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Resource Allocation in Non-Orthogonal and Hybrid Multiple Access System with Proportional Rate Constraint

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Abstract—Non-orthogonal multiple access (NOMA) has attracted a lot of attention recently due to its superior spectral efficiency and could play a vital role in improving the capacity of future networks. In this paper, a resource allocation scheme is developed for a downlink multi-user NOMA system. An optimization problem is formulated to maximize the sum rate under the total power and proportional rate constraints. Due to the complexity of computing the optimal solution, we develop a low complexity sub-optimal solution for two-user scenario and then extend it to the multi-user case by proposing a user-pairing approach as well as a number of power allocation techniques that facilitate dealing with a large number of users in NOMA system. Simulation results support the effectiveness of the proposed approaches and show the close performance to the optimal one. In addition, we propose a new hybrid multiple access technique that combines the properties of NOMA and the orthogonal frequency division multiple access (OFDMA). Simulation results show that the proposed hybrid method provides better performance than NOMA in terms of the overall achievable sum rate and the coverage probability.

Index Terms—Orthogonal Frequency Division Multiple Access (OFDMA), Non-Orthogonal Multiple Access (NOMA), hybrid multiple access, vertical pairing, sum rate maximization.

I. INTRODUCTION

DEVELOPING the air interface techniques represents one of the ways to enhance the capacity of future networks to meet the predicted high mobile data demand. In 4G networks, orthogonal multiple access is used as the air interface technique and proved its effectiveness against multi-path fading and achieving high system throughput. However, it does not make full use of the spectral resource as it restricts each user to use a limited part of the spectrum, which makes it insufficient to enhance the spectral efficiency and the capacity of future networks [1]–[5]. Recently, non-orthogonal multiple access (NOMA) is presented as a promising radio access scheme for further capacity enhancement and to accommodate the future traffic demand.

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NOMA has been presented in the literature as a mean to enhance the spectral efficiency for future networks and also confirmed to be superior to OFDMA. For instance, [6]–[9] presented a comparison between orthogonal and non-orthogonal multiple access techniques. In addition, the authors in [10] presented how NOMA could enhance the cell-edge user throughput while achieving fairness. In [11], [12] the same authors investigated the gains that relay assisted NOMA transmission could offer over the conventional NOMA system. In addition, a number of power allocation techniques have been proposed in [13]–[16], but none of the proposed techniques was in closed form. Furthermore, MIMO-NOMA system was also considered in [17]–[19], where [17] suggested that NOMA with opportunistic beamforming (OBF) offers promising results. However, the authors mentioned that the main issues related to OBF-NOMA are transmission power allocation and user-scheduling. In [18], the ergodic capacity of MIMO-NOMA is investigated. The author highlighted how NOMA significantly outperforms time division multiple access (TDMA) in terms of the ergodic capacity. The authors in [19] focused on sum rate maximization for MIMO-NOMA system. The provided results show that NOMA is much better than TDMA. However, in both [18] and [19] the presented scenario considered only two users.

Cooperative NOMA was considered in [20] and shows that it achieves maximum diversity gain for all multiplexed users. The importance of fairness for NOMA is also considered in [21], where the authors suggested that improving the performance of the worst-condition user will make NOMA significantly outperforms its orthogonal counterparts. Another work on NOMA-receiver design for downlink system is investigated in [22]. A dynamic fractional frequency reuse scheme for NOMA system was assessed in [3]. A NOMA system with random beam-forming was also presented in [23]. The author also proposed intra-beam superposition coding and intra-beam SIC for NOMA system, but the power allocation aspect was not taken into account. The authors in [24]–[26] applied NOMA with visible light communication (VLC) system to boost the throughput in VLC downlink networks. The authors in [27] investigated the performance of NOMA in relay-MIMO system. A closed form expression for the outage probability was derived to show the privilege of NOMA over orthogonal multiple access schemes. However, no power allocation was

considered in that work. Additionally, the authors in [28] addressed the optimal policy of joint subcarrier and power assignment for multi-carrier (MC) NOMA systems. They first formulated the design of resource allocation algorithm as a non-convex optimization problem to maximize the system throughput, and then solved it using monotonic optimization. The system performance is compared to conventional MC orthogonal multiple access (OMA) and the results showed the advantage of MC-NOMA over MC-OMA. However, in the considered system, a maximum of only two-users were allowed to share each subcarrier which does not exploit the full potential of NOMA.

All of these works confirm the significant performance gain that NOMA offers over orthogonal multiple access. They also show the importance of NOMA for 5G networks.

A. Main Contribution

In this paper, we investigate the resource allocation problem for downlink multi-user NOMA system. An optimization problem is formulated to maximize the sum rate under the total transmit power and proportional minimum user rate constraints. Considering the proportional rate constraint is a key contribution of our work and differentiate us from existing resource allocation methods for NOMA. Not only does this constraint ensure fairness between users, it is crucial in NOMA. Firstly for NOMA, the weaker users will have to detect their signals by treating the stronger users as interferers. The stronger users will also need to detect the weaker users' signals first and remove them before they detect their own signals. In practice, this requires sufficient power allocated to the weaker users for such detection to be successful. This can be achieved by a proportional rate constraint. Secondly, according to the rate boundary of NOMA [29], NOMA achieves the highest performance gain over orthogonal multiple access (OMA) when the weaker users achieve a good rate. Therefore, simply achieving a high rate for the stronger users as in conventional minimum rate constraints will not fully utilize the potential of NOMA and may also be impractical. Thus, we considered proportional fairness constraint in our work. We first derive two closed-form sub-optimal solutions for a two-user case as obtaining the optimal solution for NOMA requires high complexity numerical operations. The closed-form solution is shown to achieve performance that is close to the optimal one and better performance than all existing techniques. However, the solution is restricted to two users only. We then extend the obtained solution for a larger number of users by proposing a subband-based approach whereby two users are multiplexed into each subband. However, splitting the whole bandwidth into subbands cannot fully utilize the potential of NOMA, where the entire bandwidth can be occupied by all users. Thus, we propose a vertical pairing concept where users are grouped in pairs and allowed to occupy the entire bandwidth. The pairs are then multiplexed in the power domain using a modified solution from the obtained two-users sub-optimal one. Moreover, a low complexity power allocation scheme is proposed that allocates power to each resource block (RB) in proportion to the sum of channel power of all

multiplexed users. This facilitates the sum rate optimization of NOMA with a large number of users. In addition, this paper discusses the idea of hybrid multiple access, which represents a combination between NOMA and OFDMA, as a good candidate for the next generation wireless networks. Simulation results are provided to confirm the superiority of the proposed NOMA-power allocation schemes over the existing ones, as well as the superiority of the proposed hybrid multiple access scheme over conventional NOMA.

The rest of this paper is organized as follows. Section II discusses the system model of the NOMA system. Section III presents the formulated sum rate maximization problem and the obtained sub-optimal solution for two-user scenario, as well as the closed-form power allocation techniques. In Section IV, extension to a multi-user scenario is presented along with the pairing concept and other power allocation techniques. Section V presents the proposed hybrid multiple access technique with RBs classification and power allocation approaches. The simulation scenarios and results are presented in Section VI, and finally, Section VII concludes the paper.

II. SYSTEM MODEL

A downlink subcarrier based NOMA system in a single cell of U users is considered. The total available bandwidth W_T is divided into S RBs; each occupying a bandwidth of B_s and has N_c subcarriers. The total transmission power is set to P_t . Fig. 1 illustrates the spectral occupancy of a two-user scenario of a) OFDMA and b) NOMA system. The horizontal axis denotes the bandwidth in terms of RBs, and the vertical axis is the power allocated for each resource. For OFDMA, the RBs are exclusively allocated to one of the users and the power allocated to user 1 and 2 at the s -th RB is $P_s^{(1)}$ and $P_s^{(2)}$ respectively. For NOMA, both users occupy all the RBs and the user with a better channel at the s -th RB will be allocated the power $P_s^{(H)}$, and the weaker one with the power $P_s^{(L)}$.

At the transmitter side of NOMA, the users are multiplexed in the power domain and are being separated by successive interference cancellation (SIC) at the receiver side. The mechanism behind SIC is that the user with weak channel conditions treats the signal of the user with the better channel as noise and decodes its own data from the received signal. On the other hand, the user with the better channel performs SIC, where it decodes the data of the weaker user and then proceeds to subtract it from the received signal and decode its own data [14].

For two users multiplexed over the s -th RB using NOMA principles, the achievable rate by the user with the higher (H) and lower (L) channel gain is respectively given by

$$R_s^{(H)} = B_s \log_2 \left(1 + \gamma_s^{(H)} \right) \quad (1)$$

$$R_s^{(L)} = B_s \log_2 \left(1 + \gamma_s^{(L)} \right) \quad (2)$$

and their sum rate over that RB is given by

$$R_s = R_s^{(H)} + R_s^{(L)} \quad (3)$$

It must be noted that the superscripts (H) and (L) are not fixed to user 1 and 2 and are assigned according to the channel gains

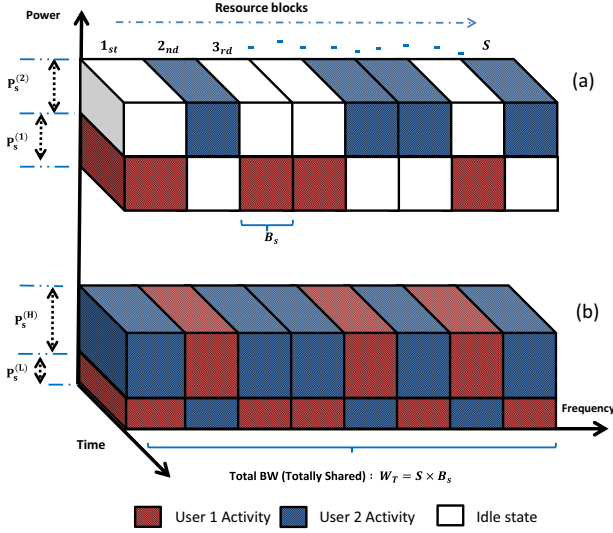


Fig. 1. Illustration of RB allocation for: (a). OFDMA (b). NOMA

of the two users in each RB. In other words, we do not assume that user 1 always have a better channel gain than user 2 over all RBs. In addition, the terms

$$\gamma_s^{(H)} = \frac{P_s^{(H)} |h_s^{(H)}|^2}{B_s N_0} \quad (4)$$

$$\gamma_s^{(L)} = \frac{P_s^{(L)} |h_s^{(L)}|^2}{P_s^{(H)} |h_s^{(L)}|^2 + B_s N_0} \quad (5)$$

represent the received SINR of the two users in the s -th RB, where $|h_s^{(H)}|^2 = \frac{\xi^{(H)} |H_s^{(H)}|^2}{PL^{(H)}}$ and $|h_s^{(L)}|^2 = \frac{\xi^{(L)} |H_s^{(L)}|^2}{PL^{(L)}}$ represent the channel powers which include the effect of fading, the Log-normal shadowing factor ξ , and the path loss effect given by $PL = PL_0 + 10v \log_{10} \left(\frac{d}{d_0} \right)$ [dB], where PL_0 represents the path loss at reference distance (d_0) and d stands for the distance between the user and the serving base station with v being the path loss exponent. In addition, N_0 denotes the noise power spectral density.

III. PROBLEM FORMULATION AND SOLUTION FOR TWO-USER SCENARIO

The optimization problem is first formulated to maximize the sum rate of the two-user NOMA system, and the solution is then generalized to the multi-user case. In order to guarantee that each user is able to achieve its target data rate, the optimization will include nonlinear constraints of proportional fairness.

Mathematically, the problem of the two-user scenario is formulated as

$$\text{maximize} \quad R \quad (6)$$

$$\text{Subject to} \quad \sum_{s=1}^S (P_s^{(H)} + P_s^{(L)}) \leq P_t \quad (7)$$

$$P_s^{(H)}, P_s^{(L)} \geq 0, \forall s \quad (8)$$

$$\sum_{s=1}^S R^{(H)} : \sum_{s=1}^S R^{(L)} = \Phi_{min}^{(1)} : \Phi_{min}^{(2)}. \quad (9)$$

where R denotes the sum rate over all the RBs, and it is given by $R = \sum_{s=1}^S R_s$, constraints (7) and (8) are to guarantee a positive allocated power and limited by the maximum allowable P_t . In addition, the minimum rate proportional fairness constraint (9) is to control the achievable throughput by all users where $\Phi_{min}^{(1)}$ and $\Phi_{min}^{(2)}$ are the minimum rate requirements for user 1 and user 2, respectively. Constraint (9) helps to maintain proportionality between the minimum achievable rates of the users. In other words, the proportionality is guaranteed for the minimum achievable rates, but the rate obtained in the solution is not restricted to this ratio, as long as the minimum rates for all users are satisfied. It is important to point out that by using the proportional fairness constraint, once the minimum rates for all users are satisfied, the remaining resources will also be allocated in a proportional manner. Such approach is important to maintain fairness in distributing the radio resources among these users and to ensure that the weak users have enough power to decode their own data from the received signal while treating the stronger users as noise, and to ensure that the stronger users have enough power to apply SIC and cancel the effect of the weak users and detect their own data. Without this constraint, the maximum sum rate could simply be achieved by allocating all the bandwidth and power to one user or a few users who have the best channel conditions and not all users will be allowed to transmit. In addition, another important property of this constraint is that it can utilize the potential advantage of NOMA over OMA [29]. The minimum rate requirement is assigned to each user based on the large scale fading factor (the distance based path loss and the Log-normal shadowing factor) experienced by that user in addition to the small scale fading effects. Since path loss and shadowing is more dominant and vary slowly, the proportionality constraint is therefore effectively more long term rather than short term.

A. Equal RB Power Allocation (ERPA)

Solving the formulated problem in (6) to (9) requires a numerical solution or some iterative algorithm for suboptimal solution. Therefore, we first propose a low complexity sub-optimal approach that allocates the power equally among all the RBs. In other words, the total transmission power in each RB is set to be

$$P_{RB} = P_s^{(L)} + P_s^{(H)} = \frac{P_t}{S}. \quad (10)$$

Using this assumption along with the optimization steps that are depicted in Appendix A, the sub-optimal power for the strong user is found to be

$$P_s^{(H)} = - \frac{(|h_s^{(H)}|^2 + |h_s^{(L)}|^2) B_s N_0}{2|h_s^{(H)}|^2 |h_s^{(L)}|^2} + \frac{\psi_3 \sqrt{B_s N_0}}{2|h_s^{(H)}|^2 |h_s^{(L)}|^2 \sqrt{\Gamma_1}} \quad (11)$$

while that of the weaker user is

$$P_s^{(L)} = \frac{2|h_s^{(H)}|^2 |h_s^{(L)}|^2 P_{RB} + (|h_s^{(H)}|^2 + |h_s^{(L)}|^2) B_s N_0}{2|h_s^{(H)}|^2 |h_s^{(L)}|^2} - \frac{\psi_3 \sqrt{B_s N_0}}{2|h_s^{(H)}|^2 |h_s^{(L)}|^2 \sqrt{\Gamma_1}} \quad (12)$$

where $\Gamma_1 = 2^{\frac{1}{\Phi_{min}^{(1)}}}$ and $\Gamma_2 = 2^{\frac{1}{\Phi_{min}^{(2)}}}$.

It is worth mentioning that the superscripts (H) and (L) are included just to distinguish the parameters of the users with the better channel gain from those with weaker channel gains at the s -th RB and not over all RBs. It also does not necessarily mean that $P_s^{(H)}$ is higher than $P_s^{(L)}$, where it could be less than or equal to $P_s^{(L)}$ depending on the final values from the proposed closed form solutions.

B. Average Channel Based Power Allocation (ACPA)

While the complexity of the proposed ERPA method is significantly lower than that of the optimal one, it still needs S times of calculations for each power allocation step. In order to achieve a further simplification, we propose the ACPA scheme that depends on the average channel power of each user across the entire bandwidth for power allocation. In other words, the average channel power of the strong and the weak user is determined by, $G_H = \frac{\sum_{s=1}^S |h_s^{(H)}|^2}{S}$ and $G_L = \frac{\sum_{s=1}^S |h_s^{(L)}|^2}{S}$, respectively, and these values will be used to determine the power to be allocated to the respective user. Applying this approach to (11) and (12), the sub-optimal power for the strong user is given by

$$P_s^{(H)} = -\frac{(G_H + G_L) B_s N_0}{2G_H G_L} + \frac{\sqrt{B_s N_0} \sqrt{4\frac{\psi_2}{B_s N_0} G_L P_t + 4\psi_2 + B_s N_0 (G_H - G_L)^2 \Gamma_1}}{2G_H G_L \sqrt{\Gamma_1}} \quad (13)$$

and for the weak user is

$$P_s^{(L)} = \frac{2G_H G_L P_t + (G_H + G_L) B_s N_0}{2G_H G_L} - \frac{\sqrt{B_s N_0} \sqrt{4\frac{\psi_2}{B_s N_0} G_L P_t + 4\psi_2 + B_s N_0 (G_H - G_L)^2 \Gamma_1}}{2G_H G_L \sqrt{\Gamma_1}}. \quad (14)$$

This method offers simplicity over the ERPA method, and will also be compared to the optimal solution in Section VI.

IV. MULTI-USER NOMA WITH VERTICAL PAIRING CONCEPT

The closed form solution obtained in Section III allows low complexity implementation of power allocation for two-user NOMA. To extend the applicability of these closed form solutions to a multi-user case, we propose a vertical pairing concept which group the users in pairs as shown in Fig. 2. We denote the total number of pairs to be Z , and the pairs are arranged in an ascending order according to their channel powers from the bottom (first pair is the weakest) to the top (the Z -th pair is the strongest). The transmitted signal at the s -th RB is given by

$$X_s = \sum_{j=1}^Z \left(\sqrt{P_{j,s}^{(H)}} X_{j,s}^{(H)} + \sqrt{P_{j,s}^{(L)}} X_{j,s}^{(L)} \right) \quad (15)$$

which includes the information intended for all users whom will share the same time-frequency resource, where $X_{j,s}^{(H)}$ and $X_{j,s}^{(L)}$ are the superposition coded information bearing

signal intended for the strong and weak user in the j -th pair respectively with $P_{j,s}^{(H)}$ and $P_{j,s}^{(L)}$ as the corresponding transmission power. The total power for this j -th pair at s -th RB is denoted as $P_{j,s} = P_{j,s}^{(H)} + P_{j,s}^{(L)}$. It is worth mentioning that the transmitted signals with vertical pairing has exactly the same form as that of the conventional NOMA, as it is effectively a summation of all superposition coded signals from all users. The received signal by the stronger user of the z -th pair at the s -th RB is given by

$$Y_{z,s}^{(H)} = X_s h_{z,s}^{(H)} + n_{z,s}^{(H)} \quad (16)$$

where $h_{z,s}^{(H)}$ represents the channel power between the BS and the strong user and $n_{z,s}^{(H)}$ represents the additive white Gaussian noise. The expression for the weaker user in this pair is similar to (16) but with the superscript (H) as (L) . From (16), it is clear that each user will receive a signal with its data and those intended for other users.

Fig. 2 illustrates NOMA structure with vertical pairing concept. Starting from the bottom of Fig. 2, the weakest user in the first pair will not perform SIC while the better user in this pair will perform SIC only to its partner in this pair. At the top of Fig. 2, on the other hand, the weaker user of the Z -th pair (the strongest pair) will perform SIC to all of the previous pairs, while the strongest user at this pair will perform SIC to all of the previous pairs and its partner as well. Assuming perfect decoding, Fig. 3 depicts the SIC process for four users (two pairs: $Z = 2$) along with the pairing concept. It must be noted that the SIC detection process has the same principles as that in conventional NOMA. The key difference is in the power allocation procedure, which is described in the following subsection. It must be noted that the purpose and the procedure of pairing in our proposed approach is different to conventional pairing in NOMA [16], [28]. Driven by the high complexity of the power allocation algorithms, conventional NOMA pairing approaches allocate two users into a RB (or a subband) such that the algorithmic complexity can be lowered. However such a horizontal approach will not fully exploit the potential of NOMA, which benefits from using all bandwidth for all users. On the other hand for our vertical pairing approach, since the power allocation solution for two-user case is in closed form, complexity is not an issue and we can expand to cases with more than two users to exploit the capacity improvements.

The proposed vertical pairing structure allows the use of the solutions in (11) and (12) to allocate power for the users within each pair. However, the power allocation across the pairs have to be determined. For the case of U users, the sum rate of all possible pairs ($Z = \frac{U}{2}$) is given by

$$R = B_s \sum_{z=1}^Z \sum_{s=1}^S \left(\log_2 \left(1 + \gamma_{z,s}^{(H)} \right) + \log_2 \left(1 + \gamma_{z,s}^{(L)} \right) \right) \quad (17)$$

where the terms

$$\gamma_{z,s}^{(H)} = \frac{P_{z,s}^{(H)} |h_{z,s}^{(H)}|^2}{I_s |h_{z,s}^{(H)}|^2 + B_s N_0} \quad (18)$$

$$\gamma_{z,s}^{(L)} = \frac{P_{z,s}^{(L)} |h_{z,s}^{(L)}|^2}{\left(I_s + P_{z,s}^{(H)} \right) |h_{z,s}^{(L)}|^2 + B_s N_0} \quad (19)$$

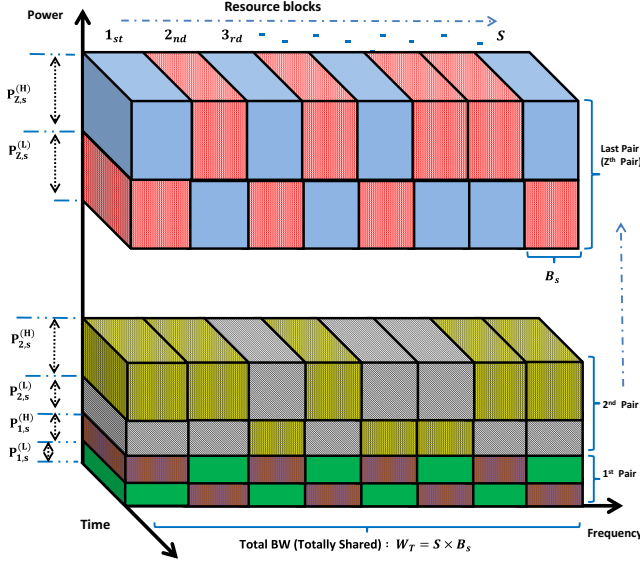


Fig. 2. NOMA structure with vertical pairing concept.

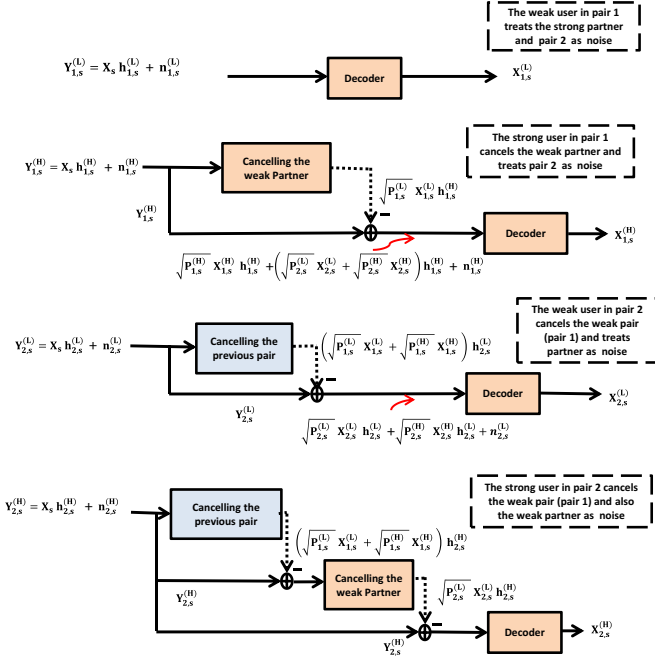


Fig. 3. Illustration of the SIC process with perfect decoding in vertically paired NOMA with four users (two pairs: $Z = 2$), where the users within the pairs are arranged in an increasing order.

are the received SINR of the strong and weak user in the z -th pair at the s -th RB, respectively, and I_s represents the power allocated to the preceding stronger pairs at the same s -th RB, and it is given by

$$I_s = \begin{cases} 0 & z = Z \\ \sum_{k=z+1}^Z P_{k,s}^{(H)} + P_{k,s}^{(L)} & 1 \leq z \leq (Z-1). \end{cases} \quad (20)$$

The optimal solution to maximize the sum rate in (17) with total power and minimum rate constraints requires complex

numerical solutions. However if the total transmission power for each pair is known, the optimization problem in (6)-(9) can be used to determine the optimal power allocation for the strong and weak user in each pair. Since there is now other pairs for each RB, the interference will affect the optimal solution. The ones in (11) and (12) will now be given by (21) and (22). In the following, we propose a low complexity power allocation approach for this pairing-based scheme, and also several simple power allocation schemes for comparisons.

A. Hierarchical Pairing Power Allocation (HPPA)

The key to the low complexity closed form solutions in (11) and (12) is to have only two candidates to allocate the power to. In here, the power will be allocated to the vertically paired users in hierarchical manner based on their channel powers. By splitting the users into two groups and combining their channel powers, the above pair-based power allocation solution can be modified to obtain the power allocated for each group. Following the concept of NOMA, the stronger half of the users will form one group, and the weaker half will form the other. In this scheme, the terms $|h_s^{(H)}|^2$ and $|h_s^{(L)}|^2$ in (21) and (22) will become the sum of all channel powers in the stronger and weaker group respectively.

Once the power for the two groups are determined, the same procedure can be repeated to each group as a second stage; i.e., two subgroups are formed in each group of the previous stage. This multiple stage approach is repeated until the subgroups become pairs of users, and the solution (11) and (12) can be used directly.

To better illustrate the procedure, we consider an example of 8 users at the s -RB and the multistage process is shown in Fig. 4. The application of this scheme is as follows:

- First Stage (2 groups each has 4 users): Divide the users into two groups with G_{g1} and G_{g2} as the sum of the users channel powers in each group, which will be used to replace $|h_s^{(H)}|^2$ and $|h_s^{(L)}|^2$, respectively, in (11) and (12). With the total available power being the same as the total transmission power per each RB (P_{RB}), apply the obtained sub-optimal solutions in (11) and (12) to find the power across the two groups to be P_{g1} and P_{g2} .
- Second Stage (4 subgroups of 2 users): By dividing each group of users further into 2 subgroups we will have four pairs of users, and each pair with a combined channel power of $G_{g1,1}$, $G_{g1,2}$, $G_{g2,1}$ and $G_{g2,2}$. Using (11) and (12) with P_{g1} and P_{g2} as the total power for each subgroup, the power across each pair could be found easily as P_1 , P_2 , P_3 , and P_4 .
- Third Stage (8 users): This is the user level stage, where the sub-optimal solution of the ERPA method in (11) and (12), along with P_1 , P_2 , P_3 , and P_4 used to replace P_{RB} , are used to allocate the power for all users within the pairs.

It must be noted that the number of users in the proposed HPPA approach can be any positive integer and not mandatory to be in the power of two. In the case of an odd number of users, the number of users per each group can be unequal and one group can have one user more than the other group. The

$$\begin{aligned}
P_{z,s}^{(L)} &= \frac{(B_s N_0 (|h_{z,s}^{(H)}|^2 + |h_{z,s}^{(L)}|^2) + 2|h_{z,s}^{(H)}|^2 |h_{z,s}^{(L)}|^2 P_{z,s})}{2|h_{z,s}^{(H)}|^2 |h_{z,s}^{(L)}|^2} \\
&\sqrt{\frac{4 \left(|h_{z,s}^{(H)}|^2 \right)^2 \left(|h_{z,s}^{(L)}|^2 \right)^2 I_s P_{z,s} \Gamma_2 + 4\psi_2 (|h_{z,s}^{(H)}|^2 I_s + |h_{z,s}^{(L)}|^2 P_{z,s}) + B_s N_0 \left(\frac{\psi_1}{\Phi_{min}^{(H)} \Phi_{min}^{(L)}} + 4\psi_2 \right)}{2|h_{z,s}^{(H)}|^2 |h_{z,s}^{(L)}|^2 \sqrt{\Gamma_1}}} \quad (21)
\end{aligned}$$

$$\begin{aligned}
P_{z,s}^{(H)} &= \frac{\left(\left(|h_{z,s}^{(H)}|^2 \right)^2 \left(|h_{z,s}^{(L)}|^2 \right)^2 I_s \Gamma_2 (I_s \Gamma_2 + 4P_{z,s}^{(L)} \Gamma_1) + (B_s N_0)^2 \left(\left(|h_{z,s}^{(H)}|^2 - |h_{z,s}^{(L)}|^2 \right) \Gamma_1 + |h_{z,s}^{(L)}|^2 \Gamma_2 \right)^2 \right)}{2|h_{z,s}^{(H)}|^2 |h_{z,s}^{(L)}|^2 \Gamma_1} + \\
&\frac{2\psi_2 \left(\left(|h_{z,s}^{(H)}|^2 - |h_{z,s}^{(L)}|^2 \right) I_s \Gamma_1 + |h_{z,s}^{(L)}|^2 I_s \Gamma_2 + 2|h_{z,s}^{(L)}|^2 P_{z,s}^{(L)} \Gamma_1 \right)^{1/2}}{2|h_{z,s}^{(H)}|^2 |h_{z,s}^{(L)}|^2 \Gamma_1} + \\
&\frac{|h_{z,s}^{(H)}|^2 (|h_{z,s}^{(L)}|^2 I_s (\Gamma_2 - 2\Gamma_1) - \Gamma_1 B_s N_0) + |h_{z,s}^{(L)}|^2 B_s N_0 (\Gamma_2 - \Gamma_1)}{2|h_{z,s}^{(H)}|^2 |h_{z,s}^{(L)}|^2 \Gamma_1} \quad (22)
\end{aligned}$$

same steps are followed repeatedly until the power is allocated to all users.

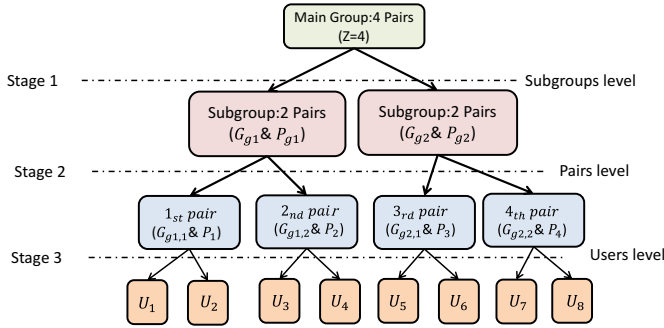


Fig. 4. Illustration of the power allocation stages in HPPA method for 8 users.

B. Equal-per-Pair Power Allocation

In order to demonstrate the advantage of the HPPA method, a number of trivial approaches will be used to compare against its performance. The first approach is to arrange the users in a descending order of channel power (the user with the highest channel power is at the top and the one with the lowest channel power at the bottom as depicted in Fig. 2) and pair every two consecutive users (i.e., users with similar channel powers are paired together). Then the power are allocated equally for all pairs so that the total transmission power is divided by the number of pairs in all RBs, and is given by

$$P_{eq} = \frac{P_{RB}}{Z}. \quad (23)$$

where P_{eq} represents the total power allocated for each pair at each RB. Next, the suboptimal approaches will be used based on (21) and (22) to obtain the power for each user within the pairs. The advantage of this approach over HPPA is simplicity because (21) and (22) are used only once per pair.

C. Proportional-per-Pair Power Allocation

The second approach is to determine the transmission power for each pair in proportion to the combined channel power of

the paired users. At the s -th RB, the sum of the channel powers at the z -th pair is denoted as $M_{z,s} = |h_{z,s}^{(H)}|^2 + |h_{z,s}^{(L)}|^2$. The power allocated to each pair at the s -th RB is denoted as P_s and it could be obtained by

$$P_s = \frac{P_{RB} M_{z,s}}{\sum_{z=1}^Z M_{z,s}}. \quad (24)$$

The benefit of using (24) is that each pair will be allocated a power that is proportional to its channel power, which would help in maximizing the user rates. After deciding the power across each pair, the power within the pairs will be allocated using (21) and (22).

D. Subband-based ERPA

This method is applied by dividing the whole spectrum into subbands and then NOMA is applied within each subband. A maximum of two users (one user pair) are multiplexed per each subband (horizontal pairing) and their power is allocated using the ERPA scheme. Due to the simplicity of this method, it is less spectral efficient than others as not all of the RBs will be shared among the users.

It must be noted that the last three approaches do not guarantee satisfying the minimum rate criterion, and so are only for comparative purposes.

V. THE PROPOSED HYBRID SYSTEM

In spite of the difference between OFDMA and NOMA in the sense of orthogonality between users, their combination could enhance the capacity. While orthogonal access assigns part of the spectrum to each user, multiplexing the users in a non-orthogonal manner offer fairness among these users in terms of the achievable throughput as they are all allowed to use the whole bandwidth regardless of their channel conditions [10], [29]. However, the difference between the channel gains of cell center and cell edge users could be significant and hence applying NOMA to the entire spectrum may not be beneficial. Thus we propose a hybrid multiple access scheme such that part of the spectrum are reserved for orthogonal

access (dedicated) and the rest (shared) are for all users by NOMA. An illustration of this hybrid scheme in a two user case is depicted in Fig. 5. As an example from this figure, the first RB is dedicated for user 1 while the S -th RB is dedicated for user 2; on the other hand, both users shared the second RB. This gives the hybrid method the advantage of, firstly, being less susceptible to interference and requiring less SIC process than NOMA, and secondly more spectral efficient than OFDMA since some users could share more spectrum as compared to the purely orthogonal case.

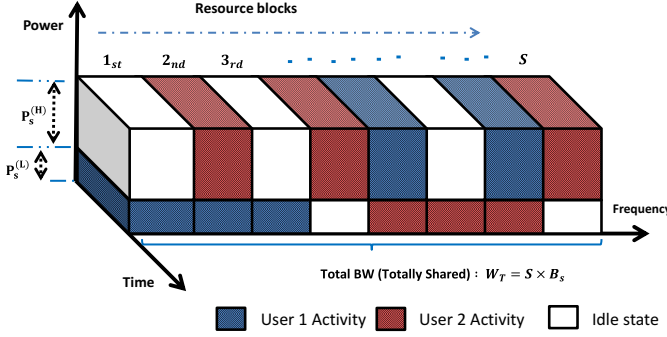


Fig. 5. Structure of the hybrid orthogonal - non orthogonal scheme.

For this hybrid scheme, the optimization problem will have to determine which RB is allocated to the orthogonal and non-orthogonal counterparts. In addition, the RBs classified for orthogonal transmission will also have to be allocated to the users. Moreover, the power allocation will also have to be optimized. The global optimal solution will involve high complexity numerical computation and thus we propose a low complexity multi-stage sub-optimal approach, where RB allocation is first performed assuming equal power allocation, followed by the power allocation approach that has been mentioned in the earlier sections.

A. RB Allocation and Classification

To classify whether a RB should be used for orthogonal or non-orthogonal transmission, the respective achievable rates are first computed and the best one will be selected. For the orthogonal case, the achievable rate of each user over the s -th RB is calculated using

$$R_{orth,s} = B_s \log_2 \left(1 + \frac{P_{RB} |h_s^{(u)}|^2}{B_s N_0} \right). \quad (25)$$

On the other hand, the non-orthogonal sum rate of all U users over the s -th RB is given by

$$R_{non,s} = B_s \log_2 \left(1 + \frac{\beta_{\tilde{u},s} P_{RB} |h_s^{(\tilde{u})}|^2}{B_s N_0} \right) + B_s \sum_{u=1, u \neq \tilde{u}}^U \log_2 \left(1 + \frac{\beta_{u,s} P_{RB} |h_s^{(u)}|^2}{B_s N_0 + \sum_{m=u+1}^U \beta_{m,s} P_{RB} |h_s^{(u)}|^2} \right) \quad (26)$$

where $\tilde{u} \in \{1, 2, \dots, U\}$ represents the index of the users who has the strongest channel power at the s -th RB, $\beta_{u,s} =$

$\frac{|h_s^{(u)}|^2}{\sum_{j=1}^U |h_s^{(j)}|^2}$ refers to the power allocation factor which al-

ways has a positive quantity and of $\sum_{u=1}^U \beta_{u,s} \leq 1$, and P_{RB} stands for the total power per each RB which will be assumed to be equal for all RBs during the allocation process and it is calculated as $P_{RB} = \frac{P_t}{S}$, and finally $|h_s^{(u)}|^2$ stands for the channel gain received by the u -th user over the s -th RB. Allocating the power in this way will guarantee that the RB classification process will be largely done based on the channel power of each RB. It is worth mentioning that after the classification process, the power allocation for the shared RBs will be using HPPA and for the orthogonal part, a water-filling based optimal solution will be used to allocate the power.

Next, by examining the RBs one by one, the RB classification process will be done by comparing the achievable $R_{orth,s}$ by each user (i.e., the individual user rate of all users over the s -th RB) against the $R_{non,s}$ (i.e., the sum rate of all users over the same s -th RB). After that, for the s -th RB, if $R_{non,s} \geq R_{orth,s}$ then the respective RB will be shared using NOMA. Otherwise, this RB will be classified as an orthogonal RB and will be allocated for dedicated use by the user who has it with the highest achievable rate $R_{orth,s}$ for the purpose of sum rate maximization. Algorithm 1 shows the RB classification and allocation steps for the proposed hybrid method.

B. Power Allocation for the Hybrid System

At the end of the classification process, all users will have non-orthogonal RBs but only some will also have the orthogonal ones. First, we will allocate the power to the dedicated and shared part proportional to the number of RBs allocated. That is, the amount of power allocated to the dedicated and shared parts are respectively $P_{orth} = \frac{P_t * S_{orth}}{S}$ and $P_{non} = \frac{P_t * S_{non}}{S}$, where S_{non} and S_{orth} are the corresponding number of RBs allocated to each part. For the shared part, since it is primarily a NOMA transmission, we use the proposed HPPA method for power allocation but with the total available power in the first stage as P_{non} instead of P_t . On the other hand, the power allocation for the orthogonal part will be applied using an optimal water-filling based approach proposed in [30]. The sum rate for all users over the non-orthogonal part (i.e., the shared RBs) is given by

$$R_a = B_s \sum_{z=1}^Z \sum_{q \in \Omega_{non}} \left(\log_2 \left(1 + \gamma_{z,q}^{(H)} \right) + \log_2 \left(1 + \gamma_{z,q}^{(L)} \right) \right). \quad (27)$$

On the other hand, the sum rate for the additional orthogonal part (i.e., the dedicated RBs) is given by

$$R_b = B_s \sum_{f \in \mathcal{O}_{orth}} \sum_{w \in \Omega_{orth}} \log_2 \left(1 + \frac{P_w^{(f)} |h_w^{(f)}|^2}{B_s N_0} \right). \quad (28)$$

where Ω_{non} and Ω_{orth} are the set of non-orthogonal and orthogonal resource blocks (RB) indices, respectively, and \mathcal{O}_{orth} represents the set of users with dedicated RBs. Finally,

Algorithm 1 Steps of RBs allocation and classification algorithm

- 1) **Initialize** : U, S
- 2) $P_{RB} = \frac{P_t}{S}$: Total power per RB which is allocated equally for the classification purpose.
- 3) **Initialize** : The sets of the indices of orthogonal and non-orthogonal RBs as $\Omega_{orth} = \emptyset$ and $\Omega_{non} = \emptyset$, respectively.
- 4) **Initialize** : The sets of the users indices as $\mathcal{O}_{orth} = \emptyset$.
 - a) Set $S_{non} = 0, S_{orth} = 0$ (RBs Counters), $u = 1$
 - i) **for** $s = 1$ **to** S
 - **find** $(u,s) = \text{argmax } |h_s^{(u)}|^2$
 - **Calculate** $R_{orth,s}$ from (25) for the user who has the best channel gain $|h_s^{(H)}|^2$ at this RB
 - **Calculate** $R_{non,s}$ from (26) for all of the users over the same s -th RB.
 - **If** $R_{non,s} > R_{orth,s}$ **do**
 - $\{\Omega_{non}\} \leftarrow \{s\}$
 - $S_{non} = S_{non} + 1$
 - **else do**
 - $\{\Omega_{orth}\} \leftarrow \{s\}$
 - $\{\mathcal{O}_{orth}\} \leftarrow \{u\}$
 - $S_{orth} = S_{orth} + 1$
 - **end if**
 - ii) **end for**

the total sum rate for the hybrid multiple access is the sum of (27) and (28).

The proposed hybrid scheme allows more than just two users to be multiplexed over the shared RBs using closed form power allocation solution, and this is the main advantage over the MC-NOMA system that was investigated by [28] in which a maximum of two users were allowed to share a single subcarrier.

VI. SIMULATION AND RESULTS

The downlink scenario consists of U cellular users uniformly distributed within a circular coverage area of diameter D with a BS at the center. The wireless channel is modeled as a six-path frequency selective fading channel using the ITU pedestrian - B model where the average power of the multi-path are [0 dB, -0.9 dB, -4.9 dB, -8 dB, -7.8 dB, -23.9 dB] [31]. In addition, channel estimation is assumed to be perfectly applied and the CSI is assumed to be perfectly known at the BS. Unless stated otherwise, Table I depicts the simulation parameters [32]–[34] that are used in all of the simulation scenarios. Unless otherwise mentioned, the optimal power allocation for NOMA are numerically solved and the performance is compared to the proposed low complexity

Table I
SIMULATION PARAMETERS [32]–[34]

Parameter Name	Value
Transmitted power(P_t)	1 W (30 dBm)
Cell diameter (D)	300 m
Path loss exponent (v)	3
Noise power density (N_0)	-174 dBm / Hz
Total bandwidth (W_T)	5 MHz
No. of RBs (S)	25
Bandwidth per RB (B_s)	200 kHz
No. of subcarriers per RB (N_c)	12
Shadowing standard deviation	8 dB
PL_0 at 2 GHz band	$15.3 + v \log_{10}(d_0)$
$\Phi_{min}^{(1)}$	1 Mbps
$\Phi_{min}^{(2)}$	0.5 Mbps

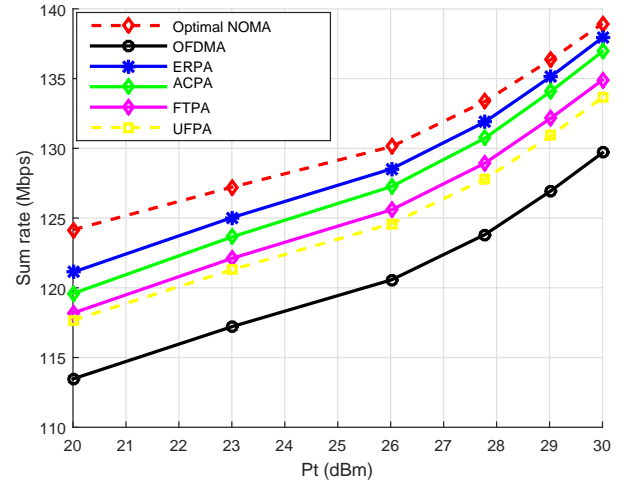


Fig. 6. Sum rate against different values of P_t .

ERPA, ACPA, HPPA, equal per pair, proportional per pair, subband based NOMA (with a total of 16 subbands) schemes, and other existing schemes such as FTPA in [16], and UFPA in [14]. In addition, The OFDMA system will also be compared to show the advantage of NOMA.

A. Two-user Scenario

In Fig. 6, a sum rate comparison for the two-user scenario is made among the proposed methods against the optimal NOMA, OFDMA, and existing schemes from literature. It shows that as the maximum transmission power P_t is increased, the sum rates of these schemes increase accordingly. Moreover, the sum rate for the two-user scenario is also evaluated for different BS to user separation as illustrated in Fig. 7. From this figure, the sum rate is decreasing because the attenuation increases correspondingly with the distance.

More importantly, both of these figures show that NOMA with optimal power allocation performs significantly better than OFDMA as it achieves higher sum rate. In addition, the performance of the proposed sub-optimal methods only have small degradation from the optimal scheme but with much

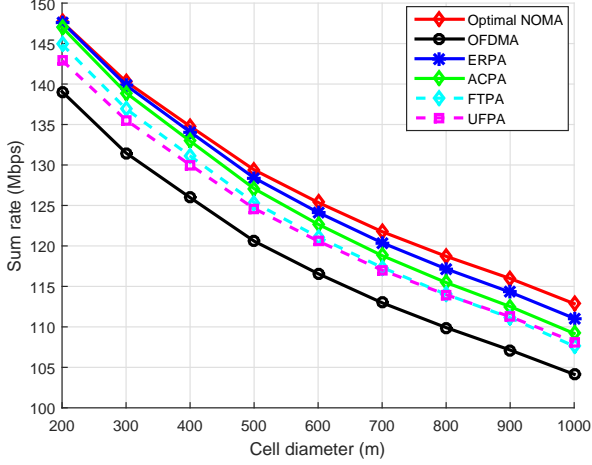


Fig. 7. Sum rate against different cell diameters with $P_t = 30$ dBm.

less complexity. The results in both figures show that the proposed methods are better than the other existing methods. In particular, ERPA provides better and closer performance to the optimal one than ACPA. This is because ERPA allocates power on a per-RB basis, while ACPA does so based on the average channel power of all RBs.

B. Multi-user Scenario

In the multi-user case, the simulations involve a comparison among the numerically optimized conventional NOMA (i.e., all users transmit in the same band), the proposed HPPA, the three simplified schemes in Section IV.B-D (Equal-per-pair, Proportional-per-pair, and Subband based ERPA), and OFDMA. Fig. 8 shows that the proposed HPPA is the best technique as compared to other schemes and is the closest to the optimal one. It is also better than the three simplified schemes, with equal-per-pair being the best amongst them. This also shows that putting all users in the same band has better performance than the subband based approach, and is made feasible by the low complexity closed-form solutions in our work. Fig. 9 compares the proposed methods against the optimal one in terms of the coverage probability. It shows that the number of users who achieve the target rate using the proposed schemes is comparable to the optimal scheme. Again, this figure shows that HPPA is the best method and closest to the optimal one which verifies the effectiveness of the hierarchical pairing concept for NOMA system with a large number of users. The gap between the HPPA and the optimal scheme is due to fairness achieved in the power allocation. In order to maintain both rate requirement satisfaction and overall sum rate maximization, the optimal scheme is allocating power among the users in a slightly fairer manner than the HPPA scheme. Moreover, this figure also shows that the equal based scheme is the closest to the HPPA and offers better performance than the proportional and the subband based approaches. However, since all of these three schemes do not guarantee the satisfaction of the minimum rate requirements for all users they perform poorly comparing to the HPPA scheme.

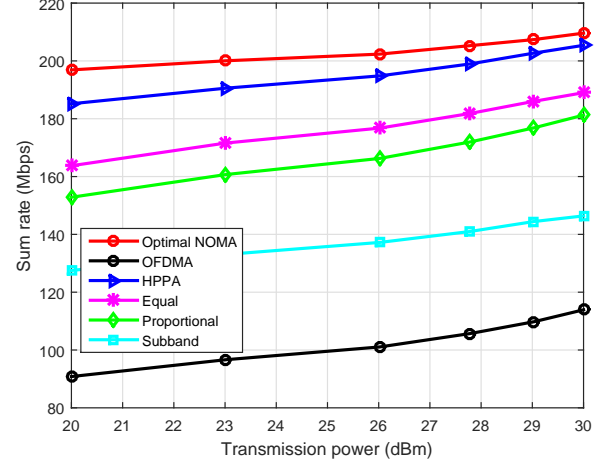


Fig. 8. Sum rate against different values of P_t for $U = 16$

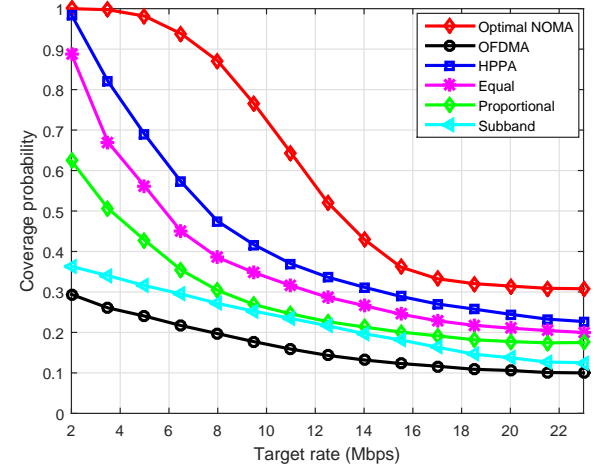


Fig. 9. Coverage probability of 10 users and $P_t = 30$ dBm against different target rates

Finally, Fig. 10 illustrates the comparison in terms of the sum rate against increasing number of users. This figure shows that, as the number of users increases, the achievable sum rate of NOMA based schemes (except the subband based NOMA) also increases accordingly. This highlights the multiuser diversity gain that NOMA offers by multiplexing the users in power domain and allowing them all to share all the available bandwidth. In addition, the performance of the subband based NOMA declines due to the fact that having only two users multiplexed per subband means that as the number of users increases, less RBs will be available to allocate for each subband which in turn affects the users' achievable rate. Similar trend is also obtained for the case of OFDMA under the same optimization problem setup and constraints as that formulated for NOMA in (6) to (9), this is because as the number of users increases, more competition occurs among the available resources because each user attempts to acquire as many RBs as it can to maintain the minimum rate requirements.

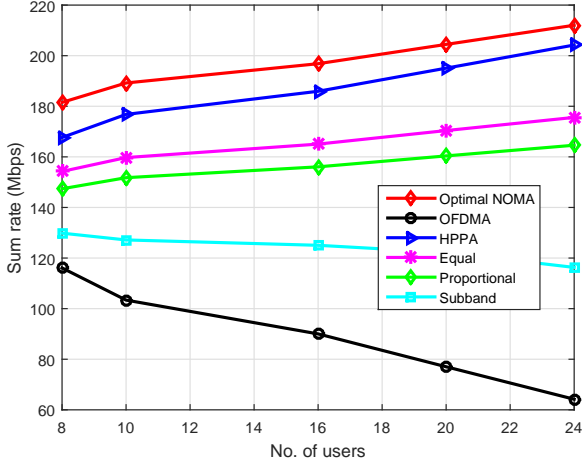


Fig. 10. Sum rate against different number of users with $P_t = 20 \text{ dBm}$.

C. Hybrid Multiple Access Against NOMA System

In Fig. 11, the proposed hybrid scheme is compared against the HPPA-based NOMA system in terms of the sum rate for increasing number of users. In order to evaluate the performance under different channel conditions, the generic PL exponent model stated in Section II is used, with v chosen at 3 and 5 in this simulation. From this figure, despite the poor channel conditions, it is clear that the sum rate of the two schemes increases in proportion to the number of users. This is because of the multiuser diversity gain that is obtained as the number of the multiplexed users increases. The advantage of the hybrid scheme over NOMA is the adaptability of the transmission scheme to the channel conditions. If the received channel power by a certain user is significantly higher than that received by other users, the hybrid scheme will allocate this RB exclusively to this user rather than sharing it among all users using NOMA. On the other hand, if the users have comparable channel gain, it is better to apply NOMA than OFDMA. In this way, the overall sum rate is further increased while the minimum rate requirements are satisfied.

Fig. 12 shows the sum rate comparison for increasing numbers of RBs at $v=2, 4$, and 6 . It is intuitive that as the number of RB increases, the overall sum rate increases due to the larger bandwidth. This figure also shows that the proposed hybrid method is again better than NOMA in terms of sum rate. In particular, the difference is larger when there are more RBs. This is due to the improved frequency diversity and that the hybrid scheme can exploit it better by optimizing the transmission methods that maximizes the rate based on the channel quality. It should also be noted that with higher path loss exponent, the performance difference is larger because with poorer channel conditions, the users are better off in having more orthogonal transmissions.

VII. CONCLUSION

This paper investigates the power allocation problem for NOMA system. Two sub-optimal power allocation methods have been proposed to allocate the transmission power to each

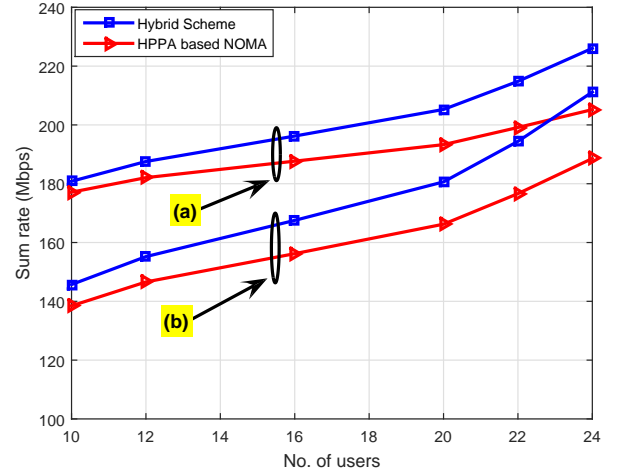


Fig. 11. Sum rate of the hybrid scheme against HPPA based NOMA system with $P_t = 20 \text{ dBm}$, and a path loss exponent of: (a). $v=3$. (b). $v=5$.

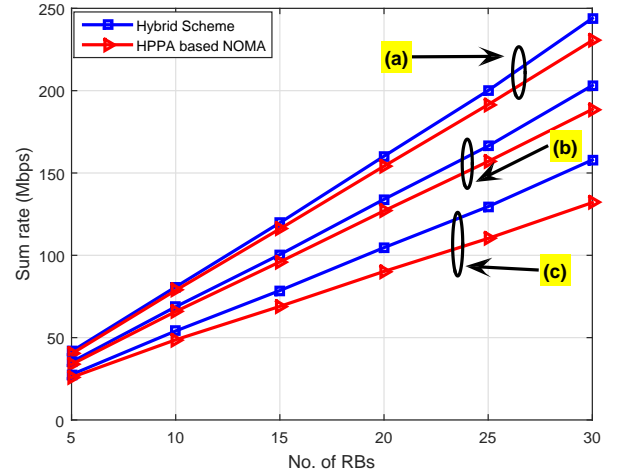


Fig. 12. Sum rate of the hybrid scheme against HPPA based NOMA system with $P_t = 20 \text{ dBm}$, $U = 10$, and a path loss exponent of: (a). $v=2$. (b). $v=4$. (c). $v=6$.

user in a two-user scenario. In addition, to optimize the sum rate for a large number of users, the proposed techniques are extended to a multi-user scenario by the vertical pairing concept. The pairs are then multiplexed in the power domain, which is obtained from a modified solution to the obtained sub-optimal ones in the two-user scenario. Furthermore, we also proposed the idea of hybrid multiple access as a combination between NOMA and OFDMA to utilize the transmission schemes for varying channel conditions. Simulation results show that NOMA provides better performance than OFDMA. Moreover, the simulations confirm that the proposed ERPA and ACPA methods achieve comparable performance to the optimal one with the advantage of lower complexity and also better than the other NOMA existing schemes. Among these two methods, ERPA method performs slightly better than ACPA at the cost of more complexity. For the multi-user scenario, the proposed HPPA showed the best performance

among the compared schemes and the closest to the optimal one. This confirms the effectiveness of the hierarchical pairing concept applicability to NOMA system with a large number of users. Finally the hybrid scheme outperforms NOMA due to the flexibility of adapting the transmission approach according to the channel condition, and is a favourable method to satisfy the future traffic demand.

APPENDIX A

Taking into account the objective function in (3) and using the Lagrangian dual decomposition approach in [35] to solve the formulated problem in (6) to (9), the Lagrangian function of the optimization problem could be expressed as

$$F = B_s \log_2 \left((1 + \gamma_s^{(H)}) (1 + \gamma_s^{(L)}) \right) - \mu \left(\frac{B_s \log_2 (1 + \gamma_s^{(H)})}{\Phi_{min}^{(1)}} - \frac{B_s \log_2 (1 + \gamma_s^{(L)})}{\Phi_{min}^{(2)}} \right) - \lambda \left(\sum_{s=1}^S (P_s^{(H)} + P_s^{(L)}) - P_t \right) \quad (29)$$

where μ and λ represent the Lagrange multipliers. Differentiating against $P_s^{(H)}$, $P_s^{(L)}$, λ , and μ , respectively, we obtain

$$\frac{dF}{dP_s^{(H)}} = \frac{B_s \left(\frac{\gamma_s^{(H)} (1 + \gamma_s^{(L)})}{P_s^{(H)}} - \frac{(\gamma_s^{(L)})^2 (1 + \gamma_s^{(H)})}{P_s^{(L)}} \right)}{(1 + \gamma_s^{(H)}) (1 + \gamma_s^{(L)})} - \lambda - \mu \left(\frac{B_s \gamma_s^{(H)}}{\Phi_{min}^{(1)} P_s^{(H)} (1 + \gamma_s^{(H)})} + \frac{B_s (\gamma_s^{(L)})^2}{\Phi_{min}^{(2)} P_s^{(L)} (1 + \gamma_s^{(L)})} \right) \quad (30)$$

$$\frac{dF}{dP_s^{(L)}} = -\lambda + \frac{\gamma_s^{(L)} \mu B_s}{\Phi_{min}^{(2)} P_s^{(L)} (1 + \gamma_s^{(L)})} + \frac{\gamma_s^{(L)} B_s}{P_s^{(L)} (1 + \gamma_s^{(L)})} \quad (31)$$

$$\frac{dF}{d\lambda} = P_t - \sum_{s=1}^S (P_s^{(H)} + P_s^{(L)}) \quad (32)$$

$$\frac{dF}{d\mu} = \frac{B_s \log_2 (1 + \gamma_s^{(L)})}{\Phi_{min}^{(2)}} - \frac{B_s \log_2 (1 + \gamma_s^{(H)})}{\Phi_{min}^{(1)}}. \quad (33)$$

Setting each of these equations to zero and solving (30) for the Lagrange variable λ we obtain

$$\lambda = \frac{B_s \left(\frac{\gamma_s^{(H)} (1 + \gamma_s^{(L)})}{P_s^{(H)}} - \frac{(\gamma_s^{(L)})^2 (1 + \gamma_s^{(H)})}{P_s^{(L)}} \right)}{(1 + \gamma_s^{(H)}) (1 + \gamma_s^{(L)})} - \mu \left(\frac{N_0 |h_s^{(H)}|^2}{\Phi_{min}^{(1)} (1 + \gamma_s^{(H)})} + \frac{(\gamma_s^{(L)})^2 B_s}{\Phi_{min}^{(2)} P_s^{(L)} (1 + \gamma_s^{(L)})} \right) \quad (34)$$

which can be used to solve (31) for $P_s^{(H)}$ as

$$P_s^{(H)} = \frac{(|h_s^{(H)}|^2 + |h_s^{(L)}|^2) \Phi_{min}^{(1)} \Phi_{min}^{(2)} B_s N_0}{|h_s^{(H)}|^2 |h_s^{(L)}|^2 \mu (\Phi_{min}^{(1)} + \Phi_{min}^{(2)})} - \frac{(|h_s^{(L)}|^2 \Phi_{min}^{(1)} + |h_s^{(H)}|^2 \Phi_{min}^{(2)}) \mu B_s N_0}{|h_s^{(H)}|^2 |h_s^{(L)}|^2 \mu (\Phi_{min}^{(1)} + \Phi_{min}^{(2)})}. \quad (35)$$

Next, solving for $P_s^{(L)}$ by using (32) would be

$$\sum_{s=1}^S P_s^{(L)} = P_t + B_s N_0 \sum_{s=1}^S \left(\frac{(|h_s^{(L)}|^2 \Phi_{min}^{(1)} + |h_s^{(H)}|^2 \Phi_{min}^{(2)}) \mu}{|h_s^{(H)}|^2 |h_s^{(L)}|^2 \mu (\Phi_{min}^{(1)} + \Phi_{min}^{(2)})} - \frac{(|h_s^{(H)}|^2 + |h_s^{(L)}|^2) \Phi_{min}^{(1)} \Phi_{min}^{(2)}}{|h_s^{(H)}|^2 |h_s^{(L)}|^2 \mu (\Phi_{min}^{(1)} + \Phi_{min}^{(2)})} \right). \quad (36)$$

Solving this requires the use of complex numerical solutions. To derive a simple closed form solution, we assume that the total transmit power of all RBs are equal, i.e., the total transmission power in each RB (P_{RB}) is obtained by simply dividing the total available power P_t by the total number of RBs S as given by (10), from which we can obtain that

$$P_s^{(L)} = P_{RB} - P_s^{(H)}. \quad (37)$$

Using (37) to solve (36) for $P_s^{(L)}$ we obtain

$$P_s^{(L)} = P_{RB} + \frac{(|h_s^{(L)}|^2 \Phi_{min}^{(1)} + |h_s^{(H)}|^2 \Phi_{min}^{(2)}) \mu B_s N_0}{|h_s^{(H)}|^2 |h_s^{(L)}|^2 \mu (\Phi_{min}^{(1)} + \Phi_{min}^{(2)})} - \frac{(|h_s^{(H)}|^2 + |h_s^{(L)}|^2) \Phi_{min}^{(1)} \Phi_{min}^{(2)} B_s N_0}{|h_s^{(H)}|^2 |h_s^{(L)}|^2 \mu (\Phi_{min}^{(1)} + \Phi_{min}^{(2)})}. \quad (38)$$

Since (35) and (38) still contain the Lagrangian variable μ , it has to be solved in order to allocate the transmission power for the strong and weak user. Using (38) to solve (33) for the second Lagrange variable μ we obtain

$$\mu = \frac{\psi_1 (\Phi_{min}^{(1)} - \Phi_{min}^{(2)}) + (\Phi_{min}^{(1)} + \Phi_{min}^{(2)}) \psi_3 \sqrt{\psi_1 (\Phi_{min}^{(1)} \Phi_{min}^{(2)})}}{2 \left(\frac{\psi_2}{B_s N_0} |h_s^{(L)}|^2 P_{RB} (\Phi_{min}^{(1)} + \Phi_{min}^{(2)})^2 + \psi_4 \right)} \quad (39)$$

where

$$\begin{cases} \psi_1 = (|h_s^{(H)}|^2 - |h_s^{(L)}|^2)^2 \Phi_{min}^{(1)} \Phi_{min}^{(2)} \Gamma_1 B_s N_0 \\ \psi_2 = B_s N_0 |h_s^{(H)}|^2 |h_s^{(L)}|^2 \Gamma_2 \\ \psi_3 = \sqrt{4 \frac{\psi_2}{B_s N_0} |h_s^{(L)}|^2 P_{RB} + 4 \psi_2 + \frac{\psi_1}{\Phi_{min}^{(1)} \Phi_{min}^{(2)}}} \\ \psi_4 = \psi_2 (\Phi_{min}^{(1)} + \Phi_{min}^{(2)})^2 + \psi_1 \\ \Gamma_1 = \frac{1}{2 \Phi_{min}^{(1)}} \\ \Gamma_2 = \frac{1}{2 \Phi_{min}^{(2)}} \end{cases}.$$

Finally, substituting (35) into (38) to obtain (11) and (12).

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