Comments and Corrections

Corrections to and Comments on "Throughput and Optimal Threshold for FFR Schemes in OFDMA Cellular Networks"

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***Abstract*—In a recent paper published in these Transactions, Xu *et al.* (see ibid., vol. 11, no. 8, pp. 2776–2785, Aug. 2012) proposed an analytical framework to calculate the throughput and optimal threshold for fractional frequency reuse (FFR) schemes in orthogonal frequency division multiple access (OFDMA) cellular networks. This correspondence points out a problem in the original derivation of Xu *et al.* and how this affects the claimed performance of the maximum signal-to-interference-plus-noise ratio (MSINR) scheduling rule. A fix to this problem is presented in this work, which turns out to extend the applicability of the analysis beyond the case studied by Xu *et al.* and at the same time shows how the performance of the round robin (RR) scheduling rule can be obtained as a special case of the MSINR scheduler. Monte Carlo simulation results demonstrate the validity and merits of our proposed approach.**

***Index Terms*—Fractional frequency reuse (FFR), orthogonal frequency division multiple access (OFDMA), scheduling.**

I. Introduction

In [1], the throughput and optimal threshold for fractional frequency reuse (FFR) schemes in orthogonal frequency division multiple access (OFDMA) cellular networks were investigated. The authors proposed a theoretical framework to derive the average cell throughput for both the round robin (RR) and the maximum SINR (MSINR) scheduling strategies when users are uniformly distributed in the cell area. Furthermore, they exploited the dependence of these analytical expressions on the radius of the circumference separating the cell-center and cell-edge regions to gain some insight on the behaviour of the optimal threshold radius versus the number of active users in the system.

This correspondence points out a problem in the rationale behind the analytical framework derived by Xu *et al.* in [1] and how this affects the claimed performance for the maximum signal-to-interference-plus-noise ratio (MSINR) scheduling rule. A new analytical framework fixing this problem is presented, that could eventually be extended to the analysis of other frequency reuse strategies such as the soft frequency reuse or other cellular deployments such as a multi-tier network consisting of macro-, micro-and/or femto-cells. Despite the flaws in [1] we aim to fix in this correspondence, it is important to stress that the conclusions that Xu *et al.* presented in their paper were all derived from simulation results and, thus, they are essentially correct. In fact, in this paper we further show that simulation results exhibit an almost perfect match with theoretical results derived using our proposed analytical framework.

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II. System Model

In order to obtain an expression as general as possible for the average throughput of the system we will start our analysis assuming a general system model and then it will be particularized to the case analyzed in [1]. Let us consider the downlink of an FFR-aided OFDMA-based multi-cell system. The BS of interest, which is referred to as BS 0, is located at the origin and provides service to a fixed number M of users that are distributed independently at random over the coverage area. Due to the use of FFR, the coverage area of BS 0 is divided in two disjoint regions: the cell-center region, referred to as region C, and the cell-edge region, referred to as region E. Consequently, user m, located at polar coordinates $(d_{0,m}, \theta_{0,m})$, is classified either as a cell-center user or as a cell-edge user depending on whether $(d_{0,m}, \theta_{0,m})$ belong to region C or region E. Non-overlapping frequency bands are allocated to cell-center and cell-edge users. Cell-center users employ a frequency reuse factor equal to one with a set of F_C subcarriers, and cell-edge users employ a higher frequency reuse factor (e.g., three or seven) with a set of F_E subcarriers in the cell of interest (cell 0).

III. Average Throughput Calculation

The instantaneous signal-to-interference-plus-noise ratio (SINR) experienced by user m in cell 0 on the nth subcarrier can be expressed as

$$\mathrm{SINR}_{m,n} = \frac{P_L(d_{0,m}) \left| H^0_{m,n} \right|^2 P_{0,n}}{N_0 \Delta f + \sum_{j \in \mathcal{J}_m} P_L(d_{j,m}) \left| H^j_{m,n} \right|^2 P_{j,n}}, \quad (1)$$

where $P_L(d_{j,m})$ is used to denote the path loss between BS j and user m, $d_{j,m}$ is the distance between BS j and user m, Δf is the subcarrier spacing, $P_{j,n}$ is the power allocated by BS j to subcarrier n, N_0 denotes the noise power spectral density, and $\mathcal{J}_m$ denotes the set of interfering cells for user m that depends on whether user m has been classified as a cell-center or a cell-edge user. In the expressions of instantaneous SINRs for all users in cell 0, both the distance variables $\{d_{j,m}\}_{\forall\, j,m}$, and small-scale channel fading gains $\left\{ \left| H^j_{m,n} \right|^2 \right\}_{\forall\, m,n,j}$ are random variables. Furthermore, the number of cell-center and cell-edge users, M_C and $M_E = M - M_C$, respectively, are also random variables because users are randomly positioned on the cell. It is worth stressing that, assuming that the position of any interfering BS in $\mathcal{J}_m$ is fixed, the random distance $d_{j,m}$ can be expressed in terms of the random position of user m and thus, the random variable $\mathrm{SINR}_{m,n}$ is a function, among others, of the random variables $d_{0,m}$ and $\theta_{0,m}$.

From this point onwards, we will use A as a token to represent either the cell-center region C or the cell-edge region E. Let γ^A_n denote the SINR experienced by the user in region A that has been allocated subcarrier n. Our goal is tho determine the average throughput on this subcarrier, in bps/Hz, given by

$$\begin{aligned}\eta_n^A \triangleq& \mathbb{E}_{\gamma_n^A}\left[\log_2\left(1+\gamma_n^A\right)\right]\\ =&\sum_{k=1}^{M}\Pr\{M_A=k\}\mathbb{E}_{\gamma_n^A|M_A}\left[\log_2\left(1+\gamma_n^A\right)|M_A=k\right]\\ =&\sum_{k=1}^{M}\binom{M}{k}p_A^k(1-p_A)^{M-k}\\ &\times\log_2 e\int_0^\infty\frac{1-F_{\gamma_n^A|M_A}(t|k)}{1+t}\mathrm{d}t, \end{aligned} \tag{2}$$

where $\mathbb{E}_x[\cdot]$ denotes the expectation operation with respect to the random variable x, p_A is the probability that a given user is located in region A, and $F_{\gamma_n^A|M_A}(t|k)$ is the conditional cumulative distribution function (CDF) of γ_n^A conditioned on the event that there are $M_A=k$ users in region A. The average throughput of the system can be obtained as $\eta=F_C\eta_n^C+F_E\eta_n^E$, hence, the problem is that of determining $F_{\gamma_n^A|M_A}(t|k)$.

A. General Assumption

Let us define the random variable $\gamma_{m,n}^A$ as

$$\gamma_{m,n}^A=\begin{cases}\mathrm{SINR}_{m,n} & \text{if } (d_{0,m},\theta_{0,m})\in A,\\ 0 & \text{otherwise.}\end{cases} \tag{3}$$

For the analysis carried on in this paper, the following assumption is needed: on each subcarrier n, the random variables $\{\gamma_{1,n}^A,\gamma_{2,n}^A,\ldots,\gamma_{M,n}^A\}$ are independent and identically distributed (i.i.d.) according to the joint probability

$$P_{S_n,A}(t)\triangleq\Pr\left\{\mathrm{SINR}_{m,n}\le t,(d_{0,m},\theta_{0,m})\in A\right\}. \tag{4}$$

In other words, we assume that on each subcarrier in region A the users are statistically equivalent in terms of SINR.

B. MSINR Scheduler

When the MSINR scheduling strategy is used, for any subcarrier n allocated to users in cell-region A we have that

$$\gamma_n^{\text{MSINR-A}}=\max\left\{\gamma_{1,n}^A,\gamma_{2,n}^A,\ldots,\gamma_{M,n}^A\right\}. \tag{5}$$

Using the general assumption introduced in Section III-A, the conditional CDF of $\gamma_n^{\text{MSINR-A}}$, conditioned on the event that there are $M_A=k$ users in region A, can be readily evaluated as

$$\begin{aligned}F_{\gamma_n^{\text{MSINR-A}}|M_A}(t|k)&=\Pr\left\{\gamma_n^{\text{MSINR-A}}\le t\,\Big|\,M_A=k\right\}\\ &=\left(\frac{P_{S_n,A}(t)}{p_A}\right)^k=\left(F_{S_n|A}(t)\right)^k, \end{aligned}\tag{6}$$

where $F_{S_n|A}(t)\triangleq P_{S_n,A}(t)/p_A=\Pr\{S_n\le t|(d,\theta)\in A\}$ is used to denote the conditional distribution of the generic SINR S_n on subcarrier n, conditioned on the event that the user is in cell region A. This expression is general for any system sticking to the general assumption introduced in Section III-A and implementing the MSINR scheduling policy.

For the sake of analytical tractability, in [1] the cell shape is modelled as a circle with a radius R and a BS with an omnidirectional antenna is located at the cell center. Users are uniformly distributed in the cell, assuming that the minimum distance of a user from its desired BS is equal to R_0. Thus, the event space of the random variable d is restricted to the range $[R_0,R]$. Users at a distance less than the threshold R_{th} from their serving BS are classified as cell-center users, while those at a distance ranging between R_{th} and R are classified as cell-edge users. Furthermore, as stated by Maqbool *et al.* in [2], it is assumed that in a multi-cell network the instantaneous SINR experienced by a generic user located at (d,θ) varies very little with the polar angle and thus, it can be approximated as a function of the distance d to the BS. In this special case we have that $p_C=F_d(R_{th})$ and $p_E=1-p_C$, where $F_d(t)$ is the CDF of the distance of a user to BS 0. Furthermore, $F_{S_n|C}(t)=\Pr(S_n\le t|R_0\le d\le R_{th})$ and $F_{S_n|E}(t)=\Pr(S_n\le t|R_{th}\le d\le R)$ are the conditional distributions of the generic SINR S_n on subcarrier n conditioned on the event that the user is in cell region C or cell region E, respectively. Substituting these results in (2) we obtain an expression that is different from that presented by Xu *et al.* in [1, eqs. (32)–(34)] and the reason is basically because in the analysis presented in [1, eqs. (9)–(16)] the conditioning is only performed on the position of the tagged user rather than on the positions of all users served by BS 0.

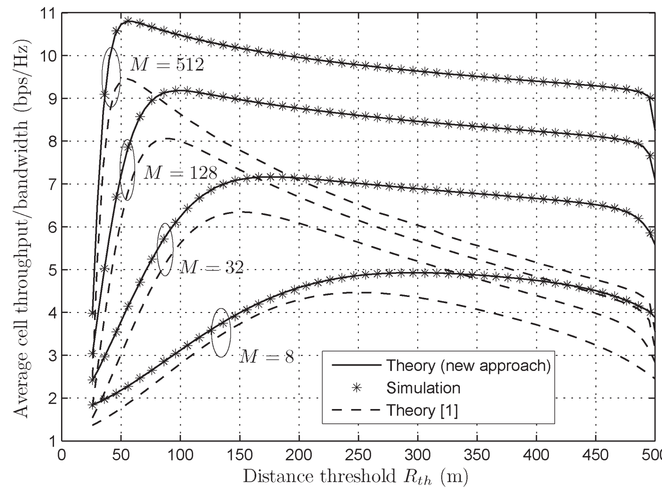

Fig. 1. Average cell throughput versus the distance threshold with the number of users per cell as parameter.

C. RR Scheduler

Since the user SINR's are statistically equivalent on each subcarrier, serving M_A users in the cell region A using a RR scheduling policy is equivalent to serving $M_A=1$ users with MSINR (even when users are selected with non uniform probability). Therefore, $F_{\gamma_n^{\text{RR-A}}|M_A}(t|k)=F_{S_n|A}(t)$ and the results in [1, (26)–(28)] follow.

IV. Results and Discussion

In this section, numerical results obtained using our analytical approach are compared to those obtained using the analytical approach presented in [1]. Monte Carlo simulation results are also provided in order to validate our analytical framework. A cellular network with 19 cells (see [1, Fig. 1]) is considered and the main system parameters are listed in [1, Table I].[1]

Fig. 1 illustrates numerical and simulation results for the average cell throughput as a function of the distance threshold and for different numbers of active users in the cell. Results have been obtained using an MSINR scheduling strategy and a cell-edge frequency reuse factor equal to three. Simulation results have been obtained by averaging over

[1]Noise amplifier gain has been neglected as it does not explicitly appear in any of the equations of the theoretical model. Furthermore, a receiver noise figure of 7 dB has been considered as is typically done in simulations based on 3GPP LTE standards.

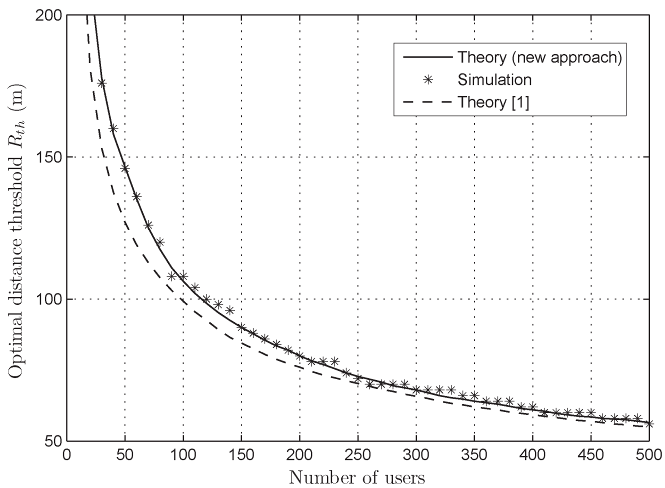

Fig. 2. Optimal distance threshold versus the number of users.

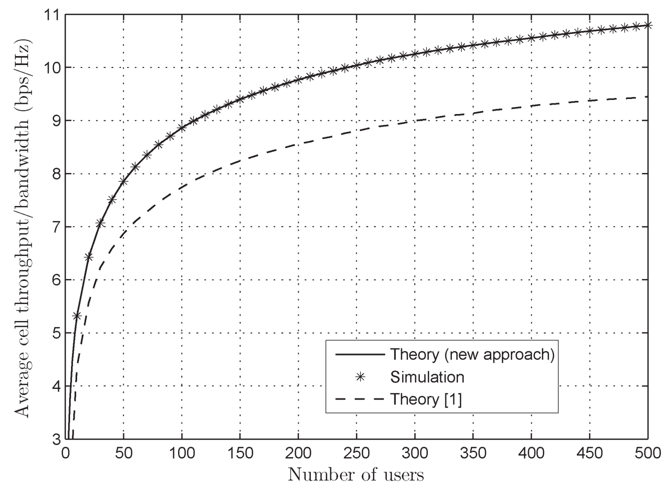

Fig. 3. Average cell throughput with the optimal distance threshold versus the number of users.

10 000 Monte Carlo trials. It can be observed, first, that both analytical approaches provide very different results and second, that they predict quite different values for the optimal FFR distance threshold R_{th}. As expected, simulation results exhibit an almost perfect match with numerical results derived using our proposed analytical framework and serve to validate the correctness of our approach in front of that proposed by Xu *et al.* in [1].

In order to stress the importance of the proposed analytical approach, Figs. 2 and 3 show, respectively, the optimal distance threshold and optimal average throughput versus the number of users per cell, for a system using an MSINR scheduler and a frequency reuse factor equal to three for the cell-edge users. Again, it can be observed that the predicted optimal distance thresholds and average throughput values using our analytical approach have an almost perfect match with results obtained through Monte Carlo simulations and differ considerably from those predicted using the analytical approach presented in [1]. In fact, differences between the optimal distance threshold obtained through simulation and theoretical analysis are basically due to that, in order to keep complexity to affordable levels, a distance threshold resolution of two meters was used to obtain the simulation results.

ACKNOWLEDGMENT

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