{R}^+}{\log_2 P}, \lim_{\substack{P \to \infty \\ \text{Fixed } N}} \frac{\mathcal{R}^-}{\log_2 P}\right) = (1, 1).$$
(7)

On the other hand, when the transmit power is fixed, the achievable rate of the selected user can increase to infinity as the number of users increases, i.e.,

$$\lim_{\substack{N \to \infty \\ \text{Fixed } P}} \mathcal{R} = \infty.$$
(8)

Then, how much DoF can be achieved when both the number of users and the transmit power increase? Obviously, non-zero DoF can be obtained by exploiting multiuser dimensions, and the achievable DoF

$$\lim_{N \to \infty} \left[ \lim_{P \to \infty} \frac{\mathcal{R}}{\log_2 P} \right] \tag{9}$$

will depend on the increasing speeds of N and P. In this case, DoF d (> 0) at the first transmitter comprises the DoF gain term  $d_1$  ( $\geq 0$ ) and the DoF loss term  $d_2$  ( $\geq 0$ ) such that  $d_1 - d_2 = d$ , i.e.,

$$(d_1, d_2) \triangleq \left(\lim_{P \to \infty} \frac{\mathcal{R}^+}{\log_2 P}, \lim_{P \to \infty} \frac{\mathcal{R}^-}{\log_2 P}\right).$$
(10)

We call  $(d_1, d_2)$  as a *target DoF achieving strategy* if  $d = d_1 - d_2$  for the target DoF d. Since each strategy requires different user scaling, we need to find the optimal DoF achieving strategy that exploits multiuser diversity most efficiently, i.e., which requires the minimum user scaling. For the *target DoF* per transmitter d (> 0), we find the optimal target DoF achieving strategy  $(d_1^*, d_2^*)$  satisfying  $d_1^* - d_2^* = d$  and derive the required user scaling. Note that the definition of DoF in this paper is extended from the conventional definition of DoF in order to properly capture multiuser diversity gain in terms of achievable rate. Achievable DoF defined in (9) depends on increasing speeds of N and P; and can have non-zero values even larger than one if the number of users properly scales with the transmit power.

## C. DoF Achieving Strategies and Reduced Set of Candidates for the Optimal Strategy

From the definitions of the rate gain term and the rate loss term given in (3) and (4), respectively, the strategies which achieve the target DoF d are given by

$$\{(d_1, d_2) \mid d_1 - d_2 = d, \ d_1 \ge d_2, \ d_1 \ge 0, \ d_2 \ge 0\}.$$
(11)

The following lemma shows that we do not need to consider all of the candidate strategies in (11) but take into account only a subset of (11) to find the optimal target DoF achieving strategy.

Lemma 1. For any non-negative target DoF, the optimal DoF achieving strategy is in the set

$$\{(d_1, d_2) \mid d_1 \in [1, \infty), \ d_2 \in [0, 1], \ d_1 - d_2 = d\}.$$
(12)

Proof: At each channel realization, the achievable DoF has the form of

$$\log_2(1 + X + Y) - \log_2(1 + Y), \tag{13}$$

where  $X \triangleq |\mathbf{v}_{n^*}^{\dagger} \mathbf{h}_{n^*,1}|^2$  and  $Y \triangleq P \sum_{k=2}^{K} |\mathbf{v}_{n^*}^{\dagger} \mathbf{h}_{n^*,k}|^2$  are its own signal power and the interfering signal power at the selected user, respectively. Since the function (13) is an increasing function of X and a decreasing function of Y, for an increase of (13), the multiuser dimension should be used for increasing X, for decreasing Y, or mixture of them. This fact results in (12).

Lemma 1 provides a basic guideline of using the multiuser dimension; multiuser diversity should not be used for either decreasing DoF gain term or increasing DoF loss term. Since the optimal target DoF achieving strategy is obtained in the reduced set of candidate strategies, we consider the DoF gain term larger than one and DoF loss term smaller than one, i.e.,  $d_1 \in [1, \infty)$  and  $d_2 \in [0, 1]$ , in the latter parts of this paper.

# III. INTERFERENCE ALIGNMENT MEASURE

### A. Where does the DoF Loss Come from?

In our system model, each user suffers from K-1 interfering channels which is larger than or equal to the number of receive antennas, i.e.,  $K-1 \ge N_r$ . Since the interfering channels are isotropic and independent of each other, they span  $N_r$ -dimensional space. Thus, the whole signal space at the receiver is corrupted by interfering signals, and hence the DoF loss term becomes one if no effort is made to align interfering signals. On the other hand, the DoF loss can be reduced by aligning interfering signals in smaller dimensional subspace. For example, if the interfering signals are perfectly aligned in  $(N_r - 1)$ dimensional subspace, they can be nullified by postprocessing so that we can make the DoF loss zero.

The transmitter can exploit the multiuser dimensions to align interfering signals by simply selecting a user whose interfering channels are most aligned. Thus, each user needs to measure how much the interfering channels are aligned in  $(N_r - 1)$ -dimensional subspace at the receiver. We call this measure as the *interference alignment measure*. In this section, we geometrically interpret the interfering channels and define the interference alignment measure at each user. The interference alignment measure will be used for computing the reducible DoF loss via multiuser diversity in Section IV.

## **B.** Preliminaries

Let  $S_0$  be the surface of the  $N_r$ -dimensional unit hypersphere centered at the origin, i.e.,

$$S_0 = \{ \mathbf{x} \in \mathbb{C}^{N_r} | \| \mathbf{x} \|^2 = 1 \}.$$

For an arbitrary unit vector  $\mathbf{c} \in \mathbb{C}^{N_r}$  and an arbitrary non-negative real number  $0 \le \lambda \le 1$ , we can divide  $S_0$  into two parts,  $S_1(\mathbf{c}, \lambda)$  and  $S_2(\mathbf{c}, \lambda)$ , given by

$$S_{1}(\mathbf{c},\lambda) \triangleq \left\{ \mathbf{x} \in \mathbb{C}^{N_{r}} \middle| |\mathbf{c}^{\dagger}\mathbf{x}|^{2} \ge \lambda, ||\mathbf{x}||^{2} = 1 \right\}$$
$$S_{2}(\mathbf{c},\lambda) \triangleq \left\{ \mathbf{x} \in \mathbb{C}^{N_{r}} \middle| |\mathbf{c}^{\dagger}\mathbf{x}|^{2} \le \lambda, ||\mathbf{x}||^{2} = 1 \right\}.$$
(14)

When  $\mathbf{x}, \mathbf{c} \in \mathbb{R}^3$ , two parts  $S_1(\mathbf{c}, \lambda)$  and  $S_2(\mathbf{c}, \lambda)$  are represented in Fig. 2. Let  $A(S_i(\mathbf{c}, \lambda))$  be the surface area of  $S_i(\mathbf{c}, \lambda)$  for i = 0, 1, 2. The surface area of an  $N_r$ -dimensional complex unit hypersphere is given by  $A(S_0) = 2\pi^{N_r}/(N_r - 1)!$ , and it was shown that [19]

$$A(S_1(\mathbf{c},\lambda)) = \frac{2\pi^{N_r}(1-\lambda)^{N_r-1}}{(N_r-1)!},$$

which is invariant with c. Therefore, we obtain

$$A(S_2(\mathbf{c},\lambda)) = \frac{2\pi^{N_r}(1-(1-\lambda)^{N_r-1})}{(N_r-1)!}$$

from the relationship  $A(S_0) = A(S_1(\mathbf{c}, \lambda)) + A(S_2(\mathbf{c}, \lambda))$ . From this fact, we obtain the following lemma.

**Lemma 2.** Let  $\mathbf{g}_1, \ldots, \mathbf{g}_m$  be independent and isotropic unit vectors in  $\mathbb{C}^{N_r}$ . For an arbitrary unit vector  $\mathbf{c} \in \mathbb{C}^{N_r}$  and  $\lambda \in [0, 1]$ , the probability that  $S_2(\mathbf{c}, \lambda)$  contains  $\{\mathbf{g}_1, \ldots, \mathbf{g}_m\}$  becomes

$$\Pr[\{\mathbf{g}_1,\ldots,\mathbf{g}_m\} \subset S_2(\mathbf{c},\lambda)] = \left(1 - (1-\lambda)^{N_r-1}\right)^m,\tag{15}$$

which is invariant with c.

*Proof:* From the ratio of  $A(S_2(\mathbf{c}, \lambda))$  and  $A(S_0)$ , we obtain

$$\Pr[\mathbf{g}_i \in S_2(\mathbf{c}, \lambda)] = \frac{A(S_2(\mathbf{c}, \lambda))}{A(S_0)} = 1 - (1 - \lambda)^{N_r - 1}, \quad \forall i.$$
(16)

Since  $g_1, \ldots, g_m$  are independent of each other, it is satisfied that

$$\Pr[\{\mathbf{g}_1,\ldots,\mathbf{g}_m\} \subset S_2(\mathbf{c},\lambda)] = \Pr[\mathbf{g}_1 \in S_2(\mathbf{c},\lambda)]^m,$$

which is given in (15).

### C. Interference Alignment Measure at Each User

In this subsection, we define the interference alignment measure at each user. The DoF loss is determined by how much the interfering channels are closely aligned in  $(N_r - 1)$ -dimensional subspace. Only if interfering channels are perfectly aligned in  $(N_r - 1)$ -dimensional subspace, we can have zero DoF loss. The interference alignment measure is used for computing the DoF loss at each user. Let  $\mathbf{g}_1, \ldots, \mathbf{g}_{K-1}$ be the K - 1 ( $\geq N_r$ ) normalized interfering channels at a user and  $q(\mathbf{g}_1, \ldots, \mathbf{g}_{K-1})$  be the interference alignment measure among them.

Consider the following optimization problem:

$$\begin{array}{ll} \underset{\mathbf{c},\lambda}{\text{minimize}} & A(S_2(\mathbf{c},\lambda)) \end{array} \tag{17} \\ \text{subject to} & S_2(\mathbf{c},\lambda) \supset \{\mathbf{g}_1,\ldots,\mathbf{g}_{K-1}\}, \\ & \|\mathbf{c}\|^2 = 1, \quad \lambda \in [0,1]. \end{array}$$

From the definition of  $S_2(\mathbf{c}, \lambda)$  given in (14), this problem is equivalent to

minimize 
$$\lambda$$
 (18)  
subject to  $|\mathbf{c}^{\dagger}\mathbf{g}_{k}|^{2} \leq \lambda$  for  $1 \leq k \leq K - 1$ ,  
 $\|\mathbf{c}\|^{2} = 1, \ \lambda \in [0, 1],$ 

which can be solved by linear programming [21], [22]. Let  $(\mathbf{c}^*, \lambda^*)$  be the solution of the above problem. Then,  $S_2(\mathbf{c}^*, \lambda^*)$  has the smallest surface area among all  $S_2(\mathbf{c}, \lambda)$  containing  $\mathbf{g}_1, \ldots, \mathbf{g}_{K-1}$ .

Using  $\mathbf{c}^*$ , we can divide an  $N_r$ -dimensional space into two subspaces which are the one-dimensional subspace spanned by  $\mathbf{c}^*$  and the  $(N_r - 1)$ -dimensional complementary subspace denoted by  $\mathcal{U}$ . If there exists  $\mathbf{c}^*$  such that  $\mathbf{c}^* \perp \{\mathbf{g}_1, \ldots, \mathbf{g}_{K-1}\}$ , it is satisfied that  $\operatorname{span}(\mathbf{g}_1, \ldots, \mathbf{g}_{K-1}) \subset \mathcal{U}$  and  $S_2(\mathbf{c}^*, 0) \supset \{\mathbf{g}_1, \ldots, \mathbf{g}_{K-1}\}$ , and hence  $\lambda^*$  becomes zero. In this case, we can say that the interfering channels are perfectly aligned in  $(N_r - 1)$ -dimensional subspace in  $\mathbb{C}^{N_r}$ . Note that  $S_2(\mathbf{c}^*, 0)$  is an  $(N_r - 1)$ -dimensional

subspace orthogonal to  $\mathbf{c}^*$ , and  $S_2(\mathbf{c}^*, 1)$  is the  $N_r$ -dimensional complex hypersphere,  $S_0$ . When  $\lambda^*$  is the smaller, the vectors are the more aligned in the  $(N_r - 1)$ -dimensional subspace,  $\mathcal{U}$ . Thus, we will use  $\lambda^*$  as an *interference alignment measure* to quantify how much the interfering channels are closely aligned in an  $(N_r - 1)$ -dimensional subspace, i.e.,

$$\mathbf{q}(\mathbf{g}_1, \dots, \mathbf{g}_{K-1}) = \min_{\|\mathbf{c}\|=1} \max_{1 \le k \le K-1} |\mathbf{c}^{\dagger} \mathbf{g}_k|^2$$
(19)

$$=\lambda^{\star} \quad (\lambda^{\star} \in [0,1]). \tag{20}$$

In other words, we use the mini-max distance of the interfering channels from an  $(N_r - 1)$ -dimensional subspace. In Fig. 3, the interference alignment measure is geometrically represented. The more the interfering channels are aligned, the smaller the interference alignment measure becomes.

Since the interference alignment measure is obtained from the optimization problem (18), the exact distribution is difficult to find. Instead, we obtain the lower bound for the cumulative distribution function (CDF) of the interference alignment measure in the following lemma.

**Lemma 3.** When  $K > N_r$ , the probability that the interference alignment measure  $q(\mathbf{g}_1, \dots, \mathbf{g}_{K-1})$  is smaller than  $\lambda \in [0, 1]$  is lower bounded on

$$\Pr\left[\mathbf{q}(\mathbf{g}_1,\ldots,\mathbf{g}_{K-1}) \le \lambda\right] \ge \left(1 - (1-\lambda)^{N_r-1}\right)^{K-N_r}.$$
(21)

*Proof:* We consider two events:

(E1): 
$$q(\mathbf{g}_1, \dots, \mathbf{g}_{K-1}) \leq \lambda$$
  
(E2):  $S_2(\bar{\mathbf{c}}, \lambda) \supset \{\mathbf{g}_1, \dots, \mathbf{g}_{K-1}\}$ 

where  $\bar{\mathbf{c}}$  is the  $N_r$ -dimensional unit vector such that  $\bar{\mathbf{c}} \perp \{\mathbf{g}_1, \dots, \mathbf{g}_{N_r-1}\}$ . By the definition of the interference alignment measure given in (19), (E1) is true whenever (E2) is true, equivalently,  $\Pr[(E1)] \geq \Pr[(E2)]$ . The probability of (E2) is obtained by

$$\Pr\left[(\text{E2})\right] = \Pr\left[S_2(\bar{\mathbf{c}}, \lambda) \supset \{\mathbf{g}_1, \dots, \mathbf{g}_{K-1}\}\right]$$

$$\stackrel{(a)}{=} \Pr\left[S_2(\bar{\mathbf{c}}, \lambda) \supset \{\mathbf{g}_{N_r}, \dots, \mathbf{g}_{K-1}\}\right]$$

$$\stackrel{(b)}{=} \left(1 - (1 - \lambda)^{N_r - 1}\right)^{K - N_r},$$
(22)

where the equality (a) is from the definition of  $\bar{\mathbf{c}}$  such that  $\bar{\mathbf{c}} \perp \{\mathbf{g}_1, \dots, \mathbf{g}_{N_r-1}\}$ . Also, the equality (b) holds from Lemma 2 and from the fact that  $\bar{\mathbf{c}}$  is independent of  $\{\mathbf{g}_{N_r}, \dots, \mathbf{g}_{K-1}\}$ . Thus, we obtain

$$\Pr\left[(E1)\right] \ge \left(1 - (1 - \lambda)^{N_r - 1}\right)^{K - N_r}.$$
(23)

# D. Achievable Value of the Interference Alignment Measure via User Selection

The remaining question is how much we can reduce the interference alignment measure via user selection. In the first user group, the *n*th user has K - 1 interfering channels,  $\mathbf{h}_{n,2}, \ldots, \mathbf{h}_{n,K}$ . The interference alignment measure at the *n*th user can be written by

$$\mathfrak{q}\big(\tilde{\mathbf{h}}_{n,2},\ldots,\tilde{\mathbf{h}}_{n,K}\big),\tag{24}$$

where  $\tilde{\mathbf{h}}_{n,k}$  is the normalized interfering channel, i.e.,  $\tilde{\mathbf{h}}_{n,k} = \mathbf{h}_{n,k}/\|\mathbf{h}_{n,k}\|$ . Thus, the achievable smallest interference alignment measure via user selection is given by

$$\min_{n} \ \mathfrak{q}(\tilde{\mathbf{h}}_{n,2},\ldots,\tilde{\mathbf{h}}_{n,K}).$$
(25)

Obviously, the smallest interference alignment measure will decrease as the number of users increases. In the following lemma, we find the relationship between (25) and the number of total users (i.e., N).

**Lemma 4.** When there are N users, the expectation of the smallest interference alignment measure is upper bounded on

$$\mathbb{E}\left[\min_{n} \ \mathfrak{q}\left(\tilde{\mathbf{h}}_{n,2},\ldots,\tilde{\mathbf{h}}_{n,K}\right)\right] < N^{-\frac{1}{K-N_{r}}}.$$
(26)

Proof: The complementary CDF of (25) is bounded on

$$\Pr\left[\min_{n} \mathfrak{q}(\tilde{\mathbf{h}}_{n,2}, \dots, \tilde{\mathbf{h}}_{n,K}) \geq \lambda\right] = \Pr\left[\mathfrak{q}(\tilde{\mathbf{h}}_{n,2}, \dots, \tilde{\mathbf{h}}_{n,K}) \geq \lambda \text{ for all } n\right]$$
$$= \prod_{n=1}^{N} \Pr\left[\mathfrak{q}(\tilde{\mathbf{h}}_{n,2}, \dots, \tilde{\mathbf{h}}_{n,K}) \geq \lambda\right]$$
$$= \left(1 - \Pr\left[\mathfrak{q}(\tilde{\mathbf{h}}_{n,2}, \dots, \tilde{\mathbf{h}}_{n,K}) \leq \lambda\right]\right)^{N}$$
$$\stackrel{(a)}{\leq} \left[1 - (1 - (1 - \lambda)^{N_{r} - 1})^{K - N_{r}}\right]^{N}, \tag{27}$$

where  $\lambda \in [0, 1]$ , and the inequality (a) holds from Lemma 3. Using this bound, we obtain (26) as

$$\mathbb{E}\left[\min_{n} \mathfrak{q}\left(\tilde{\mathbf{h}}_{n,2},\ldots,\tilde{\mathbf{h}}_{n,K}\right)\right] = \int_{0}^{1} \Pr\left[\min_{n} \mathfrak{q}\left(\tilde{\mathbf{h}}_{n,2},\ldots,\tilde{\mathbf{h}}_{n,K}\right) \geq \lambda\right] d\lambda$$

$$\leq \int_{0}^{1} \left[1 - (1 - (1 - \lambda))^{N_{r}-1}\right]^{N} d\lambda$$

$$\stackrel{(a)}{\leq} \int_{0}^{1} \left[1 - (1 - (1 - \lambda))^{K-N_{r}}\right]^{N} d\lambda$$

$$\stackrel{(b)}{=} \frac{1}{K - N_{r}} \beta\left(\frac{1}{K - N_{r}}, N + 1\right)$$

$$\stackrel{(c)}{=} \frac{\Gamma\left(1 + \frac{1}{K - N_{r}}\right) \Gamma(N + 1)}{\Gamma\left(N + 1 + \frac{1}{K - N_{r}}\right)}$$

$$\stackrel{(d)}{\leq} N^{-\frac{1}{K - N_{r}}},$$

where the inequality (a) is due to  $(1 - \lambda)^{N_r - 1} \le (1 - \lambda)$  for  $0 \le \lambda \le 1$ , and the equality (b) holds from the representation of beta function [24, p.324]

$$\int_0^1 x^{p-1} (1-x^q)^{r-1} dx = \frac{1}{q} \beta\left(\frac{p}{q}, r\right).$$

The equality (c) comes from the definition of the beta function  $\beta(p,q) = \Gamma(p)\Gamma(q)/\Gamma(p+q)$  and the property of the Gamma function  $\Gamma(p+1) = p\Gamma(p)$ . In the right-hand-side of the equality (c), it holds  $\Gamma(1 + \frac{1}{K-N_r}) < 1$  because  $0 < \Gamma(x) < 1$  for 1 < x < 2. Also, it is satisfied that

$$\frac{\Gamma(N+1)}{\Gamma\left(N+1+\frac{1}{K-N_r}\right)} \stackrel{(e)}{<} \left(N+1+\frac{1}{K-N_r}\right)^{-\frac{1}{K-N_r}}$$
$$< N^{-\frac{1}{K-N_r}},$$

where (e) is from the Gautschi's inequality [25] given by

$$\frac{\Gamma(x+s)}{\Gamma(x+1)} < (x+1)^{s-1}, \quad \text{for} \ x > 0, \quad 0 < s < 1,$$

with  $x = N + \frac{1}{K - N_r}$  and  $s = 1 - \frac{1}{K - N_r}$ . Thus, the inequality (d) holds.

# IV. OPTIMAL EXPLOITATION OF MULTIUSER DIVERSITY FOR THE TARGET DOF

In this section, we derive the optimal strategies of exploiting multiuser diversity for the target DoF d. We first decompose the target DoF d into the DoF gain term  $d_1$  and the DoF loss term  $d_2$  such that  $d = d_1 - d_2$ , and find the required user scalings for  $d_1$  and  $d_2$ , respectively. Then, the optimal target

DoF achieving strategy is derived by determining the optimal combination  $(d_1^{\star}, d_2^{\star})$  which requires the minimum user scaling for the target DoF d.

# A. Required User Scaling to Reduce the DoF Loss Term

In this subsection, we find the required user scaling to reduce the DoF loss. Via user selection, the rate loss term given in (4) can be minimized by

$$\mathbb{E}\left[\min_{n,\mathbf{v}_n} \log_2\left(1+P\sum_{k=2}^K |\mathbf{v}_n^{\dagger}\mathbf{h}_{n,k}|^2\right)\right].$$
(28)

This value is upper bounded on

$$\mathbb{E}\left[\min_{n,\mathbf{v}_{n}} \log_{2}\left(1+P\sum_{k=2}^{K}|\mathbf{v}_{n}^{\dagger}\mathbf{h}_{n,k}|^{2}\right)\right] \stackrel{(a)}{=} \mathbb{E}_{\|\mathbf{h}\|,\tilde{\mathbf{h}}}\left[\min_{n,\mathbf{v}_{n}} \log_{2}\left(1+P\sum_{k=2}^{K}\|\mathbf{h}_{n,k}\|^{2}|\mathbf{v}_{n}^{\dagger}\tilde{\mathbf{h}}_{n,k}|^{2}\right)\right] \\ \stackrel{(b)}{\leq} \mathbb{E}_{\tilde{\mathbf{h}}}\left[\min_{n,\mathbf{v}_{n}} \mathbb{E}_{\|\mathbf{h}\|}\log_{2}\left(1+P\sum_{k=2}^{K}\|\mathbf{h}_{n,k}\|^{2}|\mathbf{v}_{n}^{\dagger}\tilde{\mathbf{h}}_{n,k}|^{2}\right)\right] \\ \stackrel{(c)}{\leq} \mathbb{E}_{\tilde{\mathbf{h}}}\left[\min_{n,\mathbf{v}_{n}} \log_{2}\left(1+N_{r}P\sum_{k=2}^{K}|\mathbf{v}_{n}^{\dagger}\tilde{\mathbf{h}}_{n,k}|^{2}\right)\right] \\ \stackrel{(d)}{\leq} \mathbb{E}_{\tilde{\mathbf{h}}}\left[\min_{n} \log_{2}\left(1+N_{r}P(K-1)\mathfrak{q}(\tilde{\mathbf{h}}_{n,2},\ldots,\tilde{\mathbf{h}}_{n,K})\right)\right] \\ \stackrel{(e)}{\leq} \log_{2}\left(1+N_{r}P(K-1)\mathbb{E}_{\tilde{\mathbf{h}}}\left[\min_{n}\mathfrak{q}(\tilde{\mathbf{h}}_{n,2},\ldots,\tilde{\mathbf{h}}_{n,K})\right]\right) \\ \stackrel{(f)}{\leq} \log_{2}\left(1+N_{r}P(K-1)N^{-\frac{1}{K-N_{R}}}\right), \tag{29}$$

where the equality (a) is obtained by decomposing the channel vector into direction and magnitude independent of each other such that  $\mathbf{h}_{n,k} = \|\mathbf{h}_{n,k}\| \tilde{\mathbf{h}}_{n,k}$ . The inequality (b) holds because the minimum of the average is larger than the average of the minimum. The inequality (c) is from the Jensen's inequality and  $\mathbb{E} \|\mathbf{h}_{n,k}\|^2 = N_r$ . Also, the inequality (d) holds from the fact that

$$\min_{\mathbf{v}_{n}} \left[ \sum_{k=2}^{K} |\mathbf{v}_{n}^{\dagger} \tilde{\mathbf{h}}_{n,k}|^{2} \right] \leq \min_{\mathbf{v}_{n}} \left[ (K-1) \max_{2 \leq k \leq K} |\mathbf{v}_{n}^{\dagger} \tilde{\mathbf{h}}_{n,k}|^{2} \right] \\
= (K-1) \mathfrak{q}(\tilde{\mathbf{h}}_{n,2}, \dots, \tilde{\mathbf{h}}_{n,K}),$$
(30)

where  $q(\tilde{\mathbf{h}}_{n,2},\ldots,\tilde{\mathbf{h}}_{n,K})$  is the interference alignment measure at the user *n* given in (19). The inequality (e) is from the Jensen's inequality, and the inequality (f) holds from Lemma 4. We obtain the following theorem.

**Theorem 1.** We can obtain the DoF loss term  $d_2 \in [0, 1]$  when the number of users in each group scales as

$$N \propto P^{(1-d_2)(K-N_r)}$$

*Proof:* To obtain the DoF loss term  $d_2$ , it is enough to make (29) satisfying

$$\lim_{P \to \infty} \frac{\log_2 \left( 1 + N_r P(K-1) N^{-\frac{1}{K-N_R}} \right)}{\log_2 P} = d_2,$$
(31)

which is achieved if  $N \propto P^{(1-d_2)(K-N_r)}$ .

A tighter upper bound of the rate loss term than (29) could exist, but the derived upper bound in (29) enables us to compare the increasing speeds of the transmit power and the required number of users, which is the crucial factor of DoF calculation. The scaling law of the required number of users obtained from (29), which is derived in Theorem 1, is enough to find the optimal target DoF achieving strategy as shown in Section IV-C.

## B. Required User Scaling to Increase the DoF Gain Term

We also find the required user scaling to increase the DoF gain term. From the definition of the rate gain term given in (3), the maximum rate gain term obtained by user selection is

$$\mathbb{E}\left[\max_{n,\mathbf{v}_n}\log_2\left(1+P\sum_{k=1}^K |\mathbf{v}_n^{\dagger}\mathbf{h}_{n,k}|^2\right)\right].$$
(32)

This value is lower bounded on

$$\mathbb{E}\left[\max_{n,\mathbf{v}_{n}}\log_{2}\left(1+P\sum_{k=1}^{K}|\mathbf{v}_{n}^{\dagger}\mathbf{h}_{n,k}|^{2}\right)\right] \geq \mathbb{E}\left[\max_{n,\mathbf{v}_{n}}\log_{2}\left(1+P|\mathbf{v}_{n}^{\dagger}\mathbf{h}_{n,1}|^{2}\right)\right]$$
$$=\mathbb{E}\left[\max_{n}\log_{2}\left(1+P\|\mathbf{h}_{n,1}\|^{2}\right)\right],$$
(33)

and upper bounded on

$$\mathbb{E}\left[\max_{n,\mathbf{v}_{n}}\log_{2}\left(1+P\sum_{k=1}^{K}|\mathbf{v}_{n}^{\dagger}\mathbf{h}_{n,k}|^{2}\right)\right] \leq \mathbb{E}\left[\max_{n}\log_{2}\left(1+P\sum_{k=1}^{K}\|\mathbf{h}_{n,k}\|^{2}\right)\right]$$
$$\leq \mathbb{E}\left[\max_{n}\log_{2}\left(1+PK\max_{k}\|\mathbf{h}_{n,k}\|^{2}\right)\right].$$
(34)

Since all  $\|\mathbf{h}_{n,k}\|^2$  are i.i.d.  $\chi^2(2N_r)$  random variables, for sufficiently large N, the bounds (33) and (34) acts like [23]

$$\mathbb{E}\left[\max_{n} \log_2\left(1+P\|\mathbf{h}_{n,1}\|^2\right)\right] \sim \log_2(1+P\log N)$$
$$\mathbb{E}\left[\max_{n,k} \log_2\left(1+PK\|\mathbf{h}_{n,k}\|^2\right)\right] \sim \log_2(1+PK\log(KN)).$$

Thus, when both N and P are large enough, (32) act like  $\log_2(P \log N)$ , i.e,

$$\lim_{\substack{P \to \infty \\ N \to \infty}} \mathbb{E}\left[\max_{n, \mathbf{v}_n} \log_2\left(1 + P\sum_{k=1}^K |\mathbf{v}_n^{\dagger} \mathbf{h}_{n,k}|^2\right)\right] \sim \log_2(P \log N).$$
(35)

Therefore, we establish the following theorem.

**Theorem 2.** The DoF gain term  $d_1 \in [1, \infty)$  is achievable when the number of users in each group scales as

$$N \propto e^{P^{(d_1-1)}}.$$

Proof: We use (35). By setting

$$\lim_{P \to \infty} \frac{\log_2(P \log N)}{\log_2 P} = d_1,\tag{36}$$

we obtain the required user scaling for the DoF gain term  $d_1$  given by  $N \propto e^{P^{(d_1-1)}}$ .

# C. Target DoF Achieving Strategy

In Theorem 1 and Theorem 2, we found the required user scalings for the DoF loss term  $d_2$  and the DoF gain term  $d_1$ , respectively. In this subsection, we find the optimal target DoF achieving strategy which requires the minimum user scaling. We start with the following theorem.

**Theorem 3.** For the target DoF up to one, the whole multiuser dimensions should be devoted to minimizing the DoF loss caused by interfering signals. The optimal DoF achieving strategy for the target DoF  $d \in [0, 1]$  is  $(d_1^*, d_2^*) = (1, 1 - d)$ , and the corresponding sufficient user scaling is

$$N \propto P^{d(K-N_r)}.$$
(37)

*Proof:* In Theorem 1, we have shown that the target DoF  $d \in [0, 1]$  is achievable by reducing the DoF loss term with the user scaling  $N \propto P^{d(K-N_r)}$ . On the other hand, this user scaling cannot increase

the DoF gain term. Substituting  $N \propto P^{d(K-N_r)}$  into (35) the DoF gain term  $d_1$  becomes

$$\lim_{P \to \infty} \frac{\log_2 \left( P \log(P^{d(K-N_r)}) \right)}{\log_2 P} = 1.$$
(38)

which is the same as when there is a fixed number of users as described in (7). That is, any other combinations  $(d_1, d_2) = (1 + \Delta, 1 - d + \Delta)$  which achieve the target DoF d requires larger user scaling than  $N \propto P^{d(K-N_r)}$ , where  $\Delta > 0$  since  $d_1 > 1$ . Therefore, the optimal target DoF achieving strategy is given by  $(d_1^*, d_2^*) = (1, 1 - d)$ , and the sufficient user scaling is  $N \propto P^{d(K-N_r)}$ .

Now, we derive the target DoF achieving strategy when the target DoF d is greater than one. To find the optimal DoF achieving strategy, we firstly find the sufficient user scaling for an arbitrary strategy  $(d_1, d_2)$  achieving DoF d (=  $d_1 - d_2 > 1$ ). Then, we show that the optimal target DoF achieving strategy for the target DoF d (> 1) is  $(d_1^*, d_2^*) = (d, 0)$ .

**Lemma 5.** For the target DoF d (> 1), the sufficient user scaling for an arbitrary strategy  $(d_1, d_2)$ achieving DoF d (=  $d_1 - d_2$ ) is given by

$$N \propto e^{P^{(d_1 - 1)}} P^{(1 - d_2)(K - N_r)},\tag{39}$$

where  $d_1 > 1$  and  $d_2 \in [0, 1]$ .

*Proof:* As a target DoF achieving scheme, we consider a two-stage user selection scheme; the first stage is to increase the DoF gain term, and the second stage is to decrease the DoF loss term. The considered two stage user selection strategy is illustrated in Fig. 4. We randomly divide total N users into  $N_2$  subgroups having  $N_1$  users each such that  $N_1N_2 = N$ . Then, the user selection in each stage is performed as follows.

- Stage 1: In each subgroup, a single user having the largest channel gain is selected among  $N_1$  users. As a result, we have  $N_2$  selected users after Stage 1.
- Stage 2: Among the  $N_2$  users, the transmitter selects a single user to minimize the DoF loss term. In Stage 1, the DoF gain term  $d_1$  is obtained at each selected user when

$$N_1 \propto e^{P^{(d_1-1)}}$$
 (40)

as stated in Theorem 2. In Stage 2, we can make the DoF loss term  $d_2$  when

$$N_2 \propto P^{(1-d_2)(K-N_r)}$$
 (41)

as shown in Theorem 1. Thus, the target DoF d (> 1) with the strategy  $(d_1, d_2)$  such that  $d = d_1 - d_2$  can be obtained by the user scaling  $N_1N_2$ , which is given in (39).

From Lemma 5, we obtain the optimal DoF achieving strategy for the target DoF d (> 1) in following theorem.

**Theorem 4.** The optimal target DoF achieving strategy for  $d \in [1, \infty)$  is to increase the DoF gain term to d and to perfectly eliminate the DoF loss, i.e.,  $(d_1^*, d_2^*) = (d, 0)$ . Consequently, the sufficient user scaling for target DoF d (> 1) becomes

$$N \propto e^{P^{(d-1)}} P^{(K-N_r)}.$$
 (42)

*Proof:* The proof is similar to that of Theorem 3. From Lemma 5, we can obtain the target DoF d (> 1) by the strategy (d, 0) with the sufficient user scaling given in (42). However, this scaling cannot increase the DoF gain term larger than d even when the user scaling is only used to increase the DoF gain term. Substituting (42) into (35), we still have

$$\lim_{P \to \infty} \frac{\log_2 \left( P \log(e^{P^{(d-1)}} P^{(K-N_r)}) \right)}{\log_2 P} = d.$$
(43)

This implicates that the user scaling given in (42) is sufficient for the strategy (d, 0) but not enough for other strategies  $(d + \Delta, 1)$  as well as  $(d + \Delta, \Delta)$  which requires the higher user scaling than that of  $(d + \Delta, 1)$ , where  $\Delta \in (0, 1]$ . Therefore, the optimal strategy for the target DoF d (> 1) becomes  $(d_1^{\star}, d_2^{\star}) = (d, 0)$ .

In Fig. 5, the optimal DoF achieving strategy  $(d_1^{\star}, d_2^{\star})$  is plotted according to the target DoF  $d (= d_1 - d_2)$ .

## V. PRACTICAL USER SELECTION SCHEMES

In this section, we discuss how the optimal target DoF achieving strategy can be realized by practical user selection schemes. The practical schemes considered in this section require no cooperation and no information exchange among the transmitters. For practical scenarios, we assume that each user has knowledge of channel state information (CSI) of the direct channel and the covariance matrix of the received signal without explicit knowledge of CSI of the interfering channels. That is, the *n*th user knows CSI of its own desired channel  $\mathbf{h}_{n,1}$  and the covariance matrix of the received signal  $\mathbb{E}[\mathbf{y}_n \mathbf{y}_n^{\dagger}] =$ 

 $\mathbf{I}_{N_r} + P \sum_{k=1}^{K} \mathbf{h}_{n,k} \mathbf{h}_{n,k}^{\dagger}$ . From these values, the user *n* easily obtains the interference covariance matrix denoted by  $\mathbf{R}_n \triangleq P \sum_{k=2}^{K} \mathbf{h}_{n,k} \mathbf{h}_{n,k}^{\dagger}$  such as

$$\mathbf{R}_{n} = \mathbb{E}[\mathbf{y}_{n}\mathbf{y}_{n}^{\dagger}] - P\mathbf{h}_{n,1}\mathbf{h}_{n,1}^{\dagger} - \mathbf{I}_{N_{r}}.$$
(44)

Therefore, the achievable rate at the first transmitter given in (2) can be rewritten by

$$\mathcal{R} \triangleq \mathbb{E} \log_2 \left( 1 + rac{P |\mathbf{v}_{n^*}^{\dagger} \mathbf{h}_{n^*,1}|^2}{\mathbf{v}_{n^*}^{\dagger} (\mathbf{I}_{N_r} + \mathbf{R}_{n^*}) \mathbf{v}_{n^*}} 
ight).$$

To increase  $\mathcal{R}$ , various user selection schemes can be considered, but we focus on several popular techniques in the following subsections – to maximize the postprocessed SNR (i.e.,  $P|\mathbf{v}_{n^*}^{\dagger}\mathbf{h}_{n^*,1}|^2$ ), to minimize the postprocessed INR (i.e.,  $\mathbf{v}_{n^*}^{\dagger}\mathbf{R}_{n^*}\mathbf{v}_{n^*}$ ), and to maximize the postprocessed SINR (i.e.,  $\frac{P|\mathbf{v}_{n^*}^{\dagger}\mathbf{h}_{n^*,1}|^2}{\mathbf{v}_{n^*}^{\dagger}(\mathbf{I}_{N_r}+\mathbf{R}_{n^*})\mathbf{v}_{n^*}}$ ).

## A. The Maximum Postprocessed SNR User Selection (MAX-SNR)

In the MAX-SNR user selection scheme, each user maximizes the postprocessed SNR, and the transmitter selects the user having the maximum postprocessed SNR. Consequently, the postprocessed SNR at the selected user becomes

$$\max_{n} \left[ \max_{\mathbf{v}_{n}} P |\mathbf{v}_{n}^{\dagger} \mathbf{h}_{n,1}|^{2} \right] \stackrel{(a)}{=} \max_{n} P ||\mathbf{h}_{n,1}||^{2},$$
(45)

where the equality (a) holds when the *n*th user adopts the postprocessing vector  $\mathbf{v}_n^{\text{snr}} = \mathbf{h}_{n,1}/||\mathbf{h}_{n,1}||$ . Thus, the selected user denoted by  $n_{\text{snr}}^*$  becomes

$$n_{\mathsf{snr}}^* = \underset{n}{\operatorname{arg\,max}} \ P \|\mathbf{h}_{n,1}\|^2,\tag{46}$$

and the desired channel gain at each user ( $\|\mathbf{h}_{n,1}\|^2$  for the user *n*) should be informed to the transmitter.

Using the MAX-SNR scheme, the transmitter can only increase the DoF gain term while the DoF loss term remains one. Although the MAX-SNR scheme can achieve the target DoF d by the strategy  $(d_1, d_2) = (1 + d, 1)$ , it is not optimal target DoF achieving strategy. The required user scaling for the target DoF d (> 0) by the MAX-SNR scheme becomes

$$N \propto e^{P^{(1+d)}},$$

as shown in Theorem 2. This user scaling is of course higher than (37) for the target DoF  $d \in (0, 1]$ and (42) for the target DoF d (> 1) since the MAX-SNR does not realize the optimal target achieving strategy. In other words, one can easily find that

$$\lim_{P \to \infty} \left[ e^{P^{(1+d)}} / P^{d(K-N_r)} \right] = \infty \quad \text{for } d \in (0,1],$$
$$\lim_{P \to \infty} \left[ e^{P^{(1+d)}} / \left( e^{P^{(d-1)}} P^{(K-N_r)} \right) \right] = \infty \quad \text{for } d > 1.$$

## B. The Minimum Postprocessed INR User Selection (MIN-INR)

In the MIN-INR user selection scheme, each user minimizes the postprocessed INR, and the transmitter selects the user having the minimum postprocessed INR. Thus, the postprocessed INR at the selected user becomes

$$\min_{n} \left[ \min_{\mathbf{v}_{n}} \mathbf{v}_{n}^{\dagger} \mathbf{R}_{n} \mathbf{v}_{n} \right] \stackrel{(a)}{=} \min_{n} \left[ \Lambda_{\min} \left( \mathbf{R}_{n} \right) \right], \tag{47}$$

where the equality (a) is obtained by the postprocessing vector of the *n*th user

$$\mathbf{v}_{n}^{\mathsf{inr}} = V_{\min}\left(\mathbf{R}_{n}\right). \tag{48}$$

The required feedback information from the *n*th user is  $\Lambda_{\min}(\mathbf{R}_n)$ , and index of the selected user denoted by  $n_{\inf}^*$  becomes

$$n_{\inf}^{*} = \arg\min_{n} \left[ \Lambda_{\min} \left( \mathbf{R}_{n} \right) \right].$$
(49)

Note that this scheme minimizes the rate loss term defined in (4).

Using the MIN-INR scheme, the transmitter can decrease the DoF loss term while the DoF gain term remains to be one. Therefore, the MIN-INR scheme realizes the optimal target DoF achieving strategy  $(d_1, d_2) = (1, 1 - d)$  for the target DoF  $d \in [0, 1]$ . The required number of users by the MIN-INR scheme for the target DoF  $d \in [0, 1]$  scales like

$$N \propto P^{d(K-N_r)}$$
.

which is the required user scaling of the optimal target DoF achieving strategy when the target DoF is  $d \in [0, 1]$  as shown in Theorem 3.

The MAX-SINR user selection scheme is known to maximize the achievable rate at the transmitter although it requires additional complexity for postprocessing at the receivers. The achievable rate by the MAX-SINR scheme denoted by  $\mathcal{R}_{sinr}$  becomes

$$\mathcal{R}_{\text{sinr}} \triangleq \mathbb{E}\left[\max_{n, \mathbf{v}_n} \log_2\left(1 + \frac{P|\mathbf{v}_n^{\dagger}\mathbf{h}_{n,1}|^2}{\mathbf{v}_n^{\dagger}(\mathbf{I}_{N_r} + \mathbf{R}_n)\mathbf{v}_n}\right)\right].$$
(50)

At each channel realization, the postprocessed SINR at the selected user is given by

$$\max_{n} \left[ \max_{\mathbf{v}_{n}} \frac{P |\mathbf{v}_{n}^{\dagger} \mathbf{h}_{n,1}|^{2}}{\mathbf{v}_{n}^{\dagger} (\mathbf{I}_{N_{r}} + \mathbf{R}_{n}) \mathbf{v}_{n}} \right].$$
(51)

To maximize the postprocessed SINR, the *n*th user adopts the postprocessing vector given by

$$\mathbf{v}_n^{\mathsf{sinr}} = \frac{(\mathbf{I}_{N_r} + \mathbf{R}_n)^{-1} \mathbf{h}_{n,1}}{\|(\mathbf{I}_{N_r} + \mathbf{R}_n)^{-1} \mathbf{h}_{n,1}\|}$$

which is identical with the MMSE-IRC in [26]. The corresponding postprocessed SINR at user n becomes  $P\mathbf{h}_{n,1}^{\dagger}(\mathbf{I}_{N_r} + \mathbf{R}_n)^{-1}\mathbf{h}_{n,1}$  [27], and hence the selected user at the transmitter denoted by  $n_{sinr}^*$  is given by

$$n_{\mathsf{sinr}}^* = \arg\max_n \, \mathbf{h}_{n,1}^{\dagger} (\mathbf{I}_{N_r} + \mathbf{R}_n)^{-1} \mathbf{h}_{n,1}.$$
(52)

**Lemma 6.** To obtain the target  $DoF \ d \in [0, 1]$ , the required user scaling of the MAX-SINR scheme is exactly the same as that of the MIN-INR scheme.

*Proof:* From the fact that

$$\mathcal{R}_{\rm inr} \le \mathcal{R}_{\rm sinr} \le \mathcal{R}_{\rm sinr}^+ - \mathcal{R}_{\rm inr}^-,\tag{53}$$

we obtain

$$\lim_{P \to \infty} \frac{\mathcal{R}_{\text{inr}}}{\log_2 P} \le \lim_{P \to \infty} \frac{\mathcal{R}_{\text{sinr}}}{\log_2 P} \le \lim_{P \to \infty} \frac{\mathcal{R}_{\text{sinr}}^+}{\log_2 P} - \lim_{P \to \infty} \frac{\mathcal{R}_{\text{inr}}^-}{\log_2 P} \stackrel{(a)}{=} \lim_{P \to \infty} \frac{\mathcal{R}_{\text{inr}}}{\log_2 P},$$
(54)

where the equality (a) is because  $\lim_{P \to \infty} \frac{\mathcal{R}_{sinr}}{\log_2 P} = 1$  as shown in the proof of Theorem 3. Therefore, the required user scaling for  $\lim_{P \to \infty} \frac{\mathcal{R}_{sinr}}{\log_2 P} = d$  is exactly the same as the required user scaling for  $\lim_{P \to \infty} \frac{\mathcal{R}_{inr}}{\log_2 P} = d$ , equivalently, for  $\lim_{P \to \infty} \frac{\mathcal{R}_{inr}}{\log_2 P} = 1 - d$ .

Lemma 6 indicates that the MAX-SINR scheme realizes the optimal DoF achieving strategy  $(d_1^{\star}, d_2^{\star}) = (1, 1 - d)$  for the target DoF  $d \in [0, 1]$ .

**Lemma 7.** The MAX-SINR scheme realizes the DoF achieving strategy  $(d_1^{\star}, d_2^{\star}) = (d, 0)$  whenever the target DoF d is greater than 1.

*Proof:* Since the MAX-SINR scheme is the optimal user selection scheme, it achieves DoF d (> 1) with the user scaling  $N \propto e^{P^{(d-1)}} P^{(K-N_r)}$  as stated in Theorem 4. From the definition of (32), we obtain  $\mathcal{R}_{sinr}^+ < (32)$ , and hence we have  $\lim_{P \to \infty} \frac{\mathcal{R}_{sinr}^+}{\log_2 P} \leq \lim_{P \to \infty} \frac{(32)}{\log_2 P}$ . As shown in (43), the sufficient user scaling for the MAX-SINR scheme to obtain the target DoF d cannot increase the DoF gain term larger than d even if the whole user scaling is only devoted to increasing the DoF gain term. This implicates that when we obtain the target DoF d (> 1) by the MAX-SINR scheme with the user scaling  $N \propto e^{P^{(d-1)}} P^{(K-N_r)}$ , we obtain  $\lim_{P \to \infty} \frac{\mathcal{R}_{sinr}^-}{\log_2 P} = 0$  and have the DoF gain d at most (i.e.,  $\lim_{P \to \infty} \frac{\mathcal{R}_{sinr}^+}{\log_2 P} = d$ ). Therefore, the MAX-SINR scheme can only have  $\left(\lim_{P \to \infty} \frac{\mathcal{R}_{sinr}^-}{\log_2 P}, \lim_{P \to \infty} \frac{\mathcal{R}_{sinr}^-}{\log_2 P}\right) = (d, 0)$  if  $\lim_{P \to \infty} \frac{\mathcal{R}_{sinr}}{\log_2 P} = d (> 1)$ .

## D. Two-stage User Selection Scheme

For the target DoF d (> 1), the two-stage user selection scheme described in the proof of Lemma 5 can be adopted. More specifically, the transmitter selects the users by the MAX-SNR scheme in the first stage. Then, in the second stage, the transmitter selects a single user by the MIN-INR scheme or the MAX-SINR scheme. As shown in the proof of Lemma 5, the two-stage user selection scheme can realize the optimal target DoF achieving strategy for the target DoF d (> 1).

## VI. EXTENSION TO K-TRANSMITTER INTERFERING MIMO BROADCAST CHANNELS

In this section, we extend our system to interfering MIMO BC cases. More specifically, each transmitter with  $N_t$  antennas sends  $N_t$  independent streams over  $N_t$  orthonormal random beams using equal power allocation. Similar to the user selection procedure in [7], each transmitter broadcasts  $N_t$  orthonormal random beams, and each user feeds  $N_t$  scalar values corresponding to all beams back to the transmitter. The feedback information corresponding to each stream such as SNR, INR, and SINR can be easily found in a similar way to the SIMO case. A single user is selected for each beam, but the same user can be selected for different beams. However, it rarely occurs that multiple streams are transmitted for a single user as the number of users increases. When multiple streams are transmitted for a single user, the user is assumed to decode each stream treating the other streams as interferences. We denote the orthonormal random beams by  $\mathbf{u}_1, \ldots, \mathbf{u}_{N_t}$  which satisfies that  $\|\mathbf{u}_1\|^2 = \cdots = \|\mathbf{u}_{N_t}\|^2 = 1$  and  $\mathbf{u}_i^{\dagger} \mathbf{u}_j = 0$  for all  $i \neq j$ . We start from the following remark.

**Remark 1.** Let  $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$  be the channel matrix whose elements are i.i.d. Gaussian random variables. Then, for an arbitrary unitary matrix  $\mathbf{U} \in \mathbb{C}^{N_t \times N_t}$  (i.e.,  $\mathbf{U}^{\dagger}\mathbf{U} = \mathbf{U}\mathbf{U}^{\dagger} = \mathbf{I}_{N_t}$ ), the distributions of  $\mathbf{H}$  and  $\mathbf{H}\mathbf{U}$  are identical. Since the  $N_r$  columns of  $\mathbf{H}$  are independent and isotropic random vectors in  $\mathbb{C}^{N_r}$ , so are the  $N_r$  columns of  $\mathbf{H}\mathbf{U} \in \mathbb{C}^{N_r \times N_t}$ .

Owing to Remark 1, the K-transmitter MIMO interfering BC is statistically identical with the  $KN_t$ transmitter SIMO interfering BC. Let  $\mathbf{H}_{k,n} \in \mathbb{C}^{N_r \times N_t}$  be the channel matrix from the kth transmitter to
the *n*th user in the first user group. Since the random beams satisfy that  $[\mathbf{u}_1, \ldots, \mathbf{u}_{N_t}]^{\dagger}[\mathbf{u}_1, \ldots, \mathbf{u}_{N_t}] =$   $[\mathbf{u}_1, \ldots, \mathbf{u}_{N_t}][\mathbf{u}_1, \ldots, \mathbf{u}_{N_t}]^{\dagger} = \mathbf{I}_{N_t}$ , the user *n* in the first user group has  $KN_t$  independent and isotropic
channel vectors

$$\mathbf{H}_{k,n}\mathbf{u}_i \in \mathbb{C}^{N_r \times 1} \qquad k \in [K] \quad i \in [N_t]$$

formed by the random beams and channel matrices from all transmitters.

If the *n*th user in the first group is served by the *i*th random beam, the user has desired channel  $\mathbf{H}_{n,1}\mathbf{u}_i \in \mathbb{C}^{N_t \times 1}$  and the  $(KN_t - 1)$  interfering channels, which correspond to  $(N_t - 1)$  inter-stream interfering channels  $\{\mathbf{H}_{n,1}\mathbf{u}_j\}_{j \neq i}$  and  $(K - 1)N_t$  inter-transmitter interfering channels

$$\bigcup_{j \in [N_t]} \{ \mathbf{H}_{n,k} \mathbf{u}_j \}_{k=2}^K$$

Consequently, each random beam can be regarded as a single antenna transmitter with the transmit power  $P/N_t$ . This fact leads to the following theorems.

**Theorem 5** (MIMO BC). In a MIMO BC where a transmitter with  $N_t$  antennas supports  $N_t$  users among N users with  $N_r(< N_t)$  antennas each, the optimal DoF achieving strategy for the target DoF  $d \ (\in [0, N_t])$  is  $(N_t, N_t - d)$  and requires the number of users to scale as  $N \propto P^{(d/N_t)(N_t - N_r)}$ . For the target DoF  $d \ (> N_t)$ , the optimal DoF achieving strategy is (d, 0) and requires the number of users to scale as  $N \propto e^{P^{(d/N_t-1)}}P^{(N_t-N_r)}$ .

*Proof:* The DoF gain term  $d_1(>N_t)$  is obtained when each stream achieves DoF gain term  $d_1/N_t$  (> 1). Thus, with the same procedure given in Section IV-B, we can easily show that the DoF gain term

 $d_1(>N_t)$  is obtained when  $N \propto e^{P^{(d_1/N_t-1)}}$ . On the other hand, the DoF loss term  $d_2 (\le N_t)$  is obtained when each stream achieves DoF loss term per stream  $d_2/N_t$  ( $\le 1$ ). As stated earlier, using  $N_t$  orthonormal random beams at the transmitter each of which independently supports a single user, the MIMO BC can be translated into an  $N_t$ -transmitter SIMO IC where each transmitter supports one of N users with  $N_r$ antennas. In this case, DoF loss term  $d_2$  ( $\le N_t$ ), i.e., DoF loss  $d_2/N_t$  ( $\le 1$ ) per stream, is obtained when  $N \propto P^{(1-d_2/N_t)(N_t-N_r)}$ . Therefore, we can conclude that the optimal DoF achieving strategy for the target DoF d ( $\in [0, N_t]$ ) is ( $N_t, N_t - d$ ) and requires the number of users to scale as  $N \propto P^{(d/N_t)(N_t-N_r)}$ . Also, for the target DoF d ( $> N_t$ ), the optimal DoF achieving strategy is (d, 0) and requires the number of users to scale as  $N \propto e^{P^{(d/N_t-1)}}P^{(N_t-N_r)}$ .

**Theorem 6** (Interfering MIMO BC). Consider a K-transmitter interfering MIMO BC where the kth transmitter with  $N_t^{(k)}$  antennas supports  $N_t^{(k)}$  users among  $N^{(k)}$  users with  $N_r^{(k)}(< T \triangleq \sum_{k=1}^K N_t^{(k)})$ antennas each. At the kth transmitter, the optimal DoF achieving strategy for the target DoF  $d (\in [0, N_t^{(k)}])$ is  $(N_t^{(k)}, N_t^{(k)} - d)$  and requires the number of users to scale as  $N \propto P^{(d/N_t^{(k)})(T-N_r^{(k)})}$ . For the target DoF  $d (> N_t)$ , the optimal DoF achieving strategy becomes (d, 0) and requires the number of users to scale as  $N^{(k)} \propto e^{P^{(d/N_t^{(k)}-1)}}P^{(T-N_r^{(k)})}$ .

Proof: The proof is similar to that of Theorem 5. The kth transmitter obtains DoF gain term  $d_1$  (>  $N_t^{(k)}$ ) when each stream obtains DoF gain term  $d_1/N_t^{(k)}(>1)$ , and the required user scaling is exactly given by  $N^{(k)} \propto e^{P^{(d_1/N_t^{(k)}-1)}}$ . On the other hand, the kth transmitter obtains  $d_2$  ( $\leq N_t^{(k)}$ ) when the DoF loss term per stream becomes  $d_2/N_t^{(k)}$  ( $\leq 1$ ). Using  $N_t^{(k)}$  orthonormal random beams at each transmitter each of which independently supports a single user, the interfering MIMO BC can be translated into an  $T(=\sum_{k=1}^{K} N_t^{(k)})$ -transmitter SIMO IC where each transmitter supports a single user among  $N^{(k)}$  users with  $N_r^{(k)}$  antennas. Thus, the kth transmitter obtains the DoF loss term  $d_2$  ( $\leq N_t^{(k)}$ ) when  $N^{(k)} \propto P^{(1-d_2/N_t^{(k)})(T-N_r^{(k)})}$ . Therefore, we can conclude that the optimal DoF achieving strategy of the kth transmitter for the target DoF d ( $\in [0, N_t^{(k)}]$ ) is  $(N_t^{(k)}, N_t^{(k)} - d)$  and obtained when the number of users scales as  $N^{(k)} \propto P^{(d/N_t^{(k)})(T-N_r^{(k)})}$ . Also, for the target DoF d ( $> N_t^{(k)}$ ), the optimal DoF achieving strategy is (d, 0) and requires the number of users to scale as  $N^{(k)} \propto e^{P^{(d/N_t^{(k)}-1)}}P^{(T-N_r^{(k)})}$ .

#### VII. NUMERICAL RESULTS

In this section, we first compare achievable rates of the practical user selection schemes for given number of users. Then, we check if the target DoF can be achievable with increasing number of users by showing achievable rates per transmitter for the practical user selection schemes. We have also considered two time division multiple access (TDMA) schemes. In the first TDMA scheme (TDMA1), a single transmitter operates at each time so that 1/K DoF is achieved at each transmitter. In the second TDMA scheme (TDMA2), only  $N_r$  of K transmitters operate at each time so that  $N_r/K$  DoF is achieved at each transmitter.

Fig. 6 shows the achievable rates of each transmitter for various user selection schemes in IBC when there are 4 transmitters and each transmitter has 10 users with three receive antennas each. It is confirmed that the achievable rates are saturated in the high SNR region and the achievable DoF per transmitter becomes zero for the fixed number of users.

Now, we show that the target DoF can be achievable if the number of users properly scales. In Fig. 7, the number of users scales as  $N \propto P^{d(K-N_r)}$ , i.e.,  $N \propto P$  for the target DoF one. Specifically, two user scaling N = P and N = 0.5P are considered, and other configurations except the number of users are the same as those in Fig. 6. Fig. 7 verifies that the MIN-INR and the MAX-SINR schemes achieve DoF one per transmitter as predicted in Theorem 3 and Lemma 6.

In Fig. 8, we consider two different user scaling  $N = P^{0.5}$  and  $N = P^1$  from those in Fig. 7. According to Theorem 3 and Lemma 6, the achievable DoF at each transmitter by either the MAX-SINR scheme or the MIN-INR scheme is d when the number of users scales as  $N \propto P^d$ . As predicted, Fig. 8 shows that the achieved DoF per transmitter is 0.5 and 1 when  $N = P^{0.5}$  and  $N = P^1$ , respectively, by either the MIN-INR scheme or the MAX-SINR scheme.

#### VIII. CONCLUSIONS

We first studied the optimal way of exploiting multiuser diversity in the *K*-transmitter SIMO IBC where each transmitter with a single antenna selects a user and the number of transmitters is larger than the number of receive antennas at each user. We proved that the multiuser dimensions should be used first for decreasing the DoF loss caused by interfering signals; the whole multiuser dimensions should

be exploited to reduce the DoF loss term to 1 - d for the target DoF  $d \in [0, N_t]$ , while the multiuser dimensions should be devoted to making the DoF loss zero and then to increasing the DoF gain term to dfor the target DoF  $d \in [N_t, \infty)$ . We also derived the sufficient user scaling for the target DoF. The DoF per transmitter  $d \in [0, N_t]$  is obtained when the number of users scales as  $N \propto P^{(d/N_t)(KN_t-N_r)}$ , and the DoF per transmitter  $d \in [N_t, \infty)$  is achieved when the number of users scales as  $N \propto e^{P^{(d/N_t)(KN_t-N_r)}}$ . Also, we extended the results to the K-transmitter MIMO IBC where each transmitter having the multiple antennas supports the multiple users.

#### REFERENCES

- H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 1748–1759, May 2010.
- [2] D. Gesbert, S. Hanly, H. Huang, S. Shitz, O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE J. Sel. Areas Commun.*, vol. 28, pp. 1380–1408, Sep. 2010.
- [3] V. R. Cadambe and S. A. Jafar, "Interference alignment and the degree of freedom for the K user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, pp. 3425–3441, Aug. 2008.
- [4] T. Gou and S. A. Jafar, "Degrees of freedom of the K user M × N MIMO interference channel," *IEEE Trans. Inf. Theory*, vol. 56, no. 12, pp. 6040–6057, Dec. 2010.
- [5] K. Gomadam, V. R. Cadambe, S. A. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," in *Proc. of IEEE Global Telecommunications Conference*, pp. 1–6, Dec. 2008.
- [6] C. Suh and D. Tse, "Interference alignment for cellular networks," in Proc. of Allerton Conference on Communication, Control, and Computing, pp. 1037–1044, Sep. 2008.
- [7] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channels with partial side information," *IEEE Trans. Inf. Theory*, vol. 51, no. 2, pp. 506–522, Feb. 2005.
- [8] Y. Huang and B. Rao, "Random beamforming with heterogeneous users and selective feedback: individual sum rate and individual scaling laws", *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2080–2090, May 2013.
- [9] J. H. Lee and W. Choi, "Opportunistic interference aligned user selection in multi-user MIMO interference channels," in *Proc. of IEEE Global Telecommunications Conference*, Miami, FL, USA, Dec. 2010.
- [10] J. H. Lee and W. Choi, "Interference alignment by opportunistic user selection in 3-user MIMO interference channels," in Proc. of IEEE International Conference on Communications, Kyoto, Japan, June, 2011.
- [11] J. H. Lee and W. Choi, "On the achievable DoF and user scaling law of opportunistic interference alignment in 3-transmitter MIMO interference channels," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2743-2753, June 2013.
- [12] J. H. Lee and W. Choi, "Opportunistic interference alignment by receiver selection in a K-user 1 x 3 SIMO interference channel," in Proc. of IEEE Global Telecommunications Conference, Houston, TX, USA, Dec. 2011.

- [13] T. Kim, D. J. Love, and B. Clerckx, "Instantaneous degrees of freedom of downlink interference channels with multiuser diversity," in Proc. of Asilomar Conference on Signals, Systems, and Computers, pp.1293–1297, Nov. 2011.
- [14] B. C. Jung, D. Park, and W.-Y. Shin, "Opportunistic interference mitigation achieves optimal degrees-of-freedom in wireless multi-cell uplink networks," *IEEE Trans. Commun.*, vol. 60, no. 7, July 2012.
- [15] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory*, vol. 48, no.
  6, pp. 1277–1294, June 2002.
- [16] W. Choi, A. Forenza, J. G. Andrews, and R. W. Heath, "Opportunistic space division multiple access with beam selection," *IEEE Trans. Wireless Commun.*, vol. 6, no. 12, pp. 2371–2380, Dec. 2007.
- [17] W. Choi and J. G. Andrews, "The capacity gain from intercell scheduling in multi-antenna systems," *IEEE Trans. Wireless Commun.*, vol. 7, no. 2, pp. 714-725, Feb. 2008
- [18] J. Joung and A. H. Sayed, "User selection methods for multiuser two-way relay communications using space division multiple access," *IEEE Trans. Wireless Commun.*, vol. 9, no. 7, pp. 2130–2136, July 2010.
- [19] K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On beamforming with finite rate feedback in multiple-antenna systems," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2562–2579, Oct. 2003.
- [20] C. K. Au-Yeung, and D. J. Love, "On the performance of random vector quantization limited feedback beamforming in a MISO system," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 458–462, Feb. 2007.
- [21] S. Boyd and L. Vandenberghe, Convex Optimization. New York: Cambridge University Press, 2004.
- [22] K. G. Murty, "Ball Centers of Special Polytopes," Mar. 2009. [Online] Available:http://www-personal.umich.edu/~murty/
- [23] M. Sharif and B. Hassibi, "A comparison of time-sharing, DPC and beamforming for MIMO broadcast channels with many users," *IEEE Trans. Commun.*, vol. 55, no. 1, pp. 11–15, Jan. 2007.
- [24] I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products, 7th ed. Burington, MA: Academic, 2007.
- [25] W. Gautschi, "Some elementary inequalities relating to the gamma and incomplete gamma function," J. Math. Phys., vol. 38, pp. 77–81, 1959.
- [26] Y. Ohwatari, N. Miki, T. Asai, T. Abe, and H. Taoka, "Performance of advanced receiver employing interference rejection combining to suppress inter-cell interference in LTE-advanced downlink", in *Proc. IEEE Vehicular Technology Conference (VTC-Fall)*, Sep. 2011.
- [27] Y. Tokgoz, B. D. Rao, M. Wengler, and B. Judson, "Performance analysis of optimum combining in antenna array systems with multiple interferers in flat Rayleigh fading," *IEEE Trans. Commun.*, vol. 52, no. 7, pp. 1047–1050, July 2004.

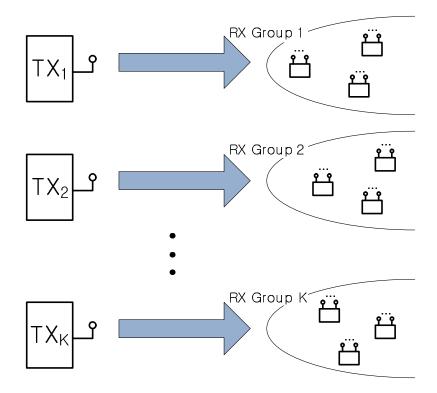


Fig. 1. System model. Each transmitter selects and serves a single user in each group.

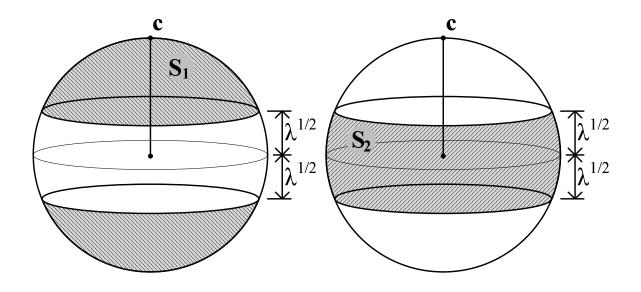


Fig. 2. Illustration of  $S_1(\mathbf{c}, \lambda)$  and  $S_2(\mathbf{c}, \lambda)$  in  $\mathbb{R}^3$  case.

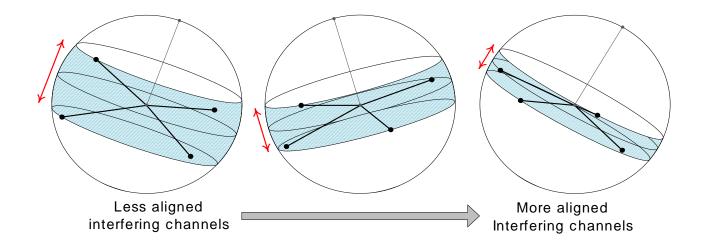


Fig. 3. Graphical representations of the interfering channels and interference alignment measures.

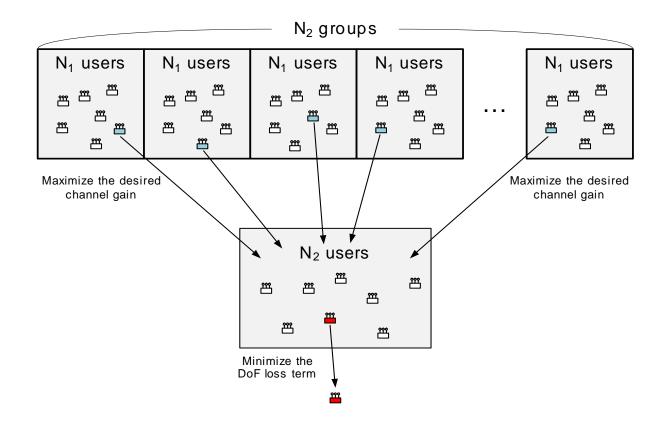


Fig. 4. Two-stage user selection scheme.

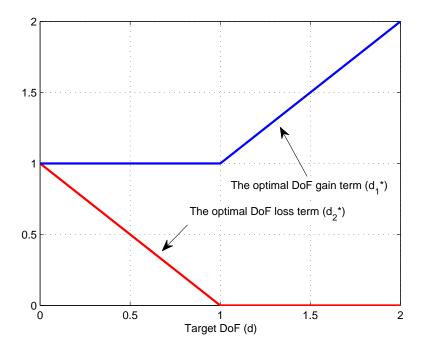


Fig. 5. The optimal DoF achieving strategy  $(d_1^\star, d_2^\star)$  for the target DoF d

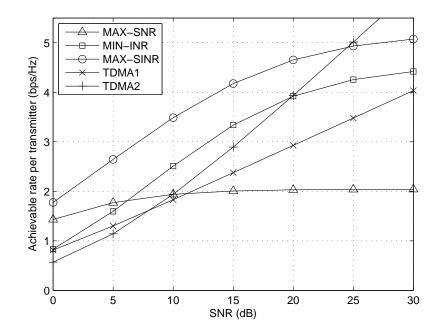


Fig. 6. Achievable rates per transmitter using various schemes in IBC with  $(K, N_r) = (4, 3)$  and N = 10.

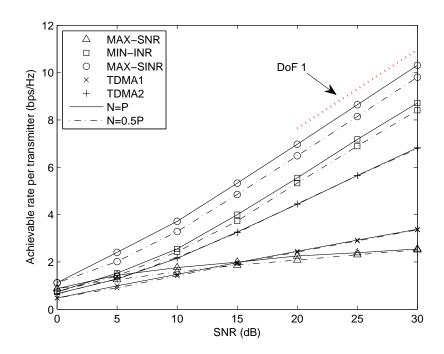


Fig. 7. Achievable rates per transmitter using various schemes when the number of users in each group scales as N = P and N = 0.5P, respectively.

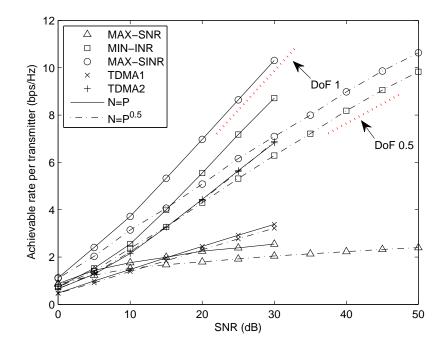


Fig. 8. Achievable rates per transmitter using various schemes when the number of users in each group scales as  $N = P^{0.5}$  and  $N = P^1$ , respectively.