

Mobile Data Offloading through A Third-Party WiFi Access Point: An Operator's Perspective

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Abstract—WiFi offloading is regarded as one of the most promising techniques to deal with the explosive data increase in cellular networks due to its high data transmission rate and low requirement on devices. In this paper, we investigate the mobile data offloading problem through a third-party WiFi access point (AP) for a cellular mobile system. From the cellular operator's perspective, by assuming a usage-based charging model, we formulate the problem as a utility maximization problem. In particular, we consider three scenarios: (i) successive interference cancellation (SIC) available at both the base station (BS) and the AP; (ii) SIC available at neither the BS nor the AP; (iii) SIC available at only the BS. For (i), we show that the utility maximization problem can be solved by considering its relaxation problem, and we prove that our proposed data offloading scheme is near-optimal when the number of users is large. For (ii), we prove that with high probability the optimal solution is One-One-Association, i.e., one user connects to the BS and one user connects to the AP. For (iii), we show that with high probability there is at most one user connecting to the AP, and all the other users connect to the BS. By comparing these three scenarios, we prove that SIC decoders help the cellular operator maximize its utility. To relieve the computational burden of the BS, we propose a threshold-based distributed data offloading scheme. We show that the proposed distributed scheme performs well if the threshold is properly chosen.

Index Terms—WiFi offloading, utility maximization, user association, integer programming, schur convex.

I. INTRODUCTION

The rapid development of mobile phones and mobile internet services in recent years has generated a lot of data usage over the cellular network [1]. The unprecedented explosion of mobile data traffic has led to overloaded cellular networks. For example, in metro areas and during peak hours, most 3G networks are overloaded [2]. Mobile users in overloaded areas will have to experience degraded cellular services, such as low data transmission rate and low quality phone calls.

A straightforward approach to address the above problem is to upgrade the cellular network to the more advanced 4G network. Another approach is to deploy more base stations (BSs) with smaller cell size such as femtocells [3], [4]. However, these approaches incur increase in infrastructure cost. A more cost-effective approach is to offload some of the mobile traffic to WiFi networks, which is often referred to as WiFi offloading. It has a few advantages: (i). No user equipment upgrading is required. This is because most of the mobile data services are created by smartphones which already have built-in WiFi modules. (ii). No licensed spectrum

is required. WiFi devices operate in unlicensed and world-unified 2.4GHz and 5GHz bands. (iii). High data rates. IEEE 802.11n WiFi can deliver data rates as high as 600Mbps and IEEE 802.11ac can deliver up to 6.933Gbps [5], which is much faster than 3G. (iv). Low infrastructure cost. The WiFi routers are much cheaper than the cellular BSs.

For the aforementioned reasons, WiFi offloading becomes a hot research topic and has attracted the attention of many researchers all over the world [6]–[20]. The feasibility of augmenting 3G using WiFi was investigated in [6]. The performance of 3G mobile data offloading through WiFi networks for metropolitan areas was studied in [7]. The numbers of APs needed for WiFi offloading in large metropolitan area was studied in [8]. Different approaches to implement WiFi offloading and to improve the performance of WiFi offloading were investigated in [9]–[14]. The load-balancing and user-association problem for offloading in heterogeneous networks with cellular networks and small cells are investigated in [15]–[18]. In [15], the authors investigated the outage probability and ergodic rate when a flexible cell association scheme is adopted. In [16], the authors developed a general and tractable model for data offloading in heterogeneous networks with different tiers of APs. In [17], the authors investigated the downlink user association problem for load balancing in a heterogeneous cellular networks. In [18], the authors investigated the data offloading schemes for load coupled networks, and showed that the optimal loading is tractable when proportional fairness is considered. Recent works [19]–[22] investigated the network economics of data offloading through WiFi APs using game theory [23].

Different from the above work, in this paper, we consider the scenario that there is a third-party WiFi AP providing data offloading service with a usage-based charging policy. We investigate the data offloading problem through such a third-party WiFi AP for a cellular mobile communication system. From business perspective, the cellular operator aims to maximize its revenue. Thus, in this paper, we investigate the data offloading problem from the economic point of view. We formulate the problem as a utility maximization problem and derive the corresponding data offloading schemes for the cellular operator. In particular, we consider three scenarios, namely, SIC available at both the BS and the AP, SIC available at only the BS, and SIC available at neither the BS nor the AP. We study the different utility functions and propose different data offloading schemes.

The main contribution and results of this paper are summarized as follows.

- *SIC available at both the BS and the AP*: The utility

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maximization problem for this case is solved by considering its relaxation problem. We show that the relaxation problem is a convex optimization problem. By using the convex optimization techniques, we prove that there is at most one user with fractional indicator function. A data offloading scheme is then obtained by rounding the fractional indicator function to its nearest integer. It is strictly proved that the proposed data offloading scheme is near-optimal when the number of users is large.

- *SIC available at neither the BS nor the AP:* For this case, we rigorously prove that when the number of users is large, the optimal solution is One-One-Association, i.e., the user with the best user-to-BS channel connects to the BS and that with the best user-to-AP channel connects to the WiFi AP.
- *SIC available at only the BS:* For this case, we show that when the number of users is large, there is at most one user connecting to the WiFi AP, and all the other users connect to the BS. A polynomial-time algorithm is developed to find the optimal offloading scheme.
- *SIC is beneficial for the cellular operator:* We rigorously prove that SIC decoders are beneficial for the cellular operator in terms of maximizing its utility.
- *Distributed data offloading scheme:* We propose a threshold-based distributed data offloading scheme for the case when SIC decoders are available at both the BS and the AP. We prove that the proposed distributed scheme can achieve the same performance as the centralized data offloading scheme once the threshold is properly chosen.

The rest of this paper is organized as follows: In Section II, we describe the system model and the problem formulation. In Section III, we present the results obtained for the case when SIC decoders are available at both the BS and the WiFi AP. In Section IV, we present the results obtained for the case when SIC decoders are not available at both the BS and the WiFi AP, and the results for the case when the SIC decoder is available at the BS side are given in Section V. Then, in Section VI, we show that SIC decoders are beneficial for the cellular operator. We also present a high-efficiency distributed data offloading scheme for the case when SIC decoders are available at both the BS and the WiFi AP. Simulation results are given in Section VII. Section VIII concludes the paper.

II. SYSTEM MODEL

In this paper, as shown in Fig. 1, we consider a cellular network with N users served by a base station (BS). We assume that there is a third-party WiFi access point (AP) within the coverage area of the BS. The WiFi AP and the BS use orthogonal frequencies. Thus, there is no inter-network interference between WiFi and cellular network. To maximize the network throughput and improve the overall network performance, the cellular operator may direct some of its users to be served by the WiFi AP. Since the WiFi AP belongs to a third-party operator, data offloading through AP is thus not for free. The cellular operator has to reward the AP operator an incentive while guaranteeing an optimized utility.

In this paper, we focus on the uplink scenario. We assume that all the users adopt fixed power transmission, i.e., P_i

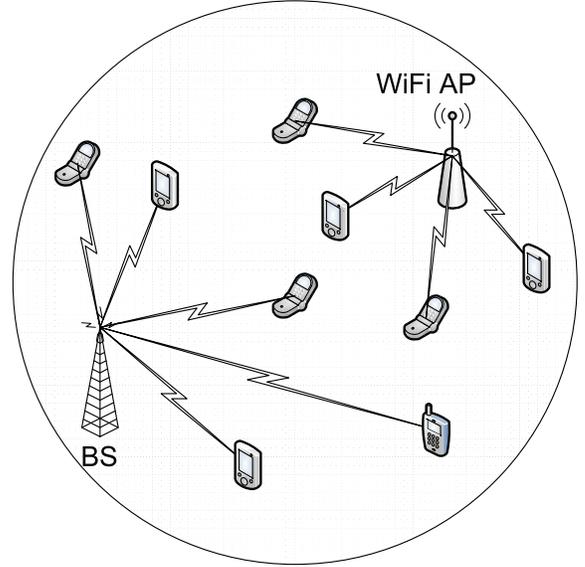


Fig. 1. System Model

for user i . For the convenience of analysis, we assume that $P_i = P, \forall i$. We also assume the users are uniformly distributed in the coverage area. The channel power gain between user i and the BS is denoted by $g_{i,B}$, and that between user i and the WiFi AP is denoted by $g_{i,A}$. Unless otherwise specified, we assume that $g_{i,B}$'s and $g_{i,A}$'s are strictly positive, mutually independent, and have continuous probability distribution function (pdf). The power of the additive Gaussian noises at the BS and the AP are denoted by σ_B^2 and σ_A^2 , respectively. We also assume that all the channel state information (CSI) and users' transmit power are known at the BS. Now, we define $x_i \in \{0, 1\}$ and $y_i \in \{0, 1\}$ as two indicator functions to indicate user i 's connection to BS and AP, respectively. If user i connects to BS, $x_i = 1$; otherwise, $x_i = 0$. Similarly, if user i connects to AP, $y_i = 1$; otherwise, $y_i = 0$. Besides, at any time, user i is only allowed to connect to either BS or AP, but not to both of them simultaneously, i.e., $x_i + y_i \leq 1, \forall i$.

In this paper, we assume that the cellular operator charges its users at λ per nat of data usage, and it pays the third-party WiFi operator at μ per nat of data usage over the AP. For convenience, throughout the paper, we use the natural logarithm. Hence, the data is measured in nats rather than in bits. Then, the utility function of the operator is defined as

$$U(x, y) = \lambda R_B(x) + (\lambda - \mu) R_A(y), \quad (1)$$

where $R_B(x)$ is the sum-rate at the BS, and $R_A(y)$ is the sum-rate at the WiFi AP. The exact form of the sum-rate depends on whether the SIC decoder is available. As implied by the name, in a receiver with a SIC decoder, users' signals are extracted from the composite received signal successively, rather than in parallel. The SIC decoder is able to remove the interference of the most recently decoded user from the current composite received signal by subtracting it out. According to [24], if a SIC decoder is available at the BS, the sum-rate at the BS

can be written as $R_B^w(\mathbf{x}) = \ln\left(1 + \sum_{i=1}^N \frac{g_{i,B}P}{\sigma_B^2} x_i\right)$; on the other hand, if the SIC decoder is not available at the BS, $R_B^o(\mathbf{x}) = \sum_{i=1}^N \ln\left(1 + \frac{x_i g_{i,B}P}{\sum_{j=1, j \neq i}^N x_j g_{j,B}P + \sigma_B^2}\right)$. Similarly, at the WiFi AP, we have $R_A^w(\mathbf{y}) = \ln\left(1 + \sum_{i=1}^N \frac{g_{i,A}P}{\sigma_A^2} y_i\right)$ and $R_A^o(\mathbf{y}) = \sum_{i=1}^N \ln\left(1 + \frac{y_i g_{i,A}P}{\sum_{j=1, j \neq i}^N y_j g_{j,A}P + \sigma_A^2}\right)$, with or without SIC decoder.

Depending on whether SIC decoder is available at the BS/AP, we have the following four possible utility functions

$$U^{ww}(\mathbf{x}, \mathbf{y}) = \lambda R_B^w(\mathbf{x}) + (\lambda - \mu) R_A^w(\mathbf{y}), \quad (2)$$

$$U^{oo}(\mathbf{x}, \mathbf{y}) = \lambda R_B^o(\mathbf{x}) + (\lambda - \mu) R_A^o(\mathbf{y}), \quad (3)$$

$$U^{wo}(\mathbf{x}, \mathbf{y}) = \lambda R_B^w(\mathbf{x}) + (\lambda - \mu) R_A^o(\mathbf{y}), \quad (4)$$

$$U^{ow}(\mathbf{x}, \mathbf{y}) = \lambda R_B^o(\mathbf{x}) + (\lambda - \mu) R_A^w(\mathbf{y}). \quad (5)$$

In the rest of the paper, we study the optimal data offloading schemes for the above four cases.

III. WITH SIC DECODERS AT BOTH SIDES

In this Section, we investigate the case that both the BS and the WiFi AP are equipped with a SIC decoder. Thus, the utility maximization problem of the cellular operator can be formulated as

Problem 3.1:

$$\max_{\{x_i, y_i, \forall i\}} \lambda \ln\left(1 + \sum_{i=1}^N S_{i,B} x_i\right) + (\lambda - \mu) \ln\left(1 + \sum_{i=1}^N S_{i,A} y_i\right), \quad (6)$$

$$\text{s.t. } x_i \in \{0, 1\}, \forall i, \quad (7)$$

$$y_i \in \{0, 1\}, \forall i, \quad (8)$$

$$x_i + y_i \leq 1, \forall i, \quad (9)$$

where $S_{i,B} \triangleq \frac{g_{i,B}P}{\sigma_B^2}$ and $S_{i,A} \triangleq \frac{g_{i,A}P}{\sigma_A^2}$.

It is observed from this problem formulation that the third-party operator's pricing strategy μ has a great influence on the optimal solution of the above problem. When μ is larger than λ , the cellular operator will not assign any user to the AP. This is rigorously proved by the following proposition.

Proposition 3.1: When $\lambda \leq \mu$, the optimal solution of Problem 3.1 is $\mathbf{x}^* = \mathbf{1}_N, \mathbf{y}^* = \mathbf{0}_N$, where $\mathbf{1}_N$ and $\mathbf{0}_N$ denote the N-dimension all-one vector and all-zero vector, respectively.

Proof: To prove $\mathbf{x}^* = \mathbf{1}_N$ and $\mathbf{y}^* = \mathbf{0}_N$ is the optimal solution of Problem 3.1, we have to show that $f(\mathbf{x}^*, \mathbf{y}^*)$ is larger than $f(\mathbf{x}, \mathbf{y})$, where $f(\mathbf{x}, \mathbf{y})$ denotes the objective function of Problem 3.1 and (\mathbf{x}, \mathbf{y}) is any feasible solution of Problem 3.1. Suppose $(\tilde{\mathbf{x}}, \tilde{\mathbf{y}})$ is a feasible solution of Problem 3.1, then it follows that

$$\begin{aligned} f(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}) &= \lambda R_B^w(\tilde{\mathbf{x}}) + (\lambda - \mu) R_A^w(\tilde{\mathbf{y}}) \\ &\stackrel{a}{\leq} \lambda R_B^w(\tilde{\mathbf{x}}) + (\lambda - \mu) R_A^w(\mathbf{0}_N) \\ &\stackrel{b}{\leq} \lambda R_B^w(\mathbf{1}_N) + (\lambda - \mu) R_A^w(\mathbf{0}_N), \end{aligned} \quad (10)$$

where "a" follows from the fact that $\lambda - \mu \leq 0$ and $R_A^w(\mathbf{y})$ is always nonnegative, and "b" follows from the fact that $R_B^w(\mathbf{x})$

is an increasing function of \mathbf{x} , and thus the equality holds only when $\mathbf{x}^* = \mathbf{1}_N$. \square

Proposition 3.1 indicates that the cellular operator will not offload any mobile data to the WiFi AP if the third-party operator charges at a price higher than its revenue, i.e., $\mu \geq \lambda$. On the other hand, from the third-party operator's perspective, if the cellular operator does not offload mobile data through its WiFi AP, it will earn nothing, which is a lose-lose situation. Thus, a reasonable third-party operator will charge a price lower than λ , which is the scenario we consider in the following studies, i.e., $\mu < \lambda$.

Proposition 3.2: The optimal solution of Problem 3.1 is obtained when (9) holds with equality for arbitrary i .

Proof: This can be proved by contradiction. Suppose $(\mathbf{x}^*, \mathbf{y}^*)$ is the optimal solution of Problem 3.1, and it has an element (x_k^*, y_k^*) satisfying $x_k^* + y_k^* < 1$. Then, from (7) and (8), it follows that $x_k^* = 0, y_k^* = 0$. Now, we show that we can always find a feasible solution $(\tilde{\mathbf{x}}, \tilde{\mathbf{y}})$ with its elements satisfying $\tilde{x}_i^* + \tilde{y}_i^* = 1, \forall i$ with a higher value of (6). We let $\tilde{x}_{-k} = \mathbf{x}_{-k}^*, \tilde{y}_{-k} = \mathbf{y}_{-k}^*$, where the minus sign before the letter k in the subscript of a vector refers to all the elements of the vector except the k th element. Then, since the logarithm function is an increasing function, it is clear that if we set $\tilde{x}_k^* = 1, \tilde{y}_k^* = 0$ or $\tilde{x}_k^* = 0, \tilde{y}_k^* = 1$ will result in a higher value of (6) than that resulted by $x_k^* = 0, y_k^* = 0$. This contradicts with our presumption. Proposition 3.2 is thus proved. \square

With the results given in Proposition 3.2, we can reduce the complexity of Problem 3.1 by setting $y_i = 1 - x_i$. Problem 3.1 can be converted to the following problem.

Problem 3.2:

$$\max_{x_i, \forall i} \lambda \ln\left(1 + \sum_{i=1}^N S_{i,B} x_i\right) + (\lambda - \mu) \ln\left(1 + \sum_{i=1}^N S_{i,A} (1 - x_i)\right), \quad (11)$$

$$\text{s.t. } x_i \in \{0, 1\}, \forall i. \quad (12)$$

This is a nonlinear integer programming problem. When the number of users is small, it can be solved by exhaustive search. However, when the number of users is large, exhaustive search is not applicable due to the high complexity. In this paper, we solve Problem 3.2 by solving its relaxation problem, and rigorously prove that the gap between the relaxation problem and Problem 3.2 is negligible when the number of the users is large.

The **relaxation problem** of Problem 3.2 is given as follows:

Problem 3.3:

$$\max_{x_i, \forall i} \lambda \ln\left(1 + \sum_{i=1}^N S_{i,B} x_i\right) + (\lambda - \mu) \ln\left(1 + \sum_{i=1}^N S_{i,A} (1 - x_i)\right), \quad (13)$$

$$\text{s.t. } 0 \leq x_i \leq 1, \forall i. \quad (14)$$

Problem 3.3 is a convex optimization problem. To show its convexity, we only need to show that the objective function is convex or concave since all the constraints are linear. Denote the objective function of the relaxation problem as f_r , then f_r is convex/concave if its Hessian is positive/negative

semidefinite. Denote the Hessian of f_r as \mathbf{H} , we show that \mathbf{H} is negative semidefinite by the following proposition.

Proposition 3.3: The Hessian \mathbf{H} is negative semidefinite.

Proof: The Hessian of f can be written as

$$\mathbf{H} = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_N \partial x_1} & \cdots & \frac{\partial^2 f}{\partial x_N^2} \end{pmatrix}, \quad (15)$$

where the diagonal elements and off-diagonal elements can be obtained as $\frac{\partial^2 f}{\partial x_k^2} = -\frac{\lambda S_{k,B}^2}{(1+\sum_{i=1}^N S_{i,B}x_i)^2} - \frac{(\lambda-\mu)S_{k,A}^2}{(1+\sum_{i=1}^N S_{i,A}(1-x_i))^2}$, and $\frac{\partial^2 f}{\partial x_k \partial x_j} = -\frac{\lambda S_{k,B}S_{j,B}}{(1+\sum_{i=1}^N S_{i,B}x_i)^2} - \frac{(\lambda-\mu)S_{k,A}S_{j,A}}{(1+\sum_{i=1}^N S_{i,A}(1-x_i))^2}$.

It is observed \mathbf{H} can be rewritten as

$$\mathbf{H} = -\frac{\lambda}{\left(1+\sum_{i=1}^N S_{i,B}x_i\right)^2} \mathbf{B} - \frac{(\lambda-\mu)}{\left(1+\sum_{i=1}^N S_{i,A}(1-x_i)\right)^2} \mathbf{A}, \quad (16)$$

where matrices \mathbf{B} and \mathbf{A} have the same structure as the following matrix \mathbf{X}

$$\mathbf{X} = \begin{pmatrix} S_{1,X}^2 & \cdots & S_{1,X}S_{N,X} \\ \vdots & \ddots & \vdots \\ S_{N,X}S_{1,X} & \cdots & S_{N,X}^2 \end{pmatrix}. \quad (17)$$

It can be shown that for any vector $\mathbf{c} = [c_1 \cdots c_N]^T$, $\mathbf{c}^T \mathbf{X} \mathbf{c}$ can be obtained as

$$\mathbf{c}^T \mathbf{X} \mathbf{c} = (c_1 S_{1,X} + \cdots + c_N S_{N,X})^2 \geq 0. \quad (18)$$

Thus, it is clear that both \mathbf{B} and \mathbf{A} are positive semidefinite. Then, since both λ and $\lambda - \mu$ are non-negative, it is easy to see that \mathbf{H} is negative semidefinite. Therefore, the objective function is strictly concave. \square

Problem 3.3 is shown to be convex, and it can be easily verified that Slater's condition holds for this problem. Thus, the duality gap between Problem 3.3 and its dual problem is zero, and solving its dual problem is equivalent to solving the original problem.

Now, we consider its dual problem. The Lagrangian of Problem 3.3 is

$$L(\mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\beta}) = (\lambda - \mu) \ln \left(1 + \sum_{i=1}^N S_{i,A}(1-x_i) \right) + \lambda \ln \left(1 + \sum_{i=1}^N S_{i,B}x_i \right) - \sum_{i=1}^N \alpha_i (x_i - 1) + \sum_{i=1}^N \beta_i x_i, \quad (19)$$

where $\boldsymbol{\alpha} = [\alpha_1 \cdots \alpha_N]^T$ and $\boldsymbol{\beta} = [\beta_1 \cdots \beta_N]^T$ are the nonnegative dual variables associated with the constraints.

The dual function is $q(\boldsymbol{\alpha}, \boldsymbol{\beta}) = \max_{\mathbf{x}} L(\mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\beta})$. The Lagrange dual problem is then given by $\min_{\alpha_i \geq 0, \beta_i \geq 0} q(\boldsymbol{\alpha}, \boldsymbol{\beta})$. Therefore, the optimal solution needs to satisfy the following

Karush-Kuhn-Tucker (KKT) conditions [25]:

$$\alpha_k (x_k^* - 1) = 0, \quad (20)$$

$$\beta_k x_k^* = 0, \quad (21)$$

$$0 \leq x_k^* \leq 1, \quad (22)$$

$$\alpha_k^* \geq 0, \beta_k^* \geq 0, \quad (23)$$

$$\frac{\partial L(\mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\beta})}{\partial x_k^*} = -\frac{(\lambda - \mu)S_{k,A}}{1 + \sum_{i=1}^N S_{i,A}(1-x_i^*)} + \frac{\lambda S_{k,B}}{1 + \sum_{i=1}^N S_{i,B}x_i^*} - \alpha_k + \beta_k = 0, \quad (24)$$

Due to the complexity of the problem, solving the above KKT conditions will not render us a closed-form solution. However, from these KKT conditions, we are able to gain some significant features of the optimal solution.

Theorem 3.1: The optimal solution of the relaxation problem has at most one user indexed by k ($k \in \{1, 2, \dots, N\}$), with a fractional x_k satisfying $0 < x_k < 1$.

Proof: This proposition can be proved by contradiction. Suppose that there are two arbitrary users denoted by m and n having fractional x_m and x_n , respectively, i.e., $0 < x_m < 1$ and $0 < x_n < 1$. From (20) and (21), it follows that $\alpha_m = 0$, $\alpha_n = 0$, $\beta_m = 0$, and $\beta_n = 0$. Then, applying these facts to (24), it follows that

$$\frac{\lambda S_{m,B}}{1 + \sum_{i=1}^N S_{i,B}x_i^*} - \frac{(\lambda - \mu)S_{m,A}}{1 + \sum_{i=1}^N S_{i,A}(1-x_i^*)} = 0, \quad (25)$$

$$\frac{\lambda S_{n,B}}{1 + \sum_{i=1}^N S_{i,B}x_i^*} - \frac{(\lambda - \mu)S_{n,A}}{1 + \sum_{i=1}^N S_{i,A}(1-x_i^*)} = 0. \quad (26)$$

Then, for these two users, the following equality must hold

$$\frac{S_{m,B}}{S_{m,A}} = \frac{S_{n,B}}{S_{n,A}} = \frac{(\lambda - \mu)}{\lambda} \frac{1 + \sum_{i=1}^N S_{i,B}x_i^*}{1 + \sum_{i=1}^N S_{i,A}(1-x_i^*)}. \quad (27)$$

It is easy to observe that $\frac{S_{m,B}}{S_{m,A}} = \frac{S_{n,B}}{S_{n,A}}$ is equivalent to $\frac{g_{m,B}}{g_{m,A}} = \frac{g_{n,B}}{g_{n,A}}$. However, it can be verified that the equality $\frac{g_{m,B}}{g_{m,A}} = \frac{g_{n,B}}{g_{n,A}}$ is satisfied with a zero probability since the channel power gains are mutually independent and have continuous pdf. This result contradicts with our presumption. Thus, it is concluded that there is at most one user with a fractional x_k , i.e., $0 < x_k < 1$. Theorem 3.1 is thus proved. \square

From Theorem 3.1, it is observed that there is at most one user with a fractional indicator for the optimal solution of Problem 3.3. This indicates that the optimal solution of Problem 3.3 is either equal to or just one-user away from that of Problem 3.2. Thus, the following scheme is proposed to find the optimal solution of Problem 3.2.

TABLE I

Proposed Centralized Data Offloading Scheme for Problem 3.1	
1)	Solve Problem 3.3 by standard convex optimization algorithms, such as interior-point method [26], or existing solvers such as CVX [27].
2)	Convert the obtained solution into a feasible solution of Problem 3.2 by rounding the fractional indicator function to its nearest integer (0 or 1).

In general, the above algorithm provides a sub-optimal solution to Problem 3.2. However, due to the special feature

presented in Theorem 3.1, we are able to prove that the proposed solution given in the Table I is near-optimal when the number of users is large.

Theorem 3.2: The gap between the optimal solution of Problem 3.2 and the proposed solution given in Table I is negligible when the number of users is large.

Proof: For the convenience of exposition, we denote the maximum values of Problem 3.2 attained at the optimal solution and at the proposed solution given in Table I as f_o^* and f_s^* , respectively. Since the solution given in Table I is also a feasible solution of Problem 3.2. Thus, it follows that

$$f_s^* \leq f_o^*. \quad (28)$$

On the other hand, it is clear that the maximum value of Problem 3.2 is upper bounded by its relaxation problem. Thus, if we denote the maximum values of the relaxation problem attained at the optimal solution as f_r^* , it follows that

$$f_o^* \leq f_r^*. \quad (29)$$

Combining the above facts together, we have

$$f_s^* \leq f_o^* \leq f_r^*. \quad (30)$$

Thus, if we are able to show that the gap between f_s^* and f_r^* is negligible when the number of users is large, it is clear that the gap between f_s^* and f_o^* will also be negligible when the number of users is large.

Now, we show that the gap between f_s^* and f_r^* is negligible when the number of users is large. Suppose x^* is the optimal solution of the relaxation problem, and user k is the user with a fractional indicator function x_k^* . Then, the value of f_r^* is $\lambda \ln\left(1 + \sum_{i=1}^N S_{i,B} x_i^*\right) + (\lambda - \mu) \ln\left(1 + \sum_{i=1}^N S_{i,A} (1 - x_i^*)\right)$, while the value of f_s^* is obtained by either setting $x_k = 0$ when $x_k < 0.5$ or setting $x_k = 1$ when $x_k \geq 0.5$. Obviously, it follows that $f_s^* > \tilde{f}_s^*$, where $\tilde{f}_s^* \triangleq \lambda \ln\left(1 + \sum_{i=1, i \neq k}^N S_{i,B} x_i^*\right) + (\lambda - \mu) \ln\left(1 + \sum_{i=1, i \neq k}^N S_{i,A} (1 - x_i^*)\right)$, which corresponds to the scenario that user k connects to neither the BS nor the AP.

Then, the gap Δ between f_s^* and f_r^* satisfies

$$\begin{aligned} \Delta < f_r^* - \tilde{f}_s^* &= \lambda \ln\left(1 + \frac{S_{k,B} x_k^*}{1 + \sum_{i=1, i \neq k}^N S_{i,B} x_i^*}\right) \\ &+ (\lambda - \mu) \ln\left(1 + \frac{S_{k,A} (1 - x_k^*)}{1 + \sum_{i=1, i \neq k}^N S_{i,A} (1 - x_i^*)}\right). \end{aligned} \quad (31)$$

Since the users are uniformly distributed in the area, thus when the number of users is large, it is inferred that the denominators of the above equation will be very large. Consequently, the value of Δ is close to zero. Theorem 3.2 is thus proved. \square

IV. WITHOUT SIC DECODERS AT BOTH SIDES

In this section, we consider the scenario that neither the BS nor the WiFi AP implements the SIC decoder. The utility maximization problem of the cellular operator for this case can be formulated as

Problem 4.1:

$$\begin{aligned} \max_{\{x_i, y_i, \forall i\}} \quad & \lambda \sum_{i=1}^N \ln\left(1 + \frac{x_i g_{i,B} P}{\sum_{j=1, j \neq i}^N x_j g_{j,B} P + \sigma_B^2}\right) \\ & + (\lambda - \mu) \sum_{i=1}^N \ln\left(1 + \frac{y_i g_{i,A} P}{\sum_{j=1, j \neq i}^N y_j g_{j,A} P + \sigma_A^2}\right), \end{aligned} \quad (32)$$

$$\text{s.t.} \quad x_i \in \{0, 1\}, \forall i, \quad (33)$$

$$y_i \in \{0, 1\}, \forall i, \quad (34)$$

$$x_i + y_i \leq 1, \forall i. \quad (35)$$

Problem 4.1 is a nonlinear integer programming problem which is difficult to solve directly due to its high complexity. Besides, it is not difficult to verify that the relaxation problem of Problem 4.1 is non-convex. Thus, we are not able to solve Problem 4.1 in the same way as Problem 3.1. To solve Problem 4.1, we first consider the following two subproblems.

Subproblem 4.1a:

$$\max_{\{x_i, \forall i\}} \quad \lambda \sum_{i=1}^N \ln\left(1 + \frac{x_i g_{i,B} P}{\sum_{j=1, j \neq i}^N x_j g_{j,B} P + \sigma_B^2}\right), \quad (36)$$

$$\text{s.t.} \quad x_i \in \{0, 1\}, \forall i. \quad (37)$$

Subproblem 4.1b:

$$\max_{\{y_i, \forall i\}} \quad (\lambda - \mu) \sum_{i=1}^N \ln\left(1 + \frac{y_i g_{i,A} P}{\sum_{j=1, j \neq i}^N y_j g_{j,A} P + \sigma_A^2}\right), \quad (38)$$

$$\text{s.t.} \quad y_i \in \{0, 1\}, \forall i. \quad (39)$$

Denote the optimal solution of Subproblem 4.1a as x_i^* , $\forall i \in \{1, 2, \dots, N\}$ and that of Subproblem 4.1b as y_i^* , $\forall i \in \{1, 2, \dots, N\}$. Then, it is clear that if x_i^* and y_i^* satisfy the constraints (35) for all $i \in \{1, 2, \dots, N\}$, x_i^* and y_i^* will be the optimal solution for Problem 4.1. In the following, we will show that when the number of users is large, Problem 4.1 can be solved by individually solving Subproblem 4.1a and Subproblem 4.1b. It is seen that Subproblem 4.1a and Subproblem 4.1b have the same structure. As a result, the optimal solutions of these two subproblems should also have the same structure. In the following, using Subproblem 4.1b as an example, we present the optimal solution of the two subproblems.

Lemma 4.1: i) Sort the users according to their channel power gains in descending order: $g_{1,A} \geq g_{2,A} \geq \dots \geq g_{N,A}$. At an optimal solution, only the first k^* ($\leq N$) users transmit, and $k^* = \arg\max_k \sum_{i=1}^k \ln\left(1 + \frac{g_{i,A} P}{\sum_{j=1, j \neq i}^k g_{j,A} P + \sigma_A^2}\right)$.

ii) Further, if $g_{1,A} \geq \frac{(e-1)\sigma_A^2}{P}$, $k^* = 1$. That is, only the user with the largest channel gain transmits.

Proof: To solve Subproblem 4.1b, we first consider its relaxation problem, which is given as follows.

Problem 4.2

$$\max_{\{y_i, \forall i\}} \quad \sum_{i=1}^N \ln\left(1 + \frac{y_i g_{i,A} P}{\sum_{j=1, j \neq i}^N y_j g_{j,A} P + \sigma_A^2}\right), \quad (40)$$

$$\text{s.t.} \quad 0 \leq y_i \leq 1, \forall i. \quad (41)$$

Let $P_i \triangleq y_i P, \forall i$, it is not difficult to observe that Problem 4.2 can be converted to the following problem,

Problem 4.3

$$\max_{\{P_i, \forall i\}} \sum_{i=1}^N \ln \left(1 + \frac{g_{i,A} P_i}{\sum_{j=1, j \neq i}^N g_{j,A} P_j + \sigma_A^2} \right), \quad (42)$$

$$\text{s.t. } 0 \leq P_i \leq P, \forall i. \quad (43)$$

This problem is shown to be Schur convex in [28]. By using the Schur convex properties, it is shown in [28] that the optimal power allocation is binary power allocation, i.e., either 0 or P for all i . This indicates that the optimal solution for Problem 4.2 is either 0 or 1 for all i . Thus, it can be observed that Problem 4.2 is actually equivalent to Subproblem 4.1b. Then, the results obtained for Problem 4.3 can be directly applied to Subproblem 4.1b. Based on the results in [28] (Theorem 1 and 4), it is not difficult to obtain the results presented in this lemma. Details are omitted here for brevity. \square

With the results given in Lemma 4.1, we are now ready for the following theorem.

Theorem 4.1: When the number of users N is large, with high probability, the optimal solution for Problem 4.1 is as follows.

- Only two users are active in the network: one connects to the BS and the other connects to the WiFi AP.
- Denote the index of the users connecting to the BS and the WiFi AP as m and n , respectively. Then, user m has the best user-to-BS channel, i.e., $m = \operatorname{argmax}_i g_{i,B}$; and user n has the best user-to-AP channel, i.e., $n = \operatorname{argmax}_i g_{i,A}$.

Proof: Let $m = \operatorname{argmax}_i g_{i,B}$ and $n = \operatorname{argmax}_i g_{i,A}$. From Lemma 4.1 Part (ii), (a) if $g_{n,A} \geq \frac{(e-1)\sigma_A^2}{P}$, then at the optimal solution for Subproblem 4.1b, only the user n transmits. Similarly, (b) if there exists $g_{m,B} \geq \frac{(e-1)\sigma_A^2}{P}$, then at the optimal solution for Subproblem 4.1a, only user m transmits. Finally, (c) if $m \neq n$, then the optimal solution of Problem 4.1 is the one is given in Theorem 4.1.

We now show that, when the number of users is large, these three conditions ((a) - (c)) hold with high probability. Define

- Event A: There is no user satisfying $g_{i,A} \geq \frac{(e-1)\sigma_A^2}{P}$.
- Event B: There is no user satisfying $g_{i,B} \geq \frac{(e-1)\sigma_B^2}{P}$.
- Event C: There exists one user having the best user-to-BS channel, and simultaneously having the best user-to-AP channel.

Hence, the probability that at least one of the three conditions ((a) - (c)) is violated can be written as

$$\operatorname{Prob}\{A \cup B \cup C\} \leq \operatorname{Prob}\{A\} + \operatorname{Prob}\{B\} + \operatorname{Prob}\{C\}, \quad (44)$$

where the inequality results from the well-known union bound.

In the following, we show that $\operatorname{Prob}\{A\} \rightarrow 0$, $\operatorname{Prob}\{B\} \rightarrow 0$, and $\operatorname{Prob}\{C\} \rightarrow 0$ go to zero as $N \rightarrow \infty$. First, we look at $\operatorname{Prob}\{A\}$, which is given by

$$\begin{aligned} \operatorname{Prob}\{A\} &= \operatorname{Prob} \left\{ g_{i,A} < \frac{(e-1)\sigma_A^2}{P}, \forall i \right\} \\ &\stackrel{a}{=} \left(\operatorname{Prob} \left\{ g_A < \frac{(e-1)\sigma_A^2}{P} \right\} \right)^N \end{aligned}$$

$$= \left(\int_0^{(e-1)\sigma_A^2/P} dF(g_A) \right)^N, \quad (45)$$

where the equality ‘‘a’’ results from the fact that the channel power gains are i.i.d., and $F(g_A)$ denotes the CDF of the channel power gain. Since $\int_0^{(e-1)\sigma_A^2/P} dF(g_A)$ is strictly less than 1, $\operatorname{Prob}\{A\} \rightarrow 0$ as $N \rightarrow \infty$.

Using the same approach, $\operatorname{Prob}\{B\} \rightarrow 0$ as $N \rightarrow \infty$.

Now, we consider $\operatorname{Prob}\{C\}$.

$$\operatorname{Prob}\{C\} = \binom{N}{1} \operatorname{Prob} \left\{ \begin{array}{l} j = \operatorname{argmax}_i g_{i,A}, \\ \text{and } j = \operatorname{argmax}_i g_{i,B} \end{array} \right\} \quad (46)$$

$$\stackrel{a}{=} N * \operatorname{Prob} \{j = \operatorname{argmax}_i g_{i,A}\} * \operatorname{Prob} \{j = \operatorname{argmax}_i g_{i,B}\} \quad (47)$$

$$= \frac{1}{N} \quad (48)$$

where the equality ‘‘a’’ results from the fact that the channel power gains are i.i.d. and have continuous pdf. From (48), $\operatorname{Prob}\{C\} \rightarrow 0$ as $N \rightarrow \infty$.

Combining the above results, $1 - \operatorname{Prob}\{A \cup B \cup C\} \rightarrow 1$ as $N \rightarrow \infty$, which completes the proof of Theorem 4.1. \square

V. WITH A SIC DECODER AT ONE SIDE

In this section, we consider the scenario that the SIC decoder is only available at one side. Particularly, we only study the case that the SIC decoder is only available at the BS. The case that the SIC decoder is only available at the WiFi AP is a symmetric case, and thus can be solved in the same way.

Problem 5.1:

$$\begin{aligned} \max_{\{x_i, y_i, \forall i\}} \lambda \ln \left(1 + \sum_{i=1}^N \frac{x_i g_{i,B} P}{\sigma_B^2} \right) \\ + (\lambda - \mu) \sum_{i=1}^N \ln \left(1 + \frac{y_i g_{i,A} P}{\sum_{j=1, j \neq i}^N y_j g_{j,A} P + \sigma_A^2} \right), \quad (49) \end{aligned}$$

$$\text{s.t. } x_i \in \{0, 1\}, \forall i, \quad (50)$$

$$y_i \in \{0, 1\}, \forall i, \quad (51)$$

$$x_i + y_i \leq 1, \forall i. \quad (52)$$

Similar to Problem 4.1, we are not able to solve this problem directly or by solving its relaxation problem. To solve Problem 5.1, we first consider the following two subproblems.

Subproblem 5.1a:

$$\max_{\{x_i, \forall i\}} \lambda \ln \left(1 + \sum_{i=1}^N \frac{x_i g_{i,B} P}{\sigma_B^2} \right), \quad (53)$$

$$\text{s.t. } x_i \in \{0, 1\}, \forall i. \quad (54)$$

Subproblem 5.1b:

$$\max_{\{y_i, \forall i\}} (\lambda - \mu) \sum_{i=1}^N \ln \left(1 + \frac{y_i g_{i,A} P}{\sum_{j=1, j \neq i}^N y_j g_{j,A} P + \sigma_A^2} \right), \quad (55)$$

$$\text{s.t. } y_i \in \{0, 1\}, \forall i. \quad (56)$$

Denote the optimal solution of Subproblem 5.1a as x_i^* , $\forall i \in \{1, 2, \dots, N\}$ and that of Subproblem 5.1b as y_i^* , $\forall i \in$

$\{1, 2, \dots, N\}$. Subproblem 5.1a is easy to solve, and the optimal solution is $x_i^* = 1, \forall i$. Subproblem 5.1b is exactly the same as Subproblem 4.1b, and thus the optimal solution of Subproblem 5.1b can be obtained from Lemma 4.1. It is obvious that x_i^* and y_i^* cannot satisfy the constraints (52) for all $i \in \{1, 2, \dots, N\}$. Thus, Problem 5.1 cannot be solved by directly solving Subproblem 5.1a and Subproblem 5.1b. This makes Problem 5.1 more challenging than Problem 4.1.

To solve Problem 5.1, we need the following lemma.

Lemma 5.1: The optimal solution of Problem 5.1 is obtained when (52) holds with equality for all i .

Proof: This can be proved by contradiction. Suppose $(\mathbf{x}^*, \mathbf{y}^*)$ is the optimal solution of Problem 5.1, and it has an element (x_k^*, y_k^*) satisfying $x_k^* + y_k^* < 1$. Then, from (50) and (51), it follows that $x_k^* = 0, y_k^* = 0$. Now, we show that we can always find a feasible solution $(\tilde{\mathbf{x}}, \tilde{\mathbf{y}})$ with its elements satisfying $x_i^* + y_i^* = 1, \forall i$, will result in a higher value of (49). We let $\tilde{\mathbf{x}}_{-k} = \mathbf{x}_{-k}^*, \tilde{\mathbf{y}}_{-k} = \mathbf{y}_{-k}^*$. Clearly, if we set $\tilde{x}_k^* = 1, \tilde{y}_k^* = 0$ will result in a higher value of (49) than that resulted by $x_k^* = 0, y_k^* = 0$ since the logarithm function is an increasing function. This contradicts with our presumption. Lemma 5.1 is thus proved. \square

Based on Lemma 5.1 and the optimal solutions of Subproblems 5.1a and 5.1b, we are now able to obtain the following lemma, Lemma 5.2, which will be used to prove Theorem 5.1. Proof of Lemma 5.2 requires assuming the path loss model for the users' channel gains. That is, the channel gain is given by $g = \alpha z^{-\gamma}$, where $\gamma = 2$ is the path loss exponent, z is the distance to either the AP or the BS and $\alpha \geq 0$ is a constant factor. Consequently, the results in Lemma 5.2 and Theorem 5.1 rely on the path loss model, a geometry for the users, BS and AP and a probability distribution of the users over the specified geometry. For the convenience of exposition, we consider a 1 by 1 square area with the BS at coordinate (0,0) and the WiFi AP at (1,1). We will assume that the users are uniformly distributed. For simplicity, we give the proof of Lemma 5.2 based on the geometry specified in Fig. 2, but it is worth pointing out that the proof extends to more general geometries with minor modifications.

Lemma 5.2: Let z^* be the solution to the equation $\alpha z^{-\gamma} = \frac{(e-1)\sigma_A^2}{P}$. Let D , as specified in Fig. 2, be $D = \min\{z^*, 0.67\}$ (the constant, 0.67, is derived in the proof of Lemma 5.2). Suppose there is at least one user in the quarter circle with radius D and centered at the AP, as shown in Fig. 2, then at the optimal solution to Problem 5.1, at most one user connects to the AP and all the other users connect to the BS.

Proof: We first consider subproblem 5.1b. Since there is at least one user in the stated quarter circle, the user with the strongest channel gain to the AP has a channel gain of at least $\frac{(e-1)\sigma_A^2}{P}$. From Lemma 4.1 part (ii), at the optimal solution to subproblem 5.1b, only the user with the strongest channel gain transmits. We denote the transmitting user as user k^* , and refer to the transmitting user as the *dominant* user.

Next, returning back to Problem 5.1, let \mathcal{S}^* be the set of users connected to the WiFi AP under the optimal solution. Based on Lemma 5.1, all users in \mathcal{S}^{*C} will connect to the BS. Let where $|\cdot|$ denote the cardinality of a set. We now show that $|\mathcal{S}^*| \leq 1$, where $|\cdot|$ by contradiction.

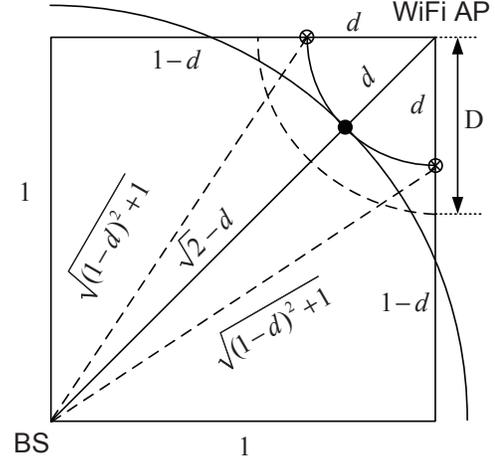


Fig. 2. Path loss Model. \otimes denotes the non-dominant user, and \bullet denotes the dominant user.

Suppose first that $|\mathcal{S}^*| = 2$. We have two possible cases:

- Case 1: *With a dominant user in \mathcal{S}^* .* That is, user $k^* \in \mathcal{S}^*$. In this case, if we assign the non-dominant user to the BS, the utility at the BS side $\lambda R_B^u(\mathbf{x})$ will increase. On the other hand, from Lemma 4.1 part (ii), the utility at the AP side is maximized when only user k^* connects to the AP. Thus, by assigning the non-dominant user to the BS, we can also increase the utility at the AP side $(\lambda - \mu)R_B^o(\mathbf{y})$. Hence, the total utility of the operator increases if we assign only user k^* to the AP, and the rest to the BS. This contradicts the assumption that $|\mathcal{S}^*| = 2$.
- Case 2: *Dominant user not in \mathcal{S}^* .* That is, user $k^* \notin \mathcal{S}^*$. Denote the channel power gain of the channel between the dominant user and the BS as $h_{k^*,B}$, and the channel power gains of the channels between the two non-dominant users and the BS as $h_{m,B}$ and $h_{n,B}$, respectively. Now, consider the case where we switch the connections of k^* , and m and n . In this case, the utility of the AP clearly increases by Lemma 4.1 part (ii), but the utility at the BS may not increase. However, it is straightforward to verify that the utility at the BS increases if the following condition holds.

$$h_{m,B} + h_{n,B} \geq h_{k^*,B}. \quad (57)$$

Now, referring to Fig. 2, let the dominant user be a distance of $\sqrt{2} - d$ away from the BS, where $d \leq D$ under the conditions stated in the Lemma. The two users in \mathcal{S}^* have to be at least distance d away from the AP, since their channel gains to the AP is weaker than the dominant user's. Considering the worst case scenario as given in Fig. 2, we have

$$\begin{aligned} h_{m,B} + h_{n,B} &\geq \frac{2\alpha}{(1-d)^2 + 1} \\ &\stackrel{a}{\geq} \frac{\alpha}{(\sqrt{2}-d)^2} \\ &\geq h_{k^*,B}, \end{aligned} \quad (58)$$

where “a” follows from $d \leq D \leq 0.67$. Hence, inequality (57) holds under the conditions stated in Lemma 5.2. Therefore, the total utility increases by switching the two non-dominant users with the dominant user, which contradicts our assumption that $|\mathcal{S}^*| = 2$.

Using the same arguments, we can show that any $|\mathcal{S}^*| > 2$ results in a contradiction under the conditions stated in Lemma 5.2. Hence, $|\mathcal{S}^*| \leq 1$, which completes the proof of Lemma 5.2. \square

We are now ready to prove Theorem 5.1.

Theorem 5.1: When the number of users N is large, with high probability the optimal solution for Problem 5.1 under path loss model is: At most one user connects to the AP and all the other users connect to the BS.

Proof: Since the users are uniformly distributed over the square of area one given in Fig. 2, the probability that there is at least one user in the quarter circle with radius D and centered at the AP is $\text{Prob}(AP) = 1 - (1 - \pi D^2/4)^N$. Since $D > 0$, $\text{Prob}(AP) \rightarrow 1$ as $N \rightarrow \infty$. Hence, the condition in Lemma 5.2 holds with high probability, which implies that the assertion in Theorem 5.1 holds with high probability. \square

Based on the result given in Theorem 5.1, the optimal solution of Problem 5.1 can be easily found by the following algorithm, which is given in Table II.

TABLE II

Proposed Data Offloading Scheme for Problem 5.1	
1. For $k = 1 : N$;	
initialize $\mathbf{x} = [1, 1, \dots, 1]^T$; $\mathbf{y} = [0, 0, \dots, 0]^T$;	
set $\mathbf{x}(k) = 0$; $\mathbf{y}(k) = 1$;	
compute $F(k) = U(\mathbf{x}, \mathbf{y})$;	
end	
2. Find the optimal allocation and the maximum value of F , $[F_{max}, \text{index}] = \max F$;	
3. Compare F_{max} with the utility without offloading $U(\mathbf{1}_N, \mathbf{0}_N)$.	

VI. RELATED SCENARIOS

A. Benefit of SIC Decoders

In this subsection, we investigate the role of SIC decoders in the utility maximization of the cellular operator. We rigorously prove that the SIC decoder is beneficial for the cellular operator in terms of maximizing its utility.

Theorem 6.1: Let $(\mathbf{x}^*, \mathbf{y}^*)$, $(\hat{\mathbf{x}}^*, \hat{\mathbf{y}}^*)$, $(\tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*)$ be the optimal solutions of Problem 3.1, 4.1, and 5.1, respectively. In general, the following inequality always holds,

$$U^{ww}(\mathbf{x}^*, \mathbf{y}^*) \geq U^{wo}(\tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*) \geq U^{oo}(\hat{\mathbf{x}}^*, \hat{\mathbf{y}}^*). \quad (59)$$

Proof: To prove Theorem 6.1, we first show that $U^{ww}(\mathbf{x}^*, \mathbf{y}^*) \geq U^{wo}(\tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*)$. It can be observed that $U^{ww}(\mathbf{x}^*, \mathbf{y}^*) \geq U^{ww}(\tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*)$. This is due to the fact that $(\tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*)$ is a feasible solution of Problem 3.1, while $(\mathbf{x}^*, \mathbf{y}^*)$ is the optimal solution of Problem 3.1. Thus, if we can show that $U^{ww}(\tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*) \geq U^{wo}(\tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*)$ holds, $U^{ww}(\mathbf{x}^*, \mathbf{y}^*) \geq U^{wo}(\tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*)$ will hold. Since $U^{ww}(\tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*) = \lambda R_B^w(\tilde{\mathbf{x}}^*) + (\lambda - \mu) R_A^w(\tilde{\mathbf{y}}^*)$ and $U^{wo}(\tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*) = \lambda R_B^w(\tilde{\mathbf{x}}^*) + (\lambda - \mu) R_A^o(\tilde{\mathbf{y}}^*)$, we only need to show that $R_A^w(\tilde{\mathbf{y}}^*) \geq R_A^o(\tilde{\mathbf{y}}^*)$ always holds, which is presented as below.

Assume that K elements of $\tilde{\mathbf{y}}^*$ are equal to 1, where $K \in \{1, 2, \dots, N\}$. Then, it follows that

$$\begin{aligned} R_A^w(\tilde{\mathbf{y}}^*) &= \ln \left(1 + \sum_{i=1}^K \frac{g_{i,AP}}{\sigma_A^2} \right) \\ &= \ln \left(\frac{\sigma_A^2 + \sum_{i=1}^K g_{i,AP}}{\sigma_A^2} \right) \\ &= \ln \left[\left(\frac{\sigma_A^2 + \sum_{i=1}^K g_{i,AP}}{\sigma_A^2 + \sum_{i=2}^K g_{i,AP}} \right) \left(\frac{\sigma_A^2 + \sum_{i=2}^K g_{i,AP}}{\sigma_A^2 + \sum_{i=3}^K g_{i,AP}} \right) \right. \\ &\quad \left. \dots \left(\frac{\sigma_A^2 + \sum_{i=K}^K g_{i,AP}}{\sigma_A^2} \right) \right] \\ &\stackrel{a}{=} \sum_{j=1}^K \ln \left(\frac{\sigma_A^2 + \sum_{i=j}^K g_{i,AP}}{\sigma_A^2 + \sum_{i=j+1}^K g_{i,AP}} \right) \\ &= \sum_{j=1}^K \ln \left(1 + \frac{g_{j,AP}}{\sigma_A^2 + \sum_{i=j+1}^K g_{i,AP}} \right) \\ &\stackrel{b}{\geq} \sum_{j=1}^K \ln \left(1 + \frac{g_{j,AP}}{\sigma_A^2 + \sum_{i=1, i \neq j}^K g_{i,AP}} \right) = R_A^o(\tilde{\mathbf{y}}^*), \end{aligned} \quad (60)$$

where we introduce a dumb item $\sum_{i=K+1}^K g_{i,AP} = 0$ in the equality “a” for notational convenience. The inequality “b” follows from the fact that $\sum_{i=1, i \neq j}^K g_{i,AP} \geq \sum_{i=j+1}^K g_{i,AP}, \forall j$.

Then, it is clear that $U^{ww}(\mathbf{x}^*, \mathbf{y}^*) \geq U^{wo}(\tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*)$ always holds. Using the same approach, we can easily show that $U^{wo}(\tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*) \geq U^{oo}(\hat{\mathbf{x}}^*, \hat{\mathbf{y}}^*)$ always holds. Theorem 6.1 is thus proved. \square

From Theorem 6.1, it is observed that SIC decoder plays an important role in the utility maximization of the cellular operator. It is beneficial for the operator to equip the BS with SIC decoders in terms of maximizing its utility.

B. Distributed Data Offloading

In the previous sections, we have obtained the optimal data offloading schemes for Problem 3.1, 4.1, and 5.1 when the number of users is large. However, the proposed data offloading schemes are centralized schemes, which needs the users to send the user-to-AP and user-to-BS channel power gains to the BS, and then the BS has to compute the optimal user association and feedback the decisions to the users. For Problem 4.1 and 5.1, due to the special structure of the problems, the proposed centralized algorithms can find the optimal solution in polynomial time. However, for Problem 3.1, due to the complexity of the problem, the proposed algorithm puts a heavy computational burden on the BS. Thus, to relieve the computational burden on BS and reduce the overhead for CSI and decision transfer, in this section, we propose a simple but highly efficient distributed data offloading scheme for Problem 3.1, which is given in Table III.

It is observed from Table III that the BS does not have to collect the CSI from the users, and it only needs to broadcast a predetermined threshold T to the users. Thus, the network overhead of the distributed algorithm is much lower than

TABLE III

Proposed Distributed Data Offloading Scheme for Problem 3.1
1). The cellular operator broadcasts an offloading threshold T to each user.
2). User i computes its value of $\frac{S_{i,B}}{S_{i,A}}$, $\forall i$. If $\frac{S_{i,B}}{S_{i,A}} \geq T$, it connects to the BS; Otherwise, it connects to the WiFi AP.

that of the centralized algorithm. On the other hand, the computational complexity of the distributed algorithm is much lower than that of the centralized algorithm. For the centralized algorithm, the BS has to solve a relaxed integer programming problem to decide the optimal association for each user, whose worst-case computational complexity is $O(N^3)$ [29]. While for the distributed scheme, the computational complexity is $O(N)$, since each user only has to compute a ratio ($\frac{S_{i,B}}{S_{i,A}}$ for user i) to decide its association. However, it is worth pointing out that the performance of the distributed algorithm greatly depends on the value of the threshold T .

In the following, we show that the distributed data offloading scheme can achieve the same performance as the centralized one given in Table I if the threshold T is properly chosen.

Theorem 6.2: There exists an optimal threshold T^* , for any user i other than the user with fractional indicator function, the following equality holds.

$$x_i^* = \begin{cases} 1, & \text{if } \frac{S_{i,B}}{S_{i,A}} > T^*, \\ 0, & \text{if } \frac{S_{i,B}}{S_{i,A}} < T^*, \end{cases} \quad (61)$$

where $T^* = \frac{(\lambda-\mu)}{\lambda} \frac{1+\sum_{i=1}^N S_{i,B}x_i^*}{1+\sum_{i=1}^N S_{i,A}(1-x_i^*)}$, and x_i^* , $\forall i$ is the optimal solution of Problem 3.3.

Proof: This proof is based on the KKT conditions given out in Section III. It is observed from (24) that if $\frac{S_{i,B}}{S_{i,A}} > T^*$, where $T^* = \frac{(\lambda-\mu)}{\lambda} \frac{1+\sum_{i=1}^N S_{i,B}x_i^*}{1+\sum_{i=1}^N S_{i,A}(1-x_i^*)}$, it follows that $\alpha_i - \beta_i > 0$. From (20) and (21), it is also observed that $\alpha_i \neq 0$ and $\beta_i \neq 0$ can not hold simultaneously. Since α_i and β_i are nonnegative, thus if $\beta_i > 0$, α_i must be equal to zero. Consequently, we have $\alpha_i - \beta_i < 0$, which contradicts with the fact that $\alpha_i - \beta_i > 0$. Thus, it clear that $\alpha_i > 0$ and $\beta_i = 0$. Then, from (20), it follows that $x_i = 1$. Similarly, when $\frac{S_{i,B}}{S_{i,A}} < T^*$, it can be shown that $\alpha_i = 0$ and $\beta_i > 0$, which indicates that $x_i = 0$.

Theorem 6.2 is thus proved. \square

C. Fading Scenarios

In this paper, we consider three cases: (1) With SIC decoders at both sides; (2) Without SIC decoders at both sides; (3) With a SIC decoder at one side. It is worth pointing out that we do not assume any specific distribution of the channel power gains for Cases (1) and (2). Thus, the results obtained for Cases (1) and (2) can be directly applied to the block-fading scenario [30], where the channel remains constant during each fading block but possibly changes from one block to another. For the block-fading scenario, we can solve the utility maximization problem for each fading block, and update the user association scheme every fading block. This is due to the fact that there are no coupling constraints between the fading blocks, and thus maximizing the utility function

for each fading block is equivalent to maximizing the long-term utility function [31], i.e., $\mathbb{E}[U(\mathbf{x}, \mathbf{y})]$, where $U(\mathbf{x}, \mathbf{y})$ is given by equation (1) and the expectation is taken over the probability distribution of all the involved channel power gains. Since we did not assume any specific distribution of the channel power gains, the result holds for block-fading channels with any fading distributions, such as Rayleigh fading, Rician fading, Nakagami fading. However, for Case (3), we assumed the path loss model when deriving the results. This is due to the following reason. Using other fading channel models instead of the path loss model makes the utility maximization problem for this case mathematically intractable. Thus, the offloading scheme proposed for this case may not be optimal if fading channel models are adopted. However, according to the simulation results presented in Section VII, the offloading scheme proposed for this case also works well when fading channel models are considered.

D. Downlink Scenarios

In this paper, we focus on the uplink scenario. In this subsection, we show how to extend the obtained results to the downlink scenario. For the downlink scenario, the system model becomes broadcast channels. For broadcast channels, there are usually two implementation ways:

- Superposition coding with SIC. The transmitter encodes the messages for all the receivers using superposition coding. Each receiver decodes the received message using SIC. This case is similar to the uplink scenario with SIC. If we assume both BS and AP adopt this scheme, and both of them adopt equal power allocation, the resultant utility maximization problem can be obtained by letting $S_{i,B} = \frac{g_{i,B}P}{\sigma_{i,B}^2}$ and $S_{i,A} = \frac{g_{i,A}P}{\sigma_{i,A}^2}$, $\forall i$ in Problem 3.1. Then, we can solve this problem using the same approach as Problem 3.1 by introducing $g'_{i,B} \triangleq \frac{g_{i,B}}{\sigma_{i,B}^2}$ and $g'_{i,A} \triangleq \frac{g_{i,A}}{\sigma_{i,A}^2}$.
- Orthogonal schemes. If SIC decoders are not available at the receivers, for the broadcast channel, the transmitter will not encode the message for all users together. Instead, they will use orthogonal schemes, such as TDMA. For this case, the resultant user association is trivial, i.e., in each time slot, one user is selected to connect to the BS, and one user is selected to connect to the AP.

VII. NUMERICAL RESULTS

In this section, numerical results are provided to evaluate the performance of the proposed data offloading schemes.

A. Simulation Parameters

The simulation setup is as follows. We consider a 1 by 1 square area with the base station at coordinate (0,0) and the WiFi AP at (1,1). The number of users is denoted by N , and these N users are uniformly scattered in the square. For simplicity, we assume that the transmit power of each user is the same and given by 1. Unless otherwise stated, the path loss model is adopted to model the channel power gain. Let $(\text{pos}_{x_i}, \text{pos}_{y_i})$ denote the position of user i , then the channel power gain between it and the BS can

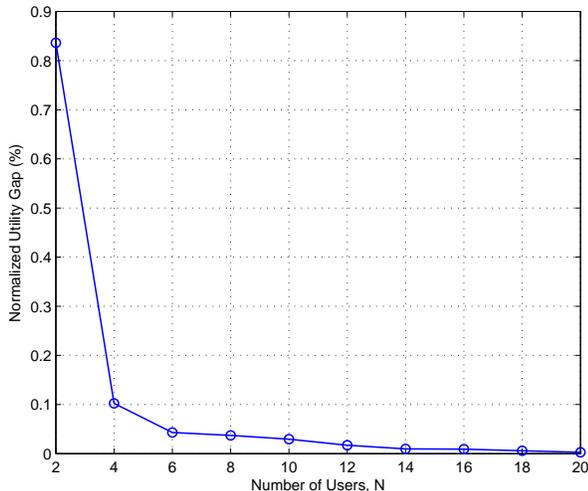


Fig. 3. With SIC decoders at both sides: normalized utility gap vs. the number of users.

be modeled as $g_{i,B} = (\sqrt{\text{pos}x_i^2 + \text{pos}y_i^2})^{-\gamma}$, where γ is the path loss coefficient. Similarly, the channel power gain between user i and the AP can be modeled as $g_{i,A} = (\sqrt{(1 - \text{pos}x_i)^2 + (1 - \text{pos}y_i)^2})^{-\gamma}$. In this paper, we consider the free space path loss model where $\gamma = 2$. The power of the additive Gaussian noises at the BS and the AP are set to 1, i.e., $\sigma_B^2 = 1$ and $\sigma_A^2 = 1$. The revenue coefficient λ of the BS is set to 1, and the cost coefficient μ is set to 0.5. Matlab is used for running all the simulations.

B. With SIC Decoders at Both Sides

1) Performance of the Centralized Data Offloading Scheme:

In Fig. 3, we investigate the gap between the proposed centralized data offloading scheme given in Table I and the optimal solution. The optimal solution is obtained by the exhaustive search. For the purpose of illustration, the gap is normalized by the utility of the optimal solution. The result presented in Fig. 3 is averaged over 1000 channel realizations for each N . It is observed from Fig. 3 that the normalized utility gap decreases with the increase of the number of users. When there are only two users in the network, the normalized utility gap is as large as 0.85%. When the number of users goes up to 16, the normalized utility gap is almost zero. This is in accordance with the results presented in Theorem 3.2.

2) Performance of the Distributed Data Offloading Scheme:

In Fig. 4, we investigate the system performance of the proposed distributed data offloading scheme given in Table III. The results presented in this figure is averaged over 1000 channel realizations. The red dashed lines represent the values obtained by the centralized data offloading schemes. The green dotted dashed lines represent the values obtained by the exhaustive search. In this figure, we study how the value of the threshold T affects the performance of the proposed distributed data offloading scheme.

It is observed from Fig. 4 that the centralized algorithm can achieve almost the same performance as the exhaustive

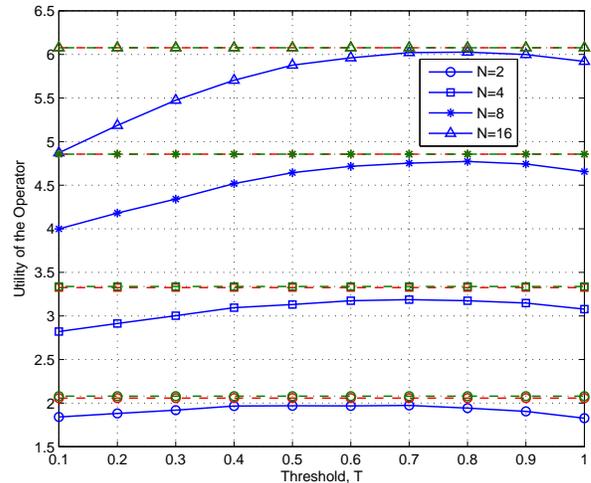


Fig. 4. With SIC decoders at both sides: performance of the distributed data offloading scheme.

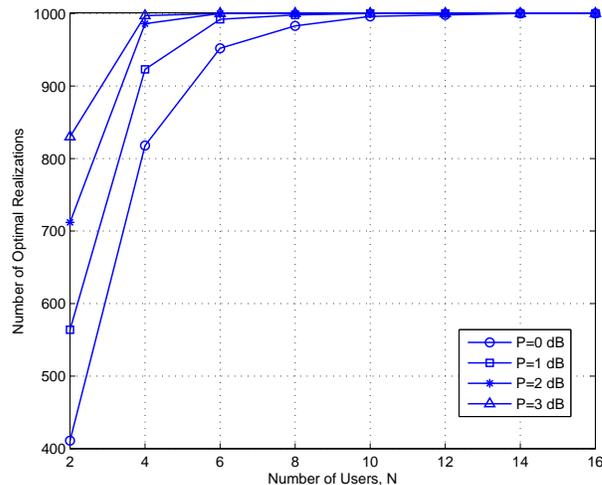


Fig. 5. Without SIC decoder at both sides: performance of the data offloading scheme.

search, especially when N is large. This is in accordance with our theoretical results. It is observed that the threshold T plays a significant role in the distributed algorithm when the number of users is large. It is observed that the utility gap between the distributed algorithm and the exhaustive search is as large as 1.2 when $N = 16$ if T is not properly chosen. However, when $N = 2$, the largest utility gap is less than 0.2. It is also observed from Fig. 4 that for each N , there does exist an optimal T which produces a utility which is almost the same as the centralized data offloading scheme. This is in accordance with the results presented in Theorem 6.2.

C. Without SIC Decoders at Both Sides

In Fig. 5, we investigate the performance of the proposed data offloading scheme for the case that SIC decoders are not available at both BS and the AP side. In Fig. 5, we

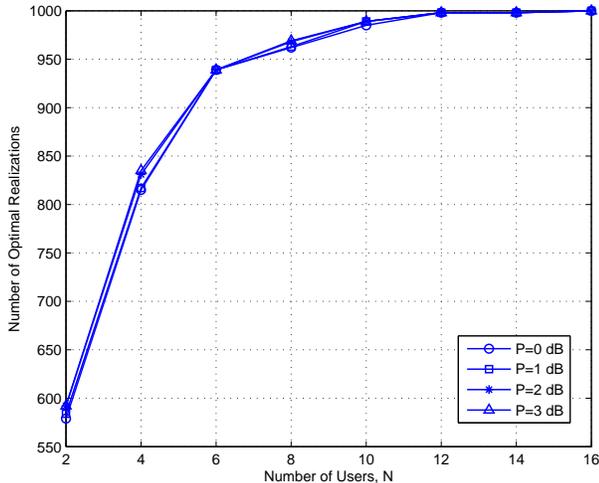


Fig. 6. With SIC decoder at only BS: performance of the data offloading scheme.

generate 1000 channel realizations for each N . We count the number of realizations, in which the proposed data offloading scheme is optimal. First, it is observed that for all the curves, the number of realizations that the proposed data offloading scheme is optimal increases with the increasing number of users. This is in accordance with our theoretical analysis given in Section IV. Secondly, it is observed that the transmit power of the users also plays an important role in the performance of the proposed data offloading scheme. For the same number of users, when the transmit power of the users is large, the number of realizations that the proposed data offloading scheme is optimal is large. This is due to the fact that when P is large, the value of $\frac{(e-1)\sigma_A^2}{P}$ is small, and thus the probability that $g_{1,A} \geq \frac{(e-1)\sigma_A^2}{P}$ is large for the same number of users. Thirdly, it is observed that when the number of users is larger than 10, the proposed data offloading scheme is always optimal for all the cases. This indicates that the proposed data offloading scheme can achieve a satisfactory performance even when the number of users is not very large.

D. With A SIC Decoder at One Side

In Fig. 6, we investigate the performance of the proposed data offloading scheme for the case that a SIC decoder is only available at the BS side. In Fig. 6, we generate 1000 channel realizations for each N . We count the number of realizations, in which the proposed data offloading scheme is optimal. It is observed that for all the curves, the number of realizations that the proposed data offloading scheme is optimal increases with the increasing number of users. This is in accordance with our theoretical analysis in Section V. Secondly, it is observed that the transmit power of the users almost does not affect the performance of the proposed data offloading scheme. This is quite different from the results obtained in Fig. 5. This is due to the fact that for this case, the proposed data offloading scheme is optimal only when both $g_{1,A} \geq \frac{(e-1)\sigma_A^2}{P}$ and $d < 0.67$ are satisfied simultaneously. For the case considered here, the

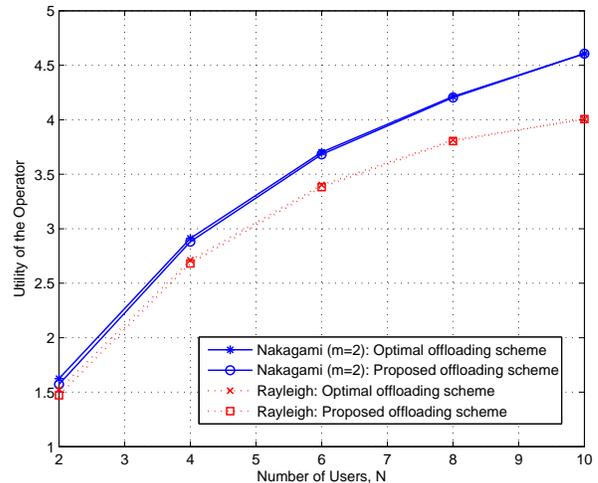


Fig. 7. With SIC decoder at only BS: fading scenario.

condition that $d < 0.67$ always dominates. Since this condition is irrelevant with the transmit power, the performance of the proposed data offloading scheme is not affected by the transmit power of the users. Finally, it is observed that when the number of users is larger than 12, the proposed data offloading scheme is always optimal. This indicates that the proposed data offloading scheme can achieve a good performance even when the number of users is not large.

In Fig. 7, we investigate the performance of the proposed data offloading scheme for different fading channel models. For the Rayleigh fading model, the channel power gains are exponentially distributed [25], and we assume that the mean of the channel power gains is one. For the Nakagami- m fading model, we consider the case that $m = 2$, and we assume that the mean of the channel power gains is one. The transmit power of each user is assumed to be the same and equal to 1. The results are averaged over 1000 channel realizations. The optimal offloading schemes are obtained by exhaustive search. It is observed from Fig. 7 that when the number of users is small, there is a small gap between the proposed offloading scheme and the optimal offloading scheme. However, when the number of users is larger than six, the proposed offloading scheme can achieve same performance as the optimal offloading scheme. This is due to the fact that when the number of users is large, the condition given in (57) holds with a high probability, and thus the proposed offloading scheme is optimal with a high probability. Overall, the proposed offloading scheme works well under different fading channel models.

E. Benefit of SIC Decoders

In Fig. 8, we compare the utility of the cellular operator for the three cases studied in this paper. The utility values for each case are obtained under their respective optimal data offloading schemes. The results presented in Fig. 8 are averaged over 1000 channel realizations for each N . It is observed that the utility increases with the increasing of N for all three cases.

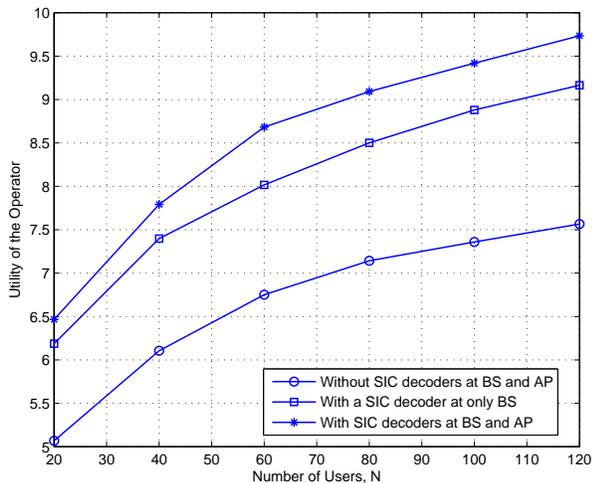


Fig. 8. Benefit of the SIC decoders.

This is in accordance with the theoretical results presented in previous sections. It is also observed that $U^{ww} > U^{wo} > U^{oo}$ for the same N . This indicates that SIC decoders have a significant effect on the utility of the cellular operator. It is always beneficial for the operator to equip the BS and/or AP with SIC decoders so as to maximize its utility. This is in accordance with the results obtained in Theorem 6.1.

VIII. CONCLUSIONS

In this paper, we have investigated the mobile data offloading problem through a third-party WiFi AP for a cellular mobile system. From the cellular operator's perspective, we have formulated the problem as a utility maximization problem. By considering whether SIC decoders are available at the BS and/or the WiFi AP, different cases are considered. When the SIC decoders are available at both the BS and the WiFi AP, the utility maximization problem can be solved by considering its relaxation problem. It is strictly proved that the proposed data offloading scheme is near-optimal when the number of users is large. We also propose a threshold-based distributed data offloading scheme which can achieve the same performance as the centralized data offloading scheme if the threshold is properly chosen. When the SIC decoders are not available at both the BS and the WiFi AP, we have rigorously proved that the optimal solution is One-One-Association, i.e., one user connects to the BS and the other user connects to the WiFi AP. When the SIC decoder is only available at the BS, we have shown that there is at most one user connecting to the WiFi AP, and all the other users connect to the BS. We also have rigorously proved that SIC decoders are beneficial for the cellular operator in terms of maximizing its utility.

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REFERENCES

- [1] Cisco Systems Inc., "Cisco visual networking index: Global mobile data traffic forecast update, 2011-2016," 2012.
- [2] Wavion Ltd., "Metro Zone Wi-Fi for Cellular Data Offloading," White paper, 2011.
- [3] V. Chandrasekhar, J. G. Andrews, and A. Gatherer, "Femtocell networks: a survey," *IEEE Commun. Mag.*, vol. 46, no. 9, pp. 59–67, 2008.
- [4] X. Kang, R. Zhang, and M. Motani, "Price-based resource allocation for spectrum-sharing femtocell networks: A stackelberg game approach," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 538–549, Apr. 2012.
- [5] Cisco Systems Inc., "802.11ac - The Fifth Generation of Wi-Fi," Technical white paper, 2012.
- [6] A. Balasubramanian, R. Mahajan, and A. Venkataramani, "Augmenting Mobile 3G Using WiFi," *Proc. ACM MobiSys*, pp. 209–222, 2010.
- [7] K. Lee, J. Lee, Y. Yi, I. Rhee, and S. Chong, "Mobile Data Offloading: How Much Can WiFi Deliver?," *IEEE/ACM Trans. Networking*, vol. 21, no. 2, pp. 536–550, Apr. 2013.
- [8] S. Dimatteocy, P. Huiy, B. Hanyz, and V. Lix, "Cellular Traffic Offloading through WiFi Networks," *Proc. IEEE MASS*, pp. 192–201, 2011.
- [9] C. B. Sankaran, "Data offloading techniques in 3GPP Rel-10 networks: A tutorial," *IEEE Commun. Mag.*, vol. 50, no. 6, pp. 46–53, June 2012.
- [10] M. H. Qutqut, F. M. Al-Turjman, and H. S. Hassanein, "MFW: Mobile femtocells utilizing WiFi: A data offloading framework for cellular networks using mobile femtocells," *Proc. IEEE ICC 2013*, pp. 6427–6431, Jun. 2013.
- [11] A. Aijaz, H. Aghvami, and M. Amani, "A survey on mobile data offloading: technical and business perspectives," *IEEE Wireless Commun.*, vol. 20, no. 2, pp. 104–112, Apr. 2013.
- [12] B. Han, P. Hui, V. Kumar, M. Marathe, J. Shao, and A. Srinivasan, "Mobile Data Offloading through Opportunistic Communications and Social Participation," *IEEE Trans. on Mobile Comput.*, vol. 11, no. 5, pp. 821–834, May 2012.
- [13] H. Elsayy, E. Hossain, and S. Camorlinga, "Traffic offloading techniques in two-tier femtocell networks," *Proc. IEEE ICC 2013*, pp. 6086–6090, June 2013.
- [14] H. Dong, P. Wang, and D. Niyato, "A dynamic offloading algorithm for mobile computing," *IEEE Trans. on Wireless Commun.*, vol. 11, no. 6, pp. 1991–1995, June 2012.
- [15] H. Jo, Y. Sang, P. Xia, and J. G. Andrews, "Heterogeneous cellular networks with flexible cell association: A comprehensive downlink SINR analysis," *IEEE Trans. on Wireless Commun.*, vol. 11, no. 10, pp. 3484–3495, Oct. 2012.
- [16] S. Singh, H. S. Dhillon, and J. G. Andrews, "Offloading in Heterogeneous Networks: Modeling, Analysis, and Design Insights," *IEEE Trans. on Wireless Commun.*, vol. 12, no. 5, pp. 2484–2497, May 2013.
- [17] Q. Ye, B. Rong, Y. Chen, M. Al-Shalash, C. Caramanis and J. G. Andrews, "User Association for Load Balancing in Heterogeneous Cellular Networks," *IEEE Trans. on Wireless Commun.*, vol. 12, no. 6, pp. 2706–2716, June 2013.
- [18] C. K. Ho, D. Yuan, and S. Sun, "Data Offloading in Load Coupled Networks: Solution Characterization and Convexity," *Proc. IEEE ICC 2013*, pp. 1161–1165, Jun. 2013.
- [19] L. Gao, G. Iosifidis, J. Huang, and L. Tassiulasy, "Economics of Mobile Data Offloading," *Proc. Infocom workshop on SDP 2013*, pp. 3303–3308, Apr. 2013.
- [20] J. Lee, Y. Yi, S. Chong, and Y. Jin, "Economics of WiFi Offloading: Trading Delay for Cellular Capacity," *Proc. Infocom workshop on SDP 2013*, pp. 3309–3314, Apr. 2013.
- [21] S. Paris, F. Martignon, I. Filippini, and L. Chen, "A bandwidth trading marketplace for mobile data offloading," *IEEE Proc. INFOCOM 2013*, pp. 430–434, Apr. 2013.
- [22] X. Zhuo, W. Gao, G. Cao, and S. Hua, "An Incentive Framework for Cellular Traffic Offloading," to appear in *IEEE Trans. on Mobile Comput.*, available in IEEE explore digital library.
- [23] D. Fudenberg and J. Tirole, *Game Theory*. MIT Press, 1993.
- [24] T. Cover and J. Thomas, *Elements of Information Theory*. John Wiley, 1991.
- [25] X. Kang, Y. -C. Liang, A. Nallanathan, H. K. Garg, and R. Zhang, "Optimal power allocation for fading channels in cognitive radio networks: Ergodic capacity and outage capacity," *IEEE Trans. on Wireless Commun.*, vol. 8, no. 2, pp. 940–950, Feb. 2009.
- [26] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.
- [27] CVX Research, Inc. CVX: Matlab software for disciplined convex programming, version 2.0 beta. <http://cvxr.com/cvx>, Sept. 2012.

- [28] H. Inaltekin, and S. V. Hanly, "Optimality of Binary Power Control for the Single Cell Uplink," *IEEE Trans. on Inform. Theory*, vol. 58, no. 10, pp. 6484–6498, Oct. 2012
- [29] C. H. Papadimitriou, and K. Steiglitz, *Combinatorial optimization : algorithms and complexity*. Dover Publications Inc., 1998.
- [30] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: informationtheoretic and communications aspects," *IEEE Trans. Inform. Theory*, vol. 44, no. 6, pp. 2619–2692, Oct. 1998.
- [31] G. Caire, G. Taricco, and E. Biglieri, "Optimum power control over fading channels," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1468–1489, Jul. 1999.