Iterative Receiver Design for Faster than Nqyuist Signaling-Spare Code Multiple Access System

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Abstract—**The sparse code multiple access (SCMA) which is a promising candidate for the next generation wireless communications can support more users using the same resource elements. On the other hand, faster than Nyquist (FTN) signaling can also be used to improve the spectral efficiency by transmitting more data information in the same time period. In this paper, we consider a combined uplink FTN-SCMA system that the data symbols corresponding to a user are further packed using FTN signaling. As a result, a higher spectral efficiency is achieved at the cost of introducing intentional inter-symbol interference (ISI). To perform the joint channel estimation and decoding, we design a low complexity iterative receiver based on the factor graph framework. In addition, to reduce the signaling overhead and transmission latency of SCMA system, we further consider a grant free scheme. Consequently, the active and inactive users should be distinguished. To address the problem, we extend the aforementioned receiver and develop a new algorithm that jointly estimates the channel state, detects the user activity and performs decoding. In order to further reduce the complexity, an energy minimization based approximation is employed to restrict the user state to Gaussian and hybrid message passing algorithm is performed. Simulation results show the considered FTN-SCMA system with the proposed receiver design is capable of transmitting more data bits than conventional SCMA scheme with negligible performance loss.**

Index Terms—**Sparse code multiple access, faster-than-Nyquist signaling, grant free, channel estimation, hybrid message passing, high spectral efficiency**

I. Introduction

The rapid growth of wireless applications requires to increase the spectral efficiency since the available bandwidth becomes limited. Conventional orthogonal multiple access (OMA) schemes such as time division multiple access (TDMA), code division multiple access (CDMA) and orthogonal frequency-division multiple access (OFDMA) assign orthogonal resource elements to different users [1]–[4]. Although OMA avoids multiuser interference, the challenges of large throughput and massive connections make it inferior for the next generation wireless communications. By contrast, the newly developed non-orthogonal multiple access (NOMA) is capable of increasing the spectral efficiency and addressing the aforementional problems [5]. Amongst several NOMA technologies [6]–[10], the sparse code multiple access (SCMA) has attracted significant attention, due to its capability of achieving extra shaping gain [11].

SCMA encoder maps the bits to sparse codewords directly. After multi-dimensional modulation and low density spreading, the bits streams corresponding to different users are encoded to generate sparse codewords from predesigned codebook directly and then multiplexed over several orthogonal resource elements. Several researches considered the signal design of SCMA at the transmitter side. In [12], the authors investigated the SCMA codebook design based on systematic construction methods. To maximize the minimum codeword distance, a multi-dimensional codebook is designed based on the constellation rotation and interleaving method [13]. In [14], the capacity based codebook design is proposed to achieve maximum sum rate. However, more users supporting by the same number of resource elements result in a rank-deficient system, which makes the complexity of the optimal receiver increases exponentially with the number of interfering users. To tackle this problem, several factor graph (FG)and message passing algorithm (MPA) based multiuser detectors were proposed by exploiting the low density codewords of SCMA. In [15], a low-complexity detection algorithm is proposed based on discretization and fast Fourier transform (FFT). A list-sphere-decoding-based algorithm is devised in [16], but it only considered signal within a hypersphere. The authors in [17] developed a partial marginalization based message passing detector for uplink SCMA. In [18], a Monte Carlo Markov Chain (MCMC) based SCMA decoder was proposed for large size SCMA codebook. In [19], the authors proposed a convergence guaranteed message passing algorithm for MIMO-SCMA systems by convexifying the Bethe free energy. The authors in [20] proposed a modified MPA receiver namely max-log MPA that uses messages updating in the log domain to avoid multiplication operations.

On the other hand, higher spectral efficiency can also be achieved via transmitting data symbols beyond the Nyquist rate, that is the faster-than-Nyquist (FTN) signaling. Mazo proved that with appropriate packing ratio, FTN transmission is capable of preserving the same bit error rate (BER) performance [21]. This makes it become a promising candidate for modulation schemes in the future communications applications. However, due to the nonorthogonality of the shaping pulse with respect to the symbol interval, long intersymbol interference (ISI) as well as colored noise at the receiver side are unavoidable [22]. As a result, a prohibitively high complexity is associated with performing optimal detection. To this end, a reduced BCJR detector for FTN signaling in AWGN channel is developed in [23], which considered only M states based on a minimum phase model. Nevertheless, the complexity still increases exponentially with the number of ISI taps. By taking the advantages of the single carrier frequency-domain equalization (FDE), the authors in [24] added cyclic prefix (CP) to tackle the ISI imposed by FTN. However, the inherent colored noise was not considered in [24] and the CP will also degrade the efficiency. [25] proposed a Forney-style factor graph based detector to handle the colored noise in FTN transmission over AWGN channels. An extension to doubly selective channels is considered in [26] and [27], where Gaussian message passing and variational inference are employed to detect the symbols, respectively. Nevertheless, the detection of data symbols for an FTN-SCMA system is still challenging due to the interferences imposed by non-orthogonal waveforms and nonorthogonal multiple access.

Note that in the aforementioned receiver design works, the channel information is assumed to be perfectly known. The problem of detecting data symbols emerges when the channel information is unknown in practical applications. Generally, training sequences can be used to obtain accurate channel information [28]. Considering that the observations contain both channel coefficients and data symbols, a joint channel estimation and detection method can avoid the use of long training sequences and improve the BER performance [29]. From this perspective, researchers developed several suboptimal joint estimation method with low complexity [30]–[32]. Motivated by the iterative structure of receiver, joint channel estimation and decoding based on MPA becomes more attractive. A virtual zero-padding aided belief propagation (BP) algorithm was devised in [33] for iterative joint channel estimation, detection and decoding. In [34], a merging belief propagation variational expectation maximization based method was derived for MIMO-OFDM system. Both algorithms are designed based on Nyquist signaling. For FTN signaling, in [35] and [36], a time domain based BP and a frequency domain based generalized approximate message passing (GAMP) joint channel estimation and decoding algorithms were proposed. Nevertheless, existing approaches do not consider the system that uses both nonorthogonal waveform and nonorthogonal multiple access.

In this work, we study a low-complexity joint channel estimation and decoding algorithm based on factor graph (FG) and message passing algorithm for an uplink FTN-SCMA systems. To tackle the colored noise induced by FTN signaling, we employ the auto regressive (AR) process to model the noise. Then the joint distribution of data symbols, channel taps and noise samples can be factorized into several local functions and represented by a factor graph. Even with FG, conventional MPA is still impractical for implementation due to exponential complexity order. To this end, we resort to the expectation propagation (EP) method that restricts the message from channel decoder as Gaussian distribution. Compared to direct approximation via moment matching, EP method aims to minimize a specified relative entropy related to the true marginal and the trail distribution [37]. In EP, the extrinsic information to channel decoder is also considered in the approximation, which can enhance the BER performance. However since the modulus of channel coefficient does not equal to 1, the Gaussian form of messages is unavailable. To tackle this problem, we commence from the variational framework and reform a modified factor node, then variational message passing can be employed. Correspondingly, only means and variances need to be updated iteratively and the complexity order of the proposed receiver for FTN-SCMA systems only scales linearly with the number of users.

Moreover, we consider the grant-free transmission scheme. It has been shown that even in busy hours, only a small percentage of users in wireless networks are active [38]. In current OMA uplink scenarios, a request-grant procedure is used: the base station (BS) schedule the uplink transmission after receiving the request from users [39]. This procedure leads to a large communication overhead, especially for massive connectivity with a huge number of devices. Therefore the uplink grant-free transmission scheme is highly expected to significantly reduce both communication overhead and transmission latency [11]. In grant-free transmission, the active users directly send signals to the BS without grants. In order to decode information bits from users that are connected simultaneously, BS has to detect user activity based on the received signal. Motivated by the sparsity of active users, compressive sensing (CS) based multiuser detection method was proposed in [40]. Considering channel estimation, a two-stage algorithm which detects user activity using CS first and then perform channel estimation and detection was proposed in [41]. An AMP-expectation maximization (EM) was proposed in [42] and solved the active user detection and channel estimation problem jointly. In [43] and [44], the authors used the precision parameter of channel coefficient variable to describe the user activity and constructed a factor graph to perform joint detection and channel estimation. Different from [43] and [44], in this paper, we use a binary variable to represent active/inactive users. By formulating the corresponding factor graph, we propose a modified message passing algorithm to iteratively calculate the distribution of active users. In addition, to further lower the receiver complexity, we use EP to approximate the binary variable by Gaussian. Accordingly, the proposed receiver still experience low complexity.

In summary, the main contributions of this paper are as follows.

- We propose to use FTN signaling in the SCMA system to transmit more data symbols using the same resource elements. As a result, a higher spectral efficiency is achieved.
- To tackle the colored noise and ISI induced by FTN signaling and inter user interference induced by SCMA, we design a novel receiver based on an AR model and a message passing algorithm that jointly perform channel estimation and detection. Since all messages are represented in Gaussian closed form, the proposed receiver only has a linear complexity with the number of users.
- Moreover, considering a grant free system which requires detection of the active users, we develop a joint user activity detection, channel estimation and decoding algorithm. With the use of EP approximation of discrete variables indicating user states, we reconstruct a specific factor node, which enables us to keep representing all messages in parametric forms.

Simulation results show that the combined FTN-SCMA system with the proposed receiver is capable of increasing the data rate while not affecting the BER performance. Also, in grant-free SCMA system, the proposed algorithm is effective for distinguishing the active/inactive users.

The remainder of this paper are organized as follows. In section II, we introduce the model of the considered FTN-SCMA system. Section III presents the proposed low complexity algorithm for joint channel estimation and decoding. In Section IV, the grant free transmission is introduced and the proposed joint user activity detection, channel estimation and decoding algorithm is described. Simulation results are provided in Section V. Finally, we draw conclusions in Section VI.

Notations: We use a boldface letter to denote a vector. The superscript T and -1 denote the transpose and the inverse operations, respectively; $\mathcal{G}(m_x, v_x)$ denotes a Gaussian distribution of variable x with mean m_x and variance v_x; $\mathbb{B}^N$ denotes a N-dimensional binary number space and $\mathbb{C}^N$ denotes a N-dimensional complex number space; $\odot$ denotes the componentwise product; $|\cdot|$ denotes the modulus of a complex number or the cardinality of a set; $\|\cdot\|_2$ denotes the ℓ^2 norm; $\propto$ represents equality up to a constant normalization factor; $\mathbf{x}\backslash x$ denotes all variables in $\mathbf{x}$ except x.

II. System Model

We consider an uplink SCMA system with K users and J resource elements. In a NOMA system, $K > J$ is assumed and we set $\lambda = \frac{K}{J}$ as the normalized user-load. In SCMA encoding, the coded bit streams of different users are mapped to J-dimensional SCMA codewords directly, i.e. $\varphi: \ \mathbf{c}_k \in \mathbb{B}^{\log_2 M} \rightarrow \mathbf{x}_k \mathbb{C}^J$, where M is the size of the predefined SCMA codebook. For brevity we denote the codeword of user k at time instant n as $\mathbf{x}_k^n = [x_{k1}^n, ..., x_{kJ}^n]^T$. Due to the sparse structure of SCMA codewords, only $D < J$ elements in $\mathbf{x}_k^n$ are non-zero. Usually we use a matrix $\mathbf{F} = [\mathbf{f}_1, ..., \mathbf{f}_K]$ to capture the sparse structure of SCMA codewords. For the kth user, $\mathbf{f}_k$ is a J dimensional vector with binary entries, e.g. $f_{kj} = 1$ if and only if the jth resource element is occupied by user k. Given this definition, the nonzero entries in the jth column of F represent the users who occupy the jth resource element, while the nonzero entries in the kth row denote the resource elements that are used by user k.

After SCMA encoding, the SCMA codewords are passed through a shaping filter $q(t)$ with symbol period $T = \tau T_0$, where T_0 is the symbol interval of the Nyquist signaling and τ is the FTN packing factor. The modulated signal corresponding to user k over the jth resource elements is formulated as

$$s_{kj}(t) = \sum_n x_{kj}^n q(t - n\tau T_0). \tag{1}$$

In Nyquist signaling, $\tau = 1$ guarantees inter symbol interference (ISI) free transmission. In FTN signaling, we use $0 < \tau < 1$ to transmit more data symbols in the same time period at the cost of introducing intentional ISI. Then the signal corresponding to user k is transmitted through channel $\mathbf{h}_k = [h_{k1}, ..., h_{kJ}]^T$. The block diagram of the transmitter is shown in Fig. 1.

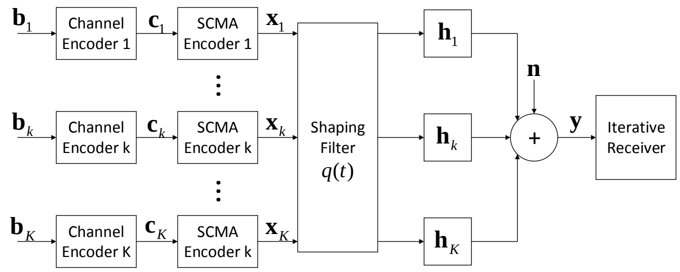

Fig. 1. Transmitter of the considered FTN-SCMA system.

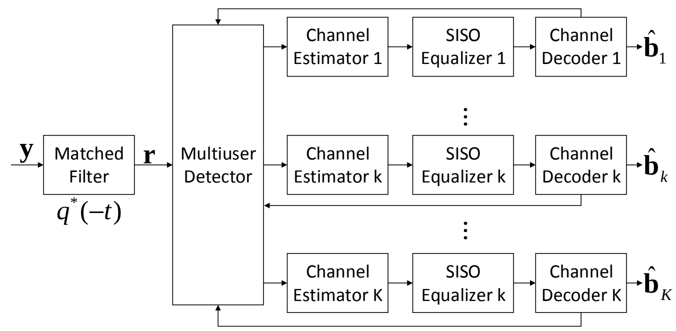

Fig. 2. Receiver structure of the considered FTN-SCMA system.

Assuming perfect synchronization between users and the base station, the received signal at the base station can be expressed as,

$$\mathbf{y}(t) = \sum_{k=1}^{K} \mathbf{h}_k \odot \mathbf{s}_j(t) + \mathbf{n}(t), \tag{2}$$

where $\mathbf{s}_j(t) = [s_{1j}(t), ..., s_{Kj}(t)]^T$ is the modulated signals of all users transmitting over the jth resource and $\mathbf{n}_t$ is the additive white Gaussian noise with power spectral density N_0. As shown in Fig. 2, the received signal is filtered by a matched filter $q^*(-t)$. Without loss of generality, we denote $g(t) = q(t) * q^*(-t)$. Then the signal is given by

$$\mathbf{r}(t) = \sum_{k=1}^{K} \mathbf{h}_k \odot \sum_n x_{kj}^n g(t - n\tau T_0) + \boldsymbol{\omega}(t). \tag{3}$$

After sampling at rate $1/\tau T_0$, the samples at the nth time slot is expressed as

$$\mathbf{r}^n = \sum_{k=1}^{K} \mathbf{h}_k \odot \tilde{\mathbf{s}}_k^n + \boldsymbol{\omega}_n, \tag{4}$$

where the jth entry in $\tilde{\mathbf{s}}_k^n$ is given as[1]

$$\tilde{s}_{kj}^n = \sum_{i=-L}^{L} g_i x_{kj}^{n-i}, \tag{5}$$

and $g_{n-i} = \int q(t - n\tau T_0) q^*(t - i\tau T_0) \mathrm{d}t$. In (4), ω_n denotes the noise samples for all resource elements at time n, formulating as $\omega_n = \int \mathbf{n}(t) q^*(t - n\tau T_0)$. Since the signal rate is above the

[1] In theory, the number of ISI taps induced by FTN is infinite. However in practice, we can choose sufficiently large number of taps, i.e. $2L + 1$ taps.

Nyquist rate, the autocorrelation function of the noise sample $\omega_j^n, \forall\ j$ is

$$\mathbb{E}[\omega_j^n \omega_j^m] = N_0 g_{n-m}, \tag{6}$$

which indicates in FTN system, the noise at the receiver side is colored. To avoid to increase the receiver complexity by using the whitening process, in the following section, we will propose an autoregressive model aided factor graph approach to overcome the colored noise and perform channel estimation and decoding.

III. Joint Channel Estimation and Decoding Algorithm for FTN-SCMA Systems

A. Approximation of Colored Noise

According to [45], the colored noise can be approximated by a P-order autoregressive (AR) model as

$$\omega_j^n = \sum_{p=1}^{P} a_p \omega_j^{n-p} + \delta_j^n, \tag{7}$$

where a_p denotes the AR process parameter and δ_j^n is the noise term with zero mean and variance σ_δ^2. The values of $\{a_p\}$ are determined by solving the following Yule-Walker equations [46].

B. Probabilistic Model and Factor Graph Representation

Assuming that each user transmits a total of N SCMA codewords and N samples are received at the base station. Our goal is to determine the *a posteriori* distribution (marginal) of the transmitted symbol x_{kj}^n based on all observations at the base station $\mathbf{r}$. Then such marginal is transformed into extrinsic log likelihood ratio (LLR) and fed to the channel decoder. The marginal distribution of x_{kj}^n is given by

$$p(x_{kj}^n|\mathbf{r}) \propto \int_{\mathbf{h},\boldsymbol{\omega},\mathbf{X}\backslash x_{kj}^n} p(\mathbf{X},\mathbf{h},\boldsymbol{\omega}|\mathbf{r}), \tag{8}$$

where $\mathbf{X}$, $\mathbf{h}$ and $\mathbf{n}$ denote all transmitted symbols, channel taps and colored noise samples, respectively. Instead of direct marginalization, here we further factorize the joint distribution $p(\mathbf{X},\mathbf{h},\boldsymbol{\omega}|\mathbf{r})$ and resort to a low-complexity factor graph approach to solve the problem.

According to the Bayesian theorem, $p(\mathbf{X},\mathbf{h},\boldsymbol{\omega}|\mathbf{r})$ is factorized as

$$p(\mathbf{X},\mathbf{h},\boldsymbol{\omega}|\mathbf{r}) \propto p(\mathbf{X})p(\mathbf{h})p(\boldsymbol{\omega})p(\mathbf{r}|\mathbf{X},\mathbf{h},\boldsymbol{\omega}). \tag{9}$$

Since the transmitted symbols and channel coefficients are independent of each other, $p(\mathbf{X})p(\mathbf{h})$ reads

$$p(\mathbf{X})p(\mathbf{h}) = \prod_{k,j} \left[p(h_{kj}) \prod_n p(x_{kj}^n) \right], \tag{10}$$

where $p(x_{kj}^n)$ is obtained from the output LLR of the channel decoder. The prior distribution $p(\boldsymbol{\omega})$ can be factorized based on the AR model as

$$p(\boldsymbol{\omega}) \propto \prod_j \prod_n \underbrace{\exp(-\frac{\omega_j^n - \sum_{p=1}^{P} a_p \omega_j^{n-p}}{2\sigma_\delta^2})}_{\psi_j^n}, \tag{11}$$

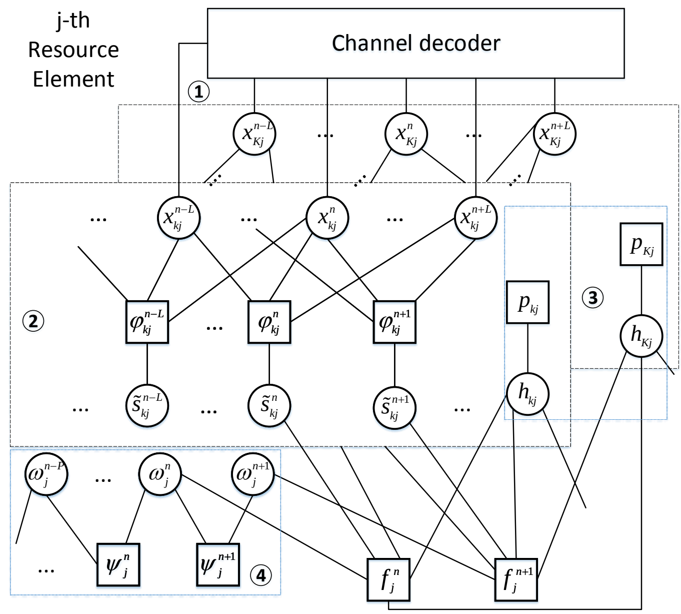

Fig. 3. Factor graph representation of the jth resource element, where the shorthand notations $p_{kj} = p(h_{kj})$. The factor graph is separated into four parts, i.e. decoding part denoted by ①, equalization part denoted by ②, channel estimation part denoted by ③, and colored noise part denoted by ④.

Conditioned on ω_j^n, the observations r_j^n at different time n are independent. As shown in [19], using auxiliary variable can help to reduce the computation load. Therefore we factorize $p(\mathbf{r}|\mathbf{X},\mathbf{h},\boldsymbol{\omega})$ as

$$p(\mathbf{r}|\mathbf{X},\mathbf{h},\boldsymbol{\omega}) \propto \tag{12}$$
$$\prod_{j,n} \underbrace{\delta(r_j^n - \sum_{k=1}^{K} \left[h_{kj}\tilde{s}_{kj}^n\right] - \omega_j^n)}_{f_j^n} \cdot \underbrace{\delta(\tilde{s}_{kj}^n - \sum_{i=-L}^{L} g_i x_{kj}^{n-i})}_{\phi_{kj}^n}.$$

Based on the factorization (10)-(12), the joint distribution $p(\mathbf{X},\mathbf{h},\boldsymbol{\omega}|\mathbf{r})$ can be represented by a factor graph, as shown in Fig. 3, on which the message passing algorithm is executed to determine the unknown variables.

C. Message Passing Receiver Design

The conventional message passing algorithm (MPA) consists of two kinds of messages. Following the sum product algorithm, the message from factor vertex f to variable node x is given by

$$\mu_{f\to x}(x) \propto \int f(\mathbf{x}) \prod_{x' \in \mathcal{S}(f)\backslash\{x\}} \mu_{x'\to f}(x')\mathrm{d}x', \tag{13}$$

and the message from x to f is defined as

$$\mu_{x\to f}(x) \propto \prod_{f' \in \mathcal{S}(x)\backslash\{f\}} \mu_{f'\to x}(x) \tag{14}$$

where $\mathcal{S}(f)$ and $\mathcal{S}(x)$ denotes the set of variable vertices connected to f and the set of factor vertices connected to x. The belief (marginal) of variable x is then given by $b(x) = \prod_{f\in\mathcal{S}(x)} \mu_{f\to x}(x)$. Next we consider the derivations of messages on the factor graph in Fig. 3.

In the decoding part, the channel decoder and the equalizer exchange extrinsic information iteratively. Since the decoding is out of the scope of this paper, we assume optimal BCJR decoding [47] is utilized in the channel decoder. After decoding, the output LLR is

$$L^a(c_{n,m}) = \frac{p(c_{n,m}=0)}{p(c_{n,m}=1)}, \tag{15}$$

where the subscripts n and m denote the nth coded bit and the mth constellation point, respectively. Then the LLRs are transformed to the prior distribution of $p(x_{kj}^n) = \sum_{i=1}^{M} p_i \delta(x_{kj}^n - \chi_i)$, where χ_i is the constellation point in SCMA encoder and p_i is the associated probability. Although the discrete distribution $p(x_{kj}^n)$ can be used as the incoming message in the MPA receiver, the complexity will increases exponentially with the number of interfered symbols. Here we resort to a Kullback-Leibler divergence based method, also known as expectation propagation (EP) to approximate the incoming message by a Gaussian distribution. We aim for finding the Gaussian distribution that minimizes the Kullback-Leibler divergence, i.e.

$$b_G(x_{kj}^n) = \arg\min_{b_G} \int b_G(x_{kj}^n) \ln \frac{b_G(x_{kj}^n)}{b(x_{kj}^n)} \mathrm{d}x_{kj}^n, \tag{16}$$

where b_G belongs to the family of Gaussian distributions and $b(x_{kj}^n)$ is the marginal distribution of the variable x_{kj}^n. The minimization in (16) is equivalent to matching the moments of $b(x_{kj}^n)$. Assuming the outgoing message has mean and variance $m_{x_{kj}^n}^e$ and $v_{x_{kj}^n}^e$ it is easy to obtain the mean and variance of $b_G(x_{kj}^n)$ as $m_{x_{kj}^n}$ and $v_{x_{kj}^n}$. Then the Gaussian approximation of $p(x_{kj}^n)$ is determined with the mean and variance

$$v_{x_{kj}^n}^0 = \left(\frac{1}{v_{x_{kj}^n}} - \frac{1}{v_{x_{kj}^n}^e} \right)^{-1}, \tag{17}$$

$$m_{x_{kj}^n}^0 = v_{x_{kj}^n}^0 \left(\frac{m_{x_{kj}^n}}{v_{x_{kj}^n}} - \frac{m_{x_{kj}^n}^e}{v_{x_{kj}^n}^e} \right). \tag{18}$$

Having $m_{x_{kj}^n}^0$ and $v_{x_{kj}^n}^0$, we can calculate the message in the equalization part. Again, we assume the message $\mu_{\tilde{s}_{kj}^n \to \phi_{k,j}^n} = \mu_{f_j^n \to \tilde{s}_{kj}^n}$ has been obtained as

$$\mu_{\tilde{s}_{kj}^n \to \phi_{k,j}^n} = \mathcal{G}(m_{\tilde{s}_{kj}^n \to \phi_{k,j}^n}, v_{\tilde{s}_{kj}^n \to \phi_{k,j}^n}). \tag{19}$$

Then the message $\mu_{\phi_{k,j}^n \to x_{kj}^{n+l}}$ can be written in Gaussian with

$$m_{\phi_{k,j}^n \to x_{kj}^{n+l}} = m_{\tilde{s}_{kj}^n \to \phi_{k,j}^n} - \sum_{i=-L, i\neq l}^{L} g_i m_{x_{kj}^{n+i} \to \phi_{k,j}^n}, \tag{20}$$

$$v_{\phi_{k,j}^n \to x_{kj}^{n+l}} = v_{\tilde{s}_{kj}^n \to \phi_{k,j}^n} + \sum_{i=-L, i\neq l}^{L} g_i^2 v_{x_{kj}^{n+i} \to \phi_{k,j}^n}. \tag{21}$$

Usually, calculating $\mu_{x_{kj}^n \to \phi_{k,j}^n}$ to different factor nodes $\phi_{k,j}^{n+l}|_{l=-L}^{L}$ following (14) requires to calculate the product of messages for $2L+1$ times. Motivated by the fact that $\mu_{x_{kj}^n \to \phi_{k,j}^n} \cdot \mu_{\phi_{k,j}^n \to x_{kj}^n} = b_G(x_{kj}^n)$, the objective message can be calculated at a linear complexity as $\mu_{x_{kj}^n \to \phi_{k,j}^n} = b_G(x_{kj}^n)/\mu_{\phi_{k,j}^n \to x_{kj}^n}$ with

$$v_{x_{kj}^n \to \phi_{k,j}^n} = \left(\frac{1}{v_{x_{kj}^n}} - \frac{1}{v_{\phi_{k,j}^n \to x_{kj}^n}} \right)^{-1}, \tag{22}$$

$$m_{x_{kj}^n \to \phi_{k,j}^n} = v_{x_{kj}^n \to \phi_{k,j}^n} \left(\frac{m_{x_{kj}^n}}{v_{x_{kj}^n}} - \frac{m_{\phi_{k,j}^n \to x_{kj}^n}}{v_{\phi_{k,j}^n \to x_{kj}^n}} \right). \tag{23}$$

After obtaining all messages $\mu_{\phi_{k,j}^{n+l} \to x_{kj}^n}|_{l=-L}^{L}$, the mean and variance of the extrinsic message to the channel decoder are given by

$$v_{x_{kj}^n}^e = \left(\sum_{l=-L}^{L} 1/v_{\phi_{k,j}^{n+l} \to x_{kj}^n} \right)^{-1}, \tag{24}$$

$$m_{x_{kj}^n}^e = v_{x_{kj}^n}^e \left(\sum_{l=-L}^{L} \frac{m_{\phi_{k,j}^{n+l} \to x_{kj}^n}}{v_{\phi_{k,j}^{n+l} \to x_{kj}^n}} \right). \tag{25}$$

Based on $m_{x_{kj}^n}^e$ and $v_{x_{kj}^n}^e$, the extrinsic LLRs are calculated and fed to the channel decoder to determine the data bits of users $\hat{\mathbf{b}}_k$.

Next, let us consider the message updating in the colored noise part. Since the nonorthogonality of FTN signaling does not affect the first order moment of noise samples, the mean of noise sample $\mathbb{E}[\omega_j^n] = 0$ holds and we only focus on the evolution of its variance. According to (11), the variance $v_{\psi_j^n \to \omega_j^n}$ is expressed as

$$v_{\psi_j^n \to \omega_j^n} = \sigma_\delta^2 + \sum_{p=1}^{P} (a^p)^2 v_{\omega_j^{n-p} \to \psi_j^n}. \tag{26}$$

It should be noted that the colored noise represents a causal system where the sample at time n only depends previous noise samples. Therefore the message from ω_j^n to f_j^n is identical with $\mu_{\psi_j^n \to \omega_j^n}$, i.e. $v_{\omega_j^n to f_j^n} = v_{\psi_j^n \to \omega_j^n}$.

For the channel estimation part, the message $\mu_{h_{kj} \to f_j^n}$ is readily determined according to the SPA rules as

$$\mu_{h_{kj} \to f_j^n} = p(h_{kj}) \prod_{n' \neq n} \mu_{f_j^{n'} \to h_{kj}}. \tag{27}$$

$p(h_{kj})$ is usually coarsely evaluated by using a sequence of pilot symbols, which can be modeled as a Gaussian distributed variable with mean $m_{h_{kj}}^0$ and variance $v_{h_{kj}}^0$. We assume $\mu_{f_j^{n'} \to h_{kj}}$ has also been obtained in the Gaussian form as $\mu_{f_j^{n'} \to h_{kj}} = (m_{f_j^{n'} \to h_{kj}}, v_{f_j^{n'} \to h_{kj}})$. Hence $\mu_{h_{kj} \to f_j^n}$ has mean and variance as

$$m_{h_{kj} \to f_j^n} = v_{h_{kj} \to f_j^n} \left(\frac{m_{h_{kj}}^0}{v_{h_{kj}}^0} + \sum_{n' \neq n} \frac{m_{f_j^{n'} \to h_{kj}}}{v_{f_j^{n'} \to h_{kj}}} \right) \tag{28}$$

$$v_{h_{kj} \to f_j^n} = \left(\frac{1}{v_{h_{kj}}^0} + \sum_{n' \neq n} \frac{1}{v_{f_j^{n'} \to h_{kj}}} \right)^{-1} \tag{29}$$

The belief $b(h_{kj})$ is obtained by adding the terms with index $n' = n$ into (28) and (29). And the maximum *a posteriori* (MAP) estimator can be used to determine the estimate of channel coefficient by $\hat{h}_{kj} = \arg\max_{h_{kj}} b(h_{kj})$. Since $b(h_{kj})$

is Gaussian distribution, the MAP estimate $\hat{h}_{kj}$ is the mean of $b(h_{kj})$

In the above, we have derived closed form Gaussian messages in four parts of the factor graph. However, they are based on the fact that the messages from f_j^n to its connected variable vertices are Gaussian distributions. In what follows, we will calculate the messages related to vertex f_j^n. Following (13), the message $\mu_{f_j^n\to\tilde{s}_{kj}^n}$ is expressed as

$$\begin{aligned}\mu_{f_j^n\to\tilde{s}_{kj}^n} \propto & \int \delta\Big(r_j^n - \sum_{k=1}^{K}[h_{kj}\tilde{s}_{kj}^n] - \omega_j^n\Big)\mu_{\omega_j^n\to f_j^n}\prod_k \mu_{h_{kj}\to f_j^n} \\ & \prod_{k'\neq k}\mu_{\tilde{s}^n_{k'j}\to f_j^n}\mathrm{d}h_{kj}\mathrm{d}\omega_j^n\mathrm{d}\tilde{s}^n_{k'j} \\ & \propto \int \exp\left(-\frac{|r_j^n-\sum_{k=1}^{K}[h_{kj}\tilde{s}_{kj}^n]|^2}{v_{\omega_j^n\to f_j^n}}\right)\prod_k \mu_{h_{kj}\to f_j^n} \\ & \prod_{k'\neq k}\mu_{\tilde{s}^n_{k'j}\to f_j^n}\mathrm{d}h_{kj}\mathrm{d}\tilde{s}^n_{k'j} \\ & \propto \int \exp\left(-\frac{|r_j^n-\sum_{k=1}^{K}[m_{h_{kj}\to f_j^n}\tilde{s}_{kj}^n]|^2}{v_{\omega_j^n\to f_j^n}+\sum_{k=1}^{k}|\tilde{s}_{kj}^n|^2 v_{h_{kj}\to f_j^n}}\right) \\ & \prod_{k'\neq k}\mu_{\tilde{s}^n_{k'j}\to f_j^n}\mathrm{d}\tilde{s}^n_{k'j}. \end{aligned} \tag{30}$$

From (30), we can see when calculating message $\mu_{f_j^n\to\tilde{s}_{kj}^n}$, the variable $\tilde{s}_{kj}^n$ appears in both numerator and denominator of the exponential term, which makes the conventional MPA unavailable. Here we resort to the variational message passing (VMP) method in [48] where the message from factor vertex f to variable vertex x is formulated as

$$\mu_{f\to x}(x) \propto \exp\left(\int \ln f(\mathbf{x})\prod_{x'\in\mathcal{S}(f)\backslash\{x\}}\mu_{x'\to f}(x')\mathrm{d}x'\right). \tag{31}$$

Consequently, the message (30) can be obtained in the Gaussian form with mean

$$m_{f_j^n\to\tilde{s}_{kj}^n} = \frac{(r_j^n-\sum_{k'=1,k\neq k}^{K} m_{h_{k'j}\to f_j^n} m_{\tilde{s}^n_{k'j}\to f_j^n})m_{h_{kj}\to f_j^n}}{|m_{h_{kj}\to f_j^n}|^2+v_{h_{kj}\to f_j^n}}, \tag{32}$$

and variance

$$v_{f_j^n\to\tilde{s}_{kj}^n} = \frac{v_{\omega_j^n\to f_j^n}}{|m_{h_{kj}\to f_j^n}|^2+v_{h_{kj}\to f_j^n}}. \tag{33}$$

The message $\mu_{f_j^n\to h_{kj}}$ can be calculated by VMP likewise, whose mean and variance are

$$m_{f_j^n\to h_{kj}} = \frac{(r_j^n-\sum_{k'=1,k\neq k}^{K} m_{h_{k'j}\to f_j^n} m_{\tilde{s}^n_{k'j}\to f_j^n})m_{\tilde{s}^n_{kj}\to f_j^n}}{|m_{\tilde{s}^n_{kj}\to f_j^n}|^2+v_{\tilde{s}^n_{kj}\to f_j^n}}, \tag{34}$$

$$v_{f_j^n\to h_{kj}} = \frac{v_{\omega_j^n\to f_j^n}}{|m_{\tilde{s}^n_{kj}\to f_j^n}|^2+v_{\tilde{s}^n_{kj}\to f_j^n}}. \tag{35}$$

D. Algorithm Summary

Using appropriate approximations, all messages on the factor graph are represented in parametric forms, which reduce the computational complexity of the conventional MPA receiver significantly. Compared to existing advanced MPA receiver, the computational complexity associated with the introduction of auxiliary variable and modified message updating rules only increases linearly with the number of users and resource elements. The details of the proposed receiver for joint channel estimation and decoding in FTN-SCMA system are presented in Table. I.

Algorithm 1 Joint Channel Estimation and Decoding Algorithm for FTN-SCMA System

Initialization:
At the first turbo iteration, initialize all undetermined messages as Gaussian distribution with zero mean and unit variance;
Using pilot sequence to coarsely estimate the mean $m^0_{h_{kj}}$ and variance $v^0_{h_{kj}}$ of channel coefficient.
for iter=1:N_{iter} **do**
  Compute the means and variances of messages in equalization part according to (20)-(23);
  Compute the message from factor vertex f_j^n to variable vertices x_{kj}^n and h_{kj} according to (32)-(35);
  Compute the variance $v_{\psi_j^n\to\omega_j^n}$ according to (26);
  Compute the message from h_{kj} to factor vertex f_j^n via (28) and (29);
  Compute the mean and variance of message to channel decoder according to (24) and (25);
  Convert the outgoing messages to LLR and feed them to the channel decoder;
  Perform BCJR decoding;
  Convert the extrinsic LLRs to Gaussian messages using (17) and (18);
end for
Determine the estimate of channel coefficient by MAP estimator.

IV. User Activity Detection in Grant-free FTN-SCMA Systems

In a grant-free system, the users do not need grant before sending signals to the base station. In existing works, a precision parameter is used as a hyper-prior to capture the channel sparsity. However, this causes more short loops in the factor graph and increases the receiver complexity. In this section, we will propose an algorithm for FTN-SCMA systems that determines the user activity directly while performing channel estimation and decoding.

Let us use a binary variable $\xi_k=\{0,1\}$ to denote the user activity, i.e. $\xi_k=1$ indicates that user k is active and vice versa. Then the nth sample at the jth resource element is expressed by

$$r_j^n = \sum_{k=1}^{K} h_{kj}\xi_k\tilde{s}_{kj}^n + \omega_j^n. \tag{36}$$

The prior distribution of ξ_k is a Bernoulli distribution given by

$$p^0(\xi_k) = p_1^{\xi_k}(1-p_1)^{1-\xi_k}, \tag{37}$$

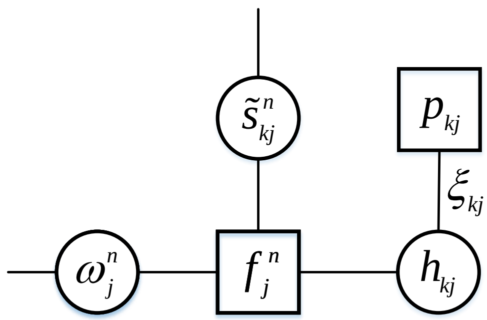

Fig. 4. Modified factor graph structure including user activity.

where p_1 is the prior knowledge of user activity based on existing data.

A. Probability based Active User Detection Algorithm

To determine the activity of user k, we need to calculate its probability of being active γ_k based on the received samples. To this end, we modify the factor graph structure 4 by including ξ_k. Since only the function f is affected, we illustrate the corresponding parts of the factor graph, as shown in Fig. 4. Here we use an auxiliary variable ξ_{kj} on the edge connected to vertices h_{kj}.[2] Then it is possible to formulate the message passing algorithm to calculate the probability of $\xi_{kj} = 1$, namely γ_{kj}.

Based on Fig. 4, the message from f^n_j to h_{kj} is obtained with mean $m_{f^n_j \to h_{kj}}$ and variance $v_{f^n_j \to h_{kj}}$ according to (34) and (35). Hence we have the intrinsic message for $\xi_{kj} h_{kj}$ with mean and variance

$$\overrightarrow{m}_{\xi_{kj} h_{kj}} = \overrightarrow{v}_{\xi_{kj} h_{kj}} \sum_n \frac{m_{f^n_j \to h_{kj}}}{v_{f^n_j \to h_{kj}}} \tag{38}$$

$$\overrightarrow{v}_{\xi_{kj} h_{kj}} = \left(\sum_n \frac{1}{v_{f^n_j \to h_{kj}}} \right)^{-1}. \tag{39}$$

The distribution of ξ_{kj} is obtained by integrating h_{kj} over the joint distribution, formulated as

$$\begin{aligned} p(\xi_{kj}) \propto & \int \exp\left(-\frac{(\xi_{kj} h_{kj} - \overrightarrow{m}_{\xi_{kj} h_{kj}})^2}{\overrightarrow{v}_{\xi_{kj} h_{kj}}} \right) \\ & \cdot \exp\left(-\frac{(h_{kj} - m^0_{h_{kj}})^2}{v^0_{h_{kj}}} \right) \mathrm{d}h_{kj} \\ & \propto \exp\left(-\frac{(\xi_{kj} m^0_{h_{kj}} - \overrightarrow{m}_{\xi_{kj} h_{kj}})^2}{\xi^2_{kj} v^0_{h_{kj}} + \overrightarrow{v}_{\xi_{kj} h_{kj}}} \right). \end{aligned} \tag{40}$$

Then the probability γ_{kj} is updated as

$$\begin{aligned} \gamma_{kj} &= \frac{p(\xi_{kj} = 1)}{p(\xi_{kj} = 0) + p(\xi_{kj} = 1)} \\ &= \frac{1}{1 + \frac{p(\xi_{kj}=0)}{p(\xi_{kj}=1)}}. \end{aligned} \tag{41}$$

[2]Alternatively, we can put the variable ξ on the edge connecting $\tilde{s}^n_{kj}$ and f^n_j or the edge connecting h_{kj} and f^n_j. However, these two means will increase N times number of variables.

Algorithm 2 User Activity Detection Algorithm I

1: Run Algorithm 1;
2: Calculate the intrinsic message to h_{kj} according to (38) and (39);
3: Determine the probability γ_{kj} by (41);
4: Calculate γ_k according to (42) and decide ξ_k;
5: Approximate the message $\mu_{h_{kj} \to f^n_j}$ to Gaussian and continue running algorithm 1.

After getting the probability $\xi_{kj} = 1$, it is readily to obtain the probability γ_k as

$$\gamma_k = \frac{\prod_j \gamma_{kj} p_1}{\prod_j \gamma_{kj} p_1 + \prod_j (1 - \gamma_{kj})(1 - p_1)}. \tag{42}$$

To determine the value of ξ_k, we set a threshold β according to empirical evidence. Then we say that user k is active if $\gamma_k \geq \beta$ and vice versa.

The extrinsic message from h_{kj} to f^n_j is still obtained by $\mu_{h_{kj} \to f^n_j} = \mu_{p_{kj} \to h_{kj}} \prod_{n' \neq n} \mu_{f^{n'}_j \to h_{kj}}$. Specifically when calculating $\mu_{p_{kj} \to h_{kj}}$, we combine ξ_{kj} and h_{kj} as a new variable,

$$\mu_{p_{kj} \to h_{kj}} \propto \left(\gamma_{kj} e^{-\frac{(h_{kj} - m^0_{h_{kj}})^2}{v^0_{h_{kj}}}} + (1 - \gamma_{kj}) e^{-\frac{[m^0_{h_{kj}}]^2}{v^0_{h_{kj}}}} \right). \tag{43}$$

Obviously, $\mu_{p_{kj} \to h_{kj}}$ is a Gaussian mixture distribution (GMD) and $\mu_{h_{kj} \to f^n_j}$ is aslo a GMD. In conjunction with the message passing receiver in Section III, we approximate $\mu_{h_{kj} \to f^n_j}$ to Gaussian and determine its mean and variance as

$$m_{h_{kj} \to f^n_j} = \mathbb{E}_{\mu_{h_{kj} \to f^n_j}} [h_{kj}] \tag{44}$$

$$v_{h_{kj} \to f^n_j} = \mathbb{E}_{\mu_{h_{kj} \to f^n_j}} [h^2_{kj}] - m^2_{h_{kj} \to f^n_j}. \tag{45}$$

In Algorithm 2, the user activity detection based on the message passing algorithm is described. To sum up, we can see that the algorithm can be readily extended from the proposed algorithm in Section III and we only need to do small modification on the factor graph. However, since ξ_{kj} has to be calculated separately, the receiver complexity increases. Also, the derivation of messages is not straightforward from the perspective of probabilistic factorization. In the following subsection, we will propose another active user detection method with reduced complexity .

B. Message Passing based Active User Detection Algorithm

To achieve a concise form of message passing receiver in the factor graph framework, we add ξ_k as a new variable vertex on the factor graph. According to (36), we use a dirac Delta function $\delta(\bar{s}^n_{kj} - \xi_k \tilde{s}^n_{kj})$ to represent the multiplication

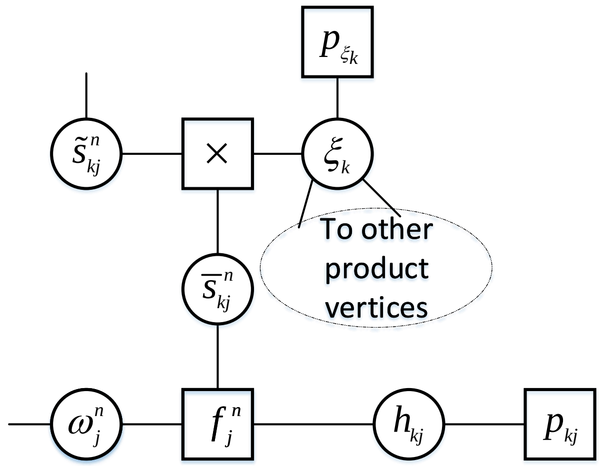

Fig. 5. Modified factor graph structure. The product node $\times^n_{kj}$ represents the constraint $\delta(\bar{s}^n_{kj} - \xi_k \tilde{s}^n_{kj})$.

relationship of $\bar{s}^n_{kj} = \xi_k \tilde{s}^n_{kj}$. Accordingly, the joint likelihood function (12) is revised as

$$p(\mathbf{r}|\mathbf{X}, \mathbf{h}, \boldsymbol{\omega}, \boldsymbol{\xi}) \propto \prod_{j,n} \underbrace{\delta(r^n_j - \sum_{k=1}^{K} \left[h_{kj} \bar{s}^n_{kj}\right] - \omega^n_j)}_{f^n_j} \tag{46}$$
$$\cdot\, \delta(\bar{s}^n_{kj} - \xi_k \tilde{s}^n_{kj}) \cdot \underbrace{\delta(\tilde{s}^n_{kj} - \sum_{i=-L}^{L} g_i x^{n-i}_{kj})}_{\phi^n_{kj}},$$

and the factor graph is modified as shown in Fig. 5. Since ξ_k is a binary variable, which follows a discrete distribution. Then in message updating, when the messages from different product verteices to ξ_k are Gaussian, the message $\mu_{\xi_k \to \times^n_{kj}}$ follows a Gaussian mixture distribution, which makes it unavailable to derive Gaussian messages. To tackle this problem, we approximate the message from ξ_k to the product vertex $\times^n_{kj}$ by Gaussian via expectation propagation.

Following SPA rules, the belief of ξ_k is $b(\xi_k) = \mu_{\xi_k \to p_{\xi_k}} p(\xi_k)$. Assuming $\mu_{\xi_k \to p_{\xi_k}}$ is Gaussian with mean $m_{\xi_k \to p_{\xi_k}}$ and variance $v_{\xi_k \to p_{\xi_k}}$, the mean and variance of $b(\xi_k)$ is

$$m_{\xi_k} = \frac{p_1 \exp(-\frac{1-2m_{\xi_k \to p_{\xi_k}}}{v_{\xi_k \to p_{\xi_k}}})}{p_1 \left[\exp(-\frac{1-2m_{\xi_k \to p_{\xi_k}}}{v_{\xi_k \to p_{\xi_k}}}) - 1\right] + 1}, \tag{47}$$
$$v_{\xi_k} = m_{\xi_k} - m^2_{\xi_k}. \tag{48}$$

An obvious observation is that in (47) the absolute value of the exponential term dominates the value of m_{ξ_k}, and v_{ξ_k} becomes smaller when ξ_k approaches 0 or 1. That is to say after running several iterations, the belief of ξ_k becomes more 'concentrated'. Having m_{ξ_k} and v_{ξ_k}, we can easily determine the Gaussian approximation of message $\mu_{\xi_k \to \times^n_{kj}}$ as

$$\mu_{\xi_k \to \times^n_{kj}} \sim \mathcal{G}\left(\frac{m_{\xi_k} v_{\xi_k} - m_{\times^n_{kj} \to \xi_k} v_{\times^n_{kj} \to \xi_k}}{v_{\xi_k} - v_{\times^n_{kj} \to \xi_k}}, \frac{v_{\xi_k} v_{\times^n_{kj} \to \xi_k}}{v_{\xi_k} - v_{\times^n_{kj} \to \xi_k}}\right). \tag{49}$$

For the product vertex, as we have $\mu_{\xi_k \to \times^n_{kj}}$ and $\mu_{\tilde{s}^n_{kj} \to \times^n_{kj}} = \mu_{\phi^n_{kj} \to \tilde{s}^n_{kj}}$, the mean and variance of message $\mu_{\bar{s}^n_{kj} \to f^n_j}$ are given as

$$m_{\bar{s}^n_{kj} \to f^n_j} = m_{\xi_k \to \times^n_{kj}} m_{\phi^n_{kj} \to \tilde{s}^n_{kj}} \tag{50}$$
$$m_{\bar{s}^n_{kj} \to f^n_j} = v_{\xi_k \to \times^n_{kj}} m^2_{\phi^n_{kj} \to \tilde{s}^n_{kj}} + (m^2_{\xi_k \to \times^n_{kj}} + v_{\xi_k \to \times^n_{kj}}) v_{\phi^n_{kj} \to \tilde{s}^n_{kj}} \tag{51}$$

The detailed derivations for (50) and (51) are given in Appendix A. The message $\mu_{\bar{s}^n_{kj} \to \times^n_{kj}}$, conversely, is the same as the message calculated in (32) and (33). Next we calculate the messages from $\times^n_{kj}$ to $\tilde{s}^n_{kj}$ and ξ_k. Again, a similar problem as in Section III occurs: even $\mu_{\tilde{s}^n_{kj} \to \times^n_{kj}}$ is Gaussian, it is not possible to formulate Gaussian form messages for ξ_k. To overcome this challenge, we commence from the Kullback-Leibler divergence [49]. By grouping the message $\mu_{m_{\bar{s}^n_{kj}} \to \times^n_{kj}}$ and the constraint as a new soft node, the joint distribution $p(m_{\bar{s}^n_{kj}}, \xi_k)$ is formulated as

$$p(m_{\bar{s}^n_{kj}}, \xi_k) \propto \exp\left(-\frac{(m_{\bar{s}^n_{kj} \to \times^n_{kj}} - \xi_k \tilde{s}^n_{kj})^2}{v_{\bar{s}^n_{kj} \to \times^n_{kj}}}\right) \mu_{\xi_k \to \times^n_{kj}} \mu_{\tilde{s}^n_{kj} \to \times^n_{kj}}. \tag{52}$$

According to the variational inference framework, we consider to use $b(\xi_k) b(\tilde{s}^n_{kj})$ to approximate (52). The Kullback Leibler divergence is given by

$$\begin{aligned} \mathrm{KLD}(\xi_k, \tilde{s}^n_{kj}) &= \int b(\xi_k) b(\tilde{s}^n_{kj}) \ln \frac{b(\xi_k) b(\tilde{s}^n_{kj})}{p(\tilde{s}^n_{kj}, \xi_k)} \mathrm{d}\xi_k \mathrm{d}\tilde{s}^n_{kj} \\ &= -\int b(\xi_k) \left[\int \ln p(\tilde{s}^n_{kj}, \xi_k) b(\tilde{s}^n_{kj}) \mathrm{d}\tilde{s}^n_{kj}\right] \mathrm{d}\xi_k \\ &\quad + \int b(\xi_k) \ln b(\xi_k) \mathrm{d}\xi_k + C, \end{aligned} \tag{53}$$

where C denotes a constant. To minimize the KLD, it is easy yo see

$$b(\xi_k) = \exp\left(\int \ln p(\tilde{s}^n_{kj}, \xi_k) b(\tilde{s}^n_{kj})\right). \tag{54}$$

Substituting (52) into (54) yields

$$\frac{b(\xi_k)}{\mu_{\xi_k \to \times^n_{kj}}} \propto \exp\left(-\xi_k^2 \frac{m^2_{\tilde{s}^n_{kj}} + v_{\tilde{s}^n_{kj}}}{v_{\bar{s}^n_{kj} \to \times^n_{kj}}} + 2\xi_k \frac{m_{\bar{s}^n_{kj} \to \times^n_{kj}} m_{\tilde{s}^n_{kj}}}{v_{\bar{s}^n_{kj} \to \times^n_{kj}}}\right), \tag{55}$$

where $v_{\tilde{s}^n_{kj}} = (v^{-1}_{\tilde{s}^n_{kj} \to \times^n_{kj}} + v^{-1}_{\times^n_{kj} \to \tilde{s}^n_{kj}})^{-1}$ and $m_{\tilde{s}^n_{kj}} = v_{\tilde{s}^n_{kj}} (m_{\tilde{s}^n_{kj} \to \times^n_{kj}} v^{-1}_{\tilde{s}^n_{kj} \to \times^n_{kj}} + m_{\times^n_{kj} \to \tilde{s}^n_{kj}} v^{-1}_{\times^n_{kj} \to \tilde{s}^n_{kj}})$. Therefore the message $\mu_{\times^n_{kj} \to \xi_k}$ is determined to be Gaussian with mean and variance

$$m_{\times^n_{kj} \to \xi_k} = \frac{m_{\bar{s}^n_{kj} \to \times^n_{kj}} m_{\tilde{s}^n_{kj}}}{m^2_{\tilde{s}^n_{kj}} + v_{\tilde{s}^n_{kj}}} \tag{56}$$
$$v_{\times^n_{kj} \to \xi_k} = \frac{v_{\bar{s}^n_{kj} \to \times^n_{kj}}}{m^2_{\tilde{s}^n_{kj}} + v_{\tilde{s}^n_{kj}}}. \tag{57}$$

Algorithm 3 User Activity Detection Algorithm II

1: Run Algorithm 1;
2: Approximate the message from ξ_k to the product vertex to Gaussian by EP according to (47) and (49);
3: Calculate the mean $m_{\tilde{s}_{kj}^n \to f_j^n}$ and variance $v_{\tilde{s}_{kj}^n \to f_j^n}$ using (50) and (51);
4: Determine the messages from the product vertex to ξ_k and $\tilde{s}_{kj}^n$ using (56)-(58);
5: Calculate the message $\mu_{\xi_k \to p_{\xi_k}}$ and estimate ξ_k using (47);
6: Continue running Algorithm 1.

Similarly, we have the Gaussian message $\mu_{\times kj^n \to \tilde{s}_{kj}^n}$ as

$$\mu_{\times kj^n \to \tilde{s}_{kj}^n} \propto \mathcal{G}\left(\frac{m_{\xi_k \to \times_{kj}^n} m_{\xi_k}}{m_{\xi_k}^2 + v_{\xi_k}}, \frac{v_{\xi_k \to \times_{kj}^n}}{m_{\xi_k}^2 + v_{\xi_k}}\right), \tag{58}$$

where m_{ξ_k} and v_{ξ_k} are the mean and variance of $b(\xi_k)$. Having $\mu_{\times_{kj}^n \to \xi_k}$ in the Gaussian form, the mean and variance of Gaussian message $\mu_{\xi_k \to p_{\xi_k}}$ can be calculated by straightforward manipulations. The value of ξ_k is given by the MAP estimate of $b(\xi_k)$, which is shown in (47). We also set a threshold β and compare it with m_{ξ_k} to decide whether user k is active or inactive. The details of the proposed user activity detection algorithm is summarized in Algorithm. 3.

V. Simulation Results

In this section, we evaluate the performance of the proposed algorithm via simulations. We consider a SCMA system with $J = 4$ resource elements that supports $K = 6$ users. The codebook is defined according to [14] with size $M = 4$ and an indicator matrix $\mathbf{F}$ as

$$\mathbf{F} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}. \tag{59}$$

Each user transmits a sequence of data bits, which is coded using a rate-5/7 low density parity code (LDPC) and then mapped to a sequence of SCMA codewords. We set the number of transmitted symbols corresponding to each user as $N = 2048$. The transmitted symbols pass through root raised cosine shaping filters with roll-off factor $\alpha = 0.5$ and packing factor $\tau = 0.8$.[3] The number of interfered symbols is assumed to be $L = 10$ on both sides. The channel is set to be Rayleigh fading whose impulse response is generated according to the Jake's model. The coarse estimate of channel coefficients is obtained by using 5 pilots symbols. The maximum number of iterations between the detector and the channel decoder is $N_{iter} = 10$. All results are averaged from 1000 independent Monte Carlo trials.

In Fig. 6, we compare the proposed algorithm with the MPA-Gauss, and MMSE-MPA methods in terms of bit error ratio (BER). As reference, the performance of conventional MPA receiver under the maximum *a posteriori* criterion

[3]We assume the same shaping filter is employed for different resource elements at the transmitter side.

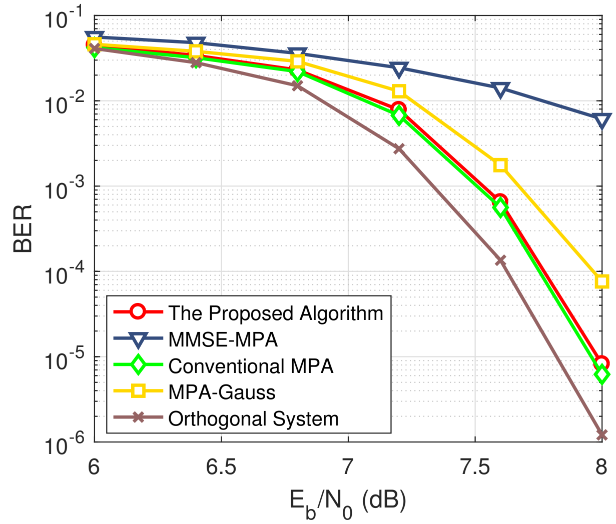

Fig. 6. BER performance of different algorithms for FTN-SCMA system.

is illustrated. The 'MPA-Gauss' method refers to directly approximating the prior distribution of $p(x_{kj}^n)$ by Gaussian distribution. The 'MMSE-MPA' method is a combination of MMSE equalizer and SCMA decoder. It is observed that the proposed algorithm outperforms all the other three algorithms and has almost the same performance as the conventional MPA receiver. MMSE-MPA method suffers from significant performance loss due to error propagation. Moreover, using MMSE equalizer imposes a cubic order of complexity, which is prohibitively high in practical applications. Compared to MPA-Gauss, the proposed algorithm achieves performance gain since EP further uses the extrinsic information fed to the channel decoder. Also, the performance based on an OMA system using Nyquist signaling is plotted. We see the performance loss of the proposed algorithm is as small as 0.2 dB. Meanwhile, 50% more users are supported and 25% higher data rate is achieved. That is to say, using the same resources, a total of more than 87.5% information can be transmitted via the considered FTN-SCMA system with negligible performance loss.

Fig. 7 depicts the BER versus E_b/N_0 of the proposed algorithm parameterized by different packing factor τ, where $\tau = 1$ corresponds to the Nyquist signaling case. It is seen that the proposed iterative receiver for FTN-SCMA system is capable of achieving similar performance to the Nyquist signaling case when $\tau \geq 0.8$. Moreover, as the packing ratio becomes smaller, severer interference is introduced and the performance gap between the FTN signaling and Nyquist signaling becomes larger. Since the actual number of ISI taps induced by FTN is infinite, the number of interfered symbols L used model (5) may be not enough to describe the ISI induced by FTN and causes performance loss. In Fig. 8, we illustrate the BER curves with various values of L while the packing ratio τ is fixed as 0.7. It is observed when L increases to 20, the performance gap between FTN and Nyquist signaling schemes becomes negligible, which means using a smaller packing ratio is still possible at the cost

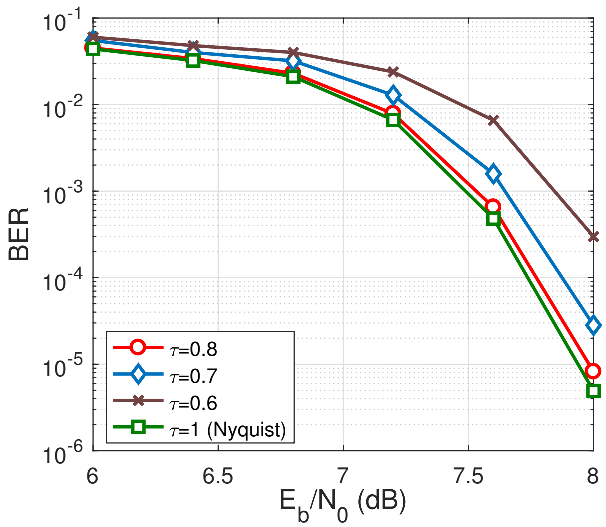

Fig. 7. BER performance of the proposed algorithm with different τ.

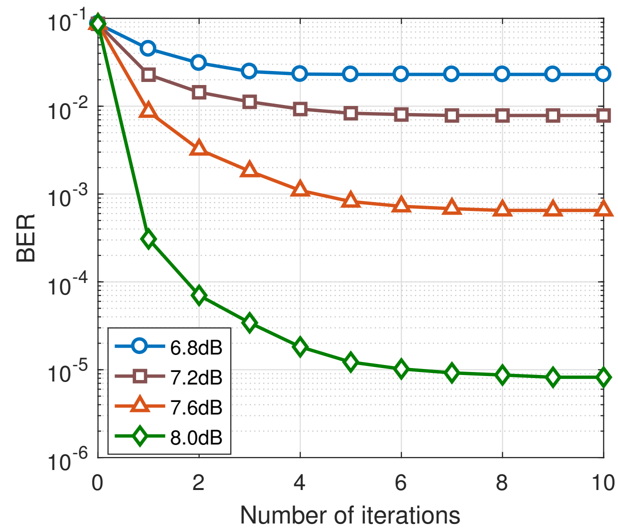

Fig. 9. BER performance of the proposed algorithm versus the number of iterations.

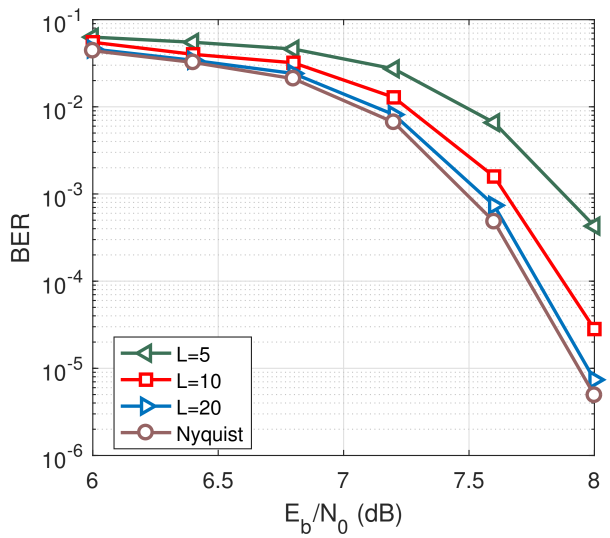

Fig. 8. BER performance of the proposed algorithm with different L.

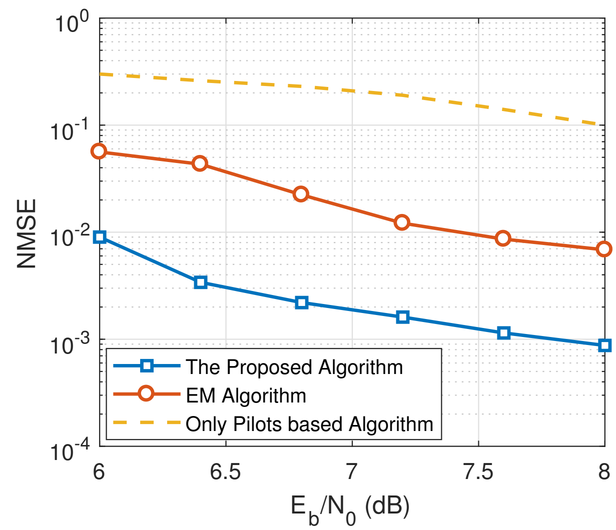

Fig. 10. NMSEs of different algorithms versus E_b/N_0.

of more complex receiver. This implies we can compromise between the transmission rate and the receiver complexity. Nevertheless, there is a lower bound for the packing factor due to the Mazo limit.

We present the BER performance of the proposed algorithm versus the number of iterations in Fig. 9 to illustrate the convergence behavior. It can be seen that for different values of E_b/N_0, the proposed algorithm always converges after several iterations. Moreover, it is noted that for larger E_b/N_0, the proposed algorithm requires more iterations to guarantee the convergence.

In Fig. 10, the normalized mean squared error (NMSE) of the estimated channel coefficients versus E_b/N_0 is illustrated. The NMSE is defined as

$$\mathrm{NMSE}_h = \frac{\sum_{k=1}^{K} \|\mathbf{h}_k - \hat{\mathbf{h}}_k\|^2}{\sum_{k=1}^{K} \|\mathbf{h}_k\|^2}, \tag{60}$$

where $\hat{\mathbf{h}}_k$ is the channel estimate obtained in Section III. The NMSEs of the least square channel estimation method using 5 pilot symbols are depicted for comparison. From Fig. 10, we see that the proposed algorithm is efficient in channel estimation, which can attain the performance of the LS algorithm based on all pilot symbols. Compared to the coarse estimate udinh only 5 pilot symbols, the proposed algorithm significantly improves the channel estimation performance. Also, the performance of an advanced joint channel estimation and decoding algorithm based on expectation maximization (EM) is presented here. Since EM discards uncertainties of variables in the iterative process, it suffers from performance loss.

Next, we evaluate the performance of the proposed two active user detection algorithms in a grant-free system. In Fig. 11, the BER performance of the proposed algorithm versus

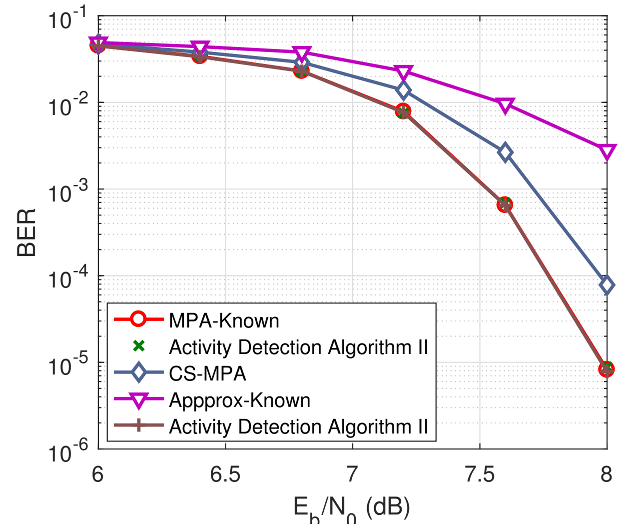

Fig. 11. BER performance of the proposed active user detection algorithms and existing method.

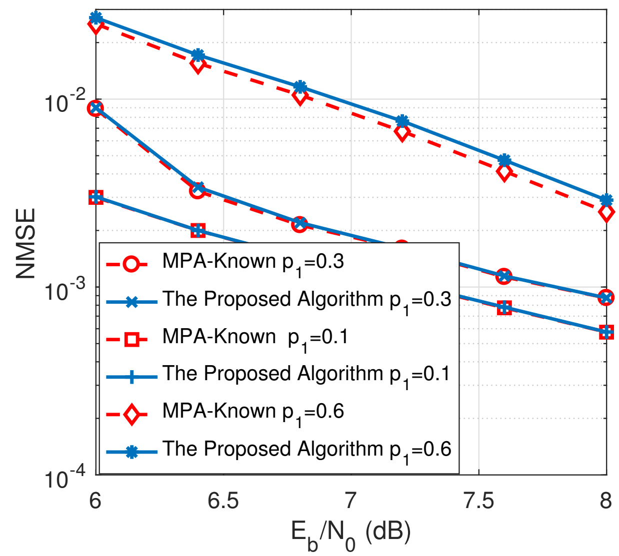

Fig. 12. NMSE of channel estimate with different active probability p_1.

E_b/N_0 is illustrated, where the probability that user is active is $p_1 = 0.3$. For comparison, we also present the performance for the proposed algorithm in Section III with known active users (denoted by 'MPA-Known'), the algorithm that regards all users as active users (denoted by 'Approx-known') and the two-stage CS-MPA algorithm [50] that first uses compressive sensing for active user detection and then performs MPA multiuser detector. We can observe that Approx-known suffers from significant performance degradation. Since the two-stage method only provides hard decision of active users to the equalization part, it also experiences considerable performance loss. Compared to the optimal case that all users' activities are known, the proposed algorithms designed under factor graph framework can achieve nearly optimal performance. Since the user activity detection algorithm II has lower complexity than Algorithm I, it is more attractive in practical grant-free systems.

Fig. 12 depicts the NMSE of channel estimates based on the joint channel estimation, decoding and active user detection algorithm parameterized by the occurrence probability of active users p_1. We see that the performance degrades as p_1 becomes larger. This can be explained by the fact that a larger p_1 leads to more active users in FTN-SCMA systems and both inter-user and inter-symbol interferences become severer. Also, we illustrate the performance of MPA-Known algorithm with different p_1 as a performance bound. It can be observed when p_1 is small, the proposed joint estimation algorithm is capable of attaining the bound. When p_1 increases, although a small performance gap emerges, the proposed algorithm is still efficient in channel estimation.

VI. Conclusions

In this paper, we considered an uplink SCMA system that utilized FTN signaling to further increase the spectral efficiency. Using AR model, the correlated noise samples are approximated by an AR process. Then based on the factorization of the joint posterior distribution, a factor graph based hybrid message passing receiver was proposed for estimating channel coefficients and FTN data symbols. Moreover, considering a grant-free transmission scheme, we extended the factor graph model and proposed two novel user activity detection methods. Consequently, the proposed receiver can deal with joint channel estimation, decoding and active user detection problem in FTN-SCMA systems. Simulation results showed that the combined FTN-SCMA system with the proposed receiver increase the data rate by more than 80% than the orthogonal communications systems.

Appendix A
Derivations of (50) and (51)

For the product vertex $\times^n_{kj}$, the message $\mu_{\bar{s}^n_{kj}\to f^n_j}$ can be regarded as the distribution of $\bar{s}^n_{kj} = \xi_k \tilde{s}^n_{kj}$ with random variables ξ_k and $\tilde{s}^n_{kj}$ following distributions $\mu_{\xi_k\to\times^n_{kj}}$ and $\mu_{\tilde{s}^n_{kj}\to\times^n_{kj}}$. Since $\mu_{\xi_k\to\times^n_{kj}}$ and $\mu_{\tilde{s}^n_{kj}\to\times^n_{kj}}$ are both Gaussian distribution, we can calculate the density of $\bar{s}^n_{kj}$ as

$$\begin{aligned} f(\bar{s}^n_{kj}) &= \int f(\tilde{s}^n_{kj}) f(\xi_k)\delta(\bar{s}^n_{kj} - \tilde{s}^n_{kj}\xi_k)\mathrm{d}\tilde{s}^n_{kj}\mathrm{d}\xi_k \\ &= \int \frac{1}{|\xi_k|} f\left(\frac{\bar{s}^n_{kj}}{\xi_k}\right) f(\xi_k)\mathrm{d}\xi_k \\ \propto \int \frac{1}{|\xi_k|} &\exp\left(-\frac{(\frac{\bar{s}^n_{kj}}{\xi_k} - m_{\tilde{s}^n_{kj}\to\times^n_{kj}})^2}{v_{\tilde{s}^n_{kj}\to\times^n_{kj}}} - \frac{(\xi_k - m_{\xi_k\to\times^n_{kj}})^2}{v_{\xi_k\to\times^n_{kj}}}\right)\mathrm{d}\xi_k. \end{aligned} \tag{61}$$

However, the above integral does not have an analytical expression. As the goal is to derive a Gaussian message, we in turn aim for determining the mean and variance of $\mu_{\bar{s}^n_{kj}\to f^n_j}$ based on incoming messages.

It is well known for two independent random variables x and y, based on the Mellin Transform [51], the nth-order moment of xy satisfies

$$\mathbb{E}[(xy)^n] = \mathbb{E}(x^n)\mathbb{E}(y^n). \tag{62}$$

Thus the first two order moments of $\mu_{\bar{s}_{kj}^n \to f_j^n}$ are given as

$$\mathbb{E}[\bar{s}_{kj}^n] = \mathbb{E}[\tilde{s}_{kj}^n]\mathbb{E}[\xi_k] = m_{\xi_k \to \times_{kj}^n} m_{\phi_{kj}^n \to \tilde{s}_{kj}^n}, \tag{63}$$

$$\begin{aligned}\mathbb{E}[(\bar{s}_{kj}^n)^2] &= \mathbb{E}[(\tilde{s}_{kj}^n)^2]\mathbb{E}[\xi_k^2] \\ &= (m^2_{\xi_k \to \times_{kj}^n} + v_{\xi_k \to \times_{kj}^n})(m^2_{\phi_{kj}^n \to \tilde{s}_{kj}^n} + v_{\phi_{kj}^n \to \tilde{s}_{kj}^n}),\end{aligned} \tag{64}$$

and the variance $v_{\bar{s}_{kj}^n \to f_j^n} = \mathbb{E}[(\bar{s}_{kj}^n)^2] - \mathbb{E}^2[\bar{s}_{kj}^n]$, which are given as (50) and (51).

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