# Integer Forcing Interference Management for the MIMO Interference Channel

Sung Ho Chae, Member, IEEE and Sang-Woon Jeon, Member, IEEE

#### Abstract

A new interference management scheme based on integer forcing (IF) receivers is studied for the two-user multiple-input and multiple-output (MIMO) interference channel. The proposed scheme employs a message splitting method that divides each data stream into common and private sub-streams, in which the private stream is recovered by the dedicated receiver only while the common stream is required to be recovered by both receivers. Specifically, to enable IF sum decoding at the receiver side, all streams are encoded using the same lattice code. Additionally, the number of common and private streams of each user is carefully determined by considering the number of antennas at transmitters and receivers, the channel matrices, and the effective signal-to-noise ratio (SNR) at each receiver to maximize the achievable rate. Furthermore, we consider various assumptions of channel state information at the transmitter side (CSIT) and propose low-complexity linear transmit beamforming suitable for each CSIT assumption. The achievable sum rate and rate region are analytically derived and extensively evaluated by simulation for various environments, demonstrating that the proposed interference management scheme strictly outperforms the previous benchmark schemes in a wide range of channel parameters due to the gain from IF sum decoding.

#### **Index Terms**

Interference channel, interference management, integer forcing (IF), linear beamforming, multipleinput multiple-output (MIMO).

S. H. Chae is with the Department of Electronic Engineering, Kwangwoon University, Seoul 01897, South Korea (e-mail: sho.chae00@gmail.com).

S.-W. Jeon, *corresponding author*, is with the Department of Military Information Engineering, Hanyang University, Ansan 15588, South Korea (e-mail: sangwoonjeon@hanyang.ac.kr).

## I. INTRODUCTION

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In the fifth generation (5G) and the future sixth generation (6G) communication systems, it is expected that a variety of emerging applications such as virtual reality (VR), augmented reality (AR), real-time ultra high definition (UHD) video streaming, autonomous driving, and the Internet of everything (IoE) will begin to be utilized in earnest [1]–[5]. To provide these new services any time, anywhere, regardless of the user's location, it is important to significantly increase the achievable data rate of cell edge users compared to the current level. More specifically, the target data rates for edge users (bottom 5% of users) are 100 Mbps and 50 Mbps for 5G downlink and uplink, respectively, and moreover, it is predicted that in 6G communication, it will be necessary to achieve data rates that are 10 times higher than at present [1], [4], [5]. To achieve such challenging goal, unprecedented technology is required to push the limits of current technologies, and advanced interference management based on multiple-input multiple-output (MIMO) antennas is being considered as one of the key technologies to significantly improve data rates of edge users in 5G and 6G communications [5]–[10].

#### A. Related Works

Currently, linear MIMO receivers such as zero-forcing (ZF) receivers and minimum mean square error (MMSE) receivers have been widely used in practice [11]–[13] due to their low complexity. These conventional linear receivers first separate the transmitted streams by applying a linear filter to the received signal vector and then decode each stream individually. Specifically, a ZF receiver uses the pseudo-inverse of the channel matrix to convert a given MIMO channel into interference-free parallel single-input and single-output (SISO) channels while an MMSE receiver uses the regularized channel inversion matrix to maximize the signal-to-noise ratio (SNR) of each individual stream. It is well known that MMSE receivers outperform ZF receivers regardless of the SNR regime, and the performance gap becomes larger at lower SNR [11]. In addition, an MMSE receiver can be combined with successive interference cancellation (SIC) operation to achieve better performance, called an MMSE-SIC receiver [11]. The MMSE-SIC receiver sequentially recovers each stream one by one by applying a linear MMSE filter, and in each sequential decoding, the contribution of the decoded stream is subtracted from the received signal vector to increase the SNR of the remaining streams that have not yet been decoded. Furthermore, ZF, MMSE, and MMSE-SIC receivers can be employed for intra-cell

and/or inter-cell interference management in multi-user communications as well as point-topoint communications [12], [13].

Although the aforementioned linear receivers clearly have an advantage in terms of complexity, they have an inherent limitation that decoupling with linear filtering can cause significant noise amplification, and as a result, the gap between the achievable rate and the capacity can be large, especially when the channel becomes near singular [11], [14]–[16]. On the other hand, nonlinear receivers such as maximum likelihood (ML) detection receiver and sphere decoding receiver [17]–[19], which exhaustively search for the most likely transmitted signal vector based on the received signal vector, can significantly reduce noise amplification compared to linear receivers. However, the complexity of most nonlinear receivers is much higher than that of linear receivers especially when supporting large numbers of antennas and/or high order modulations [20], and hence nonlinear receivers are still rarely used in practical communication systems [12], [13].

Recently, a novel linear MIMO receiver, namely, integer forcing (IF) receiver, has been proposed [21]. Note that in the previous work [22], the compute-and-forward (CF) relaying scheme has been developed for Gaussian relay networks, in which the integer-linear sum of transmitted codewords is first computed at each relay and then the result is forwarded to the destination. The IF MIMO receiver can be viewed as a receiver to which the CF scheme is applied in a Gaussian MIMO point-to-point channel. Unlike the previous linear receivers that first separate the transmitted streams by applying a linear filter and recover each stream individually with each SISO decoder, the IF receiver first creates an integer-valued full-rank effective channel by applying a linear filter and directly decodes the integer-linear combination of the transmitted codewords with each SISO decoder. To enable this sum decoding, the integer-linear sum of the transmitted codewords should be itself a codeword, and hence, the transmitter uses the same lattice code for all streams. After integer-linear sums are decoded, the original streams can be simply recovered by multiplying the decoded outputs of the linear combinations by the inverse of the effective integer channel matrix in the absence of noise. Since the IF receiver has the freedom to determine the effective integer channel matrix in a way that minimizes noise amplification in contrast to the previous linear receivers that always constrain the integer matrix by the identity matrix regardless of the channel matrix, IF receivers can significantly reduce noise amplification compared to the previous linear receivers. Consequently, it has been shown that IF receivers can

achieve rate close to the channel capacity in Rayleigh fading channels [21].

In the literature, several follow-up studies have been conducted to improve and extend the basic IF proposed in [21]. In [23], it has been shown that using a linear dispersion space-time code in conjunction with IF equalization at the receiver side can achieve the capacity within a constant gap for general MIMO channels, assuming that the transmitter only knows whiteinput mutual information. More recently, precise performance characteristics of parallel MIMO channels have been studied when applying precoding with the full-diversity rotation matrix at the transmitter side and IF equalization at the receiver side [24]. In addition, similar to MMSE-SIC, SIC operations can be combined with IF sum decoding, namely, successive IF [25], to improve the performance of the basic IF. Additionally, similar to ZF and MMSE receivers, IF receivers can be employed to manage interference in multi-user communications as well as point-to-point communications, i.e., it can be extended to multiple-access channels [25], [26], broadcast channels [26]–[29], interference channels [30]–[33], and relay networks [34]–[36]. For example, in [36], extended CF and successive CF have been proposed for multi-user multi-relay networks, and it has been shown that the proposed schemes can solve the rank failure problem and outperform the original CF scheme of [22]. Furthermore, instead of using lattice codes, IF transceivers built on off-the-shelf binary codes such as turbo and low-density parity-check (LDPC) codes have been recently developed and evaluated in practical channel environments [6], [37]. In [38], spatially modulated IF (SM-IF) that combines generalized spatial modulation with IF has been developed based on practical binary codes.

## B. Our Contributions

In this paper, we propose a low-complexity interference management scheme based on IF for the two-user MIMO interference channel. It is well known that when interference is strong, decoding interference can enlarge the achievable rate region [39]–[42]. On the other hand, when interference is weak enough, treating interference as noise has proven to be an optimal strategy to achieve the capacity region [43]–[45]. Inspired by these facts, we consider a message splitting method that splits each data stream into common and private sub-streams, as in the Han–Kobayashi scheme [46]. Each receiver then attempts to recover the desired streams, that is, the intended common and private streams, and also the other user's common streams, while treating the private streams of the other user as noise. The main difference between our work and previous message splitting schemes is that unlike previous studies, all common and private streams are encoded with the same lattice code to enable IF sum decoding at the receiver side in this paper. Note that a lattice based message splitting scheme for IF receivers has been studied in [32] as in our work, but in the earlier work [32], the SISO interference channel was considered rather than the MIMO interference channel. Our work can be viewed as a follow-up study extending the results of [32] to MIMO networks.

In the proposed scheme, novel interference management and rank adaptation based on IF sum decoding are performed by controlling the number of common and private streams for each user. For a given channel realization and SNR, the optimal choice that maximizes the achievable rate of the proposed scheme can be found in a finite search space that only depends on the number of antennas at transmitters and receivers. In addition, we propose low-complexity linear transmit beamforming suitable for various channel state information at the transmitter side (CSIT) assumptions. The achievable sum rate and rate region of the proposed scheme are analytically derived and also numerically evaluated for various channel environments. The results demonstrate that the proposed scheme significantly improves the achievable rate compared to conventional previous schemes for a wide range of channel parameters. To the best of our knowledge, our results are the first to demonstrate such improvement by combining IF sum decoding with a message-splitting method for the MIMO interference channel.

# C. Paper Organization and Notation

The remainder of the paper is organized as follows. In Section II, we describe the channel model and assumptions considered in this paper. In Section III, the proposed interference management scheme based on IF is specifically described and the resulting achievable rate is derived. In Section IV, we numerically evaluate the achievable sum rate and rate region for various environments and discuss the results. Finally, we conclude the paper in Section V.

*Notation*: Boldface lowercase and uppercase letters are used to denote vectors and matrices, respectively. Let  $\mathbf{A}^{\dagger}$ ,  $\mathbf{A}^{-1}$ ,  $\|\mathbf{A}\|$ , and rank( $\mathbf{A}$ ) denote the transpose, the inverse, the norm, and the rank of  $\mathbf{A}$ , respectively. We denote the  $n \times n$  identity matrix by  $\mathbf{I}_n$ . The real and imaginary parts of  $\mathbf{A}$  are denoted by  $\operatorname{Re}(\mathbf{A})$  and  $\operatorname{Im}(\mathbf{A})$ , respectively. Let  $[\mathbf{A}]_{i:j}$  denote the submatrix of  $\mathbf{A}$  consisting of the *i*th to *j*th column vectors of  $\mathbf{A}$ . Let us denote  $\log^+(x) = \max\{\log_2(x), 0\}$  and  $[1:n] = \{1, 2, \dots, n\}$ . We denote the circularly symmetric complex Gaussian distribution with

mean 0 and variance  $\sigma^2$  by  $\mathcal{CN}(0, \sigma^2)$  and the continuous uniform distribution on the interval [a, b] by Unif[a, b]. The expectation is denoted by  $\mathbb{E}(\cdot)$ .

# II. SYSTEM MODEL

Consider the two-user MIMO interference channel, in which transmitter *i* attempts to communicate with receiver *i* while interfering to receiver  $j \neq i$ , where  $i, j \in \{1, 2\}$ . Each transmitter and receiver are equipped with  $M_{\rm T}$  antennas and  $M_{\rm R}$  antennas, respectively. It is assumed that communication takes place over *n* time slots. Then the received signal vector of receiver *i* at time slot *t*, denoted by  $\mathbf{y}_i(t) \in \mathbb{C}^{M_{\rm R} \times 1}$ , is given by

$$\mathbf{y}_{i}(t) = \sum_{j=1}^{2} \mathbf{H}_{i,j}(t) \mathbf{x}_{j}(t) + \mathbf{z}_{i}(t), \ t \in [1:n],$$
(1)

where  $\mathbf{x}_j(t) \in \mathbb{C}^{M_{\mathrm{T}} \times 1}$  is the complex-valued transmit signal vector from transmitter j at time slot t,  $\mathbf{H}_{i,j}(t) \in \mathbb{C}^{M_{\mathrm{R}} \times M_{\mathrm{T}}}$  is the complex-valued channel matrix from transmitter j to receiver iat time slot t, and  $\mathbf{z}_i(t)$  is the complex additive white Gaussian noise vector of receiver i at time slot t with  $\mathbf{z}_i(t) \sim \mathcal{CN}(\mathbf{0}_{M_{\mathrm{R}} \times M_{\mathrm{R}}}, \mathbf{I}_{M_{\mathrm{R}}})$ . Note that the complex-valued input–output relation (1) can be equivalently represented in the form of a real-valued expression as

$$\bar{\mathbf{y}}_{i}(t) = \sum_{j=1}^{2} \bar{\mathbf{H}}_{i,j}(t) \bar{\mathbf{x}}_{j}(t) + \bar{\mathbf{z}}_{i}(t), \ t \in [1:n],$$
(2)

where

$$\begin{split} \bar{\mathbf{y}}_{i}(t) &= \begin{bmatrix} \operatorname{Re}(\mathbf{y}_{i}(t)) \\ \operatorname{Im}(\mathbf{y}_{i}(t)) \end{bmatrix} \in \mathbb{R}^{2M_{\mathsf{R}} \times 1}, \\ \bar{\mathbf{H}}_{i,j}(t) &= \begin{bmatrix} \operatorname{Re}(\mathbf{H}_{i,j}(t)) & -\operatorname{Im}(\mathbf{H}_{i,j}(t)) \\ \operatorname{Im}(\mathbf{H}_{i,j}(t)) & \operatorname{Re}(\mathbf{H}_{i,j}(t)) \end{bmatrix} \in \mathbb{R}^{2M_{\mathsf{R}} \times 2M_{\mathsf{T}}}, \\ \bar{\mathbf{x}}_{j}(t) &= \begin{bmatrix} \operatorname{Re}(\mathbf{x}_{j}(t)) \\ \operatorname{Im}(\mathbf{x}_{j}(t)) \end{bmatrix} \in \mathbb{R}^{2M_{\mathsf{T}} \times 1}, \\ \bar{\mathbf{z}}_{i}(t) &= \begin{bmatrix} \operatorname{Re}(\mathbf{z}_{i}(t)) \\ \operatorname{Im}(\mathbf{z}_{i}(t)) \end{bmatrix} \in \mathbb{R}^{2M_{\mathsf{R}} \times 1}. \end{split}$$

For notational convenience, we consider the real-valued channel stated in (2) hereafter. In addition, each transmitter should satisfy the average transmit power constraint P, i.e.,  $\frac{1}{n} \sum_{t=1}^{n} ||\bar{\mathbf{x}}_{i}(t)||^{2} \leq P$ ,  $\forall i \in \{1, 2\}$ .

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We assume that channel matrices are static during communication, i.e.,  $\bar{\mathbf{H}}_{i,j}(t) = \mathbf{H}_{i,j}, \forall t \in [1:n]$ , and thus the time index t is omitted from the notation of channel matrices hereafter. In addition, we assume global channel state information at the receiver side (CSIR), i.e.,  $\bar{\mathbf{H}}_{i,j}$  is known to each receiver for all  $i.j \in \{1, 2\}$ . On the other hand, regarding CSIT, we consider three different scenarios: 1) no CSI is available at the transmitter side, i.e., all channel matrices are unknown to both transmitters; 2) only the CSI of its own desired link is available at each transmitter, i.e., transmitter  $i \in \{1, 2\}$  knows the coefficients of the desired channel matrix  $\bar{\mathbf{H}}_{i,i}$  but does not know those of the other channel matrices; 3) each transmitter knows the CSI of both its desired link and interfering link, i.e., the coefficients in  $\bar{\mathbf{H}}_{1,i}$  and  $\bar{\mathbf{H}}_{2,i}$  are known to transmitter  $i \in \{1, 2\}$ . In the next section, we will propose a novel transmission scheme using IF that adequately mitigates interference with low complexity for each of the aforementioned CSI assumptions.

## III. PROPOSED IF-BASED TRANSMISSION SCHEME

#### A. Transmitter Side

1) Nested lattice codes: We adopt a lattice coding scheme for IF in [21]–[25], [32] and extend it in a way suitable for the two-user MIMO interference channel. For completeness, we briefly review the codebook construction of nested lattice codes here and refer to [47] for more detailed definitions and properties of lattice codes.

Consider an *n*-dimensional lattice  $\Lambda \in \mathbb{R}^n$ , which is a discrete subgroup of  $\mathbb{R}^n$  closed to any integer-linear combinations. The modulo- $\Lambda$  operation on a length-*n* vector **c** is defined as

$$\mathbf{c} \mod \Lambda = \mathbf{c} - Q_{\Lambda}(\mathbf{c}),\tag{3}$$

where  $Q_{\Lambda}(\mathbf{c})$  is the nearest neighbor quantizer that calculates

$$Q_{\Lambda}(\mathbf{c}) = \underset{\mathbf{t}\in\Lambda}{\operatorname{argmin}} \|\mathbf{c} - \mathbf{t}\|.$$
(4)

The Voronoi region of  $\Lambda$  is defined as  $\mathcal{V}_{\Lambda} = \{ \mathbf{c} \in \mathbb{R}^n : Q_{\Lambda}(\mathbf{c}) = \mathbf{0} \}$ . Additionally, the second moment of  $\Lambda$  per dimension is defined as

$$P_{\Lambda} = \frac{1}{n} \frac{1}{\operatorname{Vol}(\mathcal{V}_{\Lambda})} \int_{\mathbf{c} \in \mathcal{V}_{\Lambda}} \|\mathbf{c}\|^2 d\mathbf{c},$$
(5)

where  $Vol(\mathcal{V}_{\Lambda})$  denotes the volume of  $\mathcal{V}_{\Lambda}$ .

Now consider *n*-dimensional lattices  $\Lambda_c$  and  $\Lambda_f$ . If  $\Lambda_c \subseteq \Lambda_f$ , then  $\Lambda_c$  is said to be nested in  $\Lambda_f$ , where  $\Lambda_c$  and  $\Lambda_f$  are referred to as the coarse and fine lattices, respectively. From the nested pair  $\Lambda_c$  and  $\Lambda_f$ , a nested lattice codebook C can be constructed as  $C = \Lambda_f \cap \mathcal{V}_{\Lambda_c}$ , where the associated code rate is given by

$$R = \frac{1}{n} \log |\mathcal{C}| = \frac{1}{n} \log |\Lambda_f \cap \mathcal{V}_{\Lambda_c}| \quad \text{(bits/channel use)}.$$
(6)

Finally, since the transmit power constraint is given by P, the codebook C should be scaled to satisfy  $P_{\Lambda_c} = P$ , where  $P_{\Lambda_c}$  can be obtained by substituting  $\mathcal{V}_{\Lambda_c}$  for  $\mathcal{V}_{\Lambda}$  in (5).

2) Encoding: For a non-negative integer  $d_i$  satisfying that  $d_i \leq \min\{2M_T, 2M_R\}$ , let  $\mathbf{w}_i \in \mathbb{Z}_2^{1 \times d_i nR}$  denote the binary data stream for transmitter *i* of length  $d_i nR$ , which is assumed to be independently and uniformly drawn from  $\mathbb{Z}_2^{1 \times d_i nR}$ . To send  $\mathbf{w}_i$  using multiple transmit antennas, it is equally partitioned into  $d_{[c],i} + d_{[p],i}$  sub-streams of length nR each. Specifically, denote

$$\mathbf{w}_{i} = \begin{bmatrix} \mathbf{w}_{[c],i,1} & \mathbf{w}_{[c],i,2} & \dots & \mathbf{w}_{[c],i,d_{[c],i}} & \mathbf{w}_{[p],i,1} & \mathbf{w}_{[p],i,2} & \dots & \mathbf{w}_{[p],i,d_{[p],i}} \end{bmatrix},$$
(7)

where  $\mathbf{w}_{[c],i,j} \in \mathbb{Z}_2^{1 \times nR}$  and  $\mathbf{w}_{[p],i,k} \in \mathbb{Z}_2^{1 \times nR}$  for all  $j \in [1 : d_{[c],i}]$ ,  $k \in [1 : d_{[p],i}]$ , and  $i \in \{1, 2\}$ . That is,  $d_i = d_{[c],i} + d_{[p],i} \leq \min\{2M_T, 2M_R\}$ . Note that we here split  $\mathbf{w}_i$  into common and private streams. Specifically,  $\{\mathbf{w}_{[p],i,j}\}_{j \in [1:d_{[p],i}]}$  is the set of private streams intended to be recovered by receiver i only, while  $\{\mathbf{w}_{[c],i,k}\}_{i \in \{1,2\},k \in [1:d_{[c],i}]}$  is the set of common streams required to be recovered by both receivers 1 and 2, although streams in  $\{\mathbf{w}_{[c],j,k}\}_{k \in [1:d_{[c],j}]}$  are not the intended streams for receiver  $i \neq j$ .

Then  $\mathbf{w}_{[c],i,j}$  and  $\mathbf{w}_{[p],i,k}$  are encoded by the nested lattice code  $\mathcal{C} \subset \mathbb{R}^{1 \times n}$  explained above, i.e.,  $\mathbf{w}_{[c],i,j}$  and  $\mathbf{w}_{[p],i,k}$  are mapped into lattice points  $\mathbf{b}_{[c],i,j} \in \mathcal{C}$  and  $\mathbf{b}_{[p],i,k} \in \mathcal{C}$ , respectively, for all  $j \in [1 : d_{[c],i}]$ ,  $k \in [1 : d_{[p],i}]$ , and  $i \in \{1, 2\}$ . Notice that the same lattice code is assumed to be used for all streams  $\{\mathbf{w}_{[c],i,j}\}_{i \in \{1,2\}, j \in [1:d_{[c],i}]}$  and  $\{\mathbf{w}_{[p],i,k}\}_{i \in \{1,2\}, k \in [1:d_{[p],i}]}$  to enable IF sum decoding at each receiver.

Next, a random dither  $\mathbf{d}_{[c],i,j} \in \mathbb{R}^{1 \times n}$ , which is uniformly distributed over  $\mathcal{V}_{\Lambda_c}$  and independent of  $\mathbf{b}_{[c],i,j}$ , is employed to generate  $\mathbf{s}_{[c],i,j} = (\mathbf{b}_{[c],i,j} - \mathbf{d}_{[c],i,j}) \mod \Lambda_c$ ,  $\forall i \in \{1,2\}$  and  $\forall j \in [1 : d_{[c],i}]$ . In the same manner, we can obtain  $\mathbf{s}_{[p],i,k} = (\mathbf{b}_{[p],i,k} - \mathbf{d}_{[p],i,k}) \mod \Lambda_c$ ,  $\forall i \in \{1,2\}$  and  $\forall k \in [1 : d_{[p],i}]$ , where  $\mathbf{d}_{[p],i,k} \in \mathbb{R}^{1 \times n}$  is a random dither uniformly distributed over  $\mathcal{V}_{\Lambda_c}$ . It is assumed that all random dithers are known to all transmitters and receivers before communication. Note that due to the Crypto Lemma [48, Lemma 1],  $\mathbf{s}_{[c],i,j} \in \mathbb{R}^{1 \times n}$  and

 $\mathbf{s}_{[p],i,k} \in \mathbb{R}^{1 \times n}$  are uniformly distributed over  $\mathcal{V}_{\Lambda_c}$  and independent of  $\mathbf{b}_{[c],i,j}$  and  $\mathbf{b}_{[p],i,j}$ , and their variances are given by  $\frac{1}{n}\mathbb{E}(\|\mathbf{s}_{[p],i,k}\|^2) = \frac{1}{n}\mathbb{E}(\|\mathbf{s}_{[c],i,k}\|^2) = P_{\Lambda_c} = P$ .

Consequently,  $\mathbf{s}_{[c],i,j}$  and  $\mathbf{s}_{[p],i,k}$  are used as input signals to send streams  $\mathbf{w}_{[c],i,j}$  and  $\mathbf{w}_{[p],i,k}$ , respectively. Since there are  $d_{[c],i}+d_{[p],i}$  independent data streams with equal rate R for transmitter  $i \in \{1, 2\}$ , the total transmission rate of transmitter i is  $R_i = (d_{[c],i} + d_{[p],i})R$  and the sum rate is given by

$$R_{\text{sum}} = R_1 + R_2$$
  
=  $(d_{[c],1} + d_{[p],1} + d_{[c],2} + d_{[p],2})R.$  (8)

For notional convenience, we define the set  $d_{\text{stream}}$  as  $d_{\text{stream}} = \{d_{[c],1}, d_{[p],1}, d_{[c],2}, d_{[p],2}\}$  in the rest of the paper.

3) Transmit beamforming: To send multiple data streams simultaneously with multiple transmit antennas, we design the transmit signal vector of transmitter  $i \in \{1, 2\}$  at time slot  $t \in [1 : n]$  as

$$\bar{\mathbf{x}}_{i}(t) = \mathbf{V}_{[c],i} \mathbf{s}_{[c],i}(t) + \mathbf{V}_{[p],i} \mathbf{s}_{[p],i}(t),$$
(9)

where

$$\mathbf{s}_{[c],i}(t) = \begin{bmatrix} s_{[c],i,1}(t) & s_{[c],i,2}(t) & \cdots & s_{[c],i,d_{[c],i}}(t) \end{bmatrix}^{\dagger} \in \mathbb{R}^{d_{[c],i} \times 1}, \\ \mathbf{s}_{[p],i}(t) = \begin{bmatrix} s_{[p],i,1}(t) & s_{[p],i,2}(t) & \cdots & s_{[p],i,d_{[p],i}}(t) \end{bmatrix}^{\dagger} \in \mathbb{R}^{d_{[p],i} \times 1}, \end{cases}$$

 $s_{[c],i,j}(t)$  and  $s_{[p],i,j}(t)$  are the *t*th elements in  $s_{[c],i,j}$  and  $s_{[p],i,j}$ , respectively, and  $\mathbf{V}_{[c],i} \in \mathbb{R}^{2M_{\mathrm{T}} \times d_{[c],i}}$ and  $\mathbf{V}_{[p],i} \in \mathbb{R}^{2M_{\mathrm{T}} \times d_{[p],i}}$  are beamforming matrices that convey common and private streams of transmitter *i*, respectively, where all column vectors in  $\mathbf{V}_{[c],i}$  and  $\mathbf{V}_{[p],i}$  are linearly independent, i.e.,

$$\operatorname{rank}\left(\left[\begin{array}{cc} \mathbf{V}_{[\mathbf{c}],i} & \mathbf{V}_{[\mathbf{p}],i} \end{array}\right]\right) = d_{[\mathbf{c}],i} + d_{[\mathbf{p}],i}$$
$$= d_i$$
$$\leq \min\{2M_{\mathrm{T}}, 2M_{\mathrm{R}}\}.$$
(10)

In addition, we choose  $d_{\text{stream}}$  to satisfy

$$d_{[c],1} + d_{[c],2} + d_{[p],i} \le 2M_{\rm R} \tag{11}$$

for all  $i \in \{1, 2\}$ . Note that this condition is required to perform IF sum decoding for the intended streams and the other user's common streams at each receiver.

Furthermore, the column vectors in  $V_{[c],i}$  and  $V_{[p],i}$  are properly normalized to satisfy the transmit power constraint P, i.e., their norms are given by one. The detailed design of beam-forming matrices  $V_{[c],i}$  and  $V_{[p],i}$  suitable for each CSIT assumption will be discussed later in Section III-C.

# B. Receiver Side

1) Linear filtering: Since the transmit signal vector of transmitter i is set to (9), for given  $\mathbf{V}_{[c],i}$  and  $\mathbf{V}_{[p],i}$ , the received signal vector of receiver i at time slot  $t \in [1:n]$  is given as

$$\begin{split} \bar{\mathbf{y}}_{i}(t) &= \mathbf{H}_{i,i}\bar{\mathbf{x}}_{i}(t) + \mathbf{H}_{i,j}\bar{\mathbf{x}}_{j}(t) + \bar{\mathbf{z}}_{i}(t) \\ &= \bar{\mathbf{H}}_{i,i}\left(\mathbf{V}_{[c],i}\mathbf{s}_{[c],i}(t) + \mathbf{V}_{[p],i}\mathbf{s}_{[p],i}(t)\right) + \bar{\mathbf{H}}_{i,j}\left(\mathbf{V}_{[c],j}\mathbf{s}_{[c],j}(t) + \mathbf{V}_{[p],j}\mathbf{s}_{[p],j}(t)\right) + \bar{\mathbf{z}}_{i}(t) \\ &= \tilde{\mathbf{H}}_{[d],i}\left[\begin{array}{c} \mathbf{s}_{[c],i}(t) \\ \mathbf{s}_{[p],i}(t) \\ \mathbf{s}_{[c],j}(t) \end{array}\right] + \tilde{\mathbf{H}}_{[i],i}\mathbf{s}_{[p],j}(t) + \bar{\mathbf{z}}_{i}(t) \\ &= \tilde{\mathbf{H}}_{[d],i}\mathbf{s}_{[d],i}(t) + \tilde{\mathbf{H}}_{[i],i}\mathbf{s}_{[p],j}(t) + \bar{\mathbf{z}}_{i}(t), \end{split}$$
(12)

where  $i, j \in \{1, 2\}$  and  $i \neq j$ . Here

$$\tilde{\mathbf{H}}_{[\mathsf{d}],i} = \left[ \begin{array}{cc} \bar{\mathbf{H}}_{i,i} \mathbf{V}_{[\mathsf{c}],i} & \bar{\mathbf{H}}_{i,i} \mathbf{V}_{[\mathsf{p}],i} & \bar{\mathbf{H}}_{i,j} \mathbf{V}_{[\mathsf{c}],j} \end{array} \right] \in \mathbb{R}^{2M_{\mathsf{R}} \times \left( d_{[\mathsf{c}],i} + d_{[\mathsf{p}],i} + d_{[\mathsf{c}],j} \right)}$$

represents the effective desired channel matrix observed at receiver *i*,

$$ilde{\mathbf{H}}_{[\mathrm{i}],i} = ar{\mathbf{H}}_{i,j} \mathbf{V}_{[\mathrm{p}],j} \in \mathbb{R}^{2M_{\mathrm{R}} imes d_{[\mathrm{p}],j}}$$

represents the effective interfering channel matrix observed at receiver i, and

$$\mathbf{s}_{[\mathbf{d}],i}(t) = [\mathbf{s}_{[\mathbf{c}],i}^{\dagger}(t) \ \mathbf{s}_{[\mathbf{p}],i}^{\dagger}(t) \ \mathbf{s}_{[\mathbf{c}],j}^{\dagger}(t)]^{\dagger} \in \mathbb{R}^{\left(d_{[\mathbf{c}],i}+d_{[\mathbf{p}],i}+d_{[\mathbf{c}],j}\right) \times 1}.$$

In addition, for notational simplicity, let  $d_{[\mathbf{d}],i} = d_{[\mathbf{c}],i} + d_{[\mathbf{p}],i} + d_{[\mathbf{c}],j}$ , which is the total number of streams that receiver *i* attempts to recover. Then, based on  $\mathbf{\bar{Y}}_i = [ \mathbf{\bar{y}}_i(1) \ \mathbf{\bar{y}}_i(2) \ \cdots \ \mathbf{\bar{y}}_i(n) ] \in \mathbb{R}^{2M_{\mathsf{R}} \times n}$ , receiver *i* tries to recover the data streams associated with

$$\mathbf{S}_{[\mathbf{d}],i} = \begin{bmatrix} \mathbf{s}_{[\mathbf{d}],i}(1) & \mathbf{s}_{[\mathbf{d}],i}(2) & \cdots & \mathbf{s}_{[\mathbf{d}],i}(n) \end{bmatrix} \in \mathbb{R}^{d_{[\mathbf{d}],i} \times n}$$

by applying IF sum decoding while treating

$$\mathbf{S}_{[\mathbf{p}],j} = [\mathbf{s}_{[\mathbf{p}],j}(1) \ \mathbf{s}_{[\mathbf{p}],j}(2) \ \cdots \ \mathbf{s}_{[\mathbf{p}],j}(n) ] \in \mathbb{R}^{d_{[\mathbf{p}],j} \times n}$$

as noise. To this end, receiver *i* applies a linear filter  $\mathbf{F}_{\mathrm{IF},i} \in \mathbb{R}^{d_{[d],i} \times 2M_{\mathrm{R}}}$  to the received vector  $\bar{\mathbf{y}}_{i}(t)$  for each time slot  $t \in [1:n]$  to obtain

$$\begin{split} \tilde{\mathbf{y}}_{i}(t) &= \mathbf{F}_{\mathrm{IF},i} \bar{\mathbf{y}}_{i}(t) \\ &= \mathbf{F}_{\mathrm{IF},i} \left( \tilde{\mathbf{H}}_{[\mathrm{d}],i} \mathbf{s}_{[\mathrm{d}],i}(t) + \tilde{\mathbf{H}}_{[\mathrm{i}],i} \mathbf{s}_{[\mathrm{p}],j}(t) \right) + \mathbf{F}_{\mathrm{IF},i} \bar{\mathbf{z}}_{i}(t) \\ &= \mathbf{A}_{i} \mathbf{s}_{[\mathrm{d}],i}(t) + \underbrace{\left( \mathbf{F}_{\mathrm{IF},i} \tilde{\mathbf{H}}_{[\mathrm{d}],i} - \mathbf{A}_{i} \right) \mathbf{s}_{[\mathrm{d}],i}(t) + \mathbf{F}_{\mathrm{IF},i} \tilde{\mathbf{H}}_{[\mathrm{i}],i} \mathbf{s}_{[\mathrm{p}],j}(t) + \mathbf{F}_{\mathrm{IF},i} \bar{\mathbf{z}}_{i}(t)}_{\mathrm{Effective noise}} \\ &= \mathbf{A}_{i} \mathbf{s}_{[\mathrm{d}],i}(t) + \tilde{\mathbf{z}}_{i}(t), \end{split}$$
(13)

where

$$\mathbf{F}_{\mathrm{IF},i} = P\mathbf{A}_{i}\tilde{\mathbf{H}}_{[\mathrm{d}],i}^{\dagger} \Big( \mathbf{I}_{2M_{\mathrm{R}}} + P\tilde{\mathbf{H}}_{[\mathrm{d}],i}\tilde{\mathbf{H}}_{[\mathrm{d}],i}^{\dagger} + P\tilde{\mathbf{H}}_{[\mathrm{i}],i}\tilde{\mathbf{H}}_{[\mathrm{i}],i}^{\dagger} \Big)^{-1},$$
(14)

$$\tilde{\mathbf{z}}_{i}(t) = \left( \left( \mathbf{F}_{\mathrm{IF},i} \tilde{\mathbf{H}}_{[\mathrm{d}],i} - \mathbf{A}_{i} \right) \mathbf{s}_{[\mathrm{d}],i}(t) + \mathbf{F}_{\mathrm{IF},i} \tilde{\mathbf{H}}_{[\mathrm{i}],i} \mathbf{s}_{[\mathrm{p}],j}(t) + \mathbf{F}_{\mathrm{IF},i} \bar{\mathbf{z}}_{i}(t) \right),$$
(15)

and  $\mathbf{A}_i \in \mathbb{Z}^{d_{[d],i} \times d_{[d],i}}$  is a full-rank integer matrix chosen such that the effective noise in (13) is minimized. By extending the results in [21, Theorem 4 and Section VII-B], it can be seen that near-optimal  $\mathbf{A}_i$  can be found by solving the following optimization problem [21], [49] with approximate search algorithms such as Lenstra–Lenstra–Lovasz (LLL) algorithm [50],

$$\underset{\mathbf{A}_{i}\in\mathbb{Z}^{d_{[\mathbf{d}],i}\times d_{[\mathbf{d}],i}, \operatorname{rank}(\mathbf{A}_{i})=d_{[\mathbf{d}],i}}{\operatorname{arg\,min}} \max_{m\in[1:d_{[\mathbf{d}],i}]} \|\boldsymbol{\Gamma}_{i}^{-1/2}\mathbf{U}_{i}^{\dagger}\mathbf{a}_{i,m}\|^{2},$$
(16)

where  $U_i$  is an orthogonal matrix consisted of the eigenvectors of

$$\mathbf{Q}_{i} = \left(\mathbf{I}_{2M_{\mathsf{R}}} - P\mathbf{H}_{[\mathsf{d}],i}^{\dagger} \left(\mathbf{I}_{2M_{\mathsf{R}}} + P\tilde{\mathbf{H}}_{[\mathsf{d}],i}\tilde{\mathbf{H}}_{[\mathsf{d}],i}^{\dagger} + P\tilde{\mathbf{H}}_{[\mathsf{i}],i}\tilde{\mathbf{H}}_{[\mathsf{i}],i}^{\dagger}\right)^{-1} \mathbf{H}_{[\mathsf{d}],i}\right)^{-1}$$
(17)

as its columns,  $\Gamma_i$  is a diagonal matrix whose entries are the eigenvalues of  $\mathbf{Q}_i$ , and  $\mathbf{a}_{i,m}^{\dagger} \in \mathbb{R}^{1 \times d_{[\mathbf{d}],i}}$  is the *m*th row vector in  $\mathbf{A}_i$ . Moreover, after obtaining  $\mathbf{A}_i$ , the optimal linear filter  $\mathbf{F}_{\mathrm{IF},i}$  can be calculated in a closed form as (14).

2) IF sum decoding and achievable rate: After obtaining  $\tilde{\mathbf{Y}}_i = \begin{bmatrix} \tilde{\mathbf{y}}_i(1) & \tilde{\mathbf{y}}_i(2) & \cdots & \tilde{\mathbf{y}}_i(n) \end{bmatrix} \in \mathbb{R}^{d_{[d],i} \times n}$  over the block of *n* transmissions, receiver *i* first adds the dither matrix  $\mathbf{D}_i$  to  $\tilde{\mathbf{Y}}_i$  and performs the modulo- $\Lambda_c$  operation to get [25]

$$\tilde{\tilde{\mathbf{Y}}}_{i} = \left(\tilde{\mathbf{Y}}_{i} + \mathbf{A}_{i}\mathbf{D}_{i}\right) \mod \Lambda_{c}$$

$$= \left(\mathbf{A}_{i}\mathbf{S}_{[d],i} + \mathbf{A}_{i}\mathbf{D}_{i} + \tilde{\mathbf{Z}}_{i}\right) \mod \Lambda_{c}$$

$$= \left(\mathbf{A}_{i}\mathbf{B}_{[d],i} + \tilde{\mathbf{Z}}_{i}\right) \mod \Lambda_{c}$$

$$= \left(\mathbf{C}_{i} + \tilde{\mathbf{Z}}_{i}\right) \mod \Lambda_{c},$$
(18)

where

$$\begin{split} \tilde{\mathbf{Y}}_{i} &= \begin{bmatrix} \tilde{\mathbf{y}}_{i}(1) & \tilde{\mathbf{y}}_{i}(2) & \cdots & \tilde{\mathbf{y}}_{i}(n) \end{bmatrix} \in \mathbb{R}^{d_{[\mathbf{d}],i} \times n}, \\ \mathbf{D}_{i} &= \begin{bmatrix} \mathbf{d}_{[\mathbf{c}],i,1}^{\dagger} & \cdots & \mathbf{d}_{[\mathbf{c}],i,d_{[\mathbf{c}],i}}^{\dagger} & \mathbf{d}_{[\mathbf{p}],i,1}^{\dagger} & \cdots & \mathbf{d}_{[\mathbf{p}],i,d_{[\mathbf{p}],i}}^{\dagger} & \mathbf{d}_{[\mathbf{c}],j,1}^{\dagger} & \cdots & \mathbf{d}_{[\mathbf{c}],j,d_{[\mathbf{c}],j}}^{\dagger} \end{bmatrix}^{\dagger} \in \mathbb{R}^{d_{[\mathbf{d}],i} \times n}, \\ \tilde{\mathbf{Z}}_{i} &= \begin{bmatrix} \tilde{\mathbf{z}}_{i}(1) & \tilde{\mathbf{z}}_{i}(2) & \cdots & \tilde{\mathbf{z}}_{i}(n) \end{bmatrix} \in \mathbb{R}^{d_{[\mathbf{d}],i} \times n}, \end{split}$$

$$\mathbf{B}_{[\mathbf{d}],i} = \begin{bmatrix} \mathbf{b}_{[\mathbf{c}],i,1}^{\dagger} & \cdots & \mathbf{b}_{[\mathbf{c}],i,d_{[\mathbf{c}],i}}^{\dagger} & \mathbf{b}_{[\mathbf{p}],i,1}^{\dagger} & \cdots & \mathbf{b}_{[\mathbf{p}],i,d_{[\mathbf{p}],i}}^{\dagger} & \mathbf{b}_{[\mathbf{c}],j,1}^{\dagger} & \cdots & \mathbf{b}_{[\mathbf{c}],j,d_{[\mathbf{c}],j}}^{\dagger} \end{bmatrix}^{\dagger} \in \mathbb{R}^{d_{[\mathbf{d}],i} \times n},$$

and  $\mathbf{C}_i = (\mathbf{A}_i \mathbf{B}_{[\mathbf{d}],i}) \mod \Lambda_c$  for  $i \neq j$ . Observe that  $\mathbf{B}_{[\mathbf{d}],i} = \mathbf{S}_{[\mathbf{d}],i} + \mathbf{D}_i$  and each row of  $\mathbf{B}_{[\mathbf{d}],i}$  is a codeword from the same lattice codebook C, as described in Section III-A2. Since any integer-linear combination of lattice codewords over  $\Lambda_c$  from the same codebook C is also a codeword included in C due to the linearity property of lattice codes, each row of  $\mathbf{C}_i$  is itself a codeword in C, which can be decoded within noise tolerance. In addition, the additive noise  $\tilde{\mathbf{Z}}_i$  is independent of  $\mathbf{C}_i$  due to the fact that  $\mathbf{B}_{[\mathbf{d}],i}$  is independent of  $\mathbf{S}_{[\mathbf{d}],i}$ ,  $\mathbf{S}_{[\mathbf{p}],j}$ , and  $\bar{\mathbf{Z}}_i$ , where

$$\mathbf{\bar{Z}}_i = [\mathbf{\bar{z}}_i(1) \ \mathbf{\bar{z}}_i(2) \ \cdots \ \mathbf{\bar{z}}_i(n) ] \in \mathbb{R}^{2M_{\mathbf{R}} \times n}$$

From (18), let  $\tilde{\tilde{\mathbf{y}}}_{i,m}$ ,  $\mathbf{c}_{i,m}$ , and  $\tilde{\mathbf{z}}_{i,m}$  denote the *m*th row vector in  $\tilde{\mathbf{Y}}_i$ ,  $\mathbf{C}_i$ , and  $\tilde{\mathbf{Z}}_i$ , respectively, where  $m \in [1 : d_{[d],i}]$ . Then we have

$$\tilde{\tilde{\mathbf{y}}}_{i,m} = (\mathbf{c}_{i,m} + \tilde{\mathbf{z}}_{i,m}) \mod \Lambda_c.$$
 (19)

Based on  $\tilde{\tilde{\mathbf{y}}}_{i,m}$ , receiver *i* attempts to separately decode  $\mathbf{c}_{i,m}$  for each  $m \in [1 : d_{[d],i}]$  in the presence of the effective noise vector  $\tilde{\mathbf{z}}_{i,m}$  [21]–[23], [25], [32]. Let  $\hat{\mathbf{c}}_{i,m}$  denote the decoded version of  $\mathbf{c}_{i,m}$  and

$$\hat{\mathbf{C}}_i = \left[ egin{array}{ccc} \hat{\mathbf{c}}_{i,1}^\dagger & \hat{\mathbf{c}}_{i,2}^\dagger & \cdots & \hat{\mathbf{c}}_{i,d_{[\mathsf{d}],i}}^\dagger \end{array} 
ight]^\dagger \in \mathbb{R}^{d_{[\mathsf{d}],i} imes n}.$$

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Then the original streams in  $\mathbf{B}_{[\mathbf{d}],i}$  can be recovered from the decoded output  $\hat{\mathbf{C}}_i$  as

$$\hat{\mathbf{B}}_{[\mathbf{d}],i} = \left(\mathbf{A}_{\Lambda_c}^{-1}\hat{\mathbf{C}}_i\right) \mod \Lambda_c,\tag{20}$$

where  $\mathbf{A}_{\Lambda_c} = (\mathbf{A}) \mod \Lambda_c$  and the matrix inversion is taken over  $\Lambda_c$ . Recall that receiver *i* tries to recover all the streams in  $\mathbf{B}_{[d],i}$ , i.e., it recovers not only its intended streams sent by transmitter *i*, but also the common streams sent by transmitter *j* under the proposed scheme, where  $j \neq i$ . As in [21]–[23], [25], [32], the decoding of  $\mathbf{c}_{i,m}$  is successful with probability approaching one as  $n \to \infty$  if

$$R < \frac{1}{2}\log^+\left(\frac{P}{\sigma_{i,m}^2}\right),\tag{21}$$

where  $\sigma_{i,m}^2$  denotes the variance of  $\tilde{\mathbf{z}}_{i,m}$  given by  $\sigma_{i,m}^2 = \frac{1}{n} \mathbb{E} \|\tilde{\mathbf{z}}_{i,m}\|^2$ . Since all integer-linear combinations  $\{\mathbf{c}_{i,m}\}_{m \in [1:d_{[d],i}]}$  should be successfully decoded in order to recover  $\mathbf{B}_{[d],i}$  for all  $i \in \{1, 2\}$ , the achievable rate for each data stream is given by

$$R < \min_{i \in \{1,2\}, m \in [1:d_{[d],i}]} \frac{1}{2} \log^+ \left(\frac{P}{\sigma_{i,m}^2}\right).$$
(22)

As a result, from (8), the overall achievable rates for receivers 1 and 2 and the corresponding sum rate are given by

$$R_{1} < \left(d_{[c],1} + d_{[p],1}\right) \left(\min_{i \in \{1,2\}, m \in [1:d_{[d],i}]} \frac{1}{2} \log^{+} \left(\frac{P}{\sigma_{i,m}^{2}}\right)\right),$$

$$R_{2} < \left(d_{[c],2} + d_{[p],2}\right) \left(\min_{i \in \{1,2\}, m \in [1:d_{[d],i}]} \frac{1}{2} \log^{+} \left(\frac{P}{\sigma_{i,m}^{2}}\right)\right),$$

$$R_{1} + R_{2} < \left(d_{[c],1} + d_{[p],1} + d_{[c],2} + d_{[p],2}\right) \left(\min_{i \in \{1,2\}, m \in [1:d_{[d],i}]} \frac{1}{2} \log^{+} \left(\frac{P}{\sigma_{i,m}^{2}}\right)\right).$$
(23)

Note that we can rewrite the achievable rates (23) in a different form. Let us consider the following Cholesky decomposition:

$$\mathbf{A}_{i}\mathbf{Q}_{i}^{-1}\mathbf{A}_{i}^{\dagger} = \mathbf{L}_{i}\mathbf{L}_{i}^{\dagger},\tag{24}$$

where  $\mathbf{Q}_i$  is given as (17) and  $\mathbf{L}_i \in \mathbb{R}^{d_{[d],i} \times d_{[d],i}}$  is a lower triangular matrix. Then it can be shown that the effective noise variance  $\sigma_{i,m}^2$  is given by

$$\sigma_{i,m}^2 = P \sum_{k=1}^m l_{i,k,m}^2,$$
(25)

where  $l_{i,k,m}$  is the (k,m)th element of  $L_i$ . To avoid duplication of explanation, we refer to [25, Section III] for detailed derivation. Finally, the achievable rates (23) are rewritten as

$$R_{1} < \left(d_{[c],1} + d_{[p],1}\right) \left(\min_{i \in \{1,2\}, m \in [1:d_{[d],i}]} \frac{1}{2} \log^{+} \left(\frac{1}{\sum_{k=1}^{m} l_{i,k,m}^{2}}\right)\right),$$

$$R_{2} < \left(d_{[c],2} + d_{[p],2}\right) \left(\min_{i \in \{1,2\}, m \in [1:d_{[d],i}]} \frac{1}{2} \log^{+} \left(\frac{1}{\sum_{k=1}^{m} l_{i,k,m}^{2}}\right)\right),$$

$$+ R_{2} < \left(d_{[c],1} + d_{[p],1} + d_{[c],2} + d_{[p],2}\right) \left(\min_{i \in \{1,2\}, m \in [1:d_{[d],i}]} \frac{1}{2} \log^{+} \left(\frac{1}{\sum_{k=1}^{m} l_{i,k,m}^{2}}\right)\right).$$
(26)

*Remark 1:* Recall that  $A_1$  and  $A_2$  can be constructed by solving the optimization problem (16). Then, from (24),  $L_i$  can be obtained by using such  $A_i$  and  $Q_i$  given in (17) and, as a result, the right-hand side terms in (26), i.e.,

$$\min_{i \in \{1,2\}, m \in [1:d_{[d],i]}} \frac{1}{2} \log^+ \left( \frac{1}{\sum_{k=1}^m l_{i,k,m}^2} \right),$$

$$\min_{i \in \{1,2\}, m \in [1:d_{[d],i]}} \frac{1}{2} \log^+ \left( \frac{1}{\sum_{k=1}^m l_{i,k,m}^2} \right),$$

$$\min_{i \in \{1,2\}, m \in [1:d_{[d],i]}} \frac{1}{2} \log^+ \left( \frac{1}{\sum_{k=1}^m l_{i,k,m}^2} \right)$$

are determined. Therefore, the rest of the optimization process is to find an appropriate set of streams  $d_{\text{stream}}$  and design beamforming vectors  $\{\mathbf{V}_{[c],1}, \mathbf{V}_{[p],1}, \mathbf{V}_{[c],2}, \mathbf{V}_{[p],2}\}$  for a given CSIT assumption.

3) IF sum decoding with SIC: So far, we focused on IF sum decoding without SIC. By combining IF sum decoding with SIC, namely, successive IF [25], the achievable rates (23) or (26) can be further improved. In contrast to the basic IF receiver that recovers integer-linear combinations of codewords in parallel as described earlier, the successive IF receiver attempts to decode them one-by-one sequentially. To be specific, after linear combination  $c_{i,m}$  is recovered, the receiver estimates the effective noise corresponding to that summation, i.e., it estimates  $\tilde{z}_{i,m}$  based on  $\tilde{\tilde{y}}_{i,m}$  and the decoding output  $\hat{c}_{i,m}$ . The receiver then performs SIC of the estimated noise for all  $\{\tilde{\tilde{y}}_{i,j}\}_{j>m}$  to reduce the effective noise for the remaining integer-linear combinations that have not yet been decoded. To state the achievable rate of each user that can be obtained through the successive IF scheme, let  $L_{SIC,i} \in \mathbb{R}^{d_{[d],i} \times d_{[d],i}}$  denote the lower triangular matrix

 $R_1$ 

obtained from the following Cholesky decomposition:

$$\mathbf{A}_{\mathrm{SIC},i}\mathbf{Q}_{i}^{-1}\mathbf{A}_{\mathrm{SIC},i}^{\dagger} = \mathbf{L}_{\mathrm{SIC},i}\mathbf{L}_{\mathrm{SIC},i}^{\dagger},\tag{27}$$

where  $\mathbf{A}_{\text{SIC},i} \in \mathbb{Z}^{d_{[d],i} \times d_{[d],i}}$  is a full-rank integer matrix and  $\mathbf{Q}_i$  is given as (17). Here, a proper integer matrix of the successive IF scheme for receiver *i*, denoted by  $\mathbf{A}_{\text{SIC},i}$ , can be obtained by solving the following optimization problem [25]

$$\underset{\mathbf{A}_{\mathrm{SIC},i} \in \mathbb{Z}^{d_{[\mathrm{d}],i} \times d_{[\mathrm{d}],i}, \operatorname{rank}(\mathbf{A}_{\mathrm{SIC},i}) = d_{[\mathrm{d}],i}}{\operatorname{arg\,min}} \max_{m \in [1:d_{[\mathrm{d}],i}]} l_{\mathrm{SIC},i,m,m}^2$$
(28)

instead of solving (16), where  $l_{SIC,i.k,m}$  denotes the (k, m)th element of  $L_{SIC,i}$ . Then the achievable rates of the successive IF scheme are given by

$$R_{1} < \left(d_{[c],1} + d_{[p],1}\right) \left(\min_{i \in \{1,2\}, m \in [1:d_{[d],i}]} \frac{1}{2} \log^{+} \left(1/l_{\mathrm{SIC},i,m,m}^{2}\right)\right),$$

$$R_{2} < \left(d_{[c],2} + d_{[p],2}\right) \left(\min_{i \in \{1,2\}, m \in [1:d_{[d],i}]} \frac{1}{2} \log^{+} \left(1/l_{\mathrm{SIC},i,m,m}^{2}\right)\right),$$

$$R_{1} + R_{2} < \left(d_{[c],1} + d_{[p],1} + d_{[c],2} + d_{[p],2}\right) \left(\min_{i \in \{1,2\}, m \in [1:d_{[d],i}]} \frac{1}{2} \log^{+} \left(1/l_{\mathrm{SIC},i,m,m}^{2}\right)\right).$$
(29)

That is, the effective noise variance  $\sigma_{i,m}^2$  of the successive IF scheme becomes

$$\sigma_{i,m}^2 = P l_{\text{SIC},i,m,m}^2,\tag{30}$$

which is clearly smaller than that in (25) even when  $A_{SIC,i} = A_i$ . As a result, the successive IF receiver can outperform the basic IF receiver due to this noise reduction. For more detailed derivations, see [25].

# C. Design of Transmit Beamforming

The achievable rates (26) or (29) depend on how to design transmit beamforming matrices  $V_{[c],i}$  and  $V_{[p],i}$ ,  $i \in \{1,2\}$ . However, finding the optimal beamforming matrices that maximize the achievable rates is very difficult to solve because the considered optimization problem belongs to non-convex integer programming.<sup>1</sup> Therefore, in this paper, we focus on developing a simple universal beamforming scheme that can provide good performance over a wide range of channel matrices with low complexity suitable for each CSIT assumption.

<sup>&</sup>lt;sup>1</sup>We also note that the capacity of the two-user MIMO interference channel is not yet known in general.

1) No CSIT case: When CSI is not available at the transmitter side, we set the transmit

beamforming matrices as

$$\mathbf{V}_{[c],i} = [\mathbf{I}_{2M_{\rm T}}]_{1:d_{[c],i}},$$
  
$$\mathbf{V}_{[p],i} = [\mathbf{I}_{2M_{\rm T}}]_{d_{[c]}+1:d_{[c]}+d_{[p],i}}$$
(31)

for all  $i \in \{1, 2\}$  by utilizing the identity matrix. Then, based on the above transmit beamforming matrices (31), the set  $d_{\text{stream}} = \{d_{[c],1}, d_{[p],1}, d_{[c],2}, d_{[p],2}\}$  is numerically optimized to maximize the achievable rates (26) or (29) while satisfying the conditions (10) and (11).<sup>2</sup>

2) Partial CSIT case: Now consider the case in which only the CSI of its own desired link is available at each transmitter, i.e., only the coefficients in  $\bar{\mathbf{H}}_{i,i}$  is known to transmitter  $i \in \{1, 2\}$ . Consider the singular value decomposition (SVD) of  $\bar{\mathbf{H}}_{i,i}$  given by

$$\bar{\mathbf{H}}_{i,i} = \mathbf{G}_{i,i} \boldsymbol{\Sigma}_{i,i} \mathbf{E}_{i,i}^{\dagger} \tag{32}$$

where  $\mathbf{G}_{i,i} \in \mathbb{R}^{2M_{\mathsf{R}} \times 2M_{\mathsf{R}}}$  and  $\mathbf{E}_{i,i} \in \mathbb{R}^{2M_{\mathsf{T}} \times 2M_{\mathsf{T}}}$  are unitary matrices and  $\Sigma_{i,i} \in \mathbb{R}^{2M_{\mathsf{R}} \times 2M_{\mathsf{T}}}$  is a rectangular diagonal matrix with non-negative real diagonal values. Then, to utilize the CSI of  $\bar{\mathbf{H}}_{i,i}$ , we design the transmit beamforming matrices by using the SVD of  $\bar{\mathbf{H}}_{i,i}$  as

$$\mathbf{V}_{[c],i} = [\mathbf{E}_{i,i}]_{1:d_{[c],i}},$$
$$\mathbf{V}_{[p],i} = [\mathbf{E}_{i,i}]_{d_{[c],i}+1:d_{[c],i}+d_{[p],i}}.$$
(33)

Note that SVD-based beamforming was also considered for a point-to-point MIMO channel with an IF receiver when CSI is known to both the transmitter and receiver, i.e., the closed-loop communication scenario [51]. Specifically, it was shown that unitary precoded IF in conjunction with SVD can achieve full-diversity.<sup>3</sup>

Then, based on the transmit beamforming matrices (33), the optimal set of  $d_{\text{stream}}$  that maximizes the achievable rates (26) or (29) is numerically searched under the conditions (10) and (11).

<sup>2</sup>We assume that the optimal set  $\{d_{[c],1}, d_{[p],1}, d_{[c],2}, d_{[p],2}\}$  is searched by each receiver and reported to the corresponding transmitter before communication for all CSIT assumptions.

<sup>&</sup>lt;sup>3</sup>However, the optimal precoding for an IF receiver that maximizes the achievable rate has not yet been known even for the point-to-point channel.

3) Full CSIT case: Finally, consider the case in which full CSI is available at each transmitter, i.e., the coefficients in  $\bar{\mathbf{H}}_{1,i}$  and  $\bar{\mathbf{H}}_{2,i}$  are known to transmitter  $i \in \{1, 2\}$ . Let us consider the SVDs of  $\bar{\mathbf{H}}_{i,i}$  and  $\bar{\mathbf{H}}_{j,i}$  given by  $\bar{\mathbf{H}}_{i,i} = \mathbf{G}_{i,i} \Sigma_{i,i} \mathbf{E}_{i,i}^{\dagger}$  and  $\bar{\mathbf{H}}_{j,i} = \mathbf{G}_{j,i} \Sigma_{j,i} \mathbf{E}_{j,i}^{\dagger}$  in the same manner as in (32), where  $j \neq i$ . Additionally, we also consider the SVD of the concatenated matrix  $[\bar{\mathbf{H}}_{i,i}^{\dagger} \ \bar{\mathbf{H}}_{j,i}^{\dagger}]^{\dagger}$  given by  $[\bar{\mathbf{H}}_{i,i}^{\dagger} \ \bar{\mathbf{H}}_{j,i}^{\dagger}]^{\dagger} = \mathbf{G}_{i} \Sigma_{i} \mathbf{E}_{i}^{\dagger}$ . Then we design the transmit beamforming matrices as

$$\mathbf{V}_{[c],i} = [\mathbf{E}_i]_{1:d_{[c],i}},\tag{34}$$

$$\mathbf{V}_{[\mathbf{p}],i} = \begin{cases} [\mathbf{E}_{i,i}]_{1:d_{[\mathbf{p}],i}} & \text{if } M_{\mathrm{T}} \leq M_{\mathrm{R}}, \\ \begin{bmatrix} \gamma_{i}[\mathbf{E}_{i,i}]_{1:2M_{\mathrm{T}}-2M_{\mathrm{R}}} + (1-\gamma_{i})[\mathbf{J}_{j,i}]_{1:2M_{\mathrm{T}}-2M_{\mathrm{R}}} & [\mathbf{E}_{i,i}]_{2M_{\mathrm{T}}-2M_{\mathrm{R}}+1:d_{[\mathbf{p}],i}} \end{bmatrix} \\ & \text{if } 0 < 2M_{\mathrm{T}} - 2M_{\mathrm{R}} \leq d_{[\mathbf{p}],i}, \\ \gamma_{i}[\mathbf{E}_{i,i}]_{1:d_{[\mathbf{p}],i}} + (1-\gamma_{i})[\mathbf{J}_{j,i}]_{1:d_{[\mathbf{p}],i}} & \text{otherwise,} \end{cases}$$
(35)

where  $\mathbf{J}_{j,i} \in \mathbb{R}^{2M_{\mathrm{T}} \times (2M_{\mathrm{T}} - 2M_{\mathrm{R}})}$  denotes the null matrix of  $\mathbf{\bar{H}}_{j,i}$  that satisfies  $\mathbf{\bar{H}}_{j,i}\mathbf{J}_{j,i} = \mathbf{0}_{2M_{\mathrm{R}} \times (2M_{\mathrm{T}} - 2M_{\mathrm{R}})}$ and rank $(\mathbf{\bar{H}}_{i,i}\mathbf{J}_{j,i}) = \min\{2M_{\mathrm{R}}, 2M_{\mathrm{T}} - 2M_{\mathrm{R}}\}$  when  $M_{\mathrm{T}} > M_{\mathrm{R}}$  and  $0 \le \gamma_i \le 1$  is a real number, which is numerically optimized assuming that the other parameters are given. Recall that the column vectors in  $\mathbf{V}_{[c],i}$  and  $\mathbf{V}_{[p],i}$  are properly normalized to satisfy the transmit power constraint P for all i = 1, 2.

The motivation of the proposed beamforming construction (34) is from the fact that common streams are required to be recovered by both receivers. Hence, the right unitary matrix of the concatenated matrix consisting of the desired and interfering links are employed to convey common streams. On the other hand, since private streams are only recovered by the dedicated receiver while interfering to the other receiver, ZF beamforming is used to null out interference to the unintended receiver in addition to the SVD beamforming in (35). More specifically, a hybrid scheme, which is a simple linear combination of the ZF and SVD beamforming schemes, is proposed in (35) as a compromise between the two approaches.

Then, based on the transmit beamforming matrices (34) and (35), the set  $d_{\text{stream}}$  and  $(\gamma_1, \gamma_2)$  are optimized to maximize the achievable rates (26) or (29) while satisfying the conditions (10) and (11).

## D. Remarks

1) Comparison with previous works and implementation via practical codes in practical environments: By restricting  $A_{SIC,i}$  in (27) to an identity matrix for all  $i \in \{1, 2\}$ , the proposed successive IF scheme becomes the conventional scheme with MMSE-SIC receivers, in which the operation at the transmitter side is the same as that of the proposed successive IF while MMSE-SIC operation is performed instead of successive IF sum decoding at the receiver side. Similarly, the proposed IF scheme recovers the conventional scheme with MMSE receivers by restricting  $A_i$  in (24) to an identity matrix for all  $i \in \{1, 2\}$ . Moreover, if we set  $d_{[p],i} = d_i = \min\{2M_T, 2M_R\}$  and  $d_{[c],i} = 0$  for all  $i \in \{1, 2\}$ , i.e., sending private streams only without any rank adaptation, then the proposed IF scheme recovers the previous interference management scheme with IF receivers proposed in [21, Section VII-C]. Therefore, the proposed IF framework provides a general coding strategy including the conventional MMSE-SIC receivers, MMSE receivers, and IF receivers as special cases.

Although the proposed successive IF and IF schemes are developed based on lattice codes in this paper, it is worthwhile mentioning that both schemes can be readily implemented via practical binary codes by simply extending the recent works [6], [37]. Additionally, although channels are assumed to be fixed during the communication block in this paper, the proposed schemes can be extended to practical environments in which channels vary within a codeword by modifying (17) as explained in [6], [37], [49].

2) Receiver complexity: Note that the symbol detection complexity of the proposed successive IF scheme and the proposed IF scheme depends on the set  $d_{\text{stream}}$ . Denote the collection of all feasible  $d_{\text{stream}}$  satisfying the constraints (10) and (11) by  $\mathcal{D}$ . Then, for all  $d_{\text{stream}} \in \mathcal{D}$ , the worstcase computational complexity of the IF filter operation (14) is given by  $O(M_R M_T^2 + M_T^3)$  [6], [52], [53] for both the proposed successive IF scheme and the proposed IF scheme. In addition, the worst-case computational complexity of the proposed IF scheme to find an appropriate integer matrix  $\mathbf{A}_i$  using the LLL algorithm, i.e., the complexity of solving (16), is given by  $O(M^4 \log M)$ , where  $M = \min\{M_T, M_R\}$  [54]. In a similar vein, the worst-case computational complexity of solving (28) for the proposed successive IF scheme is given by  $O(M^5 \log M)$  [37] since the LLL algorithm needs to be applied at most M times. However, the required computational complexity to find an integer matrix is negligible considering the entire decoding process, because the LLL algorithm is required to be performed only  $|\mathcal{D}|$  times over *n* transmissions, where  $|\mathcal{D}|$  denotes the cardinality of  $\mathcal{D}$  and *n* is much greater than  $M_{\rm T}$  and  $M_{\rm R}$ .

Therefore, the overall worst-case computational complexity of the proposed successive IF scheme and the proposed IF scheme is dominated by  $O(M_R M_T^2 + M_T^3)$  per symbol, which is the same as that of MMSE receivers and MMSE-SIC receivers [52], [53].

### IV. NUMERICAL ANALYSIS AND DISCUSSIONS

In this section, we numerically evaluate the achievable sum rates and rate regions of the proposed successive IF- and IF-based interference management schemes in various environments.

## A. Simulation Environments and Evaluation Methodologies

In simulation, we assume the Rician block fading channel model, that is, the channel matrix  $\mathbf{H}_{i,j}$  in (1) is assumed to be given by

$$\mathbf{H}_{i,j} = \sqrt{\alpha_{i,j}} \left( \sqrt{\frac{1}{K+1}} \mathbf{H}_{\mathrm{NLoS},i,j} + \sqrt{\frac{K}{K+1}} \mathbf{H}_{\mathrm{LoS},i,j} \right),$$
(36)

where  $\alpha_{i,j} \geq 0$  denotes the relative channel strength of  $\mathbf{H}_{i,j}$ ,  $\mathbf{H}_{\text{NLoS},i,j}$  and  $\mathbf{H}_{\text{LoS},i,j}$  represent the non-line-of-sight (NLoS) and line-of-sight (LoS) channel matrices between transmitter j and receiver i, respectively, and  $K \geq 0$  is the Rician K factor. We further assume that each coefficient in  $\mathbf{H}_{\text{NLoS},i,j}$  follows  $\mathcal{CN}(0,1)$  and  $\mathbf{H}_{\text{LoS},i,j}$  is given by

$$\mathbf{H}_{\mathrm{LoS},i,j} = \beta_{i,j} \mathbf{q}_{M_{\mathrm{R}}}(\theta_{i,j}) \mathbf{q}^*_{M_{\mathrm{T}}}(\phi_{i,j}), \tag{37}$$

where

$$\mathbf{q}_{M}(\theta) = \begin{bmatrix} 1 \\ \exp(-\iota\pi\cos\theta) \\ \vdots \\ \exp(-\iota\pi(M-1)\cos\theta) \end{bmatrix} \in \mathbb{C}^{M\times 1},$$
(38)

 $\beta_{i,j} \sim C\mathcal{N}(0,1)$  denotes the short-term fading coefficient of the LoS path between transmitter j and receiver i,  $\theta_{i,j}$  and  $\phi_{i,j}$  denote the angles of incidence of the LoS path on the receive antenna array of the receiver j and the transmit antenna array of transmitter i, respectively, and  $\iota^2 = -1$ . We assume that uniform linear antenna arrays are used at transmitters and receivers. In addition, it is assumed that  $\phi_{i,j}$  and  $\theta_{i,j}$  follow Unif $[0, 2\pi)$  for all  $i, j \in \{1, 2\}$ . Due to the

key hole effect [11], [55], rank( $\mathbf{H}_{LoS,i,j}$ ) = 1 for all  $i, j \in \{1, 2\}$ , and hence  $\mathbf{H}_{i,j}$  becomes an ill-conditioned matrix as K increases.

Note that when evaluating sum rates and rate regions, we consider the outage rate satisfying a certain outage probability [11]. For sum rate evaluation, assuming that the sum rate  $R_{sum} = R_1 + R_2$  is achievable by a scheme for a given channel realization, the outage probability for a target sum rate  $R_{target,sum}$  is defined as

$$p_{\text{outage}}(R_{\text{target,sum}}) = \Pr\left(R_{\text{sum}} < R_{\text{target,sum}}\right).$$
(39)

Then the p% outage sum rate is defined as

$$R_{\text{outage,sum}}(p) = \sup\left\{R_{\text{target,sum}} : p_{\text{outage}}(R_{\text{target,sum}}) \le \frac{p}{100}\right\}.$$
(40)

We can generalize (39) and (40) for a rate pair as follows:

$$p_{\text{outage}}(R_{\text{target},1}, R_{\text{target},2}) = \Pr\left(R_1 < R_{\text{target},1} \text{ and } R_2 < R_{\text{target},2}\right),\tag{41}$$

$$(R_{\text{outage},1}(p), R_{\text{outage},2}(p)) = \sup\left\{(R_{\text{target},1}, R_{\text{target},2}) : p_{\text{outage}}(R_{\text{target},1}, R_{\text{target},2}) \le \frac{p}{100}\right\}.$$
 (42)

Here,  $(R_1, R_2)$  denotes an achievable rate pair for a given channel realization and  $(R_{target,1}, R_{target,2})$  denotes the target rate pair.

More specifically, for sum rate evaluation, an appropriate set  $d_{\text{stream}} \in \mathcal{D}$  that maximizes the sum rate is numerically optimized for each channel realization. The outage sum rate is then evaluated over many channel realizations. On the other hand, for rate region evaluation, we calculate an achievable rate pair  $(R_1, R_2)$  for every possible set  $d_{\text{stream}} \in \mathcal{D}$  for each channel realization. Then the outage rate pair is evaluated for each given set  $d_{\text{stream}}$  over many channel realizations. Finally, a rate region of each scheme is obtained by the convex hull of the boundary points of the outage rate pairs.

# B. Numerical Results

1) Benchmark schemes: As benchmark schemes, we consider the interference management scheme using joint ML decoding receivers and the interference management scheme using MMSE-SIC receivers explained in Section III-D. For the considered joint ML decoding, it is assumed that transmitters use i.i.d Gaussian codebooks and each receiver  $i \in \{1, 2\}$  exhaustively finds the set of stream vectors most likely sent by the transmitters based on its received signal vector  $\bar{\mathbf{Y}}_i = [\bar{\mathbf{y}}_i(1) \ \bar{\mathbf{y}}_i(2) \ \cdots \ \bar{\mathbf{y}}_i(n) ] \in \mathbb{R}^{2M_R \times n}$ . In order to numerically analyze the contribution of each technical component incorporated into the proposed successive IF, we also consider the proposed successive IF with sending common streams only, i.e.,  $d_{[c],i} = d_i \leq \min\{2M_T, 2M_R\}$  and  $d_{[p],i} = 0$  for all  $i \in \{1, 2\}$ , the proposed successive IF with sending private streams only, i.e.,  $d_{[p],i} = d_i \leq \min\{2M_T, 2M_R\}$  and  $d_{[c],i} = 0$ for all  $i \in \{1, 2\}$ , and the IF with sending private streams only without any rank adaptation, i.e.,  $d_{[p],i} = d_i = \min\{2M_T, 2M_R\}$  and  $d_{[c],i} = 0$  for all  $i \in \{1, 2\}$ .

Remark 2: As discussed in Section III-D2, the symbol detection complexity of the proposed successive IF scheme and the proposed IF scheme is determined according to the set  $d_{\text{stream}}$ , but only the worst case is considered here for ease of comparison. Then, except for the joint ML receiver, the worst-case computational complexity of all the considered receivers is given by  $O(M_R M_T^2 + M_T^3)$  per symbol, since in the worst case, all schemes have the same effective channel size. On the other hand, in case of the joint ML receiver, the worst-case computational complexity is given by  $O(\frac{2^{nRM_T}}{n})$  per symbol [21], [42], [56], which is much larger than that of the other receivers.

2) No CSIT case: First, we consider the no CSIT case. Recall that the transmit beamforming matrices are set to (31) by utilizing an identity matrix due to the lack of CSIT. In simulation, we set  $M_{\rm T} = 8$ ,  $M_{\rm R} = 4$ ,  $\alpha_{1,1} = \alpha_{2,2} = 1$ ,  $\alpha_{1,2}, \alpha_{2,1} \in \{0.25, 1\}$ , and  $K \in \{0, 20\}$ . To examine the effect of the channel strength of interfering links on the achievable rate, we consider both the relatively strong interference case ( $\alpha_{1,2} = \alpha_{2,1} = 1$ ) and the relatively weak interference case ( $\alpha_{1,2} = \alpha_{2,1} = 1$ ) and the relatively weak interference case relatively dominant (K = 10) or non-existent (K = 0) to see the effect of channel condition number on the achievable rate. Under this setting, we plot the achievable 10% outage sum rates and rate regions of the considered schemes in Fig. 1 and Fig. 2, respectively.

The results demonstrate that both the proposed successive IF scheme and the proposed IF scheme strictly outperform the MMSE-SIC scheme. In particular, the performance gap increases with increasing K due to the fact that the achievable rate of the MMSE-SIC scheme is significantly degraded compared to that of the successive IF scheme or the IF scheme for ill-conditioned channels, since in case of the MMSE-SIC scheme, severe noise amplification is caused by restricting  $A_{SIC,i}$  to an identity matrix for all  $i \in \{1,2\}$ . Figs. 1 and 2 also show that the gap between the achievable rates of the joint ML decoding and the proposed successive IF is small. Therefore, it can be seen that compared to the joint ML decoding receiver, the



Fig. 1. Achievable outage sum rates of the no CSIT case when  $M_{\rm T} = 8$ ,  $M_{\rm R} = 4$ , and  $\alpha_{i,i} = 1$ ,  $\forall i = 1, 2$ .

proposed successive IF receiver and the proposed IF receiver can significantly reduce the receiver complexity while only slightly lowering achievable rates. See Section III-D2 and Remark 2 for more detailed analysis of receiver complexity.

In Fig. 1, it is also observed that a significant rate improvement can be obtained from rank adaptation and/or message splitting (employing both common and private streams), by comparing the proposed IF scheme with the case of sending private streams only without rank adaptation, i.e.,  $d_{[p],i} = d_i = \min\{2M_T, 2M_R\}$  and  $d_{[c],i} = 0$  for all  $i \in \{1, 2\}$ . As seen in the figure, rank adaptation is essentially required to improve achievable rates. Hence, for the rest of the performance comparison in this section, we omit the case of no rank adaptation.



Fig. 2. Achievable outage rate regions of the no CSIT case when  $M_{\rm T} = 8$ ,  $M_{\rm R} = 4$ , P = 20dB, and  $\alpha_{i,i} = 1$ ,  $\forall i = 1, 2$ .

In addition, by comparing the achievable outage rate of the proposed successive IF scheme with that of the scheme with sending common or private streams only, it is shown that employing common streams becomes more beneficial as  $\alpha_{1,2}$  and  $\alpha_{2,1}$  increase, since common streams are required to be decoded by both receivers. More specifically, when  $\alpha_{1,2} = \alpha_{2,1} = 1$ , sending only common streams becomes a near-optimal strategy in terms of maximizing the sum rate under the proposed scheme in a similar vein to the results of previous works [39], [57]. On the other hand, when  $\alpha_{1,2} = \alpha_{2,1} = 0.25$ , sending only private streams becomes a near-optimal strategy.

Moreover, interestingly, the results demonstrate that employing common streams becomes more beneficial as K increases. Note that channel matrices become ill-conditioned when K is



Fig. 3. Achievable outage sum rates of the partial CSIT case when  $M_{\rm T} = 8$ ,  $M_{\rm R} = 4$ , and  $\alpha_{i,i} = 1$ ,  $\forall i = 1, 2$ .

large, resulting in an overall decrease in the proper number of transmitted streams. Hence, in this case, each receiver can more easily recover all common streams sent by both transmitters, since the total number of streams to be decoded by each receiver becomes small.

3) Partial CSIT case: Next, we perform numerical simulation of the partial CSIT case. Recall that in this case, the transmit beamforming matrices are set to (33) by utilizing the CSI of  $\bar{\mathbf{H}}_{i,i}$ . Except for the CSIT assumption, other simulation parameters are the same as in the simulation of the no CSIT case explained above. The achievable 10% outage sum rates and rate regions of the considered schemes for the partial CSIT case are plotted in Figs. 3 and 4, respectively.

By comparing Figs. 3 and 4 with Figs. 1 and 2, it is shown that the overall achievable outage



Fig. 4. Achievable outage rate regions of the partial CSIT case when  $M_{\rm T} = 8$ ,  $M_{\rm R} = 4$ , P = 20 dB, and  $\alpha_{i,i} = 1$ ,  $\forall i = 1, 2$ .

rates of the partial CSIT case increase compared to the no CSIT case thanks to the presence of CSIT of direct links. Specifically, since the singular values of the channel matrix of the desired link are preserved in the effective channel matrix obtained after applying transmit beamforming if the SVD-based beamforming (33) is employed, the use of private streams has been shown to be more advantageous for the partial CSIT case compared to the no CSIT case that simply uses an identity matrix as a transmit beamforming matrix. Additionally, it is observed that the outage rate performance tendencies with respect to  $\alpha_{i,j}$  and K are similar to those without CSIT.

4) *Full CSIT case:* Finally, we examine the full CSIT case. Recall that in this case, the transmit beamforming matrices for common and private streams are set to (34) and (35), respectively. In



Fig. 5. Achievable outage sum rates of the full CSIT case when  $M_{\rm R} = 4$  and  $\alpha_{i,i} = \alpha_{i,j} = 1$ ,  $\forall i, j = 1, 2$  where  $i \neq j$ .

simulation, we set  $M_R = 4$  and  $\alpha_{i,j} = 1$  for all  $i, j \in \{1, 2\}$ , and consider the cases where  $M_T \in \{6, 8\}$  and  $K \in \{0, 20\}$ . In addition, we consider a transmission scheme using ZF beamforming and water-filling power allocation as a conventional benchmark scheme. More specifically, in this benchmark scheme, by utilizing the CSIT of cross links, inter-user interference is completely removed by ZF beamforming of each transmitter. Each transmitter–receiver pair then performs SVD beamforming and water-filling power allocation based on its acquired interference-free single-user channel, which can achieve the capacity of the given single-user channel. Under this setting, the achievable 10% outage sum rates and rate regions of the considered schemes are depicted in Fig. 5 and Fig. 6, respectively.



Fig. 6. Achievable outage rate regions of the full CSIT case when  $M_{\rm R} = 4$  and  $\alpha_{i,i} = \alpha_{i,j} = 1$ ,  $\forall i, j = 1, 2$  where  $i \neq j$ .

From Figs. 5 and 6, it can be seen that the overall achievable outage rates of the full CSIT case are further improved compared to the no CSIT case (Figs. 1 and 2) and the partial CSIT case (Figs. 3 and 4) thanks to the presence of CSIT of both direct links and cross links. Specifically, when  $M_T$  is large enough, i.e.,  $M_T = 8$ , all interference links can be completely nulled out by transmit ZF beamforming without reducing the maximum number of streams that can be transmitted from each transmitter. Hence, when K = 0, i.e., for well-conditioned channels, it is observed that sending private streams only achieves a near-optimal outage sum rate under the proposed scheme, as shown in Fig. 6(b). On the other hand, even when  $M_T$  is large, if the channels are highly ill-conditioned, i.e., K = 20, the results demonstrate that the proposed message splitting method achieves better performance compared to the case with sending private streams only due to the gain from employing common streams as shown in Fig. 6(d), as similar to the no CSIT case and the partial CSIT case explained above. This tendency becomes more significant as  $M_{\rm T}$  is smaller, because the dimension of null space of a cross link used to send private streams becomes smaller.

Moreover, it is also observed in Figs. 6(a) and 6(c) that the achievable outage rate of the benchmark scheme using ZF beamforming and water-filling power allocation considerably decreases when  $M_{\rm T}$  is small as  $M_{\rm T} = 6$ , because the rank of the interference-free single-user channel matrix obtained after applying transmit ZF beamforming becomes only two. As a result, it can be seen that the proposed scheme combining message splitting and IF sum decoding can strictly outperform the benchmark scheme especially when  $M_{\rm T}$  is small, although equal power allocation is applied for all streams in the proposed scheme.<sup>4</sup>

# V. CONCLUSION

In this paper, we have developed a new interference management scheme based on IF and message splitting for the two-user MIMO interference channel. Combining IF sum decoding and a message-splitting method that uses both common and private streams, the proposed scheme achieves good performance while effectively managing interference with low complexity. In addition, we considered three representative CSIT assumptions widely adopted in the literature and proposed low complexity transmit beamforming suitable for each CSIT assumption. Considering the performance gain confirmed through numerical simulations and the fact that the proposed scheme can be easily implemented with practical binary codes, our work could be one of the promising techniques for interference management in beyond 5G and 6G communication systems.

Moreover, the ideas of our work can be extended in several interesting research directions. For example, extending to more than three user cases and combining the proposed scheme with interference alignment would be an interesting topic.

<sup>&</sup>lt;sup>4</sup>Note that if we restrict streams to be allocated equal power in the benchmark scheme, it is easy to see that the joint ML decoding always outperforms the benchmark scheme.

#### REFERENCES

- [1] Samsung Research, "6G white paper: The next hyper connected experience for all," pp. 1–46, [Online]. Available: https://cdn.codeground.org/nsr/downloads/researchareas/6G%20Vision.pdf, Dec. 2020.
- [2] S. Dang, O. Amin, B. Shihada, and M.-S. Alouini, "What should 6G be?" Nat. Electron., no. 3, pp. 20–29, Jan. 2020.
- [3] M. Giordani, M. Polese, M. Mezzavilla, S. Rangan, and M. Zorzi, "Toward 6G networks: Use cases and technologies," *IEEE Commun. Mag.*, vol. 58, no. 3, pp. 55–61, Mar. 2020.
- [4] ITU, "Minimum requirements related to technical performance for IMT-2020 radio interface(s)," *ITU-R Standard M.2410-0*, Nov. 2017.
- [5] W. Jiang, B. Han, M. A. Habibi, and H. D. Schotten, "The road towards 6G: A comprehensive survey," IEEE Open J. Commun. Soc., vol. 2, pp. 334–366, 2021.
- [6] S. H. Chae, S.-W. Jeon, and S. H. Lim, "Fundamental limits of spectrum sharing full-duplex multicell networks," *IEEE J. Select. Areas Commun.*, vol. 34, no. 11, pp. 3048–3061, Nov. 2016.
- [7] S. Chen, Y.-C. Liang, S. Sun, S. Kang, W. Cheng, and M. Peng, "Vision, requirements, and technology trend of 6G: How to tackle the challenges of system coverage, capacity, user data-rate and movement speed," *IEEE Wirel. Commun.*, vol. 27, no. 2, pp. 218–228, Apr. 2020.
- [8] Z. Zhang, Y. Xiao, Z. Ma, M. Xiao, Z. Ding, X. Lei, G. K. Karagiannidis, and P. Fan, "6G wireless networks: Vision, requirements, architecture, and key technologies," *IEEE Veh. Technol. Mag.*, vol. 14, no. 3, pp. 28–41, Sep. 2019.
- [9] M. U. A. Siddiqui, F. Qamar, F. Ahmed, Q. N. Nguyen, and R. Hassan, "Interference management in 5G and beyond network: Requirements, challenges and future directions," *IEEE Access*, vol. 9, pp. 68932–68965, 2021.
- [10] N. U. Saqib, K.-Y. Cheon, S. Park, and S.-W. Jeon, "Joint optimization of 3D hybrid beamforming and user scheduling for 2D planar antenna systems," in *Proc. IEEE International Conference on Information Networking (ICOIN)*, Jeju Island, Korea, Jan. 2021.
- [11] D. Tse and P. Viswanath, Fundamentals of Wireless communication. Cambridge University Press, 2005.
- [12] M. Shafi, A. F. Molisch, P. J. Smith, T. Haustein, P. Zhu, P. De Silva, F. Tufvesson, A. Benjebbour, and G. Wunder, "5G: A tutorial overview of standards, trials, challenges, deployment, and practice," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 6, pp. 1201–1221, Jun. 2017.
- [13] Q.-U.-A. Nadeem, A. Kammoun, and M.-S. Alouini, "Elevation beamforming with full dimension mimo architectures in 5G systems: A tutorial," *IEEE Commun. Surv. Tutor.*, vol. 21, no. 4, pp. 3238–3273, 2019.
- [14] D. J. Costello and G. D. Forney, "Channel coding: The road to channel capacity," *Proc. IEEE*, vol. 95, no. 6, pp. 1150–1177, Jun. 2007.
- [15] K. R. Kumar, G. Caire, and A. L. Moustakas, "Asymptotic performance of linear receivers in MIMO fading channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 10, pp. 4398–4418, Oct. 2009.
- [16] Y. Jiang, M. K. Varanasi, and J. Li, "Performance analysis of ZF and MMSE equalizers for MIMO systems: An in-depth study of the high SNR regime," *IEEE Trans. Inf. Theory*, vol. 57, no. 4, pp. 2008–2026, Apr. 2011.
- [17] M. O. Damen, H. E. Gamal, and G. Caire, "On maximum-likelihood detection and the search for the closest lattice point," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2389–2402, Oct. 2003.
- [18] B. Hassibi and H. Vikalo, "On the sphere-decoding algorithm I. expected complexity," *IEEE Trans. Signal Processing*, vol. 53, no. 8, pp. 2806–2818, Aug. 2005.
- [19] J. Jalden and B. Ottersten, "On the complexity of sphere decoding in digital communications," *IEEE Trans. Signal Processing*, vol. 53, no. 4, pp. 1474–1484, Apr. 2005.

- [20] Z. Guo and P. Nilsson, "Algorithm and implementation of the K-best sphere decoding for MIMO detection," *IEEE J. Select. Areas Commun.*, vol. 24, no. 3, pp. 491–503, Mar. 2006.
- [21] J. Zhan, B. Nazer, U. Erez, and M. Gastpar, "Integer-forcing linear receivers," *IEEE Trans. Inf. Theory*, vol. 60, no. 12, pp. 7661–7685, Dec. 2014.
- [22] B. Nazer and M. Gastpar, "Compute-and-forward: Harnessing interference through structured codes," *IEEE Trans. Inf. Theory*, vol. 57, no. 10, pp. 6463–6486, Oct. 2011.
- [23] O. Ordentlich and U. Erez, "Precoded integer-forcing universally achieves the MIMO capacity to within a constant gap," *IEEE Trans. Inf. Theory*, vol. 61, no. 1, pp. 323–340, Jan. 2015.
- [24] Y. Regev and U. Erez, "Precise performance characterization of precoded integer forcing applied to two parallel channels," *IEEE Trans. Wireless Commun.*, vol. 20, no. 12, pp. 7920–7931, Dec. 2021.
- [25] O. Ordentlich, U. Erez, and B. Nazer, "Successive integer-forcing and its sum-rate optimality," in Proc. 51st Annual Allerton Conf. on Commun., Control, and Comput., Monticello, IL, Oct. 2013, pp. 282–292.
- [26] W. He, B. Nazer, and S. Shamai Shitz, "Uplink-downlink duality for integer-forcing," *IEEE Trans. Inf. Theory*, vol. 64, no. 3, pp. 1992–2011, Mar. 2018.
- [27] D. Silva, G. Pivaro, G. Fraidenraich, and B. Aazhang, "On integer-forcing precoding for the Gaussian MIMO broadcast channel," *IEEE Trans. Wireless Commun.*, vol. 16, no. 7, pp. 4476–4488, Jul. 2017.
- [28] S.-K. Ahn and S. H. Chae, "Blind integer-forcing interference alignment for downlink cellular networks," *IEEE Commun. Lett.*, vol. 23, no. 2, pp. 306–309, Feb. 2019.
- [29] R. B. Venturelli and D. Silva, "Optimization of integer-forcing precoding for multi-user MIMO downlink," *IEEE Wirel. Commun.*, vol. 9, no. 11, pp. 1860–1864, Nov. 2020.
- [30] V. Ntranos, V. R. Cadambe, B. Nazer, and G. Caire, "Integer-forcing interference alignment," in Proc. IEEE Int. Symp. Information Theory (ISIT), Istanbul, Turkey, Jul. 2013.
- [31] S. Hong and G. Caire, "Structured lattice codes for  $2 \times 2 \times 2$  MIMO interference channel," in *Proc. IEEE Int. Symp. Information Theory (ISIT)*, Istanbul, Turkey, Jul. 2013.
- [32] O. Ordentlich, U. Erez, and B. Nazer, "The approximate sum capacity of the symmetric Gaussian K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 60, no. 6, pp. 3450–3482, Jun. 2014.
- [33] S. M. Azimi-Abarghouyi, M. Hejazi, B. Makki, M. Nasiri-Kenari, and T. Svensson, "Integer-forcing message recovering in interference channels," *IEEE Trans. Veh. Technol.*, vol. 67, no. 5, pp. 4124–4135, May 2018.
- [34] S. M. Azimi-Abarghouyi, M. Nasiri-Kenari, B. Maham, and M. Hejazi, "Integer forcing-and-forward transceiver design for MIMO multipair two-way relaying," *IEEE Trans. Veh. Technol.*, vol. 65, no. 11, pp. 8865–8877, Nov. 2016.
- [35] H. Jiang, J. Zhao, L. Shen, H. Cheng, and G. Liu, "Joint integer-forcing precoder design for MIMO multiuser relay system," *IEEE Access*, vol. 7, pp. 81 875–81 882, 2019.
- [36] M. Hejazi, S. M. Azimi-Abarghouyi, B. Makki, M. Nasiri-Kenari, and T. Svensson, "Robust successive compute-andforward over multiver multirelay networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 10, pp. 8112–8129, 2016.
- [37] S.-K. Ahn, S. H. Chae, K. T. Kim, and Y.-H. Kim, "Successive cancellation integer forcing via practical binary codes," submitted to *IEEE Trans. on Wireless Commun.*, 2021.
- [38] S. H. Chae, S.-W. Jeon, and S.-K. Ahn, "Spatially modulated integer-forcing transceivers with practical binary codes," *IEEE Trans. on Wireless Commun.*, vol. 18, no. 12, pp. 5542–5556, Dec. 2019.
- [39] M. H. M. Costa and A. E. Gamal, "The capacity region of the discrete memoryless interference channel with strong interference," *IEEE Trans. Inf. Theory*, vol. 33, no. 5, pp. 710–711, Sep. 1987.

- [40] A. B. Carleial, "A case where interference does not reduce capacity," *IEEE Trans. Inf. Theory*, vol. 21, no. 5, pp. 569–570, Sep. 1975.
- [41] H. Sato, "The capacity of the Gaussian interference channel under strong interference," *IEEE Trans. Inf. Theory*, vol. 27, no. 6, pp. 786–788, Nov. 1981.
- [42] A. El Gamal and Y.-H. Kim, Lecture Notes on Network Information Theory, 2010.
- [43] V. S. Annapureddy and V. V. Veeravalli, "Gaussian interference networks: Sum capacity in the low-interference regime and new outer bounds on the capacity region," *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3032–3050, Jul. 2009.
- [44] A. S. Motahari and A. K. Khandani, "Capacity bounds for the Gaussian interference channel," *IEEE Trans. Inf. Theory*, vol. 55, no. 2, pp. 620–643, Feb. 2009.
- [45] X. Shang, G. Kramer, and B. Chen, "A new outer bound and the noisy-interference sum-rate capacity for Gaussian interference channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 2, pp. 689–699, Feb. 2009.
- [46] T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.
- [47] R. Zamir, Lattice Coding for Signals and Networks. Cambridge, U.K.: Cambridge Univ. Press, 2014.
- [48] U. Erez and R. Zamir, "Achieving  $\frac{1}{2}\log(1 + \text{SNR})$  on the AWGN channel with lattice encoding and decoding," *IEEE Trans. Inf. Theory*, vol. 50, no. 10, pp. 2293–2314, Oct. 2004.
- [49] I. E. Bakoury and B. Nazer, "The impact of channel variation on integer-forcing receivers," in Proc. IEEE Int. Symp. Inf. Theory (ISIT), Hong Kong, Jun. 2015.
- [50] A. K. Lenstra, J. H. W. Lenstra, and L. Lovász, "Factoring polynomials with rational coefficients," *Math. Ann.*, vol. 261, no. 4, pp. 515–534, 1982.
- [51] A. Sakzad and E. Viterbo, "Full diversity unitary precoded integer-forcing," *IEEE Trans. Wireless Commun.*, vol. 14, no. 8, pp. 4316–4327, Aug. 2015.
- [52] D. Seethaler, G. Matz, and F. Hlawatsch, "An efficient MMSE-based demodulator for MIMO bit-interleaved coded modulation," in *Proc. IEEE Global Telecommun. Conf.*, (GLOBECOM), Dallas, TX, Dec. 2004.
- [53] T.-H. Liu, "Some results for the fast MMSE-SIC detection in spatially multiplexed MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 8, no. 11, pp. 5443–5448, Nov. 2009.
- [54] Y. H. Gan, C. Ling, and W. H. Mow, "Complex lattice reduction algorithm for low-complexity full-diversity MIMO detection," *IEEE Trans. Signal Processing*, vol. 57, no. 7, pp. 2701–2710, Jul. 2009.
- [55] D. Chizhik, G. J. Foschini, M. J. Gans, and R. A. Valenzuela, "Keyholes, correlations, and capacities of multielement transmit and receive antennas," *IEEE Trans. Wireless Commun.*, vol. 1, no. 2, pp. 361–368, Apr. 2002.
- [56] R. M. Legnain, R. H. M. Hafez, I. D. Marsland, and A. M. Legnain, "A novel spatial modulation using MIMO spatial multiplexing," in *Proc. International Conference on Communications, Signal Processing, and their Applications (ICCSPA)*, Sharjah, United Arab Emirates, Feb. 2013.
- [57] H. Sato, "The capacity of the Gaussian interference channel under strong interference," *IEEE Trans. Inf. Theory*, vol. IT-27, pp. 786–788, Nov. 1981.