Design of a Heterogeneous Cellular Network With a Wireless Backhaul

Meghana Bande, *Member, IEEE*, and Venugopal V. Veeravalli, *Fellow, IEEE*

***Abstract*— The downlink of a two-layered heterogeneous network is studied with macro basestations (MBs), small-cell basestations (SBs) that act as *half-duplex* analog relays, and mobile terminals (MTs). The first layer is a wireless *backhaul* layer between MBs and SBs, and the second is the *transmission* layer between SBs and MTs. The layers use the same time/frequency resources for communication, limiting the maximum per user degrees of freedom (puDoF) to half, due to the half-duplex nature of the SBs. For linear network models, it is established that the optimal puDoF can be achieved by cooperation with an appropriate number of antennas that depends on the connectivity of the network. The proposed zero-forcing schemes achieve cooperation without overloading the backhaul, through each MB sending an appropriate linear combination of MTs' message signals to the SBs in the backhaul layer. The achievable schemes exploit the half-duplexity of the SBs, and schedule the SBs and MTs to be active in different time-slots to manage interference. These results are then extended to a more realistic hexagonal cellular network and it is shown that the optimal puDoF of half can be approached using only zero-forcing schemes.**

***Index Terms*— Interference management, coordinated multipoint transmission (CoMP), half-duplex relays, interference avoidance, heterogeneous networks.**

I. Introduction

TO MEET the increasing demand for mobile traffic, heterogeneous networks are envisioned to be a key component of future cellular networks [4]. Heterogeneous networks enable flexible and low-cost deployments and provide a uniform broadband experience to users anywhere in the network [5]. Managing interference in such heterogeneous networks is crucial in order to achieve higher data rates for the users.

We consider the downlink of a cellular network as a heterogeneous network consisting of macro basestations (MBs), small cell basestations (SBs), and the mobile terminals (MTs). Heterogeneous networks that are built by complementing a macro-cell layer with additional small cells impose new challenges on the backhaul [6]. The best physical location for a small cell often precludes the option of using a wired backhaul. In such cases, deploying a wireless backhaul is both faster and more cost-effective. We consider a point-to-multipoint wireless backhaul between the MBs and the SBs where one MB serves several SBs by sharing its antenna resources. It is assumed that the MBs and the SBs operate on the same frequency band, and that the SBs act as half-duplex analog relays between the MBs and MTs.

The degrees of freedom (DoF) metric is used to quantify the performance of our proposed schemes. DoF is a high SNR approximation of the capacity of the network that captures the number of interference-free sessions in the network at high SNR. We study the dependence of the DoF in this network on several factors, such as the cluster size S and the number of antennas N at the MB. We consider a linear network model first, and then study the more practical hexagonal sectored cellular network with and without intra-cell interference in the transmission layer.

The DoF for single-layer locally connected linear networks was studied in [7]–[10] using cooperation under maximum transmit set size cooperation constraints. In the schemes of [7]–[11], the messages of multiple users are available at some of the transmitters. This work was extended to the hexagonal cellular network in [12] and [13]. In all these works, cooperation is achieved by making each message available at multiple transmitters and has been shown to significantly increase the achievable DoF in these networks.

A. Contributions

Our main contributions are as follows:

1) We model the heterogenous network with macro basestations (MBs), small-cell basestations (SBs), and mobile terminals (MTs) as a two-layered interference network with SBs that act as half-duplex analog relays between the MBs and the MTs. Prior work on two-layered relay interference networks [14], [15] has used a $K \times K \times K$ relay channel model, where

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each layer is a fully connected K-user interference channel, which we believe is not appropriate to model heterogeneous networks. We therefore use a model with a point-to-multipoint wireless backhaul between the MBs and the SBs where each MB serves several SBs, and a locally connected network for the transmission layer.

2) We provide upper bounds on the achievable puDoF for the heterogeneous network for any connectivity in the transmission layer. We first consider a linear heterogeneous network and propose simple achievable ZF schemes that can achieve the optimal puDoF using insights from prior work on cooperative transmission [7]–[12] in the downlink. Cooperative transmission in [7]–[12] imposes a high backhaul load since each message needs to be made available at multiple SBs. We avoid overloading the backhaul by sending linear combinations of the analog message signals to each SB directly so that the corresponding MT can receive its message interference-free.

3) We characterize the puDoF for the more practical heterogenous network with sectored hexagonal network in the transmission layer. Using insights from previous work on cooperative transmission for sectored cellular hexagonal network [12], we propose a novel parallelogram design for the backhaul layer clusters. We show that by extending the achievable schemes for the linear heterogeneous network, significant puDoF gains can be achieved, with and without intra-cell interference in the transmission layer.

II. System Model and Notation

We consider the downlink of a heterogeneous cellular network with MBs, SBs and MTs. It is assumed that the MBs do not directly serve the MTs and that the SBs act as half-duplex analog relays between the MBs and the MTs. There are two layers in the network, the wireless *backhaul layer* between MBs and SBs, and the *transmission layer* between SBs and MTs. We assume that the transmissions from the MBs do not cause interference at the MTs. We also assume that the SBs that are actively transmitting do not cause interference at the receiving SBs because transmission in the backhaul layer typically happens at a higher SNR than in the transmission layer and is also more localized.

In this section, we introduce the linear model for the two-layered heterogeneous network which is extended to a more practical hexagonal model in Section IV-A.

A. Backhaul Layer

For the backhaul layer, we consider a point-to-multipoint wireless backhaul where each MB is associated with S SBs. We assume that each MB is equipped with N antennas. We assume that each SB is served by only one MB. Let the channel vector between MB i and SB j at time-slot t be denoted by $\mathbf{h}_{i,j}^B(t)$. Let $\mathbf{x}_i^B(t)$ be the transmitted signal vector from MB i, and let $z_k^B(t)$ denote the additive white Gaussian noise at SB k. The received signal at k-th SB served by MB i is given by

$$y_k^B(t) = (\mathbf{h}_{i,k}^B(t))^T\mathbf{x}_i^B(t) + \sum_{j\neq i}(\mathbf{h}_{j,k}^B(t))^T\mathbf{x}_j^B(t) + z_k^B(t).$$

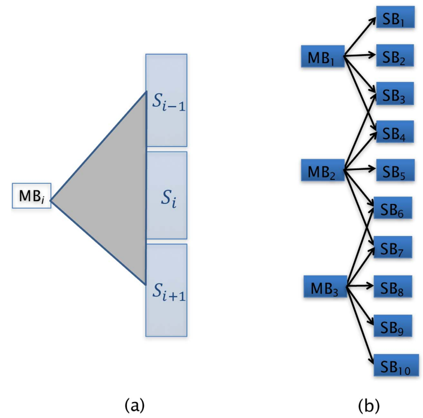


Fig. 1. In (a), MB i serves the cluster $\mathcal{S}_i$ and causes interference in the preceding and succeeding cluster. In (b), we consider a system where each MB serves a cluster of three SBs $S = 3$, and $L_B = 2$.

Local channel state information is assumed to be available at the MBs and SBs. In particular, we assume that in every cluster, the channel state information (CSI) between the MB and all the SBs in the cluster in the backhaul layer, and the CSI between all the SBs and MTs belonging to the cluster in the transmission layer is known at the cluster MB. All channel coefficients that are not identically zero are assumed to be drawn independently from a continuous joint distribution.

We consider a backhaul layer with connectivity L_B. For any MB i, let $\mathcal{S}_i$ denote the set of S consecutive SBs served by the MB where $\mathcal{S}_i = \{(i-1)S+1,\ldots,(i)S\}$.

$$\mathcal{S}_i(a:b) = \{(i-1)S+a,\ldots,(i-1)S+b\},\quad \forall i$$

where $a \leq b \leq S$, and $a:a \equiv a$. Each MB i is associated with a set $\mathcal{A}_i$ of $S+L_B$ consecutive SBs illustrated in Figure 1(a) where

$$\mathcal{A}_i = \mathcal{S}_{i-1}(S-\lfloor\frac{L_B}{2}\rfloor+1:S)\cup\mathcal{S}_i\cup\mathcal{S}_{i+1}(1:\lceil\frac{L_B}{2}\rceil).$$

Transmission from MB i to any SB in $\mathcal{S}_i$ causes interference at $\lfloor\frac{L_B}{2}\rfloor$ SBs above and at $\lceil\frac{L_B}{2}\rceil$ SBs below the set $\mathcal{S}_i$.

The channel model for the backhaul layer is given by $\mathbf{h}_{i,j}^B(t) \neq 0$ iff $j \in \mathcal{A}_i$. The backhaul layer for the linear network is illustrated in Figure 1. Let the channel gain matrix corresponding to MB i, $\mathbf{H}_i^B(t) \in \mathbb{C}^{N\times(S+L_B)}$ be defined as $\mathbf{H}_i^B(t) = [\mathbf{h}^B_{i,\mathcal{S}_{i-1}(S-\lfloor\frac{L_B}{2}\rfloor)}(t);\ldots;\mathbf{h}^B_{i,\mathcal{S}_i(S)}(t);\ldots;\mathbf{h}^B_{i,\mathcal{S}_{i+1}(\lceil\frac{L_B}{2}\rceil)}(t)]$ in the backhaul layer where each column corresponds to the channel coefficients from MB i to SB in the set $\mathcal{A}_i$. Here $[a;b]$ denotes the horizontal concatenation of two matrices a and b consisting of the same number of rows.

Let $\mathcal{R}_i(t) \subseteq \mathcal{A}_i$ denote the set of SBs receiving messages from MB i in a particular time-slot t.

B. Transmission Layer

Consider the transmission layer with K SBs and K MTs. Let $\mathcal{K}$ denote the set $\{1,\ldots,K\}$. Each SB and MT is assumed to be equipped with a single antenna. In the transmission layer, the channel gain between SB $j, \forall j \in \mathcal{K}$ and MT $i, \forall i \in \mathcal{K}$ is denoted by h_{ji}^{Tx}. At each MT i, the received signal y_i^{Tx} is given

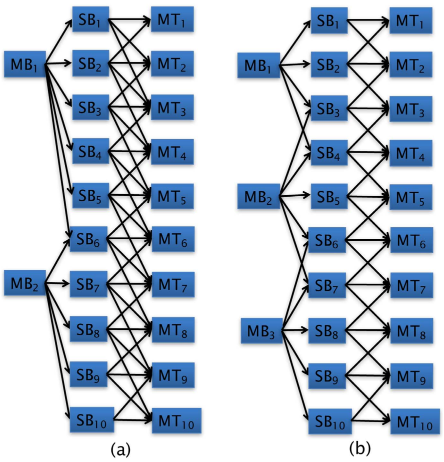


Fig. 2. Two-layered network with: (a) $S = 5$ and $L_B = 1$ in the backhaul layer, and $L_T = 3$ in the transmission layer; and (b) $S = 3$ and $L_B = 2$ in the backhaul layer, and $L_T = 2$ in the transmission layer.

by

$$y_i^{\mathrm{Tx}}(t) = h_{ii}^{\mathrm{Tx}}(t)x_i^{\mathrm{Tx}}(t) + \sum_{j\in\mathcal{I}_i^{\mathrm{Tx}}} h_{ji}^{\mathrm{Tx}}(t)x_j^{\mathrm{Tx}}(t) + z_i^{\mathrm{Tx}}(t), \qquad (1)$$

where t denotes the time-slot, $x_j^{\mathrm{Tx}}(t)$ denotes the signal transmitted by SB j under an average transmit power constraint, $z_i^{\mathrm{Tx}}(t)$ denotes the additive white Gaussian noise at MT i, $h_{ji}^{\mathrm{Tx}}(t)$ denotes the channel gain coefficient from SB j to MT i, and $\mathcal{I}_i^{\mathrm{Tx}}$ denotes the set of interferers at MT i.

We consider the linear cellular model presented by Wyner [16] and extended in [10] to a locally connected linear interference network with connectivity parameter L_T. The transmission layer is assumed to be a local L_T-Wyner model with K users. The cells are located on an infinite linear equi-spaced grid, and each transmitter is associated with a single user. Here L_T denotes the number of dominant interferers per user, where each user observes interference from $\lceil \frac{L_T}{2} \rceil$ preceding and $\lfloor \frac{L_T}{2} \rfloor$ succeeding transmitters. The channel coefficients for the L_T-Wyner model are given by

$$h_{ji}^{\mathrm{Tx}}(t) \neq 0$$
$$\text{iff } i \in \{j - \lfloor \frac{L_T}{2} \rfloor, \ldots, j-1, j, j+1, \ldots, j + \lceil \frac{L_T}{2} \rceil\}.$$

The system model is illustrated in Figure 2.

C. Capacity and Degrees of Freedom

Let P be the average transmit power constraint at each SB and the transmit power per antenna at each MB. Let $\mathcal{W}_i$ denote the alphabet for W_i, where W_i denotes the message for MT i. The rates $R_i(P) = \frac{\log|\mathcal{W}_i|}{n}$ are achievable iff the error probabilities of all messages can simultaneously be made arbitrarily small for large n, using an interference management scheme. The degree of freedom (DoF) $d_i, \forall i \in \mathcal{K}$ is defined as

$$d_i = \lim_{P\to\infty} \frac{R_i(P)}{\log P}. \qquad (2)$$

DoF corresponds to the number of interference-free sessions that can be accommodated in a multi-user channel. The maximum achievable sum DoF $\eta(K)$ in a channel with K users (MTs) is defined as

$$\eta(K) = \max_{\mathcal{D}} \sum_{i\in\mathcal{K}} d_i,$$

where $\mathcal{D}$ denotes the closure of the set of all achievable DoF tuples, and the per user DoF τ_K is defined as

$$\tau_K = \frac{\eta(K)}{K} \qquad (3)$$

with $\tau_\infty = \lim_{K\to\infty} \tau_K$.

III. Linear Networks

We consider linear networks and show that for lower connectivity in the transmission layer, i.e., $L_T = \{1, 2\}$, the optimal puDoF can be achieved without any cooperation in the network but as the transmission layer connectivity increases i.e., $L_T > 2$, cooperation in the network becomes crucial in order to achieve the optimal puDoF when $\lfloor \frac{S}{2} \rfloor \geq \lceil \frac{L_T}{2} \rceil$.

We first present an upper bound on the per user DoF for any general heterogeneous network. The result holds for any general scheme for the two-layered heterogeneous network with half-duplex SBs for any connectivity in the transmission layer. The maximum achievable puDoF is limited by the number of antennas at each MB in the backhaul layer, and also by the half-duplex nature of the SBs when there are sufficient antennas at each MB.

Theorem 1: The following upper bound holds for the asymptotic per user DoF τ_∞, for any cellular network model for the transmission layer and when each MB has N antennas and a cluster size of S,

$$\tau_\infty \leq \min(\frac{N}{S}, \frac{1}{2}). \qquad (4)$$

Proof: Consider any message W_i received at MT i. Due to the half-duplex nature of the SBs, the message W_i is transmitted over two time-slots, one in the backhaul layer and one in the transmission layer. During any T time-slots, if k is the total number of messages received interference free at all the MTs, we have $2k \leq KT$, and

$$\tau_K = \frac{\text{No. of messages received interference-free}}{KT} \leq \frac{KT}{2KT} = \frac{1}{2}.$$

The number of macro basestations in the network is $\lceil \frac{K}{S} \rceil$. After T time-slots, the maximum number of messages that can be received by the K SBs is $\lceil \frac{K}{S} \rceil TN$ messages. Hence the maximum number of messages that can be received by the MTs is $\lceil \frac{K}{S} \rceil TN$. Thus, for any scheme, the puDoF is given by

$$\tau_K \leq \frac{\lceil \frac{K}{S} \rceil TN}{KT} \leq \frac{N}{S}.$$

Thus, we have $\tau_K \leq \min(\frac{1}{2}, \frac{N}{S})$ and hence $\tau_\infty \leq \min(\frac{1}{2}, \frac{N}{S})$. ■

A. DoF Analysis for $L_T = \{1, 2\}$

We now consider the case where the connectivity in the transmission layer $L_T \leq 2$. We present achievable schemes for $L_T \leq 2$ for the simple case of $L_B = 1$. The optimal puDoF is achieved by zero-forcing the interference at the SBs in the backhaul layer using sufficient antennas at the MBs, and by deactivating SB-MT pairs appropriately in the transmission layer. We then show that the achievable schemes can be extended to the case of $L_B > 1$.

Remark 1: At any MB i', $N_1 + N_2 \leq N$ antennas are sufficient in order to send messages to N_1 SBs and to null the interference at N_2 SBs. Let $\mathcal{R}_{i'}$ denote the set of SBs that are receiving the messages and $\mathcal{Z}_{i'}$ denote the set of SBs at which interference is being zero-forced. Let $|\mathcal{R}_{i'}| = N_1$ and $|\mathcal{Z}_{i'}| = N_2$. Note that $\mathcal{R}_{i'}, \mathcal{Z}_{i'} \subseteq \mathcal{A}_{i'}$. Without loss of generality, let $\mathbf{H}_1 \in \mathbb{C}^{(N_1+N_2)\times N_1}$ and $\mathbf{H}_2 \in \mathbb{C}^{(N_1+N_2)\times N_2}$ denote the first $N_1 + N_2$ rows of the matrices $\mathbf{H}^B_{i',\mathcal{R}_{i'}}$ and $\mathbf{H}^B_{i',\mathcal{Z}_{i'}}$ respectively. Let $\mathbf{X} \in \mathbb{C}^{1\times(N_1+N_2)}$ denote the transmitted signal vector at MB i', $\mathbf{H} \in \mathbb{C}^{(N_1+N_2)\times(N_1+N_2)}$ denote $[\mathbf{H}_1; \mathbf{H}_2]$, and $\mathbf{W} \in \mathbb{C}^{1\times(N_1+N_2)}$ denote the vector containing the intended messages to $\mathcal{R}_{i'}$ appended with N_2 zeroes at the end. Here $[a; b]$ denotes the horizontal concatenation of two matrices a and b consisting of the same number of rows. Then we have $\mathbf{H}\mathbf{X}^T = \mathbf{W}^T$. From our assumptions, $\mathbf{H}$ is full rank almost surely and the solution $\mathbf{X} = (\mathbf{H}\mathbf{H}^*)^{-1}\mathbf{H}\mathbf{W}$ is obtained.

Theorem 2: The following lower bound holds for the asymptotic puDoF τ_∞, for a linear heterogeneous network when the backhaul layer connectivity $L_B = 1$ and the transmission layer connectivity $L_T \in \{1, 2\}$,

$$\tau_\infty \geq \begin{cases} \dfrac{N}{S} & \text{for } N < \dfrac{S}{2} \\ \dfrac{1}{2}(1 - \dfrac{1}{S}) & \text{for } N = \dfrac{S}{2} \text{ for } S \text{ even} \\ \dfrac{1}{2} & \text{for} N > \dfrac{S}{2} \end{cases} \tag{5}$$

Proof: In the transmission layer for the L_T-Wyner model with $L_T \in \{1, 2\}$, by deactivating alternate transceiver pairs, the remaining messages can be sent interference-free as shown in Figure 3. Thus, a puDoF of $\frac{1}{2}$ is achieved if the corresponding messages are available at the active SBs. The number of messages that can be made available at the SBs in each cluster in the backhaul layer depends on the number of antennas at the corresponding MB. We now describe how the achievable puDoF in the system changes as a function of number of antennas N at each MB.

Case 1: $N > \frac{S}{2}$.

A) When S is odd, our achievable scheme uses only $\frac{S+1}{2}$ antennas at an MB. Consider the following message assignment for each time-slot t where t is odd.

$$\mathcal{R}_i(t) = \begin{matrix} \{\mathcal{S}_i(1), \mathcal{S}_i(3), \ldots, \mathcal{S}_i(S)\} & \text{for } i \text{ odd} \\ \{\mathcal{S}_i(2), \mathcal{S}_i(4), \ldots, \mathcal{S}_i(S-1)\} & \text{for } i \text{ even.} \end{matrix}$$

When i is even, SB $\mathcal{S}_i(1)$ is not active in this time-slot. Only when i is odd, $\mathcal{S}_i(1)$ observes interference from the transmissions of MB $i-1$. MB $i-1$ needs $\frac{S-1}{2}$ antennas for sending messages and one antenna for nulling the interference at SB $\mathcal{S}_i(1)$. Thus at the end of each odd time-slot, messages are available at alternate SBs, and a puDoF of $\frac{1}{2}$ is achieved. The assignment is reversed when t is even, and the achievability follows similarly.

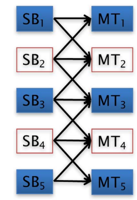


Fig. 3. Scheme achieving puDoF of $\frac{1}{2}$ in the transmission layer with $L_T = 2$. The unshaded boxes indicate deactivated transceivers.

B) When S is even, our achievable scheme uses $\frac{S}{2} + 1$ antennas at each MB. In odd and even time-slots, only the odd and even numbered SBs are served, respectively. This is possible as the cluster of any MB i contains $\frac{S}{2}$ SBs with odd indices and $\frac{S}{2}$ SBs with even indices. Only in time-slots t, where t is odd, the first SB in each active cluster observes interference. Each MB uses $\frac{S}{2}$ antennas to send messages and has an additional antenna to null interference at the first SB in the next cluster. Thus, in each time-slot, messages are available at alternate SBs and a puDoF of $\frac{1}{2}$ is achieved.

Case 2: $N < \frac{S}{2}$. In this case, $S \geq 2N+2$ or $S \geq 2N+1$ for even and odd indices, respectively. Hence in each cluster, two disjoint sets of N SBs are served in consecutive time-slots while the first SB of the cluster is inactive. Consider the following message assignment for each time-slot t when t is odd.

$$\mathcal{R}_i(t) = \begin{matrix} \{\mathcal{S}_i(3), \mathcal{S}_i(5), \ldots, \mathcal{S}_i(2N+1)\} & \text{for } i \text{ odd} \\ \{\mathcal{S}_i(2), \mathcal{S}_i(4), \ldots, \mathcal{S}_i(2N)\} & \text{for } i \text{ even.} \end{matrix}$$

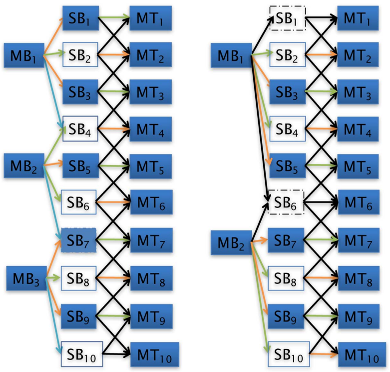
This assignment is reversed when t is even. The first SB in each cluster is not served at all and hence there is no interference in the backhaul layer. In each time-slot, N messages among every S users are sent interference-free, achieving a puDoF of $\frac{N}{S}$.

Case 3: $N = \frac{S}{2}$. This case arises only when S is even. For an even time-slot t, let an even numbered SBs be served. Only the first SB in each cluster sees interference, and hence there is no interference in the backhaul layer in this time-slot. When t is odd, all the odd numbered indices ($\frac{S}{2} - 1$) except for the first ones in each cluster are served. In the transmission layer, these messages are sent interference-free and a puDoF of $\frac{1}{2}(\frac{1}{2} + \frac{S/2-1}{S})$ is achieved. ∎

From Theorem 2 it follows that the upper bound in (4), i.e., the maximum puDoF can be achieved by simple interference avoidance schemes except for the case $N = \frac{S}{2}$ when $L_T \in \{1, 2\}$. The achievable schemes are illustrated in Figure 4.

Remark 2: We note that when $S = 1$, in Theorem 2, only the case $N > \frac{S}{2}$ is valid. A puDoF of half can be achieved with $N = 1$ at each MB. The proof follows in a similar fashion to that of Theorem 2 with alternate MBs transmitting in each time-slot in the backhaul layer interference-free since $L_B = 1$.

Remark 3: For $L_T \in \{1, 2\}$, $L_B > 1$, a puDoF of half can be achieved by using $N \geq \lceil \frac{S}{2} \rceil + L_B$ antennas at each MB. The proof follows similar to case 1 in the proof of Theorem 2



(a) (b)

Fig. 4. Achievable schemes for the network with $L_B = 1$ and $L_T = 2$: (a) puDoF of $\frac{1}{2}$ with $S = 3$ and $N = 2$; and (b) puDoF of $\frac{2}{5}$ with $S = 5$ and $N = 2$. The shaded and unshaded SBs receive messages in alternate time-slots and the dashed SBs do not receive messages.

with L_B antennas zero-forcing the interference in the backhaul layer.

B. Achievable Schemes for General L_T

The optimal puDoF for a given number of antennas cannot be achieved for higher values of L_T using only interference avoidance schemes without the use of cooperation. Cooperation here refers to the messages of each MT being available at multiple SBs. For example, when $L_T = 3$, with restriction to only ZF schemes without cooperation at the SBs, we have $\tau_\infty \leq \frac{2}{5}$ in the transmission layer even for a large N (see e.g., [10]). We consider cooperation among the SBs and show that the optimal puDoF can be achieved for $L_T \geq 3$ using only interference avoidance schemes. For cooperation, multiple messages need to be available at SBs for transmission in a particular time-slot, which requires multiple time-slots for transmission by the MBs in the backhaul layer and leads to ineffective use of resources. The SBs use the knowledge of messages available only for zero-forcing, and, thus, it suffices to have a linear combination of message signals at the SBs. For the heterogeneous network, our schemes use cooperation which refers to the message of each MT being available (as a part of a linear combination) at multiple SBs in addition to the one delivering its message. Transmission of a linear combination of message signals to the SBs requires only one time-slot in the backhaul layer. However, this would require that at each MB, the channel between the SBs in its cluster and the corresponding MTs is known. The requirement of large amount of CSI to be present at each MB is justified if the coherence time is large enough.

Claim 1: In a single layer L_T-Wyner model, if groups of A consecutive transceiver pairs are separated by F consecutive deactivated pairs where $F \geq \lceil \frac{L_T}{2} \rceil$, then there is no interference between the groups of A. Within the group of A consecutive transceiver pairs, using cooperation among the transmitters, all A messages can be sent such that the interference at each receiver is zero-forced, and a puDoF of $\frac{A}{F+A}$ is achieved in the network (see [10] for details).

In the two layer heterogeneous network, Claim 1 is applicable to the linear L_T-Wyner transmission layer, if groups of A consecutive SB-MT pairs are separated by F consecutive deactivated pairs where $F \geq \lceil \frac{L_T}{2} \rceil$. The interference can be zero-forced at the MTs if the appropriate zero-forcing linear combinations are available at the SBs and a puDoF of $\frac{A}{F+A}$ can be achieved in the transmission layer. A simple example is illustrated in Figure 5.

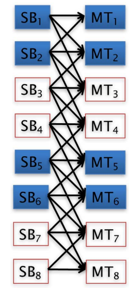


Fig. 5. Illustration of Claim 1 using the linear L_T-Wyner transmission layer with $L_T = 2$, and $A = F = 2$. In each block of two active transceivers, if the interference is zero-forced at the MTs, a puDoF of $\frac{1}{2}$ can be achieved in the transmission layer.

We present achievable schemes for $L_T \geq 3$ for the simple case of $L_B = 1$ for high connectivity in the backhaul layer, i.e., $\lfloor \frac{S}{2} \rfloor \geq \lceil \frac{L_T}{2} \rceil$. The optimal puDoF is achieved by zero-forcing interference at SBs in the backhaul layer using sufficient antennas at the MBs, and by each MB sending the appropriate linear combinations of the message signals of MTs in its cluster to the SBs in the cluster such that the interference is zero-forced at the MTs in the transmission layer. We then then show that the achievable schemes can be extended to the case of $L_B > 1$.

Theorem 3: The following lower bound holds for the asymptotic puDoF τ_∞, for a linear heterogeneous network when the backhaul layer connectivity $L_B = 1$ and the transmission layer connectivity L_T and the cluster size S are such that $\lfloor \frac{S}{2} \rfloor \geq \lceil \frac{L_T}{2} \rceil$

$$\tau_\infty \geq \begin{cases} \dfrac{N}{S} & \text{for } N < \dfrac{S}{2} \\ \dfrac{1}{2}(1 - \dfrac{1}{S}) & \text{for } N = \dfrac{S}{2} \text{ for } S \text{ even} \\ \dfrac{1}{2} & \text{for } N > \dfrac{S}{2} \end{cases} \tag{6}$$

Proof:

1) $N > \frac{S}{2}$ is equivalent to $N \geq \lfloor \frac{S}{2} \rfloor + 1$. For all i, let

$$\mathcal{R}_i(t) = \begin{array}{ll} \mathcal{S}_i(1 : \lfloor \dfrac{S}{2} \rfloor) & \text{for } t \text{ odd} \\ \mathcal{S}_i(\lfloor \dfrac{S}{2} \rfloor + 1 : S) & \text{for } t \text{ even.} \end{array}$$

In even and odd time-slots, $\lfloor \frac{S}{2} \rfloor + 1$ and $\lfloor \frac{S}{2} \rfloor$ antennas, respectively, at each MB i are used to send linear combinations to the SBs, and in an odd time-slot one antenna is used to ZF interference at $\mathcal{S}_{i+1}(1)$. From Claim 1, it follows that the puDoF is $\frac{1}{2}$.

2) $N < \frac{S}{2}$ is equivalent to $N \leq \lceil \frac{S}{2} \rceil - 1$. For all i, let

$$\mathcal{R}_i(t) = \begin{array}{ll} \mathcal{S}_i(2 : N + 1) & \text{for } t \text{ odd} \\ \mathcal{S}_i(\lceil \dfrac{S}{2} \rceil + 1 : \lceil \dfrac{S}{2} \rceil + N) & \text{for } t \text{ even.} \end{array}$$

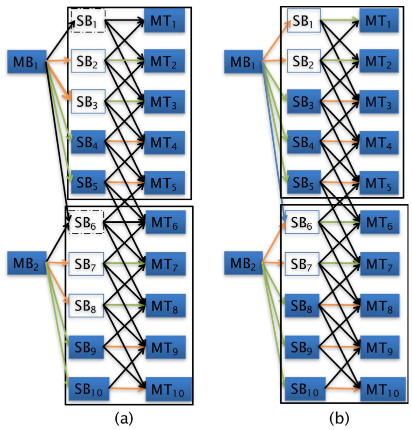


Fig. 6. Achievable schemes for the network with $L_B = 1$, $L_T = 3$ and $S = 5$. In (a), $N = 2$, $N < \frac{S}{2}$, and puDoF of $\frac{2}{5}$ is achieved. In (b), $N = 3$, $N > \frac{S}{2}$, and puDoF of $\frac{1}{2}$ is achieved. The shaded and unshaded SBs receive messages in alternate time-slots, and the dashed SBs do not receive messages.

In each time-slot, N antennas at each MB i send linear combinations to the SBs. The first SB in each cluster is always inactive. Each group of N SBs is separated by $S - N$ SBs and hence from Claim 1, the puDoF is $\frac{N}{S}$.

3) $N = \frac{S}{2}$. This case arises when S is even. For all i,

$$\mathcal{R}_i(t) = \begin{array}{ll} \mathcal{S}_i(2 : \dfrac{S}{2}) & \text{for } t \text{ odd} \\ \mathcal{S}_i(\dfrac{S}{2} + 1 : S) & \text{for } t \text{ even.} \end{array}$$

In the odd and even time-slots, $N - 1$ and N antennas, respectively, at each MB i are used to send linear combinations to the SBs. Hence we achieve a puDoF of $\frac{N}{S}$ and $N - \frac{1}{S}$ in consecutive time-slots, giving an average puDoF of $\frac{2N-1}{2S}$. ■

The achievable schemes are illustrated in Figure 6.

Remark 4: For $\lfloor \frac{S}{2} \rfloor \geq \lceil \frac{L_T}{2} \rceil$, $L_T \geq 3$, $L_B > 1$, a puDoF of half can be achieved by using $N \geq \lceil \frac{S}{2} \rceil + L_B$ antennas at each MB. The proof follows similar to case 1 in the proof of Theorem 3 with L_B antennas zero-forcing the interference in the backhaul layer.

IV. Hexagonal Cellular Network

The two-layered linear network is a much simpler interference network compared to the two-layered hexagonal network. In this section, we use these insights from the simple linear network and extend the results to the more realistic hexagonal network that has a complicated interference pattern.

A. System Model

We now present the system model for the two-layered heterogeneous network with the more practical hexagonal sectored cellular network in the transmission layer.

1) Backhaul Layer: For the backhaul layer, we consider a point-to-multipoint wireless backhaul where each MB is associated with S SBs. Designing the backhaul layer mainly involves assigning a cluster of SBs to each MB in a smart and useful manner. For an omni-directional antenna, a hexagonal cluster best approximates the radiation pattern. In our model, since we have multiple antennas at each MB which are used for beamforming and can be localized, we can choose any shape for the cluster. For the hexagonal transmission layer without intra-cell interference, a parallelogram structure arises naturally when we seek clusters with minimal inter-cluster interference [12]. Compared to a hexagonal cluster, the parallelogram structure has fewer edge nodes in neighboring clusters that observe interference. Hence, we choose a parallelogram shaped cluster of size S with each side consisting of $\sqrt{S}$ nodes. We assume that each MB is equipped with N antennas. We assume that each SB is served by only one MB.

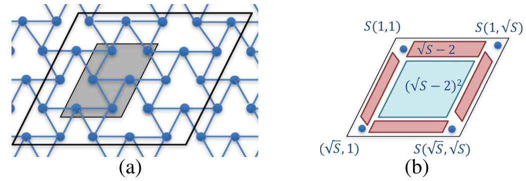


Fig. 7. In a), $\mathcal{S}_i(2:4, 2:3)$ is denoted by the shaded parallelogram. In b), the interior of a cluster is the box with $(\sqrt{S}-2)^2$ nodes, the corners are the individual nodes, and the edge nodes are the remaining nodes on the edge outside the interior.

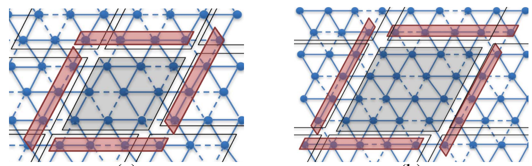
We assume that in every cluster, the channel state information (CSI) between the MB and all the SBs in the cluster in the backhaul layer, and the CSI between all the SBs and MTs belonging to the cluster in the transmission layer is known at the cluster MB. All channel coefficients that are not identically zero are assumed to be drawn independently from a continuous joint distribution.

Each MB (i, j) is associated with a cluster $\mathcal{S}_{(i,j)}$ consisting of S SBs where $\sqrt{S} \in \mathbb{Z}$ where i and j denote the row and column respectively in a two-dimensional grid. The cluster $\mathcal{S}_{(i,j)}$ with S nodes contains $\sqrt{S}$ rows containing $\sqrt{S}$ nodes each, and similarly, $\sqrt{S}$ columns containing $\sqrt{S}$ nodes each. Let $\mathcal{S}_{(i,j)}(a : b, c : d)$, $a \leq b, c \leq d, 1 \leq a, b, c, d \leq \sqrt{S}$ denote the set of nodes in the cluster belonging to rows from a to b and columns c to d. Note that here, $a : a \equiv a$. The notation is illustrated in Figure 7a.

$$\begin{aligned} &\mathcal{S}_{(i,j)}(a : b, c : d) \\ &\quad = \{((i-1)\sqrt{S} + a, (j-1)\sqrt{S} + c), \ldots, ((i-1)\sqrt{S} + a, \\ &\qquad (j-1)\sqrt{S} + d)\} \cup \{((i-1)\sqrt{S} + a + 1, (j-1)\sqrt{S} + c), \\ &\qquad \ldots, ((i-1)\sqrt{S} + a + 1, (j-1)\sqrt{S} + d)\} \cup \ldots \\ &\qquad \cup \{((i-1)\sqrt{S} + b, (j-1)\sqrt{S} + c), \ldots, \\ &\qquad ((i-1)\sqrt{S} + b, (j-1)\sqrt{S} + d)\}. \end{aligned}$$

For a cluster $\mathcal{S}_{(i,j)}$, we define the interior of the cluster as $\mathcal{S}_{(i,j)}(2 : \sqrt{S} - 1, 2 : \sqrt{S} - 1)$, the edges of the cluster as $\mathcal{S}_{(i,j)}(1, 2 : \sqrt{S} - 1), \mathcal{S}_{(i,j)}(2 : \sqrt{S} - 1, 1), \mathcal{S}_{(i,j)}(\sqrt{S}, 2 : \sqrt{S} - 1), \mathcal{S}_{(i,j)}(2 : \sqrt{S} - 1, \sqrt{S})$, and the corner nodes as $\mathcal{S}_{(i,j)}(1, 1)$, $\mathcal{S}_{(i,j)}(1, \sqrt{S})$, $\mathcal{S}_{(i,j)}(\sqrt{S}, 1)$, $\mathcal{S}_{(i,j)}(\sqrt{S}, \sqrt{S})$. This is illustrated in Figure 7b.

Each MB (i, j) causes interference at SBs belonging to the edges $\mathcal{S}_{(i-1,j)}(\sqrt{S}, 1 : \sqrt{S})$, $\mathcal{S}_{(i+1,j)}(1, 1 : \sqrt{S})$, $\mathcal{S}_{(i,j+1)}(1 : \sqrt{S}, 1)$, $\mathcal{S}_{(i,j-1)}(1 : \sqrt{S}, \sqrt{S})$ and at one corner each of the clusters $\mathcal{S}_{(i-1,j-1)}(\sqrt{S}, \sqrt{S})$, $\mathcal{S}_{(i+1,j+1)}(1, 1)$, $\mathcal{S}_{(i+1,j-1)}(1, \sqrt{S})$, $\mathcal{S}_{(i-1,j+1)}(\sqrt{S}, 1)$. This is illustrated in Figure 8a and 8b. Thus, each MB (i, j) is associated with



(a) (b)

Fig. 8. MB associated with the shaded cluster causes interference at the neighboring $4(\sqrt{S}+1)$ neighbors. In (a), $S=9$ and in (b), $S=16$.

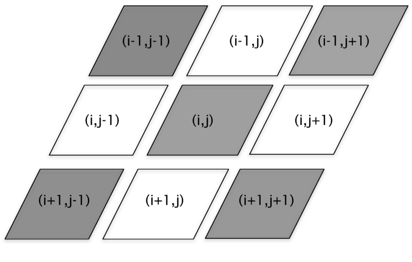
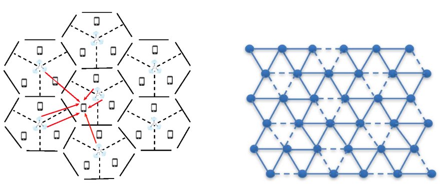


Fig. 9. Arrangement of shaded clusters and white clusters in the network.



(a) (b)

Fig. 10. (a) Cellular network and (b) interference graph. The dotted lines in (b) represent interference between sectors belonging to the same cell.

a set $\mathcal{A}_{(i,j)}$ at which its transmissions can be heard where

$$\begin{aligned}\mathcal{A}_{(i,j)} =&\mathcal{S}_{(i-1,j)}(\sqrt{S},1:\sqrt{S}) \cup \mathcal{S}_{(i+1,j)}(1,1:\sqrt{S}) \cup \mathcal{S}_{(i,j+1)}\\ &(1:\sqrt{S},1) \cup \mathcal{S}_{(i,j-1)}(1:\sqrt{S},\sqrt{S}) \cup \mathcal{S}_{(i,j)}\\ &\cup \mathcal{S}_{(i-1,j-1)}(\sqrt{S},\sqrt{S}) \cup \mathcal{S}_{(i+1,j+1)}(1,1)\\ &\cup \mathcal{S}_{(i+1,j-1)}(\sqrt{S},1) \cup \mathcal{S}_{(i-1,j+1)}(1,\sqrt{S}).\end{aligned}$$

Let N denote the number of antennas at each MB. The channel vector between MB (i,j) and SB i' is denoted by $\mathbf{h}^B_{(i,j),i'}(t)$. The channel coefficients for the backhaul layer satisfy the condition: $\mathbf{h}^B_{(i,j),i'}(t) \neq 0$ iff $i' \in \mathcal{A}_{(i,j)}$.

Let the channel gain matrix corresponding to MB (i,j), $\mathbf{H}^B_{(i,j)}(t) \in \mathbb{C}^{N\times(|\mathcal{A}_{(i,j)}|)}$ in the backhaul layer where the i'th column corresponds to the channel coefficients from MB (i,j) to SB i'.

We refer to clusters $\mathcal{S}_{i,j}$ where $i+j$ is even as *shaded* clusters, and the remaining clusters as *white* clusters. This is shown in Figure 9.

2) Transmission Layer: We consider a sectored K-user network with three sectors per cell as shown in Figure 10a. We assume that each sector is associated with one SB and one MT, and that each SB and MT is assumed to be equipped with a single antenna. A local interference model is assumed, where the interference at each receiver is only due to the basestations in the neighboring sectors. We consider two models for the transmission layer. In the first model, we assume that sectors belonging to the same cell do not interfere with each other. In the second model, we assume that sectors belonging to the same cell do interfere with each other.

Interference Graph: The cellular model is represented by an undirected interference graph $G(V,E)$ shown in Figure 10(b) where each vertex $u \in V$ corresponds to a transmitter-receiver pair. For any node a, the transmitter, receiver and intended message corresponding to the node are denoted by T_a, R_a and W_a, respectively. An edge $e \in E$ between two vertices $u, v \in V$ corresponds to interference between the transmit-receiver pairs, i.e., the transmitter at u causes interference at the receiver at v, and vice-versa. The dotted lines denote interference between sectors that belong to the same cell. Depending on the model we consider for the transmission layer, the dotted lines may or may not be present in the interference graph.

B. DoF Analysis

We now consider the puDoF in a hexagonal sectored cellular network, with and without intra-cell interference. Unless mentioned otherwise, the results hold for both the network models, i.e., with and without intra-cell interference.

We now discuss achievable schemes for the network for different number of antennas N at each MB. We use the idea of zero-forcing in the backhaul layer similar to the schemes in Section III.

We note that the achievable schemes do not require any cooperation between the MBs but do require that linear combinations be sent by the MBs to SBs to zero-force the interference at the MTs.

Remark 5: At any MB (i',j'), $N_1+N_2 \le N$ antennas are sufficient in order to send messages to N_1 SBs and to null the interference at N_2 SBs. Let $\mathcal{R}_{(i',j')}$ denote the set of SBs that are receiving the messages and $\mathcal{Z}_{(i',j')}$ denote the set of SBs at which interference is being zero-forced. Let $|\mathcal{R}_{(i',j')}| = N_1$ and $|\mathcal{Z}_{(i',j')}| = N_2$. Note that $\mathcal{R}_{(i',j')}, \mathcal{Z}_{(i',j')} \subseteq \mathcal{A}_{(i',j')}$. Without loss of generality, let $\mathbf{H}_1 \in \mathbb{C}^{(N_1+N_2)\times N_1}$ and $\mathbf{H}_2 \in \mathbb{C}^{(N_1+N_2)\times N_2}$ denote the first N_1+N_2 rows of the matrices $\mathbf{H}^B_{(i',j'),\mathcal{R}_{(i',j')}}$ and $\mathbf{H}^B_{(i',j'),\mathcal{Z}_{(i',j')}}$ respectively. Let $\mathbf{X} \in \mathbb{C}^{1\times(N_1+N_2)}$ denote the transmitted signal vector at MB (i',j'), $\mathbf{H} \in \mathbb{C}^{(N_1+N_2)\times(N_1+N_2)}$ denote $[\mathbf{H}_1;\mathbf{H}_2]$, and $\mathbf{W} \in \mathbb{C}^{1\times(N_1+N_2)}$ denote the vector containing the intended messages to $\mathcal{R}_{(i',j')}$ appended with N_2 zeroes at the end. Here $[a;b]$ denotes the horizontal concatenation of two matrices a and b consisting of the same number of rows. Then we have $\mathbf{H}\mathbf{X}^T = \mathbf{W}^T$. From our assumptions, $\mathbf{H}$ is full rank almost surely and the solution $\mathbf{X} = (\mathbf{H}\mathbf{H}^*)^{-1}\mathbf{H}\mathbf{W}$ is obtained.

We present an achievable scheme for the more practical two-layer heterogenous network with a sectored hexagonal network in the transmission layer. We show that by extending the simple ZF schemes that achieve optimal puDoF for the linear network, significant puDoF gains can be achieved for the hexagonal sectored cellular network for the cases with and without intra-cell interference.

Theorem 4: The following lower bound holds for the asymptotic puDoF τ_∞, for the hexagonal heterogeneous cellular network with or without intra-cell interference,

$$\tau_\infty \ge \begin{cases} \dfrac{N}{S} & \text{for } N < \lceil \dfrac{(\sqrt{S}-2)^2}{2} \rceil, \\ \dfrac{N + \lfloor (\sqrt{S}-2)^2/2 \rfloor}{2S} & \text{for } \lceil \dfrac{(\sqrt{S}-2)^2}{2} \rceil \le N \le c, \\ \dfrac{1}{2} - \dfrac{1}{2S} & \text{for } N \ge c+6, \end{cases}$$

where $c = \lceil \frac{(\sqrt{S}-2)^2}{2} \rceil + 4(\sqrt{S}-2)$.

Proof: We refer to clusters $\mathcal{S}_{(i,j)}$, where $i+j$ is even as shaded clusters, and the rest as white clusters, as discussed in Section IV-A.1 and shown in Figure 9. Interior, edge and corner nodes were introduced in Section IV-A.1 and shown in Figure 7b.

1) $N < \lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$

Backhaul Layer: For $N < \lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$, we show that in each time-slot, N messages are sent to SBs interference-free in the backhaul layer. In each time-slot we send linear combinations of message signals to N SBs in the interior $(\sqrt{S}-2)^2$ SBs of each cluster. Since $N < \lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$, in each time-slot we can find a new set of N SBs in the interior of the cluster which did not receive a message in the previous time-slot. In the backhaul layer, the outer nodes in each cluster observe interference from transmissions of neighboring MBs. Since there are no transmissions to the outer nodes in each cluster, there is no interference in the backhaul layer in this case.

Transmission Layer: In the transmission layer, we need to show that N messages can be sent interference-free in each cluster. There is no interference in the transmission layer across clusters because the active SBs are within the interior of each cluster. Within each cluster, linear combinations of message signals are sent by the MB in a way to ZF interference and thus the messages are sent interference-free. Hence in each time-slot, N MTs receive their messages interference-free in each cluster consisting of S MTs, achieving a per user DoF of $\frac{N}{S}$.

2) $\lceil \frac{(\sqrt{S}-2)^2}{2} \rceil \leq N \leq \lceil \frac{(\sqrt{S}-2)^2}{2} \rceil + 4(\sqrt{S}-2)$.

Note that the per user DoF in this case evaluates to $\frac{N}{S}$ if $\sqrt{S}$ is even and $\frac{N-1}{S}$ otherwise.

Backhaul Layer:

- Shaded clusters: In odd time-slots, $\lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$ SBs in the interior of the shaded clusters receive messages and $N - \lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$ antennas zero-force the interference at the edge nodes in neighboring white clusters. In even time-slots, $\lfloor \frac{(\sqrt{S}-2)^2}{2} \rfloor$ SBs corresponding to the interior of the shaded clusters receive messages and $N - \lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$ of the edge nodes in the exterior receive messages. Note that the exterior SBs do not observe interference because the MBs corresponding to white clusters zero-force the interference at these SBs in even time-slots.
- White clusters: In even time-slots, $\lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$ SBs corresponding to the interior of the white clusters receive messages and $N - \lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$ antennas zero-force the interference at the edge nodes in neighboring clusters. In odd time-slots, $\lfloor \frac{(\sqrt{S}-2)^2}{2} \rfloor$ SBs corresponding to the interior of the white clusters receive messages and $N - \lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$ of the edge nodes in the exterior receive messages. Note that the exterior SBs do not observe interference in the odd time-slots because the MBs corresponding to neighboring shaded clusters ZF the interference at these SBs.

Transmission Layer: The interior nodes of a cluster do not observe interference in the the transmission layer. In even time-slots, the $N - \lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$ edge MTs corresponding to the shaded clusters receive messages, and in odd time-slots, the $N - \lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$ edge MTs corresponding to the white clusters receive messages from their respective SBs. In each time-slot, we notice that there is no interference at the MTs. This is because edge SBs in shaded clusters cause interference only at MTs belonging to neighboring white clusters, and vice-versa. Within the cluster, linear combinations of message signals are sent by the MBs to zero force the interference, and hence the messages are received interference-free at the MTs. Thus, over two time-slots $N + \lfloor \frac{(\sqrt{S}-2)^2}{2} \rfloor$ messages are sent interference-free.

3) $N = \lceil \frac{(\sqrt{S}-2)^2}{2} \rceil + 4(\sqrt{S}-2) + 6$

Note that in this scheme, over two consecutive time-slots, the message of the MT $\mathcal{S}_{(i,j)}(\sqrt{S}, \sqrt{S})$ is sent interference-free for all clusters, and in the next two time-slots, the message of the MT $\mathcal{S}_{(i,j)}(1,1)$ is sent interference-free for all clusters. We alternate between sending the messages of MT $\mathcal{S}_{(i,j)}(\sqrt{S}, \sqrt{S})$ over two consecutive time-slots and MT $\mathcal{S}_{(i,j)}(1,1)$ over the next two consecutive time-slots.

Backhaul Layer:

- Shaded clusters: In odd time-slots, $\lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$ SBs in the interior of the shaded clusters receive messages and $4(\sqrt{S}-2)+6$ antennas zero-force interference at the edge and corner nodes in neighboring white clusters. There are eight corner nodes in the neighboring white clusters, $\mathcal{S}_{(i-1,j)}(\sqrt{S}, \sqrt{S})$, $\mathcal{S}_{(i,j-1)}(\sqrt{S}, \sqrt{S})$, $\mathcal{S}_{(i,j+1)}(1,1)$, $\mathcal{S}_{(i+1,j)}(1,1)$, $\mathcal{S}_{(i,j+1)}(\sqrt{S},1)$, $\mathcal{S}_{(i-1,j)}(\sqrt{S},1)$, $\mathcal{S}_{(i+1,j)}(1,\sqrt{S})$, $\mathcal{S}_{(i,j-1)}(1,\sqrt{S})$. For any SB (i',j'), we send only one among the nodes $\mathcal{S}_{(i',j')}(\sqrt{S}, \sqrt{S})$ and $\mathcal{S}_{(i',j')}(1,1)$. Thus, we only need to zero-force the interference at six corner SBs in the neighboring cluster.

  In the even time-slots, $\lfloor \frac{(\sqrt{S}-2)^2}{2} \rfloor$ SBs corresponding to the interior and $4(\sqrt{S}-2)$ edge nodes and three corner nodes in the exterior of the shaded clusters receive messages. For any SB (i',j'), in the even time-slots, we send only $\mathcal{S}_{(i',j')}(1,1)$. The interference at the three corner nodes of the neighboring shaded clusters is zero-forced by three additional antennas since only one among the nodes $\mathcal{S}_{(i-1,j-1)}(\sqrt{S}, \sqrt{S})$ and $\mathcal{S}_{(i+1,j+1)}(1,1)$ is not being sent. Note that the exterior SBs do not observe interference because the MBs corresponding to white clusters zero-force interference at these SBs.
- White clusters: In even time-slots, $\lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$ SBs corresponding to the interior of the white clusters receive messages, $4(\sqrt{S}-2)$ antennas zero-force interference at the edge nodes, and six antennas zero-force interference at the corner nodes in neighboring shaded clusters. In odd time-slots, $\lfloor \frac{(\sqrt{S}-2)^2}{2} \rfloor$

SBs corresponding to the interior, $4(\sqrt{S}-2)$ edge nodes, and three corner nodes in the exterior of the white clusters receive messages. The interference at the three corner nodes of the neighboring white clusters is zero-forced by three additional antennas. Note that the exterior SBs do not observe interference in the odd time-slots because the MBs corresponding to neighboring shaded clusters zero-force the interference at these SBs.

Transmission Layer: The interior nodes of a cluster do not observe interference in the the transmission layer. In even time-slots, the $4(\sqrt{S}-2)$ edge MTs corresponding to the shaded clusters receive messages, and in odd time-slots, the $4(\sqrt{S}-2)$ edge MTs corresponding to the white clusters receive messages from their respective SBs. In every shaded (or white) cluster $\mathcal{S}_{(i,j)}$, there are two corner nodes $\mathcal{S}_{(i,j)}(1,\sqrt{S}), \mathcal{S}_{(i,j)}(\sqrt{S},1)$ that do not cause interference in the transmission layer at the corner nodes in neighboring shaded (or white) clusters $\mathcal{S}_{(i-1,j+1)}(\sqrt{S},1)$, $\mathcal{S}_{(i+1,j-1)}(1,\sqrt{S})$. In every shaded (or white) cluster $\mathcal{S}_{(i,j)}$, the remaining two corner nodes $\mathcal{S}_{(i,j)}(1,1)$, $\mathcal{S}_{(i,j)}(\sqrt{S},\sqrt{S})$ cause interference at their respective corner nodes $\mathcal{S}_{(i-1,j-1)}(\sqrt{S},\sqrt{S})$, $\mathcal{S}_{(i+1,j+1)}(1,1)$ in neighboring shaded (or white) clusters. The messages to the corner nodes that do not cause interference at neighboring clusters are sent interference-free. Among the corner nodes that cause interference, only one message is sent, say $\mathcal{S}_{(i,j)}(\sqrt{S},\sqrt{S})$. There is no interference across the clusters due to the edge SBs because edge SBs in shaded clusters cause interference only at MTs belonging to neighboring white clusters, and vice-versa. Within the cluster, linear combinations of message signals are sent in a way to zero-force interference by the MBs, and hence the messages are received interference-free. Thus, over two time-slots, $S-1$ messages are sent interference-free in each cluster, giving a per user DoF of $\frac{(S-1)}{2S}$. ■

We note that the achievable schemes in Theorem 4 use only simple zero-forcing and approach the optimal per user DoF of $\frac{1}{2}$ for large S.

Now we consider the hexagonal network with no intra-cell interference and show that when $\sqrt{S}=3k$, $k\in\mathbb{Z}$, a per user DoF of $\frac{1}{2}$ is achieved. The difference in this case arises because when $\sqrt{S}$ is of the form $3k$, the corner nodes of the shaded (or white) cluster do not cause interference at the corner nodes of the neighboring shaded (or white) clusters as shown in Figure 11. We present two achievable schemes that obtain a per user DoF of $\frac{1}{2}$.

Theorem 5: For a hexagonal heterogeneous cellular network with no intra-cell interference, where the cluster size is restricted to $\sqrt{S}=3k$, $k\in\mathbb{Z}$, and where $N\geq \min\{\lceil\frac{(\sqrt{S}-2)^2}{2}\rceil+4(\sqrt{S}-2)+8, S+4\}$,

$$\tau_\infty \geq \frac{1}{2}.$$

Proof: We refer to clusters $\mathcal{S}_{(i,j)}$, where $i+j$ is even, as shaded clusters, and the rest as white clusters, as discussed

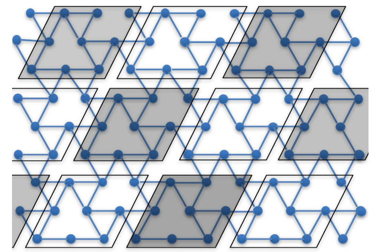


Fig. 11. The hexagonal sectored cellular network with no intra-cell interference when $\sqrt{S}=3$. The corner nodes of the same color (shaded or white) do not interfere with each other.

in Section II-A and shown in Figure 9. Interior, edge and corner nodes were introduced in Section II-A and shown in Figure 7b.

We propose two achievable schemes; one that uses $N=S+4$ antennas and one that uses $N=\lceil\frac{(\sqrt{S}-2)^2}{2}\rceil+4(\sqrt{S}-2)+8$ antennas.

1) $N=S+4$

Backhaul Layer: In odd time-slots, the S SBs corresponding to the shaded clusters receive messages and in even time-slots, the S SBs corresponding to the white clusters receive messages from their respective MBs. We note that the corner SBs in each cluster observe interference, so each MB (i,j) uses additional four antennas to zero-force interference at corner nodes of the clusters $\mathcal{S}_{i-1,j-1}(\sqrt{S},\sqrt{S})$, $\mathcal{S}_{i+1,j+1}(1,1)$, $\mathcal{S}_{i-1,j+1}(\sqrt{S},1)$, $\mathcal{S}_{i+1,j-1}(1,\sqrt{S})$. Over two consecutive time-slots all SBs receive their message interference-free.

Transmission Layer: In even time-slots, the S MTs corresponding to the shaded clusters receive messages and in odd time-slots, the S MTs corresponding to the white clusters receive messages from their respective SBs. In each time-slot, we notice that there is no interference at the MTs. This is because SBs in shaded clusters cause interference only at MTs belonging to neighboring white clusters and vice-versa. Within the cluster, since linear combinations of message signals are sent in a way to ZF interference by the MB, the messages are received interference-free. Thus over two consecutive time-slots all MTs receive their message interference-free thus giving a per user DoF of $\frac{1}{2}$.

2) $N=\lceil\frac{(\sqrt{S}-2)^2}{2}\rceil+4(\sqrt{S}-2)+8$

Backhaul Layer:

- Shaded clusters: In odd time-slots, $\lceil\frac{(\sqrt{S}-2)^2}{2}\rceil$ SBs in the interior of the shaded clusters receive messages and $4(\sqrt{S}-2)$ and eight antennas zero-force interference at the edge nodes and corner nodes in the neighboring white clusters respectively. In even time-slots, $\lfloor\frac{(\sqrt{S}-2)^2}{2}\rfloor$ SBs corresponding to the interior, $4(\sqrt{S}-2)$ edge nodes and the four corner nodes in the exterior of the shaded clusters receive messages. The interference at the four corner nodes of the neighboring shaded clusters is zero-forced by four additional antennas. Note that the exterior SBs do not observe interference because the MBs corresponding to white clusters zero-force interference at these SBs.

- White clusters: In even time-slots, $\lceil \frac{(\sqrt{S}-2)^2}{2} \rceil$ SBs corresponding to the interior of the white clusters receive messages and $4(\sqrt{S}-2)$ and eight antennas zero-force interference at the edge nodes and corner nodes in the neighboring shaded clusters respectively. In odd time-slots, $\lfloor \frac{(\sqrt{S}-2)^2}{2} \rfloor$ SBs corresponding to the interior, $4(\sqrt{S}-2)$ edge nodes, and the four corner nodes in the exterior of the white clusters receive messages. The interference at the four corner nodes of the neighboring white clusters is zero-forced by four additional antennas. Note that the exterior SBs do not observe interference in the odd time-slots because the MBs corresponding to neighboring shaded clusters zero-force the interference at these SBs.

Transmission Layer: There is no interference in the transmission layer for interior nodes of a cluster. In even time-slots, the exterior MTs corresponding to the shaded clusters receive messages, and in odd time-slots, the exterior MTs corresponding to the white clusters receive messages from their respective SBs. In each time-slot, we notice that there is no interference between the clusters. This is because SBs in shaded clusters cause interference only at MTs belonging to neighboring white clusters and vice-versa. Within the cluster, linear combinations of message signals are sent in such a way as to zero-force the interference by the MB and hence the messages are received interference-free. Over two time-slots all the messages are sent interference-free, thus giving a per user DoF of $\frac{1}{2}$. ■

C. Hexagonal Clusters

We now give some intuition about the choice of parallelogram instead of the more conventional hexagonal shape for the clusters. For comparison, consider the case where each MB has sufficient antennas to send messages to all SBs in its cluster in the backhaul layer. We try to approach a puDoF of $\frac{1}{2}$ in the transmission layer. For parallelogram clusters, we have seen in Theorem 4 that if half of the clusters are deactivated (Figure 9), then irrespective of cluster size S, the inter-cluster interference is seen at exactly 4 nodes. Now, let us consider the case of hexagonal clusters, and try to approach a puDoF of $\frac{1}{2}$ in the transmission layer. This would require that at least half the clusters stay active. We deactivate alternate clusters in each row, where the rows are shown in Figure 12. Every active cluster observes interference from two other clusters, one above and one below. In this scenario, inter-cell interference can be observed at least 4 nodes as shown in Figure 12 for a hexagonal cluster containing seven nodes. As the hexagonal cluster size increases, the number of nodes that observe inter-cell interference also increases, thus reducing the achievable puDoF using the schemes outlined in Theorem 4.

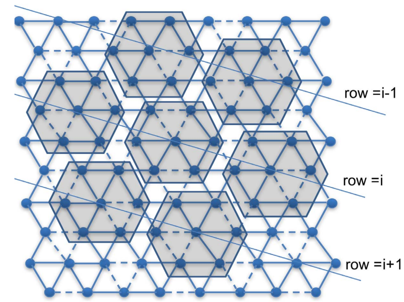


Fig. 12. Cellular network with hexagonal clusters. Inter-cluster interference between any two adjacent hexagons corresponds to interference at edge nodes.

D. Time vs. Frequency Duplexing Relays

Since the SBs are half-duplex, they cannot transmit and receive in the same frequency band at the same time. There are two strategies for accommodating this constraint. The first is a frequency-division duplexing (FDD) strategy in which the available frequency band is divided into two equal parts, with the SBs receiving in one half and transmitting in the other. In this case the backhaul and transmission layers can be treated separately, and the puDoF in the total network is half that of the DoF in the transmission layer. The puDoF in locally connected networks is strictly less than one, no matter what the cooperation order M is, and hence the puDoF achievable for the two-layered network is strictly less than $\frac{1}{2}$.

A better strategy for accommodating the half-duplex constraint at the SBs is a time-division duplex (TDD) strategy that we follow above, where the SBs receive and transmit in alternate time slots. In this case also the puDoF in the total network is half of the DoF in the transmission layer, and therefore the maximum achievable puDoF is $\frac{1}{2}$. The key difference from the FDD strategy is that in the TDD strategy we can exploit the fact that not all the SBs are active in a given cluster for the CoMP zero-forcing achievable scheme. The SBs that are inactive for zero-forcing in a given time slot can receive signals in that time slot from the MB serving the cluster, thus utilizing the shared time-frequency resources more efficiently. Using this approach we have shown that one can achieve the maximum possible puDoF of $\frac{1}{2}$ as long as there are a sufficient number of antennas at each of the MBs.

E. Uplink

For the CoMP schemes discussed in [17] for single-layer networks with a wired backhaul where cooperation is through message sharing on the backhaul, there is no uplink-downlink duality, i.e., the downlink schemes cannot be reversed to provide the same DoF in the uplink. On the other hand if we allow for the sharing of analog signals through the backhaul on the uplink, then the downlink strategies can be reversed to perform interference cancellation on the uplink to achieve the same DoF. In our design of a heterogeneous network, we have a wireless backhaul in the downlink as well as the uplink that enables us to share analog signals through the backhaul. Thus, linear combinations of analog signals can be sent over the backhaul, and uplink-downlink duality holds. In the downlink, we send linear combinations of the analog message signals to the SBs directly from the MBs and these analog signals are relayed by the SBs to zero-force the interference at the MTs. The uplink can be designed similarly to the downlink, with appropriate combinations of SBs and MTs being scheduled to transmit in different time-slots. Each MB receives a linear combination of the message signals from the active SBs and the messages can be decoded error-free by inverting the channel matrix.[1] Implementing such CoMP reception requires that at each MB, the CSI between SBs in its cluster and corresponding MTs is known.

V. Conclusion

We considered a heterogeneous cellular network consisting of MBs, SBs and MTs with a wireless backhaul layer, and with the SBs acting as half-duplex relays. We analyzed the per user DoF first for a linear heterogeneous cellular network, and then extended the results to a more general and practical heterogeneous hexagonal cellular network. We proposed simple zero-forcing schemes that use joint processing to cancel the interference at the MTs. An important feature of our approach to zero-forcing is that appropriate linear combinations of the message signals are sent to the SBs, rather than sending multiple messages, thus avoiding overloading the backhaul. In the linear network, our schemes achieve the optimal puDoF of $\frac{1}{2}$, while in the hexagonal network, our schemes approach the optimal puDoF from below. The insights from this work can also be used to design the uplink in a similar fashion, since uplink-downlink duality holds for the proposed achievable schemes.

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[1]Note that from our assumptions the channel matrix is full rank almost surely.

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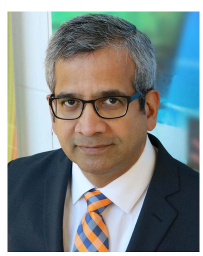

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