

Channel Estimation for Intelligent Reflecting Surface Assisted Multiuser Communications: Framework, Algorithms, and Analysis

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Abstract—In intelligent reflecting surface (IRS) assisted communication systems, the acquisition of channel state information is a crucial impediment for achieving the beamforming gain of IRS because of the considerable overhead required for channel estimation. Specifically, under the current beamforming design for IRS-assisted communications, in total $KMN + KM$ channel coefficients should be estimated, where K , N and M denote the numbers of users, IRS reflecting elements, and antennas at the base station (BS), respectively. For the first time in the literature, this paper points out that despite the vast number of channel coefficients that should be estimated, significant redundancy exists in the user-IRS-BS reflected channels of different users arising from the fact that each IRS element reflects the signals from all the users to the BS via the same channel. To utilize this redundancy for reducing the channel estimation time, we propose a novel three-phase pilot-based channel estimation framework for IRS-assisted uplink multiuser communications, in which the user-BS direct channels and the user-IRS-BS reflected channels of a typical user are estimated in Phase I and Phase II, respectively, while the user-IRS-BS reflected channels of the other users are estimated with low overhead in Phase III via leveraging their strong correlation with those of the typical user. Under this framework, we analytically prove that a time duration consisting of $K + N + \max(K - 1, \lceil (K - 1)N/M \rceil)$ pilot symbols is sufficient for perfectly recovering all the $KMN + KM$ channel coefficients under the case without receiver noise at the BS. Further, under the case with receiver noise, the user pilot sequences, IRS reflecting coefficients, and BS linear minimum mean-squared error channel estimators are characterized in closed-form.

Index Terms—Intelligent reflecting surface (IRS), channel estimation, multiple-input multiple-output (MIMO), massive MIMO.

I. INTRODUCTION

A. Motivation

Recently, intelligent reflecting surface (IRS) and its various equivalents have emerged as a promising solution to enhance

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the network throughput [2]–[5], thanks to their capability of modifying the wireless channels between the base station (BS) and users to be more favorable to communications via inducing phase shift to the incident signal at each reflecting element in real-time, as illustrated in Fig. 1. Specifically, if perfect channel state information (CSI) is available, with the aid of a smart controller, the IRS is able to properly adjust its reflection coefficients at different reflecting elements based on the CSI such that the desired signals and interfering signals are added constructively and destructively at the receivers, respectively. Along this line, the IRS reflection coefficients optimization with perfect CSI has been widely studied under various setups (see, e.g., [6]–[16]), where the effectiveness of IRS in enhancing the system throughput was verified.

However, the above throughput gain in IRS-assisted communication systems is critically dependent on the availability of CSI, the acquisition of which is quite challenging in practice. Particularly, to reduce the implementation cost, the IRS is generally not equipped with any radio frequency (RF) chains and thus not capable of performing any baseband processing functionality. Therefore, the user-IRS and IRS-BS channels cannot be separately estimated via traditional training-based approaches in general. Instead, only the concatenated user-IRS-BS channels can be estimated based on the training signals sent from the users/BS, and the corresponding number of channel coefficients can be quite large in practice. Specifically, consider a single cell consisting of a BS with M antennas, K single-antenna users, and one IRS with N reflecting elements. It can be shown that the number of channel coefficients involved for designing the IRS reflection coefficients based on the algorithms proposed in [6], [7] is $KMN + KM$. Considering the current paradigm shift towards massive multiple-input multiple-output (MIMO) [17]–[20], i.e., a large M , and massive connectivity [21], [22], i.e., a large K , it is expected that the estimation of these channel coefficients can require tremendous time. This motivates us to devote our endeavour to developing efficient channel estimation strategies for IRS-assisted multiuser communication in this paper.

B. Prior Work

Recently, several works have proposed various strategies to efficiently estimate the channels for IRS-assisted communication [1], [23]–[29]. For the single-user system, in [23], [24], an on-off state control based channel estimation strategy was proposed, where only one IRS element is switched on at each

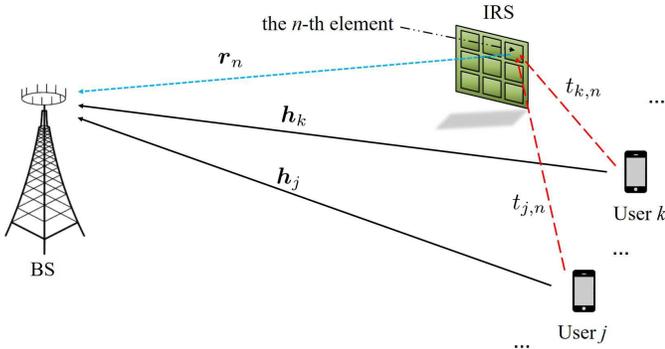


Fig. 1. An IRS-assisted multiuser communication system: for any user k and user j , the n th element of the IRS reflects their signals to the BS via the same channel.

time slot such that its reflected channel for the user can be estimated without interference from the reflected signals of the other IRS elements. Under this strategy, N time slots are sufficient to perfectly estimate all the reflected channels for the user for the case without receiver noise at the BS. On the other hand, [25], [26] proposed a novel discrete Fourier transform (DFT) based channel estimation strategy, in which all the IRS elements are on at each time slot, and their reflection coefficients are determined by the DFT matrix. Under this strategy, although still N time slots are required to perfectly estimate all the reflected channels in the case without receiver noise at the BS, the channel estimation mean-squared error (MSE) in the case with receiver noise is significantly reduced compared to the on-off state control based strategy, because the IRS elements are always on to reflect all the signal power to the BS. Further, [27] proposed a Lagrange-based estimation strategy to minimize the MSE for channel estimation. Last, [28] formulated the single-user channel estimation problem as a combined sparse matrix factorization and matrix completion problem and proposed to apply the compressed sensing technique to solve the problem. On the other hand, for the multiuser case, our conference paper [1] pointed out the possibility of reducing the channel estimation time by utilizing the fact that the IRS reflects all users' signals to the BS via the same channels. However, there lacks a systematic study of this scheme. Moreover, [29] modeled the IRS reflected channels as sparse channels, and then applied the compressed sensing technique to estimate the channels with reduced time duration. However, whether the IRS reflected channels are sparse in practice is still unknown, while this paper aims to propose a channel estimation framework that can be applied to the more general channel model without assuming any channel property.

C. Main Contributions

In this paper, we consider an IRS-assisted multiuser uplink communication system where multiple single-antenna users communicate with a multi-antenna BS with the help of an IRS. Under this setup, we investigate the passive pilot based channel estimation approach, where the IRS elements passively reflect the pilot sequences sent by the users to the BS such that the

BS is able to estimate the CSI associated with the IRS. The main contributions of this paper are summarized as follows.

First, we propose a novel *three-phase channel estimation framework* for IRS-assisted multiuser uplink communications. The foundation of this framework lies in the *correlation* among the user-IRS-BS reflected channels of different users: *each IRS element reflects the signals from different users to the BS via the same channel*, as illustrated in Fig. 1. To make the best use of this correlation, the proposed channel estimation framework works as follows. In Phase I, the IRS is switched off such that the BS can estimate its direct channels with the users. In Phase II, the IRS is switched on and only one typical user is selected to transmit non-zero pilot symbols such that its IRS reflected channels can be estimated. In Phase III, the other users transmit their pilot symbols and their IRS reflected channels can be efficiently estimated by exploiting the fact that these reflected channels are *scaled versions* of the typical user's reflected channels and thus only the *scaling factors (scalars)*, rather than the *whole channels (vectors)*, need to be estimated.

Second, for the ideal case without receiver noise at the BS, we show that the theoretically minimal pilot sequence length to perfectly estimate all the channel coefficients under the proposed three-phase framework is $K + N + \max(K - 1, \lceil (K - 1)N/M \rceil)$. Specifically, it is shown that K and N time slots are sufficient to estimate the direct channels of all the users and IRS reflected channels of the typical user in Phase I and Phase II, respectively, while $\max(K - 1, \lceil (K - 1)N/M \rceil)$ time slots are sufficient for perfect channel estimation in Phase III. Interestingly, the minimum pilot sequence length decreases with M generally. Such a result is in sharp contrast to the traditional multiuser channel estimation results without IRS, where the minimum pilot sequence length is independent of the number of receive antennas at the BS [30].

Third, for the practical case with receiver noise at the BS, we design the linear minimum mean-squared error (LMMSE) channel estimation solutions in all the three phases. In each phase, the user transmit pilot, the IRS reflection coefficients, and the BS LMMSE channel estimators are characterized in closed-form. Moreover, the corresponding MSE for channel estimation is also derived to evaluate the performance of our proposed three-phase framework.

Last but not least, this paper reveals the significant role of *massive MIMO* in channel estimation for IRS-assisted uplink communications. Especially, in the massive MIMO regime with $M > N$, the minimum pilot sequence length under our framework is $2K + N - 1$, which is scalable with the number of users: if there is one more user in the system, only 2 additional pilot symbols are sufficient to estimate the new $MN + M$ channel coefficients associated with this user.

D. Organization

The rest of this paper is organized as follows. Section II describes the system model for our considered IRS-assisted multiuser communication system. Section III introduces the three-phase channel estimation protocol. Section IV presents the minimum pilot sequence length for perfect channel estimation in closed-form for the case without noise at the

BS. Section V proposes a closed-form solution of user pilot sequences, IRS reflecting coefficients and BS LMMSE channel estimators for the case with noise at the BS. Numerical results are provided in Section VI. Finally, Section VII concludes the paper and outlines the future research directions.

Notation: \mathbf{I} and $\mathbf{0}$ denote an identity matrix and an all-zero matrix, respectively, with appropriate dimensions. For a square full-rank matrix \mathbf{S} , \mathbf{S}^{-1} denotes its inverse. For a matrix \mathbf{M} of arbitrary size, \mathbf{M}^H , \mathbf{M}^T and \mathbf{M}^* denote the conjugate transpose, transpose and conjugate of \mathbf{M} , respectively. Moreover, $\text{diag}(x_1, \dots, x_K)$ denotes a diagonal matrix with the diagonal elements given by x_1, \dots, x_K . $\lceil x \rceil$ and $\lfloor x \rfloor$ denote the smallest integer that is no smaller than x and the largest integer that is no larger than x , respectively.

II. SYSTEM MODEL

We consider a narrow-band wireless system with K single-antenna users simultaneously communicating with a BS in the uplink, where the BS is equipped with M antennas. An IRS equipped with N passive reflecting elements is deployed for enhancing the users' communication performance, as shown in Fig. 1. We assume quasi-static block-fading channels, where all the channels remain approximately constant in each fading block. Moreover, the length of each fading block is denoted by T symbols. Let $\mathbf{h}_k \in \mathbb{C}^{M \times 1}$, $k = 1, \dots, K$, denote the direct channel from the k th user to the BS. Further, let $\mathbf{r}_n \in \mathbb{C}^{M \times 1}$ and $t_{k,n} \in \mathbb{C}$ denote the channels from the n th IRS element to the BS and from the k th user to the n th IRS element, respectively, $k = 1, \dots, K$, $n = 1, \dots, N$.

In this paper, \mathbf{h}_k 's are modeled as

$$\mathbf{h}_k = (\mathbf{C}_k^{\text{B}})^{\frac{1}{2}} \tilde{\mathbf{h}}_k, \quad \forall k, \quad (1)$$

where $\mathbf{C}_k^{\text{B}} \in \mathbb{C}^{M \times M}$ with $\mathbf{C}_k^{\text{B}} > \mathbf{0}$ denotes the BS receive correlation matrix for user k , and $\tilde{\mathbf{h}}_k \sim \mathcal{CN}(\mathbf{0}, \beta_k^{\text{BU}} \mathbf{I})$ follows the independent and identically distributed (i.i.d.) Rayleigh fading channel model, with β_k^{BU} denoting the path loss of \mathbf{h}_k . Further, define $\mathbf{R} = [\mathbf{r}_1, \dots, \mathbf{r}_N]$ as the overall channel from the IRS to the BS. Then, \mathbf{R} is modeled by

$$\mathbf{R} = (\mathbf{C}^{\text{B}})^{\frac{1}{2}} \tilde{\mathbf{R}} (\mathbf{C}^{\text{I}})^{\frac{1}{2}}, \quad (2)$$

where $\mathbf{C}^{\text{B}} \in \mathbb{C}^{M \times M}$ with $\mathbf{C}^{\text{B}} > \mathbf{0}$ and $\mathbf{C}^{\text{I}} \in \mathbb{C}^{N \times N}$ with $\mathbf{C}^{\text{I}} > \mathbf{0}$ denote the BS receive correlation matrix and IRS transmit correlation matrix for \mathbf{R} , respectively, and $\tilde{\mathbf{R}} \sim \mathcal{CN}(\mathbf{0}, \beta^{\text{BI}} \mathbf{N} \mathbf{I})$ is the i.i.d. Rayleigh fading component, with β^{BI} denoting the path loss of \mathbf{R} . At last, define $\mathbf{t}_k = [t_{k,1}, \dots, t_{k,N}]^T$ as the overall channel from user k to the IRS, $\forall k$. Then, \mathbf{t}_k 's are modeled as follows:

$$\mathbf{t}_k = (\mathbf{C}_k^{\text{I}})^{\frac{1}{2}} \tilde{\mathbf{t}}_k, \quad \forall k, \quad (3)$$

where $\mathbf{C}_k^{\text{I}} \in \mathbb{C}^{N \times N}$ with $\mathbf{C}_k^{\text{I}} > \mathbf{0}$ denotes the IRS receive correlation matrix for \mathbf{t}_k , and $\tilde{\mathbf{t}}_k \sim \mathcal{CN}(\mathbf{0}, \beta_k^{\text{IU}} \mathbf{I})$ denotes the i.i.d. Rayleigh fading component, with β_k^{IU} denoting the pass loss of \mathbf{t}_k .

Thanks to the IRS controller, each element on IRS is able to dynamically adjust its reflection coefficient for re-scattering the electromagnetic waves from the users to the BS such that the useful signal and harmful interference can be added at the

BS in constructive and destructive manners, respectively [2]–[4]. Specifically, let $\phi_{n,i}$ denote the reflection coefficient of the n th IRS element at the i th time instant over the considered coherence block, $n = 1, \dots, N$, $i = 1, \dots, T$, which satisfies

$$|\phi_{n,i}| = \begin{cases} 1, & \text{if element } n \text{ is on at time instant } i, \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

Thus, if an IRS element is on, it can only change the phase of its incident signal [6], [7].

With the existence of the IRS, the received signal of the BS at time instant i , $i = 1, \dots, T$, is the superposition of the signals from the users' direct communication links and the reflected links via the IRS, which is expressed as

$$\begin{aligned} \mathbf{y}^{(i)} &= \sum_{k=1}^K \mathbf{h}_k \sqrt{p} x_k^{(i)} + \sum_{k=1}^K \sum_{n=1}^N \phi_{n,i} t_{k,n} \mathbf{r}_n \sqrt{p} x_k^{(i)} + \mathbf{z}^{(i)} \\ &= \sum_{k=1}^K \left(\mathbf{h}_k + \sum_{n=1}^N \phi_{n,i} \mathbf{g}_{k,n} \right) \sqrt{p} x_k^{(i)} + \mathbf{z}^{(i)}, \end{aligned} \quad (5)$$

where $x_k^{(i)}$ and $\mathbf{z}^{(i)} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$ denote the transmit signal of user k and additive white Gaussian noise (AWGN) at the BS at time instant i , respectively, p denotes the identical transmit power of the users, and

$$\mathbf{g}_{k,n} = t_{k,n} \mathbf{r}_n, \quad \forall n, k, \quad (6)$$

denotes the user-IRS-BS reflected channel from the k th user to the BS via the n th IRS element.

In this paper, we consider the legacy two-stage transmission protocol for the uplink communications, in which each coherence block of length T symbols is divided into the channel estimation stage consisting of $\tau < T$ symbols and data transmission stage consisting of $T - \tau$ symbols. Specifically, in the channel estimation stage, each user k is assigned with a pilot sequence consisting of τ symbols:

$$\mathbf{a}_k = [a_{k,1}, \dots, a_{k,\tau}]^T, \quad k = 1, \dots, K, \quad (7)$$

where the norm of $a_{k,i}$ is either zero or one, $\forall k, i$. At time instant $i \leq \tau$, user k transmits $x_{k,i} = a_{k,i}$ to the BS, and the received signal at the BS is

$$\mathbf{y}^{(i)} = \sum_{k=1}^K \left(\mathbf{h}_k + \sum_{n=1}^N \phi_{n,i} \mathbf{g}_{k,n} \right) \sqrt{p} a_{k,i} + \mathbf{z}^{(i)}, \quad i \leq \tau. \quad (8)$$

Define $\mathbf{Y} = [\mathbf{y}^{(1)}, \dots, \mathbf{y}^{(T)}] \in \mathbb{C}^{M \times T}$ as the overall received signal at the BS across all the T time instants of the channel estimation stage. We should properly design user pilot symbols $a_{k,i}$'s and IRS reflection coefficients $\phi_{n,i}$'s such that the BS is able to estimate the CSI that is useful for the data transmission stage (will be introduced later) based on the received signal \mathbf{Y} as well as its knowledge of $a_{k,i}$'s and $\phi_{n,i}$'s.

In the data transmission stage, the reflection coefficient of each IRS element n is fixed over different time instants, i.e., $\phi_{n,i} = \phi_n$, $i = \tau + 1, \dots, T$ [6], [7]. Moreover, to convey the information, the transmit symbol of user k in the i th time instant is modeled as a circularly symmetric complex Gaussian (CSCG) random variable with zero mean and unit variance, i.e., $x_k^{(i)} \sim \mathcal{CN}(0, 1)$, $k = 1, \dots, K$, $i = \tau + 1, \dots, T$. Then,

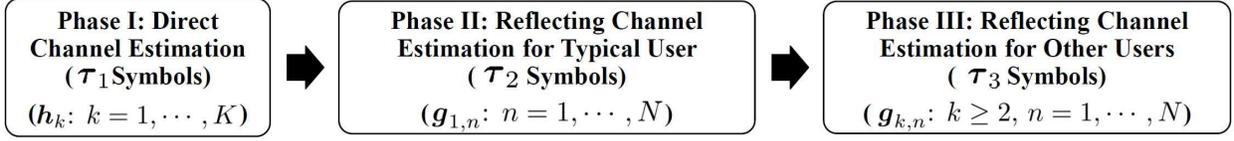


Fig. 2. A flow graph for the proposed three-phase channel estimation framework.

at each time instant $i = \tau + 1, \dots, T$, the BS applies the beamforming vector \mathbf{w}_k to decode the message of user k , i.e.,

$$\tilde{\mathbf{y}}_k^{(i)} = \sum_{j=1}^K \mathbf{w}_k^H \left(\mathbf{h}_j + \sum_{n=1}^N \phi_n \mathbf{g}_{j,n} \right) \sqrt{p} x_j^{(i)} + \mathbf{w}_k^H \mathbf{z}^{(i)}. \quad (9)$$

Therefore, the achievable rate of user k , $k = 1, \dots, K$, is

$$R_k = \frac{T - \tau}{T} \log_2(1 + \gamma_k), \quad (10)$$

where $\frac{T - \tau}{T}$ denotes the fraction of time for data transmission, and

$$\gamma_k = \frac{p \left| \mathbf{w}_k^H \left(\mathbf{h}_k + \sum_{n=1}^N \phi_n \mathbf{g}_{k,n} \right) \right|^2}{\sum_{j \neq k} p \left| \mathbf{w}_k^H \left(\mathbf{h}_j + \sum_{n=1}^N \phi_n \mathbf{g}_{j,n} \right) \right|^2 + \sigma^2 \|\mathbf{w}_k\|^2}. \quad (11)$$

It is observed from (10) and (11) that to improve the user rate by jointly optimizing the receive beamforming vectors \mathbf{w}_k 's at the BS and reflection coefficients ϕ_n 's at the IRS, in the channel training stage, the BS has to estimate $MK + MKN$ channel coefficients in \mathbf{h}_k 's, $k = 1, \dots, K$, and $\mathbf{g}_{k,n}$'s, $k = 1, \dots, K$, $n = 1, \dots, N$. Note that $MK + MKN$ is generally a very large number in next-generation cellular networks with large-scale antenna arrays at the BSs (i.e., a large M) and a massive number of connecting users (i.e., a large K). Further, to increase the time duration for data transmission in (10), very few pilot symbols can be utilized for estimating such a large number of channel coefficients. To tackle the above challenge, in the rest of this paper, we mainly focus on the channel training stage in our considered system, and propose an innovative scheme to estimate \mathbf{h}_k 's and $\mathbf{g}_{k,n}$'s accurately with low training overhead. The robust beamforming design for data transmission given the estimated channels with errors will be left to our future work.

III. THREE-PHASE CHANNEL ESTIMATION PROTOCOL

In this section, we propose a novel three-phase channel estimation protocol for IRS-assisted multiuser communications. The main idea is that although KMN unknowns need to be estimated in $\mathbf{g}_{k,n}$'s, the degrees-of-freedom (DoF) for all these channel coefficients is much smaller than KMN . Specifically, each \mathbf{r}_n appears in all $\mathbf{g}_{k,n}$'s, $k = 1, \dots, K$, according to (6), since each IRS element n reflects the signals from all the K users to the BS via the same channel \mathbf{r}_n . It is thus theoretically feasible to employ much fewer pilot symbols to estimate the KMN correlated unknowns in $\mathbf{g}_{k,n}$'s. Nevertheless, it is challenging to practically exploit the correlations among the channel coefficients arising from \mathbf{r}_n 's, since the IRS cannot estimate \mathbf{r}_n 's due to the lack of RF chains.

In the following, we propose a novel three-phase channel estimation protocol, as depicted in Fig. 2, which can exploit the channel correlations arising from \mathbf{r}_n 's to reduce the estimation time even without knowing \mathbf{r}_n 's. Specifically, in Phase I consisting of τ_1 symbols, define

$$\mathbf{a}_k^I = [a_{k,1}, \dots, a_{k,\tau_1}]^T, \quad k = 1, \dots, K, \quad (12)$$

as the pilot sequence sent by user k . In this phase, all the IRS elements are switched off, i.e.,

$$\phi_{n,i} = 0, \quad n = 1, \dots, N, \quad i = 1, \dots, \tau_1. \quad (13)$$

Then, the received signal at the BS at time slot i of Phase I is

$$\mathbf{y}^{(i)} = \sum_{k=1}^K \mathbf{h}_k \sqrt{p} a_{k,i} + \mathbf{z}^{(i)}, \quad i = 1, \dots, \tau_1. \quad (14)$$

The BS thus needs to estimate the direct channels \mathbf{h}_k 's based on the above received signals.

In Phase II consisting of τ_2 symbols, define

$$\mathbf{a}_k^{II} = [a_{k,\tau_1+1}, \dots, a_{k,\tau_1+\tau_2}]^T, \quad k = 1, \dots, K, \quad (15)$$

as the pilot sequence of user k . In this phase, all the IRS reflection elements are switched on, and merely one *typical user*, denoted by user 1 for convenience, transmits non-zero pilot symbols to the BS, i.e.,

$$\mathbf{a}_k^{II} = \mathbf{0}, \quad k = 2, \dots, K. \quad (16)$$

Then, the received signal at the BS in time slot i ($i = \tau_1 + 1, \dots, \tau_1 + \tau_2$) of Phase II is

$$\mathbf{y}^{(i)} = \sum_{n=1}^N \phi_{n,i} \mathbf{g}_{1,n} \sqrt{p} a_{1,i} + \mathbf{h}_1 \sqrt{p} a_{1,i} + \mathbf{z}^{(i)}. \quad (17)$$

Based on its received signals in Phase II as well as knowledge of \mathbf{h}_1 after Phase I, the BS estimates the IRS reflected channels of this typical user, i.e., $\mathbf{g}_{1,n}$'s, $\forall n$.

In Phase III consisting of $\tau_3 = \tau - \tau_1 - \tau_2$ symbols, define

$$\mathbf{a}_k^{III} = [a_{k,\tau_1+\tau_2+1}, \dots, a_{k,\tau_1+\tau_2+\tau_3}]^T, \quad k = 1, \dots, K, \quad (18)$$

as the pilot sequence of user k . In this phase, merely user 2 to user K transmit the non-zero pilot symbols to the BS, i.e.,

$$\mathbf{a}_1^{III} = \mathbf{0}. \quad (19)$$

As a result, the received signal at the BS in time slot i ($i = \tau_1 + \tau_2 + 1, \dots, \tau_1 + \tau_2 + \tau_3$) of Phase III is

$$\mathbf{y}^{(i)} = \sum_{k=2}^K \sum_{n=1}^N \phi_{n,i} \mathbf{g}_{k,n} \sqrt{p} a_{k,i} + \sum_{k=2}^K \mathbf{h}_k \sqrt{p} a_{k,i} + \mathbf{z}^{(i)}. \quad (20)$$

Intuitively, there are $(K - 1)MN$ unknowns to be estimated in $\mathbf{g}_{k,n}$'s, $k \geq 2$. However, this number can be significantly

reduced based on the following relationship between the user-IRS-B S reflected channels of user 1 and the other users:

$$\mathbf{g}_{k,n} = \lambda_{k,n} \mathbf{g}_{1,n}, \quad k = 2, \dots, K, \quad n = 1, \dots, N, \quad (21)$$

where

$$\lambda_{k,n} = \frac{t_{k,n}}{t_{1,n}}, \quad k = 2, \dots, K, \quad n = 1, \dots, N. \quad (22)$$

By taking (21) into (20), the received signal at the BS in the i th time slot of Phase III reduces to

$$\mathbf{y}^{(i)} = \sum_{k=2}^K \sum_{n=1}^N \phi_{n,i} \mathbf{g}_{1,n} \sqrt{p} a_{k,i} \lambda_{k,n} + \sum_{k=2}^K \mathbf{h}_k \sqrt{p} a_{k,i} + \mathbf{z}^{(i)}. \quad (23)$$

With known $\mathbf{g}_{1,n}$'s, $n = 1, \dots, N$, after Phase II, (23) reveals that each channel vector $\mathbf{g}_{k,n}$ with M unknowns, $k \geq 2$, can be efficiently recovered via merely estimating a scalar $\lambda_{k,n}$. As a result, the number of unknowns to be estimated in Phase III is significantly reduced.

For the purpose of drawing essential insights, in the rest of this paper, we first introduce this protocol for the ideal case without noise at the BS, i.e., $\mathbf{z}^{(i)} = \mathbf{0}$, $\forall i$, and characterize the *minimum pilot sequence length* to estimate all the channels perfectly. Such a result can theoretically demonstrate the performance gain brought by this new protocol for channel estimation in our considered system. Then, we will illustrate how to implement our proposed three-phase channel estimation framework in the practical case with noise at the BS and characterize the *MSE* for estimating the channels.

IV. PERFORMANCE LIMITS FOR CASE WITHOUT NOISE

In this section, we consider the ideal case without noise at the BS. In this scenario, the proposed three-phase channel estimation protocol works as follows.

A. Phase I: Direct Channel Estimation

In the case without noise, i.e., $\mathbf{z}^{(i)} = \mathbf{0}$, $\forall i$, according to [30], each user k can send

$$\tau_1 \geq \tilde{\tau}_1 = K, \quad (24)$$

pilot symbols to the BS for channel estimation. Then, based on (14), the received signal at the BS over the whole phase is

$$\begin{aligned} \mathbf{Y}^I &= [\mathbf{y}^{(1)}, \dots, \mathbf{y}^{(\tau_1)}] \\ &= \sqrt{p} [\mathbf{h}_1, \dots, \mathbf{h}_K] [(\mathbf{a}_1^I), \dots, (\mathbf{a}_K^I)]^T. \end{aligned} \quad (25)$$

As a result, the direct channels \mathbf{h}_k 's can be estimated perfectly by solving (25) if the pilot sequences of different users are orthogonal to each other, i.e.,

$$[(\mathbf{a}_1^I), \dots, (\mathbf{a}_K^I)]^T [(\mathbf{a}_1^I)^*, \dots, (\mathbf{a}_K^I)^*] = \tau_1 \mathbf{I}. \quad (26)$$

Since each pilot sequence consists of $\tau_1 \geq K$ symbols, it is feasible to design \mathbf{a}_k^I 's to satisfy (26). Then, according to (25), \mathbf{h}_k 's can be perfectly estimated as

$$[\mathbf{h}_1, \dots, \mathbf{h}_K] = \frac{1}{\tau_1 \sqrt{p}} \mathbf{Y}^I [(\mathbf{a}_1^I)^*, \dots, (\mathbf{a}_K^I)^*]. \quad (27)$$

B. Phase II: Reflecting Channel Estimation for Typical User

In the second phase, to estimate $\mathbf{g}_{1,n}$'s, we only allow the typical user, denoted by user 1, to transmit its pilot, as shown in (16). Note that \mathbf{h}_k 's have already been perfectly estimated by (27) in Phase I. Therefore, their interference for estimating $\mathbf{g}_{1,n}$'s can be canceled from the received signal at the BS in Phase II. According to (17), after interference cancellation, the effective received signal at the BS at time instant i ($i = \tau_1 + 1, \dots, \tau_1 + \tau_2$) in Phase II is

$$\bar{\mathbf{y}}^{(i)} = \mathbf{y}^{(i)} - \sum_{k=1}^K \mathbf{h}_k \sqrt{p} a_{k,i} = \sum_{n=1}^N \phi_{n,i} \mathbf{g}_{1,n} \sqrt{p} a_{1,i}. \quad (28)$$

The overall effective received signal at the BS in the second phase is then expressed as

$$\begin{aligned} \bar{\mathbf{Y}}^{\text{II}} &= [\bar{\mathbf{y}}^{(\tau_1+1)}, \dots, \bar{\mathbf{y}}^{(\tau_1+\tau_2)}] \\ &= \sqrt{p} [\mathbf{g}_{1,1}, \dots, \mathbf{g}_{1,N}] \mathbf{\Phi}^{\text{II}} \text{diag}(\mathbf{a}_1^{\text{II}}), \end{aligned} \quad (29)$$

where

$$\mathbf{\Phi}^{\text{II}} = \begin{bmatrix} \phi_{1,\tau_1+1} & \dots & \phi_{1,\tau_1+\tau_2} \\ \vdots & \ddots & \vdots \\ \phi_{N,\tau_1+1} & \dots & \phi_{N,\tau_1+\tau_2} \end{bmatrix}. \quad (30)$$

To solve (29), we can simply set

$$a_{1,i} = 1, \quad i = \tau_1 + 1, \dots, \tau_1 + \tau_2. \quad (31)$$

In this case, it can be shown that as long as

$$\tau_2 \geq \tilde{\tau}_2 = N, \quad (32)$$

we can always construct a $\mathbf{\Phi}^{\text{II}}$ such that $\text{rank}(\mathbf{\Phi}^{\text{II}}) = N$ under the constraint given in (4). The construction of such a $\mathbf{\Phi}^{\text{II}}$ can be based on the DFT matrix

$$\mathbf{\Phi}^{\text{II}} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \omega & \omega^2 & \dots & \omega^{\tau_2-1} \\ 1 & \omega^2 & \omega^4 & \dots & \omega^{2(\tau_2-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{N-1} & \omega^{2(N-1)} & \dots & \omega^{(N-1)(\tau_2-1)} \end{bmatrix}, \quad (33)$$

where $\omega = e^{-2\pi q/\tau_2}$ with $q^2 = -1$. In this case, it follows that $\mathbf{\Phi}^{\text{II}}(\mathbf{\Phi}^{\text{II}})^H = \tau_2 \mathbf{I}$. As a result, the reflected channels of user 1 can be perfectly estimated as

$$[\mathbf{g}_{1,1}, \dots, \mathbf{g}_{1,N}] = \frac{1}{\tau_2 \sqrt{p}} \bar{\mathbf{Y}}^{\text{II}} (\mathbf{\Phi}^{\text{II}})^H. \quad (34)$$

C. Phase III: Reflecting Channel Estimation for Other Users

To estimate the channels in Phase III, one straightforward approach is to allow only one user $k \geq 2$ to transmit $\tau_2 \geq N$ pilot symbols each time such that its reflected channels $\mathbf{g}_{k,n}$'s, $\forall n$, can be directly estimated based on the approach for estimating $\mathbf{g}_{1,n}$'s. Under such a scheme, we need to use at least $\tau_3 = (K-1)N$ time instants in total to estimate the reflecting channels of the remaining $K-1$ users. However, with a large number of users, the estimation of $\mathbf{g}_{k,n}$'s will take quite a long time, which leads to reduced user transmission rate due to the limited time left for data transmission as shown in (10).

$$\mathbf{V} = \begin{bmatrix} \phi_{1,\theta+1} a_{2,\theta+1} \mathbf{g}_{1,1} & \cdots & \phi_{N,\theta+1} a_{2,\theta+1} \mathbf{g}_{1,N} & \cdots & \phi_{1,\theta+1} a_{K,\theta+1} \mathbf{g}_{1,1} & \cdots & \phi_{N,\theta+1} a_{K,\theta+1} \mathbf{g}_{1,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \phi_{1,\theta+\tau_3} a_{2,\theta+\tau_3} \mathbf{g}_{1,1} & \cdots & \phi_{N,\theta+\tau_3} a_{2,\theta+\tau_3} \mathbf{g}_{1,N} & \cdots & \phi_{1,\theta+\tau_3} a_{K,\theta+\tau_3} \mathbf{g}_{1,1} & \cdots & \phi_{N,\theta+\tau_3} a_{K,\theta+\tau_3} \mathbf{g}_{1,N} \end{bmatrix}, \quad (37)$$

with $\theta = \tau_1 + \tau_2$.

We propose to exploit the channel correlations among $\mathbf{g}_{k,n}$'s to reduce the channel estimation time in Phase III. Specifically, similar to (28), after cancelling the interference caused by the direct channels \mathbf{h}_k 's from (23), the effective received signal at the BS at each time instant i , $i = \tau_1 + \tau_2 + 1, \dots, \tau_1 + \tau_2 + \tau_3$, in Phase III is

$$\bar{\mathbf{y}}^{(i)} = \mathbf{y}^{(i)} - \sum_{k=2}^K \mathbf{h}_k \sqrt{p} a_{k,i} = \sum_{k=2}^K \sum_{n=1}^N \phi_{n,i} \sqrt{p} a_{k,i} \lambda_{k,n} \mathbf{g}_{1,n}. \quad (35)$$

The overall effective received signal at the BS in Phase III is

$$\bar{\mathbf{y}}^{\text{III}} = \left[\left(\bar{\mathbf{y}}^{(\tau_1+\tau_2+1)} \right)^T, \dots, \left(\bar{\mathbf{y}}^{(\tau_1+\tau_2+\tau_3)} \right)^T \right]^T = \sqrt{p} \mathbf{V} \boldsymbol{\lambda}, \quad (36)$$

where $\boldsymbol{\lambda} = [\boldsymbol{\lambda}_2^T, \dots, \boldsymbol{\lambda}_K^T]^T \in \mathbb{C}^{(K-1)N \times 1}$ with $\boldsymbol{\lambda}_k = [\lambda_{k,1} \cdots \lambda_{k,N}]^T$, $k = 2, \dots, K$, and $\mathbf{V} \in \mathbb{C}^{M\tau_3 \times (K-1)N}$ is given in (37) on the top of this page.

Mathematically, (36) defines an equivalent linear channel estimation model consisting of $(K-1)N$ users, where each column of \mathbf{V} denotes the pilot sequence sent by each of these effective users. One interesting observation of \mathbf{V} here is that thanks to the multiple antennas at the BS, the effective channel estimation time is increased from τ_3 to $M\tau_3$. In other words, it is possible to leverage the multi-antenna technology to significantly reduce the channel training time in Phase III under our proposed strategy.

In the following, via a proper design of $\mathbf{a}_k^{\text{III}} = [a_{k,\tau_1+\tau_2+1}, \dots, a_{k,\tau_1+\tau_2+\tau_3}]^T$'s, $k = 2, \dots, K$, and $\phi_{n,i}$'s, $n = 1, \dots, N$, $i = \tau_1 + \tau_2 + 1, \dots, \tau_1 + \tau_2 + \tau_3$, we aim to find the minimum value of τ_3 to satisfy $\text{rank}(\mathbf{V}) = (K-1)N$ such that $\boldsymbol{\lambda}$ can be perfectly estimated based on (36). We start with the case of $M \geq N$.

Theorem 1: In the case of $M \geq N$, the minimum value of τ_3 to guarantee perfect estimation of $\boldsymbol{\lambda}$ according to (36) is given by

$$\tilde{\tau}_3 = K - 1. \quad (38)$$

To achieve perfect estimation of $\boldsymbol{\lambda}$ given the above minimum value of τ_3 , in the case of $M \geq N$, we can set

$$a_{k,i} = \begin{cases} 1, & \text{if } k-1 = i - \tau_1 - \tau_2, \\ 0, & \text{otherwise,} \end{cases} \quad 2 \leq k \leq K, \quad (39)$$

$$\phi_{n,i} = 1, \quad 1 \leq n \leq N, \quad \tau_1 + \tau_2 + 1 \leq i \leq \tau_1 + \tau_2 + K - 1. \quad (40)$$

Then, $\boldsymbol{\lambda}$ can be perfectly estimated as

$$\boldsymbol{\lambda}_k = [\mathbf{g}_{1,1}, \dots, \mathbf{g}_{1,N}]^\dagger \frac{\bar{\mathbf{y}}^{(\tau_1+\tau_2+k-1)}}{\sqrt{p}}, \quad k = 2, \dots, K, \quad (41)$$

where for any matrix $\mathbf{B} \in \mathbb{C}^{x \times y}$ with $x \geq y$, $\mathbf{B}^\dagger = (\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H$ denotes its pseudo-inverse matrix.

Proof: Please refer to Appendix A. ■

Next, we consider the case of $M < N$. In this case, define $\rho = \lfloor \frac{N}{M} \rfloor$, $\nu = N - M\rho$, and $\mathcal{N} = \{1, \dots, N\}$. For each user $k \geq 2$, define two sets $\Lambda_{k,1} \subset \mathcal{N}$ with cardinality $|\Lambda_{k,1}| = N - \nu$ and $\Lambda_{k,2} \subset \mathcal{N}$ with cardinality $|\Lambda_{k,2}| = \nu$, which are constructed as follows. First, define

$$\mathcal{T}_k = \{(k-2)\nu + 1, \dots, (k-1)\nu\}, \quad k = 2, \dots, K. \quad (42)$$

Then, we construct $\Lambda_{k,1}$'s and $\Lambda_{k,2}$'s as

$$\Lambda_{k,2} = \{m - (\lfloor \frac{m}{N} \rfloor - 1)N : \forall m \in \mathcal{T}_k\}, \quad (43)$$

$$\Lambda_{k,1} = \mathcal{N} \setminus \Lambda_{k,2}, \quad k = 2, \dots, K. \quad (44)$$

Moreover, for any $i = 1, \dots, (K-1)\rho$, define $\kappa_i = (i - (\lfloor \frac{i}{\rho} \rfloor - 1)\rho - 1)M$ and

$$\Omega_i = \{\Lambda_{\lfloor \frac{i}{\rho} \rfloor + 1, 1}(\kappa_i + 1), \dots, \Lambda_{\lfloor \frac{i}{\rho} \rfloor + 1, 1}(\kappa_i + M)\}, \quad (45)$$

where given any set \mathcal{B} , $\mathcal{B}(i)$ denotes its i th element. While for any $i = (K-1)\rho + 1, (K-1)\rho + 2, \dots$, define

$$\mathcal{J}_i = \{(i - (K-1)\rho - 1)M + 1, \dots, \min((i - (K-1)\rho)M, (K-1)N - (K-1)M\rho)\}. \quad (46)$$

Based on \mathcal{J}_i , given any $i > (K-1)\rho$, we define

$$\mathcal{K}_i = \{\lfloor \frac{j}{\nu} \rfloor + 1 : \forall j \in \mathcal{J}_i\}, \quad (47)$$

$$\mathcal{N}_i = \{\Lambda_{\lfloor \frac{j}{\nu} \rfloor + 1, 2}(j - (\lfloor \frac{j}{\nu} \rfloor - 1)\nu) : \forall j \in \mathcal{J}_i\}. \quad (48)$$

Then, we have the following theorem.

Theorem 2: In the case of $M < N$, the minimum value of τ_3 to guarantee perfect estimation of $\boldsymbol{\lambda}$ according to (36) is given by

$$\tilde{\tau}_3 = \left\lceil \frac{(K-1)N}{M} \right\rceil. \quad (49)$$

To perfectly estimate $\boldsymbol{\lambda}$ given the above minimum value of τ_3 , at time slot $\tau_1 + \tau_2 + i$ with $1 \leq i \leq (K-1)\rho$, we can set

$$a_{k,\tau_1+\tau_2+i} = \begin{cases} 1, & \text{if } k = \lfloor \frac{i}{\rho} \rfloor + 1, \\ 0, & \text{otherwise,} \end{cases} \quad (50)$$

$$\phi_{n,\tau_1+\tau_2+i} = \begin{cases} 1, & \text{if } n \in \Omega_i, \\ 0, & \text{otherwise,} \end{cases} \quad 1 \leq i \leq (K-1)\rho. \quad (51)$$

With the above solution, at each time instant $\tau_1 + \tau_2 + i$, we can perfectly estimate the following $\lambda_{k,n}$'s

$$\begin{aligned} & [\lambda_{\lfloor \frac{i}{\rho} \rfloor + 1, \Omega_i(1)}, \dots, \lambda_{\lfloor \frac{i}{\rho} \rfloor + 1, \Omega_i(M)}]^T \\ &= [\mathbf{g}_{1, \Omega_i(1)}, \dots, \mathbf{g}_{1, \Omega_i(M)}]^{-1} \frac{\bar{\mathbf{y}}^{(\tau_1+\tau_2+i)}}{\sqrt{p}}, \quad 1 \leq i \leq (K-1)\rho. \end{aligned} \quad (52)$$

Further, at time slot $\tau_1 + \tau_2 + i$ with $(K-1)\rho + 1 \leq i \leq \tilde{\tau}_3$, we can set

$$a_{k,\tau_1+\tau_2+i} = \begin{cases} 1, & \text{if } k \in \mathcal{K}_i, \\ 0, & \text{otherwise,} \end{cases} \quad (53)$$

$$\phi_{n,\tau_1+\tau_2+i} = \begin{cases} 1, & \text{if } n \in \mathcal{N}_i, \\ 0, & \text{otherwise,} \end{cases} \quad (K-1)\rho + 1 \leq i \leq \tilde{\tau}_3. \quad (54)$$

With the above solution, at each time instant $\tau_1 + \tau_2 + i$, we can perfectly estimate the following $\lambda_{k,n}$'s

$$\begin{aligned} & [\lambda_{\lceil \frac{j_i(1)}{v} \rceil + 1, \mathcal{N}_i(1)}, \dots, \lambda_{\lceil \frac{j_i(M_i)}{v} \rceil + 1, \mathcal{N}_i(M_i)}]^T \\ & = [\mathbf{g}_{1, \mathcal{N}_i(1)}, \dots, \mathbf{g}_{1, \mathcal{N}_i(M_i)}]^\dagger \frac{\tilde{\mathbf{y}}^{(\tau_1+\tau_2+i)}}{\sqrt{\rho}}, \quad (K-1)\rho + 1 \leq i \leq \tilde{\tau}_3, \end{aligned} \quad (55)$$

where

$$M_i = |\mathcal{N}_i|, \quad (56)$$

$$\tilde{\mathbf{y}}^{(\tau_1+\tau_2+i)} = \tilde{\mathbf{y}}^{(\tau_1+\tau_2+i)} - \sum_{k \in \mathcal{K}_i} \sum_{n \in \mathcal{N}_i \cap \Lambda_{k,1}} \sqrt{\rho} \lambda_{k,n} \mathbf{g}_{1,n}. \quad (57)$$

Proof: Please refer to Appendix B. ■

The basic idea to achieve the minimum pilot sequence length (49) under the case of $M < N$ is as follows. First, at time slot $\tau_1 + \tau_2 + i$, $1 \leq i \leq (K-1)\rho$, only one user, i.e., user $\lceil i/\rho \rceil + 1$, transmits a pilot symbol 1, and M IRS elements in the set of Ω_i are switched on such that the corresponding reflected channels can be perfectly estimated based on (52). After $(K-1)\rho$ time slots, for each user $k \geq 2$, still $v < M$ $\lambda_{k,n}$'s with $n \in \Lambda_{k,2}$ are unknown. Therefore, to guarantee that at each time slot $\tau_1 + \tau_2 + i$, $i > (K-1)\rho$ (unless the last time slot), again M $\lambda_{k,n}$'s can be estimated such that (49) is achievable, more than one user should transmit their pilots at the same time. The challenge is that if M IRS elements are switched on and more than one user transmits the pilots at one time slot, the number of $\lambda_{k,n}$'s involved in the received signal is larger than M , as shown in (96). Interestingly, we can show that if the user pilot and IRS reflection coefficients are designed based on (53) and (54), at each time slot only M $\lambda_{k,n}$'s are unknown in (96), while the other $\lambda_{k,n}$'s have already been estimated before. As a result, after interference cancellation as shown in (57), still M $\lambda_{k,n}$'s can be perfectly estimated based on (55) at each time slot.

To further explain the above procedure for channel estimation, we provide a simple example as follows.

Example 1: Consider the case when $M = 2$, $K = 3$, and $N = 3$. In this case, we have $\tilde{\tau}_3 = 3$, and $\rho = v = 1$. According to (43) and (44), we set $\Lambda_{2,1} = \{2, 3\}$, $\Lambda_{2,2} = \{1\}$, $\Lambda_{3,1} = \{1, 3\}$, and $\Lambda_{3,2} = \{2\}$. Then, it follows from (45), (47), and (48) that $\Omega_1 = \{2, 3\}$, $\Omega_2 = \{1, 3\}$, $\mathcal{K}_3 = \{2, 3\}$, and $\mathcal{N}_3 = \{1, 2\}$. Based on Theorem 2, the pilot sequence assigned to users 2 and 3 are $\mathbf{a}_2^{\text{III}} = [1, 0, 1]^T$ and $\mathbf{a}_3^{\text{III}} = [0, 1, 1]^T$, and the IRS reflecting coefficients are $\phi_{2,1} = \phi_{3,1} = \phi_{1,2} = \phi_{3,2} = \phi_{1,3} = \phi_{2,3} = 1$ and $\phi_{n,i} = 0$ otherwise. According to (52), in time instants $\tau_1 + \tau_2 + 1$ and $\tau_1 + \tau_2 + 2$, we have

$$[\lambda_{2,2}, \lambda_{2,3}]^T = [\mathbf{g}_{1,2}, \mathbf{g}_{1,3}]^{-1} \tilde{\mathbf{y}}^{\tau_1+\tau_2+1} / \sqrt{\rho}, \quad (58)$$

$$[\lambda_{3,1}, \lambda_{3,3}]^T = [\mathbf{g}_{1,1}, \mathbf{g}_{1,3}]^{-1} \tilde{\mathbf{y}}^{\tau_1+\tau_2+2} / \sqrt{\rho}. \quad (59)$$

In time instant $\tau_1 + \tau_2 + 3$, since $\lambda_{2,2}, \lambda_{2,3}, \lambda_{3,1}, \lambda_{3,3}$ are already known, their interference to estimate $\lambda_{2,1}$ and $\lambda_{3,2}$ can be

anceled from the received signal $\tilde{\mathbf{y}}^{(\tau_1+\tau_2+2)}$ as shown in (57) to get $\tilde{\mathbf{y}}^{(\tau_1+\tau_2+2)}$. Then, based on (57), we have

$$[\lambda_{2,1}, \lambda_{3,2}]^T = [\mathbf{g}_{1,1}, \mathbf{g}_{1,2}]^{(-1)} \tilde{\mathbf{y}}^{(\tau_1+\tau_2+3)} / \sqrt{\rho}. \quad (60)$$

Thereby, using $\tilde{\tau}_3 = 3$ time instants, $\lambda_{k,n}$'s, $k = 2, 3$, $n = 1, 2, 3$, are all perfectly estimated based on Theorem 2.

According to Theorems 1 and 2, we manage to reduce the channel estimation time duration in Phase III from $(K-1)N$ symbols to

$$\tilde{\tau}_3 = \max \left(K-1, \left\lceil \frac{(K-1)N}{M} \right\rceil \right), \quad (61)$$

symbols thanks to the hidden relation shown in (21). Further, the designs of user pilot and IRS reflecting coefficients shown in Theorems 1 and 2 are independent of $\mathbf{g}_{1,n}$'s. Thereby, channel estimation in Phase III does not require any channel feedback from the BS to the users and IRS.

D. Overall Channel Estimation Overhead

To summarize, for perfectly estimating all the direct channels \mathbf{h}_k 's and reflected channels $\mathbf{g}_{k,n}$'s in the case without noise at the BS, the minimum pilot sequence length is

$$\tilde{\tau} = \tilde{\tau}_1 + \tilde{\tau}_2 + \tilde{\tau}_3 = K + N + \max \left(K-1, \left\lceil \frac{(K-1)N}{M} \right\rceil \right). \quad (62)$$

Interestingly, in the massive MIMO regime [17], i.e., $M \rightarrow \infty$, $\tilde{\tau}$ reduces to

$$\tilde{\tau} = K + N + K - 1 = 2K + N - 1, \quad (63)$$

which is linear with K and N . Thereby, under the three-phase channel estimation protocol for IRS-assisted uplink communications, massive MIMO makes it possible to effectively estimate $KMN + KM$ unknown channel coefficients using a scalable number of pilot symbols. Such a result is in sharp contrast to the traditional channel estimation scenario without IRS, where the minimum channel estimation time does not depend on the number of receive antennas [30].

V. CHANNEL ESTIMATION FOR CASE WITH NOISE

In the previous section, we have shown how to perfectly estimate all the channels using at least $\tilde{\tau}$ time instants for the ideal case without noise at the BS, where $\tilde{\tau}$ is given in (62). In this section, we introduce how to estimate the channels under our proposed three-phase channel estimation protocol for the case with noise at the BS, using $\tau_1 \geq \tilde{\tau}_1$, $\tau_2 \geq \tilde{\tau}_2$, and $\tau_3 \geq \tilde{\tau}_3$ time slots in Phases I, II, and III, respectively.

A. Phase I: Direct Channel Estimation

With noise at the BS, the received signal in Phase I given in (25) is re-expressed as

$$\mathbf{Y}^I = \sqrt{\rho} [\mathbf{h}_1, \dots, \mathbf{h}_K] [\mathbf{a}_1^I, \dots, \mathbf{a}_K^I]^T + \mathbf{Z}^I, \quad (64)$$

where $\mathbf{Z}^I = [\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(\tau_1)}]$. According to [30], the optimal pilot design should guarantee that the pilot sequences of different users are orthogonal with each other, i.e., $(\mathbf{a}_k^I)^T \mathbf{a}_j^I = 0$,

$\forall k \neq j$. With $\tau_1 \geq K$, we can set the pilot sequence of user k as

$$\mathbf{a}_k^1 = [1, e^{-\frac{-2(k-1)\pi q}{\tau_1}}, \dots, e^{-\frac{-2(k-1)(\tau_1-1)\pi q}{\tau_1}}]^T, \quad \forall k, \quad (65)$$

where $q^2 = -1$. In this case, the minimum mean-squared error (MMSE) channel estimator is

$$\hat{\mathbf{h}}_k = \frac{\beta_k^{\text{BU}} \sqrt{p}}{\beta_k^{\text{BU}} p \tau_1 + \sigma^2} \mathbf{Y}^1 \mathbf{a}_k^*, \quad \forall k. \quad (66)$$

Further, the MSE for estimating \mathbf{h}_k is denoted by

$$\varepsilon_k^1 = \mathbb{E}[\mathbf{h}_k^H (\hat{\mathbf{h}}_k - \mathbf{h}_k) (\hat{\mathbf{h}}_k - \mathbf{h}_k)] = \frac{M \beta_k^{\text{BU}} \sigma^2}{\beta_k^{\text{BU}} p \tau_1 + \sigma^2}, \quad \forall k. \quad (67)$$

B. Phase II: Reflecting Channel Estimation for Typical User

In Phase II, the typical user 1 transmits the pilot symbols to the BS, which will cancel the interference caused by \mathbf{h}_k 's for estimating $\mathbf{g}_{1,n}$'s. In the case with noise the BS, the effective received signal at the BS after interference cancellation given in (28) is re-expressed as

$$\begin{aligned} \tilde{\mathbf{y}}^{(i)} &= \mathbf{y}^{(i)} - \hat{\mathbf{h}}_1 \sqrt{p} a_{1,i} \\ &= \sum_{n=1}^N \phi_{n,i} \mathbf{g}_{1,n} \sqrt{p} a_{1,i} + (\mathbf{h}_1 - \hat{\mathbf{h}}_1) \sqrt{p} a_{1,i} + \mathbf{z}^{(i)}. \end{aligned} \quad (68)$$

Note that due to the imperfect channel estimation in Phase I, $\mathbf{h}_1 - \hat{\mathbf{h}}_1 \neq \mathbf{0}$ in general. Then, the overall effective received signals over Phase II is given by

$$\begin{aligned} \tilde{\mathbf{Y}}^{\text{II}} &= \sqrt{p} [\mathbf{g}_{1,1}, \dots, \mathbf{g}_{1,N}] \Phi^{\text{II}} \text{diag}(\mathbf{a}_1^{\text{II}}) \\ &\quad + \sqrt{p} (\mathbf{h}_1 - \hat{\mathbf{h}}_1) (\mathbf{a}_1^{\text{II}})^T + \mathbf{Z}^{\text{II}}, \end{aligned} \quad (69)$$

where $\mathbf{Z}^{\text{II}} = [\mathbf{z}^{(\tau_1+1)}, \dots, \mathbf{z}^{(\tau_1+\tau_2)}]$. For convenience, define $\mathbf{C}_1^{\text{BI}} = \mathbb{E}[[\mathbf{g}_{1,1}, \dots, \mathbf{g}_{1,N}]^H [\mathbf{g}_{1,1}, \dots, \mathbf{g}_{1,N}]]$, and $\tilde{\mathbf{Z}}^{\text{II}} = \sqrt{p} (\mathbf{h}_1 - \hat{\mathbf{h}}_1) (\mathbf{a}_1^{\text{II}})^T + \mathbf{Z}^{\text{II}}$ as the overall noise consisting of channel estimation error in Phase I and AWGN. Then, we have

$$\Psi_{\text{II}} = \mathbb{E}[(\tilde{\mathbf{Z}}^{\text{II}})^H \tilde{\mathbf{Z}}^{\text{II}}] = \frac{pM \beta_1^{\text{BU}} \sigma^2}{\beta_1^{\text{BU}} p \tau_1 + \sigma^2} (\mathbf{a}_1^{\text{II}})^* (\mathbf{a}_1^{\text{II}})^T + M \sigma^2 \mathbf{I}. \quad (70)$$

Since $\mathbf{g}_{1,n}$'s do not follow the Rayleigh fading channel model, it is difficult to design the MMSE estimator based on (69). In this paper, we consider the LMMSE estimator in Phase II. By setting the pilot sequence of user 1 as

$$\mathbf{a}_1^{\text{II}} = [1, \dots, 1]^T, \quad (71)$$

and the IRS reflection coefficients Φ^{II} as (33) similar to [25], [26], the LMMSE channel estimator in Phase II is

$$\begin{aligned} &[\hat{\mathbf{g}}_{1,1}, \dots, \hat{\mathbf{g}}_{1,N}] \\ &= \sqrt{p} \tilde{\mathbf{Y}}^{\text{II}} \Psi_{\text{II}}^{-1} (\Phi^{\text{II}})^H \left(p \Phi^{\text{II}} \Psi_{\text{II}}^{-1} (\Phi^{\text{II}})^H + (\mathbf{C}_1^{\text{BI}})^{-1} \right)^{-1}. \end{aligned} \quad (72)$$

The MSE of the above LMMSE channel estimator is

$$\varepsilon^{\text{II}} = \text{tr} \left(\left(p \Phi^{\text{II}} \Psi_{\text{II}}^{-1} (\Phi^{\text{II}})^H + (\mathbf{C}_1^{\text{BI}})^{-1} \right)^{-1} \right). \quad (73)$$

C. Phase III: Reflecting Channel Estimation for Other Users

In Phase III, with the imperfect estimation of \mathbf{h}_k 's and $\mathbf{g}_{1,n}$'s, the effective received signal at time slot i , $i = \tau_1 + \tau_2 + 1, \dots, \tau_1 + \tau_2 + \tau_3$, given in (35) can be expressed as

$$\begin{aligned} \tilde{\mathbf{y}}^{(i)} &= \sum_{k=2}^K \sum_{n=1}^N \phi_{n,i} \hat{\mathbf{g}}_{1,n} \sqrt{p} a_{k,i} \lambda_{k,n} + \sum_{k=2}^K (\mathbf{h}_k - \hat{\mathbf{h}}_k) \sqrt{p} a_{k,i} + \mathbf{z}^{(i)} \\ &\quad + \sum_{k=2}^K \sum_{n=1}^N \phi_{n,i} (\mathbf{g}_{1,n} - \hat{\mathbf{g}}_{1,n}) \sqrt{p} a_{k,i} \lambda_{k,n}. \end{aligned} \quad (74)$$

However, it is difficult to design the LMMSE channel estimator based on (74) due to the fact that $\lambda_{k,n}$'s also contribute to the noise for estimating themselves with imperfect estimation of $\mathbf{g}_{1,n}$'s.

To tackle the above challenge, in the following we assume that the channel estimation error $\mathbf{g}_{1,n} - \hat{\mathbf{g}}_{1,n} = \mathbf{0}$, $\forall n$, such that there is no self-interference generated by $\lambda_{k,n}$'s. In practice, we can increase the pilot sequence length in Phase II, i.e., τ_2 , such that the estimation of $\mathbf{g}_{1,n}$'s is sufficiently accurate. Under the above assumption, the effective received signal given in (74) reduces to

$$\tilde{\mathbf{y}}^{(i)} = \sum_{k=2}^K \sum_{n=1}^N \phi_{n,i} \mathbf{g}_{1,n} \sqrt{p} a_{k,i} \lambda_{k,n} + \sum_{k=2}^K (\mathbf{h}_k - \hat{\mathbf{h}}_k) \sqrt{p} a_{k,i} + \mathbf{z}^{(i)}. \quad (75)$$

Moreover, we assume an orthogonal transmission and reflection strategy: at each time slot i , only one user, denoted by user k_i , transmits its pilot symbol to the BS, and only $M_i \leq M$ out of N IRS elements, denoted by the set Δ_i with $M_i = |\Delta_i| \leq M$, are switched on to reflect the pilot symbol, such that the BS can estimate $\mathbf{g}_{k_i,n}$'s, $\forall n \in \Delta_i$. Specifically, with $i \geq \tau_1 + \tau_2 + 1$, we define

$$\begin{aligned} k_i &= \left\lceil \frac{i - \tau_1 - \tau_2}{\lceil N/M \rceil} \right\rceil + 1, \\ \Delta_i &= \begin{cases} \{\varphi_i + 1, \dots, \varphi_i + M\}, & \text{if } \left\lfloor \frac{i - \tau_1 - \tau_2}{\lceil N/M \rceil} \right\rfloor \neq \frac{i - \tau_1 - \tau_2}{\lceil N/M \rceil}, \\ \{(\lceil \frac{N}{M} \rceil - 1)M + 1, \dots, N\}, & \text{otherwise.} \end{cases} \end{aligned} \quad (76)$$

where

$$\varphi_i = \left(i - \tau_1 - \tau_2 - \left\lfloor \frac{i - \tau_1 - \tau_2}{\lceil N/M \rceil} \right\rfloor \lceil N/M \rceil \right) M. \quad (78)$$

As a result, for user k_i , $\lceil N/M \rceil$ time slots are allocated to estimate its reflected channels. In each of the first $\lceil N/M \rceil - 1$ time slots, M IRS elements are switched on, while in the last time slot, the remaining $N - (\lceil N/M \rceil - 1)M$ IRS elements are switched on. In total, under this orthogonal transmission and reflection strategy, $\tau_3 = (K - 1) \lceil N/M \rceil$ time slots are needed to estimate $\mathbf{g}_{k,n}$'s, $\forall k \geq 2$, $\forall n$.

Next, we show how to estimate $\mathbf{g}_{k,n}$'s based on the above orthogonal transmission and reflection strategy. At time slot i , $i = \tau_1 + \tau_2 + 1, \dots, \tau_1 + \tau_2 + (K - 1) \lceil N/M \rceil$, we set

$$a_{k,i} = \begin{cases} 1, & \text{if } k = k_i, \\ 0, & \text{otherwise,} \end{cases} \quad \forall k \geq 2, \quad (79)$$

$$\phi_{n,i} = \begin{cases} 1, & \text{if } n \in \Delta_i, \\ 0, & \text{otherwise,} \end{cases} \quad \forall n. \quad (80)$$

Then, the effective received signal at time slot i given in (75) reduces to

$$\begin{aligned}\tilde{\mathbf{y}}^{(i)} &= \sum_{n \in \Delta_i} \mathbf{g}_{1,n} \sqrt{p} \lambda_{k_i,n} + (\mathbf{h}_{k_i} - \hat{\mathbf{h}}_{k_i}) \sqrt{p} + \mathbf{z}^{(i)} \\ &= \sqrt{p} \mathbf{G}_{1,i} \boldsymbol{\lambda}_{k_i,i} + \tilde{\mathbf{z}}^{(i)},\end{aligned}\quad (81)$$

where

$$\mathbf{G}_{1,i} = [\mathbf{g}_{1,\Delta_i(1)}, \dots, \mathbf{g}_{1,\Delta_i(|\Delta_i|)}], \quad (82)$$

$$\boldsymbol{\lambda}_{k_i,i} = [\lambda_{k_i,\Delta_i(1)}, \dots, \lambda_{k_i,\Delta_i(|\Delta_i|)}]^T, \quad (83)$$

$$\tilde{\mathbf{z}}^{(i)} = \sqrt{p}(\mathbf{h}_{k_i} - \hat{\mathbf{h}}_{k_i}) + \mathbf{z}^{(i)}, \quad (84)$$

with $\Delta_i(j)$ denoting the j th element of Δ_i . Next, define $\mathbf{C}_i^\lambda = \mathbb{E}[\boldsymbol{\lambda}_{k_i,i} \boldsymbol{\lambda}_{k_i,i}^H]$, and

$$\begin{aligned}\boldsymbol{\Psi}_{\text{III}} &= \mathbb{E}[\tilde{\mathbf{z}}^{(i)} (\tilde{\mathbf{z}}^{(i)})^H] \\ &= \frac{\beta_{k_i}^{\text{BU}} p \sigma^4}{\beta_{k_i}^{\text{BU}} p \tau_1 + \sigma^2} \mathbf{C}_{k_i}^{\text{B}} + \frac{(\beta_{k_i}^{\text{BU}} p)^2 \tau_1 \sigma^2}{\beta_{k_i}^{\text{BU}} p \tau_1 + \sigma^2} \mathbf{I} + \sigma^2 \mathbf{I}.\end{aligned}\quad (85)$$

Given any realization of $\mathbf{G}_{1,i}$ that is assumed to be perfectly estimated in Phase II, the LMMSE channel estimator in Phase III is then given by

$$\hat{\boldsymbol{\lambda}}_{k_i,i}(\mathbf{G}_{1,i}) = \sqrt{p} \left(p \mathbf{G}_{1,i}^H \boldsymbol{\Psi}_{\text{III}}^{-1} \mathbf{G}_{1,i} + (\mathbf{C}_i^\lambda)^{-1} \right)^{-1} \mathbf{G}_{1,i}^H \boldsymbol{\Psi}_{\text{III}}^{-1} \tilde{\mathbf{y}}^{(i)}. \quad (86)$$

The corresponding MSE for estimating $\lambda_{k_i,i}$ given $\mathbf{G}_{1,i}$ is

$$\begin{aligned}\varepsilon_{k_i,i}^{\text{III}}(\mathbf{G}_{1,i}) &= \mathbb{E}[|\hat{\boldsymbol{\lambda}}_{k_i,i}(\mathbf{G}_{1,i}) - \boldsymbol{\lambda}_{k_i,i}|^2 | \mathbf{G}_{1,i}] \\ &= \text{tr} \left(\left(p \mathbf{G}_{1,i}^H \boldsymbol{\Psi}_{\text{III}}^{-1} \mathbf{G}_{1,i} + (\mathbf{C}_i^\lambda)^{-1} \right)^{-1} \right).\end{aligned}\quad (87)$$

By averaging all the channel realizations of $\mathbf{G}_{1,i}$'s, the MSE for estimating $\lambda_{k_i,i}$ is

$$\varepsilon_{k_i,i}^{\text{III}} = \mathbb{E}_{\mathbf{G}_{1,i}} \left[\text{tr} \left(\left(p \mathbf{G}_{1,i}^H \boldsymbol{\Psi}_{\text{III}}^{-1} \mathbf{G}_{1,i} + (\mathbf{C}_i^\lambda)^{-1} \right)^{-1} \right) \right]. \quad (88)$$

At last, the overall MSE for estimating $\lambda_{k_i,i}$'s $\forall i$ is given by $\varepsilon^{\text{III}} = \sum_{i=1}^{\tau_3} \varepsilon_{k_i,i}^{\text{III}}$.

D. Overall Channel Estimation Strategy

For the case with noise at the BS, the overall channel estimation strategy is summarized in Table I.

TABLE I
ALGORITHM I: OVERALL CHANNEL ESTIMATION STRATEGY

-
1. **Phase I** ($i = 1, \dots, \tau_1$): The users transmit pilot symbols \mathbf{a}_k^1 's based on (65). The IRS is switched off, i.e., $\phi_{n,i}$'s are set to be zero. The BS estimates the direct channels \mathbf{h}_k 's according to (66);
 2. **Phase II** ($i = \tau_1 + 1, \dots, \tau_1 + \tau_2$): Only user 1 transmits non-zero pilot symbols \mathbf{a}_1^{II} based on (71). The IRS is switched on, and $\phi_{n,i}$'s are set based on (33). The BS removes the interference caused by \mathbf{h}_k 's based on (69) and estimates $\mathbf{g}_{1,n}$'s based on (72);
 3. **Phase III** ($i = \tau_1 + \tau_2 + 1, \dots, \tau_1 + \tau_2 + \tau_3$): User 2 to user K transmit pilot symbols $\mathbf{a}_k^{\text{III}}$'s, $k \geq 2$, based on (79). The IRS is switched on, and $\phi_{n,i}$'s are set as (80). The BS removes the interference caused by \mathbf{h}_k 's based on (75) and estimates $\boldsymbol{\lambda}$ based on (86).
-

VI. NUMERICAL EXAMPLES

In this section, we provide numerical examples to verify the effectiveness of our proposed three-phase channel estimation protocol in the IRS-assisted multiuser communications. We assume that the IRS is equipped with $N = 32$ reflecting elements. Moreover, the path loss of \mathbf{h}_k 's, $t_{k,n}$'s, and \mathbf{r}_n 's is modeled as $\beta_k^{\text{BU}} = \beta_0 (d_k^{\text{BU}}/d_0)^{-\alpha_1}$, $\beta_k^{\text{IU}} = \beta_0 (d_k^{\text{IU}}/d_0)^{-\alpha_2}$, and $\beta^{\text{BI}} = \beta_0 (d^{\text{BI}}/d_0)^{-\alpha_3}$, respectively, where $d_0 = 1$ meter (m) denotes the reference distance, $\beta_0 = -20$ dB denotes the path loss at the reference distance, d_k^{BU} , d_k^{IU} , and d^{BI} denote the distance between the BS and user k , between the IRS and user k , as well as between the BS and the IRS, while α_1 , α_2 , and α_3 denote the path loss factors for \mathbf{h}_k 's, $t_{k,n}$'s, and \mathbf{r}_n 's, respectively. In the numerical examples, we set $\alpha_1 = 4.2$, $\alpha_2 = 2.1$, and $\alpha_3 = 2.2$. The distance between the BS and IRS is set to be $d^{\text{BI}} = 100$ m, and all the users are assumed to be located in a circular regime, whose center is 10 m away from the IRS and 105 m away from the BS, and radius is 5 m. For the correlation matrix \mathbf{C}_k^{B} , we adopt the exponential correlation matrix model [31], where the element in the i th row and j th column is denoted by $[\mathbf{C}_k^{\text{B}}]_{i,j} = (c_k^{\text{B}})^{|i-j|}$ if $i \geq j$, and $[\mathbf{C}_k^{\text{B}}]_{i,j} = [\mathbf{C}_k^{\text{B}}]_{j,i}^*$, if $i < j$, with $|c_k^{\text{B}}| < 1$. Similarly, the correlation matrices \mathbf{C}^{B} , \mathbf{C}^{I} , and \mathbf{C}_k^{I} follow the exponential correlation matrix model as well. The identical transmit power of users is 33 dBm. The channel bandwidth is assumed to be 1 MHz, and the power spectrum density of the AWGN at the BS is -169 dBm/Hz. Moreover, to illustrate the performance gain of our proposed channel estimation framework, we consider the following benchmark scheme.

Benchmark Scheme: In Phases I and II, the estimation of \mathbf{h}_k 's and $\mathbf{g}_{1,n}$'s is the same as the proposed framework. However, in Phase III, from time slots $\tau_1 + \tau_2 + (k-2)\tau_2 + 1$ to $\tau_1 + \tau_2 + (k-1)\tau_2$, $\forall k \geq 2$, only user k transmits its τ_2 pilot symbols to the BS, while the estimation of $\mathbf{g}_{k,n}$, $\forall n$, is the same as that of $\mathbf{g}_{1,n}$, $\forall n$, for both cases without and with noise at the BS. In other words, the channel correlations in (21) is not exploited to improve the channel estimation performance.

In the following, we provide numerical examples to evaluate the performance of the proposed channel estimation framework for the cases without and with noise at the BS, respectively.

A. The Case without Noise at the BS

First, we illustrate the minimum estimation time performance of the proposed framework in the case without noise at the BS. Fig. 3 shows the minimum pilot sequence length for perfect channel estimation as given in (62) versus the number of users when the number of antennas at the BS is $M = 8$ and $M = 32$, respectively. Moreover, the minimum pilot sequence length required by the benchmark scheme, which is characterized as $K + KN$, is also shown. It is observed that thanks to the exploitation of the channel correlation (21), the minimum pilot sequence length under our proposed framework increases much more slowly with the number of users than that under the benchmark scheme. Moreover, by comparing the cases when $M = 8$ and $M = 32$, it is observed that under our proposed framework, the minimum pilot sequence length

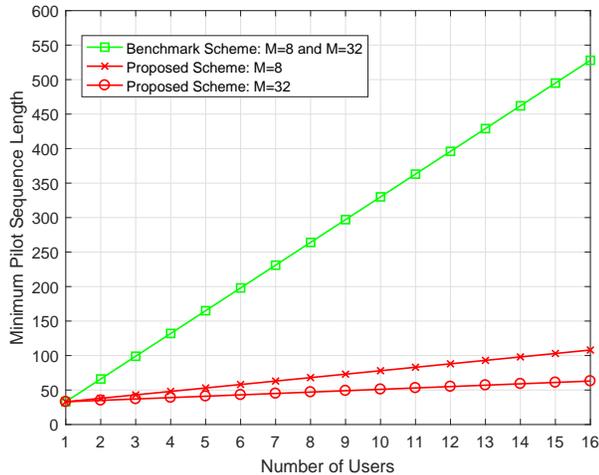


Fig. 3. Minimum pilot sequence length versus number of users.

decreases very fast as the number of BS antennas increases; while that under the benchmark strategy is independent of the number of BS antennas.

B. The Case with Noise at the BS

Next, we show the MSE performance of the proposed framework in the case with noise at the BS. In the following, we assume that the number of users is 8, and the BS is equipped with 32 antennas. First, we consider the performance of Phase II for estimating $\mathbf{g}_{1,n}$'s, $\forall n$. Besides the proposed DFT-based solution shown in (33), we also consider two more schemes for performance comparison. Specifically, in the first scheme, only one IRS element is switched on at each time slot, and all the IRS elements take turns to be in the “on” state. In the second scheme, the phase shifter of each IRS element at each time slot is randomly chosen in the range of $[0, 2\pi)$. Fig. 4 shows the normalized MSE in Phase II achieved by different schemes, which is defined as

$$e^{\text{II}} = \frac{\sum_{n=1}^N \mathbb{E}[\|\hat{\mathbf{g}}_{1,n} - \mathbf{g}_{1,n}\|^2]}{\sum_{n=1}^N \mathbb{E}[\|\mathbf{g}_{1,n}\|^2]}. \quad (89)$$

First, it is observed that our theoretical characterization of channel estimation MSE, i.e., (73), matches the Monte Carlo simulation perfectly. Second, it is observed that the normalized MSE under the DFT-based solution is much smaller than that under the other two schemes.

Next, we evaluate the performance for estimating $\lambda_{k,n}$'s in Phase III. Similar to Phase II, we define the normalized MSE for estimating $\lambda_{k,n}$'s in Phase III as

$$e^{\text{III}} = \frac{\sum_{k=2}^K \sum_{n=1}^N \mathbb{E}[|\hat{\lambda}_{k,n} - \lambda_{k,n}|^2]}{\sum_{k=2}^K \sum_{n=1}^N \mathbb{E}[|\lambda_{k,n}|^2]}. \quad (90)$$

Fig. 5 shows the normalized MSE for estimating $\lambda_{k,n}$'s versus the channel training time in Phase II. It is observed that if the channel estimation of $\mathbf{g}_{1,n}$'s is perfect, then our theoretical

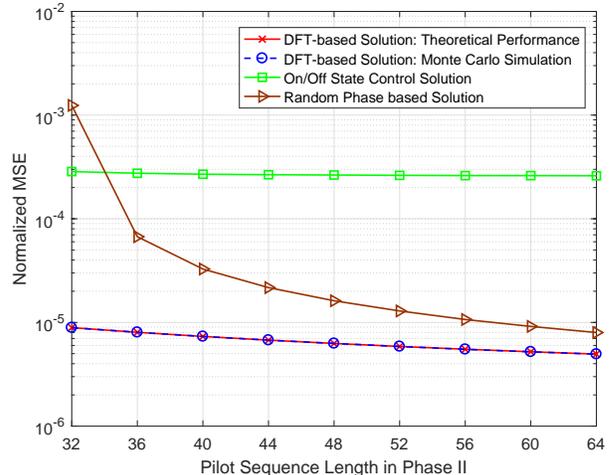


Fig. 4. Normalized MSE comparison in Phase II.

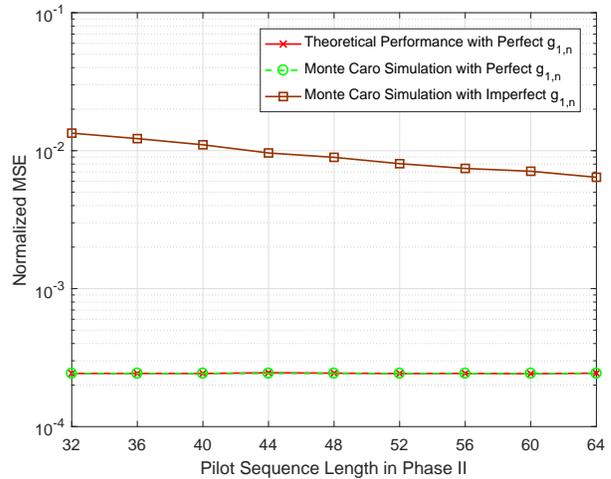


Fig. 5. Normalized MSE in Phase III: theoretical results versus Monte Carlo simulations.

characterization of the MSE for estimating $\lambda_{k,n}$'s matches the Monte Carlo simulation perfectly. In practice, the estimation of $\mathbf{g}_{1,n}$'s is imperfect. In this case, a small level of mismatch exists between our theoretical result under the assumption of perfect estimation of $\mathbf{g}_{1,n}$'s and the Monte Carlo simulation. Nevertheless, this gap can be reduced by increasing the channel estimation time in Phase II such that the estimation of $\mathbf{g}_{1,n}$'s is more accurate.

Fig. 6 shows the MSE performance in Phase III achieved by the proposed scheme and the benchmark scheme described at the beginning of this section versus the channel estimation time in Phase III. It is observed that under the benchmark scheme, the normalized MSE is above 0.3 when τ_3 ranges from 7 to 32. This indicates that the channel estimation error is almost as strong as the channel power. This is because the minimum pilot sequence length for perfect channel estimation in the case without noise at the base is 224 under the benchmark strategy. As a result, when τ_3 ranges from 7 to 32, the benchmark scheme cannot estimate the channels very well even without noise at the BS. On the other hand, under our proposed framework, the normalized MSE for estimating

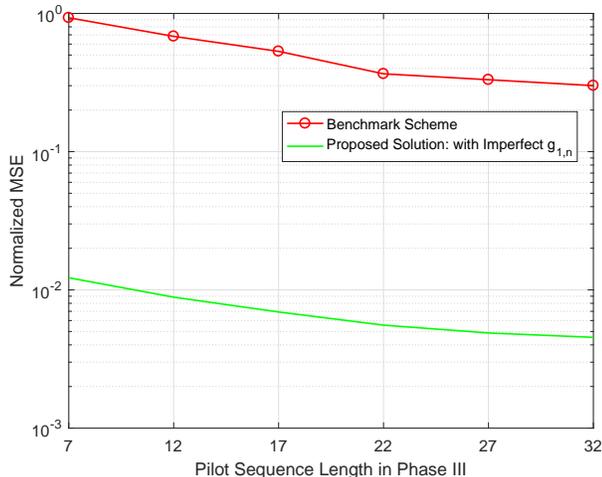


Fig. 6. Normalized MSE comparison in Phase III.

all the channels is below 10^{-2} when $\tau_3 \geq 12$.

At last, we consider the overall MSE performance for estimating both \mathbf{h}_k 's and $\mathbf{g}_{k,n}$'s. In this case, the normalized MSE is defined as

$$e = \frac{\sum_{k=1}^K \mathbb{E}[\|\hat{\mathbf{h}}_k - \mathbf{h}_k\|^2] + \sum_{k=1}^K \sum_{n=1}^N \mathbb{E}[\|\hat{\mathbf{g}}_{k,n} - \mathbf{g}_{k,n}\|^2]}{\sum_{k=1}^K \mathbb{E}[\|\mathbf{h}_k\|^2] + \sum_{k=1}^K \sum_{n=1}^N \mathbb{E}[\|\mathbf{g}_{k,n}\|^2]}. \quad (91)$$

Fig. 7 shows the normalized MSE for estimating \mathbf{h}_k 's and $\mathbf{g}_{k,n}$'s achieved by various strategies. It is observed that similar to Fig. 6, the proposed scheme significantly improves the overall channel estimation MSE performance compared to the benchmark scheme. Moreover, under the proposed framework, we also consider three schemes to allocate the extra time slots among the three phases if the overall channel estimation time is larger than (62). Under these three schemes, the extra time slots are all allocated to Phase I, all allocated to Phase II, and evenly allocated to Phases I, II, and III, respectively. It is observed in Fig. 7 that the best way is to allocate more time slots to Phase II such that the error propagation to Phase III is reduced.

VII. CONCLUSION AND FUTURE WORK

In this paper, we proposed an innovative three-phase framework to estimate a large number of channel coefficients in the IRS-assisted uplink multiuser communications accurately using merely a small number of pilot symbols. Such an interesting result is enabled by exploiting the correlations among the IRS reflected channels: each IRS reflecting element reflects the signals from different users to the BS via the same channel. Under the proposed framework, the minimum pilot sequence length for perfect channel estimation in the case without noise at the BS was characterized, and the LMMSE channel estimators in the case with noise at the BS were derived. Numerical examples were provided to verify the effectiveness of our framework compared to the benchmark scheme without taking advantage of channel correlation.

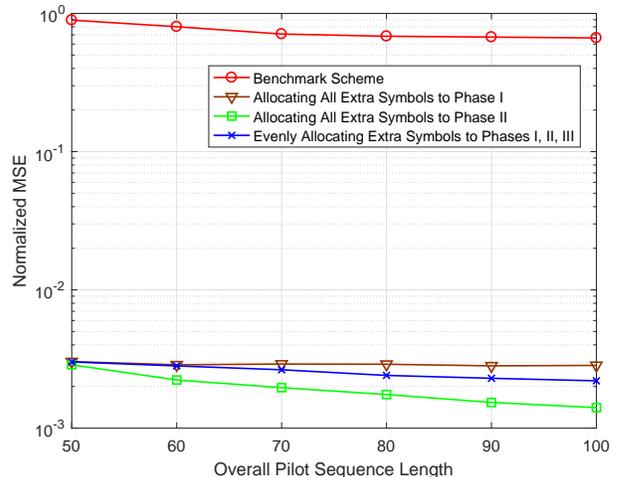


Fig. 7. Overall normalized MSE comparison between the proposed framework and the benchmark scheme.

There are a number of directions along which the channel estimation framework proposed in this paper can be further enriched. First, in Phase II of our proposed protocol, N time slots are required for channel estimation of the typical user, which can be long in practice if the IRS is equipped with a large number of IRS elements. It is thus interesting to study how to make use of the channel property in IRS-assisted communication systems to reduce the channel estimation time in Phase II. Second, the channel estimation error in Phases I and II under the proposed framework will affect channel estimation in Phase III. Intuitively, if more time slots are allocated to Phase I and Phase II, the error propagation to Phase III will be reduced, while less time is left for channel estimation. As a result, how to allocate the available time slots among the three channel estimation phases is also an open problem. Similarly, it is crucial to study the pilot power allocation strategy among the three phases of our proposed framework to improve the channel estimation performance. Moreover, it is interesting to extend our channel estimation protocol to a multi-cell communication system and study the effect of pilot contamination on the system performance. Another important direction is to study whether the channel reciprocity holds in the IRS-assisted communications. If not, new approach should be proposed to utilize the channel redundancy for reducing the channel estimation time in the downlink. Last but not least, it is crucial to characterize the user achievable rate with imperfect CSI under the proposed framework.

APPENDIX

A. Proof of Theorem 1

In the case of $M \geq N$, we first prove that there exists a unique solution to (36) only if $\tau_3 \geq K - 1$. Define

$$\eta_{n,i} = \sum_{k=2}^K \lambda_{k,n} \phi_{n,i+\tau_1+\tau_2} a_{k,i+\tau_1+\tau_2}, \quad 1 \leq n \leq N, 1 \leq i \leq \tau_3. \quad (92)$$

Then, it can be shown that (36) can be expressed as

$$\sum_{n=1}^N \eta_{n,i} \mathbf{g}_{1,n} = \bar{\mathbf{y}}^{(\tau_1+\tau_2+i)}, \quad i = 1, \dots, \tau_3. \quad (93)$$

Under the channel model of \mathbf{r}_n 's and $t_{k,n}$'s shown in (2) and (3), in the case of $M \geq N$, $\mathbf{g}_{1,n}$'s are linearly independent with each other with probability one. As a result, $\eta_{n,i}$'s, $n = 1, \dots, N$, $i = 1, \dots, \tau_3$, can be perfectly estimated based on (93). Next, we intend to solve the equations given in (92). With known $\eta_{n,i}$'s, (92) characterizes a linear system with $(K-1)N$ variables and $N\tau_3$ equations. Therefore, a unique solution to (92) exists only when the number of equations is no smaller than the number of variables, i.e., $\tau_3 \geq K-1$.

Next, we show that if $\tau_3 = K-1$, there always exists a unique solution to (36) in the case of $M \geq N$. Specifically, if $\tau_3 = K-1$, we can set $a_{k,i}$'s and $\phi_{n,i}$'s as given in (39) and (40), respectively. It then follows from (36) that

$$\bar{\mathbf{y}}^{(\tau_1+\tau_2+k-1)} = [\mathbf{g}_{1,1}, \dots, \mathbf{g}_{1,N}] \boldsymbol{\lambda}_k, \quad k = 2, \dots, K. \quad (94)$$

Since $\mathbf{g}_{1,n}$'s are linearly independent with each other with probability one in the case $M \geq N$, the pseudo-inverse matrix of $[\mathbf{g}_{1,1}, \dots, \mathbf{g}_{1,N}]$ exists. As a result, if $\tau_3 = K-1$, there exists a unique solution to (36), which is given in (41).

To summarize, in the case of $M \geq N$, there exists a unique solution to (36) only if $\tau_3 \geq K-1$. Moreover, $\tau_3 = K-1$ is sufficient to guarantee the existence of a unique solution to (36) by setting $a_{k,i}$'s and $\phi_{n,i}$'s according to (39) and (40). As a result, if $M \geq N$, $\tau_3 = K-1$ is the minimum value of τ_3 for perfectly estimating $\boldsymbol{\lambda}$ according to (36).

B. Proof of Theorem 2

Similar to the case with $M \geq N$, we first prove that in the case of $M < N$, there exists a unique solution to (36) only if $\tau_3 \geq \lceil \frac{(K-1)N}{M} \rceil$. Note that in (36), the number of variables and the number of linear equations are $(K-1)N$ and $\tau_3 M$, respectively. As a result, there exists a unique solution to (36) only if the number of equations is no smaller than that of variables, i.e., $\tau_3 \geq \lceil \frac{(K-1)N}{M} \rceil$.

Next, we show that when $\tau_3 = \lceil \frac{(K-1)N}{M} \rceil$, there always exists a solution to (36) in the case of $M < N$. The basic idea is that in each time instant $\tau_1 + \tau_2 + i$ with $i \leq (K-1)\rho$, only one user $k \geq 2$ sends a non-zero pilot symbol for estimating $\lambda_{k,n}$'s with $n \in \Lambda_{k,1}$ without any interference from other users' pilot symbols, while in each time instant $\tau_1 + \tau_2 + i$ with $(K-1)\rho + 1 \leq i \leq \tilde{\tau}_3$, multiple users transmit non-zero pilot symbols simultaneously for estimating $\lambda_{k,n}$'s with $n \in \Lambda_{k,2}$'s by eliminating the interference caused by $\lambda_{k,n}$'s with $n \in \Lambda_{k,1}$.

Specifically, at time instant $\tau_1 + \tau_2 + i$ with $i \leq (K-1)\rho$, we schedule user $k = \lfloor \frac{i}{\rho} \rfloor + 1$ to transmit a pilot symbol 1, and each of the other users transmits a pilot symbol 0, as shown in (50). Moreover, only M IRS elements in the set of Ω_i are switched on and their reflecting coefficients are set to be 1 as shown in (51). In this case, it can be shown that (36) reduces to

$$\bar{\mathbf{y}}^{(\tau_1+\tau_2+i)} = \sqrt{\rho} [\mathbf{g}_{1,\Omega_i(1)}, \dots, \mathbf{g}_{1,\Omega_i(M)}] \left[\lambda_{\lfloor \frac{i}{\rho} \rfloor + 1, \Omega_i(1)}, \dots, \lambda_{\lfloor \frac{i}{\rho} \rfloor + 1, \Omega_i(M)} \right]^T. \quad (95)$$

Under the channel model of \mathbf{r}_n 's and $t_{k,n}$'s shown in (2) and (3), in the case of $M < N$, any M out of N $\mathbf{g}_{1,n}$'s are linearly independent of each other with probability 1. Therefore, there exists a unique solution to the above equation, which is given by (52).

Next, we estimate $\lambda_{k,n}$'s with $n \in \Lambda_{k,2}$. In time instant $\tau_1 + \tau_2 + i$ with $i \geq (K-1)\rho + 1$, all the users in the set \mathcal{K}_i will transmit pilot symbols 1, while each of the other users transmits a pilot symbol 0, as shown in (53). Moreover, all the $M_i \leq M$ IRS elements in the set \mathcal{N}_i are switched on and their reflecting coefficients are set to be 1 as shown in (54). In this case, the effective received signal at this time instant given in (35) reduces to

$$\bar{\mathbf{y}}^{(\tau_1+\tau_2+i)} = \sum_{k \in \mathcal{K}_i} \sum_{n \in \mathcal{N}_i} \sqrt{\rho} \lambda_{k,n} \mathbf{g}_{1,n}. \quad (96)$$

Further, for each user $k \in \mathcal{K}_i$, $\lambda_{k,n}$'s with $n \in \Lambda_{k,1}$ have already been perfectly estimated based on (52). As a result, their interference can be canceled from (96) to get $\bar{\mathbf{y}}^{(\tau_1+\tau_2+i)}$ shown in (57). Moreover, under our construction of $\Lambda_{k,1}$'s and $\Lambda_{k,2}$'s presented prior to Theorem 2, for any two users $k_1, k_2 \in \mathcal{K}_i$, we have $\Lambda_{k_1,2} \cap \Lambda_{k_2,2} = \emptyset$ and thus $\Lambda_{k_1,2} \subset \Lambda_{k_2,1}$ and $\Lambda_{k_2,2} \subset \Lambda_{k_1,1}$. It can then be shown that $\bar{\mathbf{y}}^{(\tau_1+\tau_2+i)}$ reduces to

$$\bar{\mathbf{y}}^{(\tau_1+\tau_2+i)} = \sqrt{\rho} [\mathbf{g}_{1,\mathcal{N}_i(1)}, \dots, \mathbf{g}_{1,\mathcal{N}_i(M_i)}] \left[\lambda_{\lfloor \frac{\mathcal{J}_i(1)}{\nu} \rfloor + 1, \mathcal{N}_i(1)}, \dots, \lambda_{\lfloor \frac{\mathcal{J}_i(M)}{\nu} \rfloor + 1, \mathcal{N}_i(M)} \right]^T, \quad (97)$$

where the M_i elements in the set \mathcal{N}_i can be shown to be different if $\Lambda_{k,2}$'s are constructed based on (43). As a result, there exists a unique solution to the above equation which is given by (55).

To summarize, in the case of $M < N$, except for the last time instant, we are able to perfectly estimate M unique $\lambda_{k,n}$'s either via (52) or (55) at each time instant, while at the last time instant, the remaining $\lambda_{k,n}$'s are estimated. As a result, the minimum τ_3 for perfect channel estimation is characterized by (49). Theorem 2 is thus proved.

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