# Trust-based dynamic linear threshold models for non-competitive and competitive influence propagation

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Abstract-What are the key-features that enable an information diffusion model to explain the inherent dynamic, and often competitive, nature of real-world propagation phenomena? In this paper we aim to answer this question by proposing a novel class of diffusion models, inspired by the classic Linear Threshold model, and built around the following aspects: trust/distrust in the user relationships, which is leveraged to model different effects of social influence on the decisions taken by an individual; changes in adopting one or alternative information items; hesitation towards adopting an information item over time; latency in the propagation; time horizon for the unfolding of the diffusion process; and multiple cascades of information that might occur competitively. To the best of our knowledge, the above aspects have never been unified into the same LT-based diffusion model. We also define different strategies for the selection of the initial influencers to simulate non-competitive and competitive diffusion scenarios, particularly related to the problem of limitation of misinformation spread. Results on publicly available networks have shown the meaningfulness and uniqueness of our models.

Index Terms—information diffusion, influence propagation, trust/distrust relationships, limitation of misinformation spread

### I. Introduction

Since the early applications in viral marketing, the development of information diffusion models and their embedding in optimization methods has provided effective support to address a variety of influence propagation problems. However, one criticism that arises from existing diffusion models is the concern as to whether, and to what extent, they are adequate to explain the actual complexity of influence propagation phenomena that occur in the modern society of information. The acquisition and share of true or reliable information has often to cope with unlimited misinformation spots over the Web, e.g., fake news [12] mostly associated with consequences on the real life of individuals. A few studies on the spreading of fake news and hoaxes (e.g., [20]) have found that the difficulty for users of checking the reliability or trustworthiness of the web source generating and/or sharing the information item, can increase the likelihood of people to be deceived. Within this view, one side effect is the tendency of users to access information from like-minded sources and remain within information bubbles.

In general, in the attempt of debunking misinformation, one might intuitively recognize two main strategies: real-time

detection and correction, or delayed correction [11]. In both cases, the response time plays a crucial role into the limitation of misinformation diffusion, since users tend to reinforce their own belief — a cognitive phenomenon known as *confirmation bias*. It may also happen that such corrections do not yield the expected outcome, or they may even produce "backfire" results driving people's attention to the fake news.

In this scenario, it appears that one recipe to deal with the interleaving of information and dis/misinformation should be to educate people to be mindful of the informative source. Unfortunately, it is often difficult to understand where an information item originated from. Therefore, it turns out to be essential to capture the effects that different types of social ties, particularly trust/distrust relationships, can have on both the user behavior and propagation dynamics. Two related questions hence arise: Q1 – What are the key-features that make a diffusion model able to explain the inherent dynamic, and often competitive, nature of real-world propagation phenomena? Q2 – Do the currently used models of diffusion already incorporate such features?

To address question Q1, we recognize a number of aspects as essential constituents of a "realistic" information diffusion model, namely: (1) leveraging trust/distrust information in the user relationships to capture different effects of influence on decisions taken by a user; (2) accounting for a user's change in adopting one or alternative information items (i.e., relaxation of the diffusion progressivity assumption); (3) accounting for a user's hesitation or inclination towards the adoption of an information over time; (4) accounting for time-dependent variables, such as latency, to explain the propagation dynamics; (5) dealing with multiple cascades of information that might occur competitively.

Motivating example. To support our above hypothesis with an example, consider a typical scenario occurring in a political campaign, where two candidates want to target the audience of potential electors. Assume that every elector is initially unbiased toward one of the two candidates. The decision about which candidate to vote it will likely be the result of exogenous and endogenous influencing factors, i.e., one may be genuinely influenced by decisions taken by her/his social contacts —

Table I: Summary of related work based on optimization problem, basic diffusion model (**DM**), competitive diffusion (**C**), non-progressivity (**NP**), time-aware activation (**TA**), delayed propagation (**DP**), trust/distrust relations (**TD**).

Ref.	Problem	DM	С	NP	TA	DP	TD
[3]	rumor blocking	IC	<b>√</b>				
[22]	rumor blocking	IC	✓				
[9]	rumor blocking	LT	✓				
[7]	rumor blocking	distrib.	✓				
[5]	positive influ. max.	IC	✓	İ	İ	İ	
[17]	active time max.	IC		✓	İ	İ	
[8]	PTS min.	LT		✓			
[4]	positive influ. max.	Voter	✓	İ	İ	İ	✓
[21]	positive influ. max.	LT	✓				✓
[23]	positive influ. max.	LT	✓	İ	İ	İ	✓
[16]	time-constrain. influ. max.	IC		İ	İ	✓	
[6]	time-constrain. influ. max.	IC				✓	
[19]	positive influ. max.	IC	✓	✓	ĺ	✓	✓
[15]	rumor blocking	LT	✓	✓	✓		
[18]	positive influ. max.	IC	✓				

impact of homophily factors — but s/he may also have formed her/his own opinion outside the network of friends. However, an individual's decision can also be influenced by the behavior of neighboring *foes*. As a consequence of such negative influence received by foes, one may become more hesitant in taking a decision, which would be reflected by a *quiescence* status of the elector before being fully engaged in the promotion of the chosen candidate. Moreover, once an elector becomes active in favor of a particular candidate, it will be more difficult to change her/his mind over time, therefore a time-aware notion of *activation threshold* is needed to model the effects due to the *confirmation bias*. Finally, all decisions must be taken before the time limit (i.e., the election day) that constrains the political campaign period.

Concerning question **Q2**, a relatively large corpus of research studies has been developed in the last few years in the attempt of explaining realistic propagation phenomena, building upon classic information diffusion models, such as Independent Cascade (IC) and Linear Threshold (LT) [10]. Table I provides a schematic overview of models that incorporate one or more of the aspects mentioned before about **Q1**; it is worth noting that no existing work unifies *all* of the above aspects into the same (LT-based) diffusion model.

Contributions. In this paper, we propose a novel class of diffusion models, named Friend-Foe Dynamic Linear Threshold Models ( $F^2DLT$ ), which are based on the classic LT model and are designed to deal with non-competitive as well as competitive time-varying propagation scenarios. In our proposed models, the information diffusion graph is defined on top of a trust network, so that the strength of trust and distrust relationships is encoded into the influence probabilities. The behavior of a user in response to influencing actions is modeled with a time-varying activation function, depending on both the inherent activation-threshold of the user and her/his tendency of keeping or leaving the campaign-specific activation state over time. We also introduce a quiescence function to model the latency or delay that the influence of foes may determine in the participation of a user in the information propagation. For competitive scenarios, we define a semi-progressive model, which assumes that a user, once

activated, is only allowed to switch to a different campaign, and a *non-progressive* model, which instead requires a user to have always the support of her/his in-neighbors to keep the activation state with a certain campaign.

Another contribution of this work is the definition of four *seed selection* strategies, which mimic different, realistic scenarios of influence propagation. These strategies are central to our methodology of propagation simulation, since the development of optimization methods under our diffusion models is beyond the goals of this work. Notably, in competitive scenarios, we have focused on combinations of campaign strategies that might be reasonably considered for a misinformation spread limitation problem. Experimental evaluation conducted on four real-world networks has provided interesting findings on the meaningfulness and uniqueness of our proposed models.

#### II. FRIEND-FOE DYNAMIC LINEAR THRESHOLD MODELS

Here we describe our proposed  $F^2DLT$  models: the Non-Competitive  $F^2DLT$  (nC- $F^2DLT$ ), the Semi-Progressive  $F^2DLT$  (spC- $F^2DLT$ ), and the Non-Progressive  $F^2DLT$  (npC- $F^2DLT$ ). We first provide an overview of the framework based on  $F^2DLT$ . Next, we introduce key features common to all models, then we elaborate on each of them.

## A. Overview

Figure 1 illustrates the conceptual architecture of a framework based on our proposed models. Given a population of OSN users, the framework requires three main inputs: (i) a trust network, which is inferred from the social network of those users to model their trust/distrust relationships; (ii) user behavioral characteristics that are intrinsic to each user (i.e., exogenous to an information diffusion scenario) and oriented to express two aspects: activation-threshold, i.e., the effort needed to activate a user through cumulative influence from her/his neighbors, and quiescence, i.e., the user's hesitation in being actively committed with the propagation process; and, (iii) one or multiple competing campaigns, i.e., information cascades generated from the agent(s) having viral marketing purposes. Moreover, the information diffusion process has a time horizon, and its temporal unfolding is reflected in the evolution of the information diffusion graph: this also depends on the dynamics of the users' behaviors in response to the influence chains started by the campaign(s), which admit that users may switch from the adoption of a campaign's item to that of another one. Putting it all together, our  $F^2DLT$ based framework embeds all previously discussed aspects that are required to explain complex propagation phenomena, i.e., competitive diffusion, non-progressivity, time-aware activation, delayed propagation, and trust/distrust relations.

# B. Basic definitions

We are given a *trust network* represented by a directed graph  $G = \langle V, E, w \rangle$ , with set of nodes V, set of edges E, and weighting function  $w : E \mapsto [-1, 1]$  such that, for every edge  $(u, v) \in E$ ,  $w_{uv} := w(u, v)$  expresses how much v

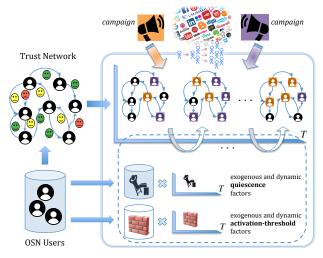


Figure 1: Illustration of the information diffusion framework based on our proposed  $F^2DLT$ .

trusts its in-neighbor u. Positive, resp. negative, value of  $w_{uv}$  corresponds to a *trust*, resp. *distrust*, relation.

For every  $v \in V$ , we denote with  $N^{in}_+(v)$  and  $N^{in}_-(v)$  the set of neighbors trusted by v (i.e., friends of v) and the set of neighbors distrusted by v (i.e., foes of v), respectively. Moreover, it holds that  $\sum_{u \in N^{in}_+(v)} w_{uv} \leq 1$  and  $\sum_{u \in N^{in}_+(v)} |w_{uv}| \leq 1$ .

Let  $\mathcal{G} = G(g,q,T) = \langle V,E,w,g,q,T \rangle$  be a directed weighted graph representing the LT-based information diffusion graph associated with trust network G, where T denotes a time interval for the diffusion process, g and q denote time-dependent activation-threshold and quiescence functions. These are introduced in  $\mathcal{G}$  to model the aspects of time-aware activation and delayed propagation, respectively. We use symbol  $S_t$  to denote the set of active nodes at time t, and symbol  $\widetilde{S}_t$  to denote the set of active nodes for which, at t, the quiescence time is not expired yet, i.e., the quiescent nodes.

**Activation-threshold function:** According to the LT model, every node  $v \in V$  is associated with an exogenous activation-threshold,  $\theta_v \in (0,1]$ , which corresponds to the apriori effort needed in terms of cumulative influence to activate the node. We enhance this concept by defining an *activation-threshold* function,  $q: V, T \mapsto \mathbb{R}^+$ , such that for  $v \in V, t \in T$ :

$$g(v,t) = \theta_v + \vartheta(\theta_v, t),$$

i.e., the activation of v at time t depends both on the user's pre-assigned threshold,  $\theta_v$ , and on a time-evolving activation term,  $\vartheta(\cdot,\cdot)$ , which models the dynamic response of a user towards the activation attempt exerted by her/his neighbors.

To specify  $\vartheta(\cdot,\cdot)$ , we might devise at least two main scenarios for  $g(\cdot,\cdot)$ : (i) non-decreasing monotone, and (ii) non-monotone function, for any  $v\in V$ . In the first scenario,  $g(\cdot,\cdot)$  would model the tendency of a user to consolidate her/his belief, according to the *confirmation-bias* principle [1]. By contrast, the second scenario would be useful to capture a user's behavior to revise her/his uncertainty to activate over

time, thus becoming more or less inclined to change her/his opinion on an information item. In this work, we focus on the confirmation bias principle, thus choosing the following form for the activation-threshold function, by which the value increases by increasing the time a node keeps staying in the same active state:

same active state: 
$$g(v,t) = \theta_v + \vartheta(\theta_v,t) = \theta_v + \delta \times \min\left\{\frac{1-\theta_v}{\delta}, t-t_v^{last}\right\} \tag{1}$$

where  $t_v^{last}$  denotes the last (i.e., most recent) time v was activated and  $\delta \geq 0^1$  represents the increment in the value of g(v,t) for consecutive time-steps. Thus, the longer a node has kept its active state for the same information cascade (campaign), the higher its activation value, and as a consequence, it will be harder to make the node change its state, or even no more possible (i.e., g(v,t) saturates to 1, as the difference  $(t-t_v^{last})$  exceeds  $(1-\theta_v)/\delta$ ).

**Quiescence function:** Each node in  $\mathcal{G}$  is also associated with a *quiescence* value, which quantifies the latency in propagation through that node. We define a *quiescence* function,  $q:V,T\mapsto T$ , non-decreasing and monotone, such that for every  $v\in V,t\in T$ , with v activated at time t:

$$q(v,t) = \tau_v + \psi(N_-^{in}(v), t),$$

where  $\tau_v \in T$  represents an exogenous term modeling the user's hesitation in being fully committed with the propagation process, and  $\psi(N^{in}_-(v),t)$  provides an additional delay proportional to the amount of v's neighbors that are distrusted and active, by the time the activation attempt is performed by the v's trusted neighbors:

$$q(v,t) = \tau_v + \psi(N_-^{in}(v), t) = \tau_v + \exp\left(\lambda \times \sum_{u \in S_{t-1}} |w_-(u, v)|\right)$$
(2)

where  $\lambda \geq 0$  is a coefficient modeling the average user sensitivity in the perceived negative influence. Intuitively, this coefficient would weight more the negative influence as the diffusing informative item is more "worth of suspicion".

Rationale for activation and propagation: Our choice of using, on the one hand, friends for the activation of a user, and on the other hand, foes to impact on delayed propagation, represents a key distinction from related work [15], [21], [23]. In our setting, we tend to reject as true in general, the principle "I agree with my friends' idea and disagree with my foes' idea", which is also close to the adage "the enemy of my enemy is my friend". Rather, we believe that any user might be provided with a self-determination capability. Therefore, in our models, the trusted connections and distrusted connections play different roles: only friends can exert a degree of influence, whereas foes can only contribute to increase the user's hesitation to commit with the propagation process.

#### C. Non-Competitive Model

We introduce the first of the three proposed models, which refers to a single-item propagation scenario. Figure 2 shows the life-cycle of a node in the diffusion graph under this model.

<sup>&</sup>lt;sup>1</sup>We assume the second additive term in Eq. (1) is zero if  $\delta = 0$ .



Figure 2: Life-cycle of a node in the nC- $F^2DLT$  model.

**Definition 1.** Non-Competitive Friend-Foe Dynamic Linear Threshold Model (nC- $F^2DLT$ ). Let  $\mathcal{G} = \langle V, E, w, g, q, T \rangle$  be the diffusion graph of Non-Competitive Friend-Foe Dynamic Linear Threshold Model (nC- $F^2DLT$ ). The diffusion process under the nC- $F^2DLT$  model unfolds in discrete time steps. At time t = 0, an initial set of nodes  $S_0$  is activated. At time  $t \geq 1$ , the following rule applies: for any inactive node  $v \in V \setminus (S_{t-1} \cup \widetilde{S}_{t-1})$ , if  $\sum_{u \in N_+^{in}(v) \cap S_{t-1}} w_{uv} \geq g(v,t)$ , then v will be added to the set of quiescent nodes  $\widetilde{S}_t$ , with quiescence time equal to  $t^* = q(v,t)$ . Once the quiescence time is expired, v will be removed from  $\widetilde{S}_t$  and added to the set of active nodes  $S_{t^*}$ . The process continues until T is expired or no more activation attempts can be performed.

## D. Competitive Models

Here we introduce the two competitive  $F^2DLT$  models. Let us first provide our motivation for developing two different competitive models: through the following example, we illustrate a particular situation that may occur when dealing with two campaigns competitively propagating through a network. Please note that, throughout the rest of this paper, we will consider only two competing campaigns for the sake of simplicity; nevertheless, our proposed models are generalizable to more than two competing campaigns.

**Example 1.** Figure 3 shows an example activation sequence in a competitive scenario between two information cascades, distinguished by colors red and green. At time t=0, nodes u and z are green-active, and their joint influence causes green-activation of node v as well (since  $0.3+0.5\geq0.6$ ). At time t=1, as fully influenced by node x, node z has switched its activation in favor of the red campaign. After this switch, at time t=2, it happens that v's activation status is no more consistent with the (joint or individual) influenced exerted by u and z. In particular, two mutually exclusive events might in principle happen at t=2: either v is deactivated or v maintains its green-activation state.

The uncertainty situation depicted in the above example prompted us to the definition of two models, namely *semi-progressive* and *non-progressive*  $F^2DLT$ : the former corresponds to the case of v keeping its current (i.e., green) activation state, whereas the latter corresponds to v returning to the inactive state. Clearly, the two models' semantics are different from each other: the semi-progressive model assumes that a user, once activated, cannot step aside, unlike the non-progressive one, which instead requires a user to have always the support of her/his in-neighbors to keep activation.

Given two information cascades, or *campaigns* C', C'', for every time step  $t \in T$  we will use symbols  $S'_t$  and  $S''_t$  to

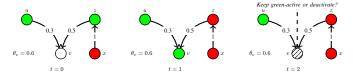


Figure 3: Uncertainty in a two-campaign activation sequence.

denote the sets of active nodes, such that that  $S'_t \cap S''_t = \emptyset$ , and analogously symbols  $\widetilde{S'}_t$  and  $\widetilde{S''}_t$  as the sets of quiescent nodes, for C' and C'', respectively. Also,  $S_t = S'_t \cup S''_t$  and  $\widetilde{S}_t = \widetilde{S'}_t \cup \widetilde{S''}_t$ .

It should also be noted that, while sharing the time interval (T) of diffusion, C' and C'' are not constrained to start at the same time  $t_0$ . Nevertheless, for the sake of simplicity, we hereinafter assume that  $t_0=t_0'=t_0''$  (with  $t_0\in T$ ), unless otherwise specified (cf. Sect. IV).

**Definition 2.** Semi-Progressive Competitive Friend-Foe Dynamic Linear Threshold Model (spC- $F^2DLT$ ). Let  $\mathcal{G} = \langle V, E, w, g, q, T \rangle$  be the diffusion graph of Semi-Progressive Competitive Friend-Foe Dynamic Linear Threshold Model (spC- $F^2DLT$ ), and C', C'' be two campaigns on  $\mathcal{G}$ . The diffusion process under the spC- $F^2DLT$  model unfolds in discrete time steps. At time t=0, two initial sets of nodes,  $S_0'$  and  $S_0''$ , are activated for each campaign. At every time step  $t \geq 1$ , the following rules apply:

**R1.** For any inactive node  $v \in V \setminus (S_{t-1} \cup \widetilde{S}_{t-1})$ , if  $\sum_{N_{t-1}^{in}(v) \cap S'_{t-1}} w_{uv} \geq g(v,t)$ , then v will be added to  $\widetilde{S}'_t$ ; analogous rule holds for C''.

**R2.** Given a node active for C'',  $v \in S''_{t-1}$ , if  $\sum_{\substack{N^{in}_+(v)\cap S'_{t-1}\\ V_{t-1}}} w_{uv} \geq g(v,t)$  and  $\sum_{\substack{N^{in}_+(v)\cap S'_{t-1}\\ V_{t-1}}} w_{uv} > \sum_{\substack{N^{in}_+(v)\cap S''_{t-1}\\ V_{t-1}}} w_{uv}$ , then v will be removed from  $S''_t$  and added to  $S'_t$ ; analogous rule holds for any node active for the first campaign.

**R3.** Every active node for which none of the above conditions is matched will keep its current state.

**R4.** When a node v is activated for the first time, it will stay in  $\widetilde{S'}_t$  or  $\widetilde{S''}_t$  until the quiescence time is expired.

**R5.** For every node that can be simultaneously activated by both campaigns, a tie-breaking rule will apply, in order to decide which campaign actually determines the node's activation.

As shown in Fig. 4, once a node becomes active, it cannot turn back to the inactive state, but it can only change the activation campaign. Moreover, switch transitions occur instantly.

**Definition 3.** Non-Progressive Competitive Friend-Foe Dynamic Linear Threshold Model (npC- $F^2DLT$ ). Let  $\mathcal{G} = \langle V, E, w, g, q, T \rangle$  be the diffusion graph of Non-Progressive Competitive Friend-Foe Dynamic Linear Threshold Model (npC- $F^2DLT$ ), and C', C'' be two campaigns on  $\mathcal{G}$ . The diffusion process in npC- $F^2DLT$  evolves according to the same rules as in spC- $F^2DLT$  plus the following rule concerning the deactivation process of an active node:

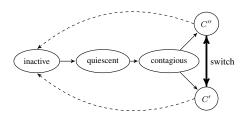


Figure 4: Life-cycle of a node in competitive models. Dashed lines are valid for  $npC-F^2DLT$  only.

**R6.** For any active node v at time t-1, if  $\sum_{N_+^{in}(v)\cap S'_{t-1}} w_{uv} < \theta_v$  and  $\sum_{N_+^{in}(v)\cap S''_{t-1}} w_{uv} < \theta_v$ , then v will turn back to the inactive state at time t.

It should be noted that a node's deactivation rule depends on  $\theta_v$  only (rather than on the whole function g(v,t)); otherwise, every node activated at a given time could deactivate itself in the next time step, due to the increase in its activation threshold. In Fig. 4, note that, unlike in  $spC-F^2DLT$ , transitions to inactive state are allowed.

## III. EVALUATION METHODOLOGY

**Data:** We used four real-world, publicly available networks, namely: *Epinions* [14], *Slashdot* [14], *Wiki-Conflict* [2], and *Wiki-Vote* [13]. The first two are "who-trust-whom" networks, Wiki-Conflict refers to Wikipedia users involved in an "editwar" (i.e., edges represent either positive or negative conflicts in editing a wikipage), and Wiki-Vote models relations between Wikipedia users that voted for/against each other in admin elections. Table II summarizes main characteristics of the networks. Note that to favor meaningful competition of campaigns based on selected pairs of strategies, we limited the diffusion context to the largest strongly connected component in each evaluation network, except for Wiki-Conflict.

All networks are originally directed and signed; in addition, the two Wikipedia-based networks also have timestamped edges. In order to derive the weighted graphs of influence probabilities, we defined the following method: for every  $(u,v) \in E$ , the edge weight  $w_{uv}$  was sampled from a binomial distribution  $\mathcal{B}(|N_{-}^{in}(v)|,p)$  if  $u \in N_{+}^{in}(v)$  (i.e., v trusts u), otherwise  $w_{uv} \sim -\mathcal{B}(|N_{-}^{in}(v)|,p)$ , where the probability of success p is equal to the fraction of trust edges in the network. We performed 1,000 samplings of edge weights, for each of the four networks. Therefore, all presented results will correspond to averages of 1,000 simulation runs.

**Seed selection strategies:** We defined four seed selection strategies, each of which mimics a different, realistic scenario of influence propagation.

Exogenous and malicious sources of information: This method, hereinafter referred to as M-Sources, aims at simulating the presence of multiple sources of malicious information within the network. Here, an exogenous source is meant as a node without incoming links, e.g., a user that is just interested in spreading her/his opinion: such a node is also regarded as

Table II: Summary of evaluation network data.

	Epinions	Slashdot	Wiki-Conflict	Wiki-Vote
#nodes	131 828	77 350	116 836	7 1 1 8
#edges	841 372	516 575	2 027 871	103 675
% distrusted/negative-edges	14.7%	23.3%	61.9%	21.6%
avg. degree	6.38	6.67	17.36	6.68
diameter	14	11	10	7
clust. coeff.	0.093	0.026	0.015	0.128
strong LCC #nodes	36 490	23 217	-	1 178
strong LCC #edges	602 722	243 600	_	31 572

malicious if a high fraction of outgoing influence exerted by the node is distrusted by out-neighbors. Formally, given a budget k, the method selects the top-k users in a ranking solution determined as  $r(v) = (\bar{W}^-/(\bar{W}^- + \bar{W}^+)) \log(|N^{out}(v)|)$ , for every v such that  $N^{in}(v) = \emptyset$ , where  $\bar{W}^+, \bar{W}^-$  are shortcut symbols to denote the sum of trust (resp. distrust) weights, respectively, outgoing from v.

Exogenous and influential trusted sources of information: Analogously to the previous method, this one, dubbed I-Sources, searches for the "best" influential trusted sources. The ranking function is as  $r(v) = (\bar{W}^+/(\bar{W}^- + \bar{W}^+))\log(|N^{out}(v)|)$ . Note that this still takes into account the negative weights, because even a highly trusted user might be distrusted by some other users (e.g., "haters").

Stress triads: This strategy is based on the notion of structural balance in triads [14]. Suppose node v has two incoming connections, the one from node z with negative weight, and the other from u with positive weight; moreover, there is a trust link from z to u. We say that z is a stress-node since, despite the distrusted link to v, it could indirectly influence v through the trusted connection with u. Our proposed Stress-Triads strategy searches for all triads containing stress-nodes and selects as seeds the first k stress-nodes with the highest number of triads they participate to.

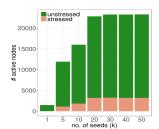
Newcomers: We call a node  $v \in V$  as a newcomer if all of its incoming edges are timestamped as less recent than its oldest outgoing edge. The start-time of v is the oldest timestamped associated with its incoming edges. We divide the set of newcomers into two groups obtained by equal-frequency binning on the temporal range specific of a network. Upon this, we distinguish between two strategies, dubbed Least-New and Most-New, which correspond to the selection of k newcomers having highest out-degree among those with the oldest start-time and with the newest start-time, respectively. Both strategies were applied to Wiki-Vote and Wiki-Conflict, due to the availability of timestamped edges.

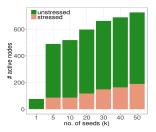
Settings of the model parameters: For every user v, the exogenous activation-threshold  $\theta_v$  and quiescence time  $\tau_v$  were chosen uniformly at random within [0,1] and [0,5]. Moreover,  $\lambda$  and  $\delta$  were varied between 0 and 5, and between 0 and 0.5, respectively.

# IV. RESULTS

# A. Evaluation of nC- $F^2DLT$

**Spread, stressed users:** We analyzed the number of final activated users (i.e., spread) by varying the size (k) of seed set, for every seed selection strategy. We initially assumed constant





- (a) Epinions, I-Sources
- (b) Wiki-Vote, Stress-Triads

Figure 5: Spread of nC- $F^2DLT$  by varying seed set size (k) and selection strategy.

activation thresholds (i.e.,  $\vartheta(\cdot,\cdot)=0$ ) and constant quiescence times (i.e.,  $\psi(\cdot,\cdot)=0$ ). Moreover, we distinguished between "stressed" and "unstressed" users, being the former regarded as active users having at least one distrusted active in-neighbor. As shown in Fig. 5 for some representative cases, we found the activation of the stressed users were activated less than the unstressed users, although with similar trend as k increases. I-Sources along with Stress-Triads revealed higher spread capability, in all networks (with the exception of Wiki-Vote). Least-New prevailed on Most-New for lower k. M-Sources led to a much lower spread than the other strategies.

**Activation loss:** We further investigated the *activation loss*, i.e., the percentage decrease of activated users due to the enabling of the time-varying quiescence factor (i.e.,  $\lambda > 0$  in Eq. 2). By setting a relatively large  $\lambda$  (set to 5) and k (set to 50), we found high percentage of activation loss for the initial time steps, especially for Stress-Triads, which might be explained since the initial influenced users tend to be subjected to a certain amount of distrusted influence. As the time horizon approaches, the activation loss tends to significantly decrease, down to nearly zero in most cases, with few exceptions including the use of I-Sources in Slashdot and Epinions, and Stress-Triads and M-Sources in Wiki-Vote.

## B. Evaluation of competitive models

To analyze the behavior of competitive models, we simulated a scenario of *limitation of misinformation spread*, i.e., we assumed that the "bad" campaign has started diffusing, and the "good" campaign is carried out in reaction to the first one. To this end, preliminarily to this evaluation, we investigated about proper combinations of seed selection strategies. Table III provides statistics about selected pairs of strategies, for two campaigns carried out independently to each other, with k=50. We observe that using Stress-Triads and I-Sources for the bad and good campaigns, respectively, is particularly significant, with sharing close to 100% in Epinions and Slashdot and above 80% in Wiki-Conflict.

In the following, we present results aimed to understand the effect of the confirmation bias factor on the users' campaign-changes/deactivations, under the case of "real-time correction" or "delayed correction" by the good campaign against the bad one (cf. Introduction). We used fixed-probability as tie-breaking rule, with probability 1 for the bad campaign, and we

Table III: Statistics about selected pairs of strategies for two campaigns: the seed set  $S_0^{(1)}$  (resp.  $S_0^{(2)}$ ) computed for the first-started or "bad" (resp. second-started or "good") campaign  $SS_1$  (resp.  $SS_2$ ), the spread  $|\Phi(S_0^{(1)})|(resp.|\Phi(S_0^{(2)})|)$ , the fraction of spread of the bad campaign shared with the good campaign (shared column), the percentage of shared users that were activated first by the bad campaign ( $SS_1$  first column), the average time of activation of the shared users, and the average time of activation of the shared users by the bad campaign before the good campaign, and vice versa. Abbreviations IS, MS, ST, LN, and MN stand for I-Sources, M-Sources, Stress-Triads, Least-New, and Most-New.

network	$SS_1$	$SS_2$	$ \Phi(S_0^{(1)}) $	$ \Phi(S_0^{(2)}) $	shared	$SS_1$	avg. activation time		
						first	any	$SS_1$	$SS_2$
								first	first
Epinions	ST	IS	10595	23321	0.99	28%	6.03	0.67	5.27
	MS	IS	59	23321	0.01	100%	4.0	3.0	0.0
Slashdot	ST	IS	3263	18671	0.98	40%	6.56	2.54	7.63
	MS	IS	58	18671	0.05	100%	4.0	5.0	0.0
Wiki-	ST	IS	344	5968	0.84	95%	2.43	1.43	4.93
WIKI-	MS	IS	203	5968	0.75	98%	3.64	0	9.5
Conflict	LN	MN	216	424	0.7	100%	3.77	0	0
Wiki-	ST	IS	727	394	0.45	78%	4.32	0.54	5.87
	MS	IS	172	394	0.41	79%	4.04	0.05	3.93
Vote	LN	MN	165	159	0.13	63%	2.72	0.0	4.37

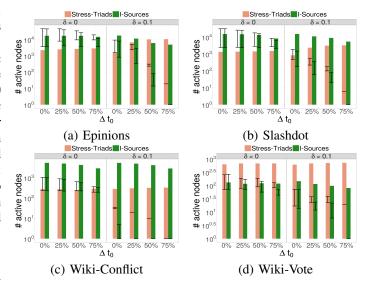


Figure 6:  $spC-F^2DLT$ : Spread, number of switched users (lower whiskers), and number of switches (lower whiskers), in log-scale, by varying start-delay ( $\Delta t_0$ ) of the "good" campaign (second bars), for  $\delta = \{0, 0.1\}$  and k = 50.

set the time horizon to the end-time of the (non-competitive) diffusion of the bad campaign.

**Evaluation of** spC- $F^2DLT$ : Figure 6 shows results on the campaign spreads, the number of users activated for one campaign that *switched* to the other campaign, and the total number of switches, by varying  $\delta$  and start-delays  $\Delta t_0$  of the good campaign (up to 75% of the time horizon).

Focusing on the campaign-switches, for  $\delta=0$ , the number of switched users follows a nearly constant trend in most networks, as the start-delay increases, while the total number of switches is subjected to a more evident decreasing trend.

Also, we observe a higher number of (unique and total) switches from the bad campaign to the good one, than vice versa. Setting  $\delta=0.1$  leads to a general decrease in the switch measurements w.r.t. the corresponding case for  $\delta=0$ .

**Evaluation of** npC- $F^2DLT$ : The spread trends observed under npC- $F^2DLT$  are similar to those corresponding to spC- $F^2DLT$  but, more importantly, the occurrence of deactivation events, which are admitted by npC- $F^2DLT$ , appeared to favor the good campaign strategy. In particular, with combinations Stress-Triads or M-Sources vs. I-Sources, the number of user-unique and total deactivations tend to increase (resp. decrease) for the bad (resp. good) campaign as  $\Delta t_0$  increases; also, for  $\delta > 0$ , the spread of the good campaign would remain higher than the spread of the bad one, due to a much larger number of deactivations from the bad campaign, up to one order of magnitude in Epinions and Slashdot, or even two orders of magnitude in Wiki networks.

#### C. Lessons Learned

The results of our evaluation revealed that the average user's sensitivity in the negative influence perceived from distrusted neighbors (which is controlled by  $\lambda$ ) makes the seed identification process more aware of the negative influence spread, thus considering the quiescence-biased contingencies by which a non-negligible fraction of users cannot be activated before the time limit.

The confirmation-bias effect underlying  $\delta$  may lead the "stronger" campaign (i.e., the one able to activate most users at the early steps of its diffusion) to increase its spread capability.

When using the semi-progressive competitive model  $(spC-F^2DLT)$ , the combined effect of increased  $\delta$  with an increase in the delay of the beginning of the secondstarted (good) campaign may reduce its capability of "saving" users from the influence of the bad campaign; therefore, to limit misinformation spread, the good campaign should concentrate its (activation) efforts in the early stage of its diffusion. Nonetheless, the non-progressive competitive model  $(npC-F^2DLT)$  appears to be more robust to the increase of  $\delta$ , in favor of the good campaign. Yet,  $npC-F^2DLT$  tends to favor deactivation events (for users previously activated by the weaker campaign) over switched events. Overall, this would suggest that the misinformation limitation problem could be more easily addressed by allowing users to "reset" their opinion when biased by the bad campaign, before eventually adopting the good campaign's choice.

#### V. CONCLUSIONS

We proposed a novel class of trust-aware, dynamic LT-based models for non-competitive and competitive influence propagation. Evaluation on real-world, publicly available networks included simulations of scenarios of misinformation spread limitation, based on realistic strategies of selection of the initial influential users. Our models pave the way for the development of sophisticated methods to solve misinformation spread limitation and related optimization problems.

Further information on this research work can be found at <a href="http://people.dimes.unical.it/andreatagarelli/ffdlt/">http://people.dimes.unical.it/andreatagarelli/ffdlt/</a>.

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