An Iterative Resource Allocation Algorithm for Multiuser OFDM with Fairness and QoS Constraints

Liang Chen and Brian Krongold
ARC Special Research Center for Ultra-Broadband Information Networks (CUBIN)
Department of Electrical and Electronic Engineering
The University of Melbourne
VIC 3010, Australia
{l.chen, bsk}@ee.unimelb.edu.au

Abstract— **This paper looks at the problem of total power minimization in multiuser OFDM systems while maintaining individual fairness (rate) and QoS (BER) requirements. A practical and efficient subchannel, power and bit allocation algorithms is described, where an average-SNR approximation is used to substantially reduce complexity. The proposed algorithm guarantees improvement through each iteration and converges quickly to a stable suboptimal solution. Numerical results and complexity analysis show that the proposed algorithm offers beneficial performance improvement compared to existing approaches.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) has emerged as perhaps one of the most promising solutions in future high-rate wireless communication services for its ability to compensate both inter-symbol interference (ISI) and inter-channel interference (ICI). The original idea of OFDM dates back to the mid 60's [1], and later technologies like the fast Fourier transform (FFT) [2] and cyclic prefix (CP) [3] have further refined OFDM's ability to combat frequency distortion and time-delay spread.

As different subchannels experience different fades, users can benefit from adaptive resource allocation to optimize the power usage and/or system throughput. The combination of OFDM and adaptive resource allocation can utilize the advantages of both. Many papers show that adaptive modulation [4] and dynamic resource allocation [5] significantly increase throughput and allow more users to transmit simultaneously. A practical and optimal bit loading algorithm to maximize throughput and/or minimize power in a single-user OFDM system is studied in [6].

As different users also experience different fades, a subchannel which appears in a deep fade to one user may not be in a deep fade for other users. Multiuser OFDM systems with an adaptive subchannel, power and bit allocation scheme can therefore, achieve higher multiplexing and diversity gains with lower power consumption. Allocation strategies for multiuser OFDM adaptive resource allocation, however, are still not fully explored. Furthermore, the sum capacity can be maximized through the simple multiuser watering-filling algorithm [7], but it does not guarantee fairness among users.

In multiuser systems, it is more important to maintain fairness and QoS among users rather than maximize the overall performance. However, existing methods devised for this problem either guarantee optimality, but are slow to converge [8], [9], or are computationally efficient, but far from optimal [10].

In this paper, we present a practical and efficient adaptive subchannel, power, and bit allocation algorithms for multiuser OFDM systems with individual fairness (rate) and QoS (BER) requirements. The algorithm uses an average-SNR approximation to perform iterative allocations which guarantees improvement in each iteration and converges to a suboptimal solution. The proposed algorithm provides a stable solution no matter what the channel correlations are. However, when adjacent subchannels have dependent fading characteristics, assigning a contiguous frequency band at a time can further reduce the algorithm overhead. The organization of this paper is as follows. We first give the system and mathematical model of this optimization problem. An overview of some previous approaches is given in Section 2. In Section 3, we give the details of the algorithm and an analysis of computational complexity, followed by the simulation results in Section 4. Finally, we draw conclusions in Section 5.

II. System Model and Problem Formulation

The multiuser adaptive OFDM system under consideration is shown in Figure 1. Channel-state information (CSI) is assumed to be known[1] through channel estimation and the help of feedback channels at both the transmitter and receiver. The system can support up to K users with individual rate requirements of R_k bits per user per OFDM symbol. There are N subchannels available within the system, each having a bandwidth that is assumed to be much smaller than the coherence bandwidth of the channel. The system does not allow sharing in either time or frequency; each subchannel is assigned to one user exclusively at any time.

At the transmitter, instantaneous CSI is used by a subchannel, power, and bit allocation algorithm to assign the data rate and corresponding power budget for each user. We define $c_{k,n}$ to be the transmit rate on subchannel n for user k. As there is no subchannel sharing, clearly we have $c_{k',n} = 0$

[1] In practice, Estimated or predicted CSI, although not perfect, can be used to provide performance gains. In this paper, however, we look at the ideal case to evaluate the methodology.

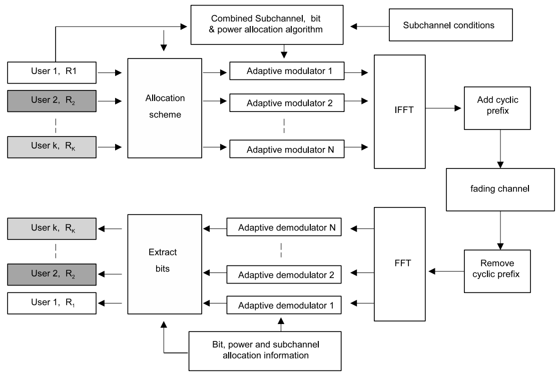

Fig. 1. Block diagram of Multiuser OFDM system

for all $k' \neq k$, if $c_{k,n} \neq 0$. We assume the allowable sets of subchannel rates (constellation sizes) are the same for all users and all subchannels, and that a maximum rate, $R_{\max}$, exists. Thus, there is no additional power constraint in our scenario.

The complex symbols on N subchannels are then transformed into a time-domain signal via the inverse fast Fourier transform (IFFT). A cyclic prefix (CP) is attached and used as a guard interval to help preserve orthogonality in the frequency-selective fading channels. Let $H_{k,n}$ be the magnitude of nth subchannel gain for user k, and $\sigma^2_{k,n}$ be the respective noise power. The resulting unit power signal-to-noise ratio (SNR) for user k on subchannel n is defined to be $\alpha^2_{k,n} = \frac{H^2_{k,n}}{\sigma^2_{k,n}}$.

Given a universal QoS constraint, let $P_{k,n}$ be the minimum transmit power to support rate $c_{k,n}$ on subchannel n for user k. The relationship between these quantities is

$$P_{k,n} = f_{k,p_e}(c_{k,n})/\alpha^2_{k,n}, \tag{1}$$

where p_e denotes some error-probability/QoS constraint and $f_{k,p_e}(\cdot)$ denotes the minimum power required for a subchannel to support a particular rate for user k on a unit-SNR subchannel. On the receiver side, we assume the subchannel and bit allocation information is known, and appropriate demodulation can be performed.

The objective is to develop a fast and efficient resource allocation algorithm, in which subchannels, power and rate are all adaptively assigned with regard to individual rate and QoS requirements.

Mathematically, we formulate this problem as

$$P_T = \min_{c_{k,n} \in \mathbf{D}} \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{1}{\alpha^2_{k,n}} f_{k,p_e}(c_{k,n}) \tag{2}$$

subject to:

$\sum_n c_{k,n} = R_k, \quad \forall\, k \in \{1, 2, \ldots, K\}$, and

For each $n \in \{1, \ldots, N\}$, if $c_{k,n} \neq 0$, then $c_{k',n} = 0$ for all $k' \neq k$,

where $\mathbf{D}$ is the set of rates (constellations) available to each subchannel.

We further note that this problem is, in general, a non-convex optimization problem, regardless of whether or not $f_{k,p_e}(c_{k,n})$ is a convex function of $c_{k,n}$.

A. Some Previous Approaches

1) Pre-determined OFDM-FDMA: System simply allocates fixed bandwidth to users proportional to their rate requirements. Although subchannels can be frequency interleaved to improve diversity, the overall performance is limited.

2) Lagrangian Relaxation: By allowing users to share subchannels, the original problem in (2) is relaxed to a convex optimization problem. A standard method in [8] searching for the optimal set of Lagrangian multipliers $\{\lambda_k\}$ that solve the dual optimization problem. Convergence accuracy and speed can be controlled by varying the minimum iteration increment $\Delta\lambda$. However, they negatively affect each other making it very difficult to reach an optimal or good suboptimal solution. The main limitations of this algorithm are

- It does not converge smoothly as it does not guarantee improvement in each iteration. Thus, even if it is forced to stop after a large number of iterations, the result is still quite unpredictable and may not be close to the optimal solution.
- It is computationally intensive, requiring a large number of iterations.

3) Bandwidth Assignment Based on SNR and Amplitude-Craving Greedy (BABS+ACG): The algorithm in [10] uses a flat fading assumption to reduce the complexity and break the problem into two parts: bandwidth allocation and subchannel allocation. Simple greedy algorithms are used on both parts to decide how many subchannels are assigned to each user as well as which subchannels are assigned to which user. The major drawbacks of this algorithm are

- The result depends on how one labels subchannels and users. Thus, it is not unique and stable.
- There is no further refinement to fix the problem brought on by the invalid flat fading assumption.

III. Sub-optimal Iterative Resource Allocation

The problem raised in (2) is computationally intractable because a joint decision of subchannel, bit and power allocation has to be made[2]. Although exclusive subchannel assignment prevents any greedy algorithm to be optimal in the multiuser environment, the single-user case gives us the idea that if we could break this joint allocation problem into a set of subproblems, that once the subchannel assignment is done, the rest of the problem becomes much easier.

In this paper, we focus on finding a low complexity, suboptimal resource allocation algorithm which has regard for *both* subchannel gains and rate/QoS requirements. Furthermore, this algorithm should give a unique, stable, suboptimal solution,

[2]Notice if there is only one user in the system, the original problem becomes the well-investigated single-user OFDM system which is generally convex and tractable.

which means it does not depend on how one labels users and subchannels and guarantees improvement throughout iteration.

The main body of this proposed iterative scheme can be intuitively separated into 2 stages:

1) *Initial Allocation*: A fast allocation that decides:
   - *Bandwidth Allocation*: Number of subchannels each user has will be decided,
   - *Subchannel Allocation*: Which subchannel goes to which user will be decided.
2) *Iterative Refinement*: Iterative rearranging to amend the problem brought in by the initial flat fading assumption, and to further reduce the total power usage.

An average SNR approximation and fast greedy searches are used in all steps. Subchannel assignment information is used as both input and output throughout each iteration.

A. Initial Allocation

1) Bandwidth Allocation: Results from [8] show that giving extra bandwidth to users with worse channel conditions usually helps to reduce total power consumption. In the first stage of the algorithm, the number of subchannels each user should have, S_k, will be decided. In order to reduce the complexity, we follow the average SNR approach in [10] and momentarily assume each user experiences flat fading, $\overline{\alpha_k^2}$, over its assigned frequency band[3].

When experiencing flat fading, the optimal power-rate allocation for each user would be equally distribute power and bits among assigned subchannels. The minimum value of S_k is $\lceil R_k/R_{\max}\rceil$ and the transmission rate on each subchannel is R_k/S_k. The new optimization problem becomes [10]

$$P_T' = \min_{S_k} \sum_{k=1}^{K} \frac{S_k}{\overline{\alpha_k^2}} f_{k,p_e}\left(\frac{R_k}{S_k}\right) \tag{3}$$

subject to:

$\sum_k S_k = N$, and

$S_k \geq \lceil R_k/R_{\max}\rceil$ for all $k \in \{1,\ldots,N\}$.

Consider the subchannels as N resource blocks, and nodes with $\overline{\alpha_k^2}$ as K orthogonal transmission carriers. This new optimization problem is very similar to a single-user discrete water-filling problem with minimum rate requirements. We assume a feasible solution exists, and the optimal, greedy algorithm, which we call "Bandwidth Allocation based on Iterative Queuing" (BAIQ) goes through the potential power saving list $\mathcal{A}$ and assigns one subchannel each time to user with maximum potential power saving:

$S_k = \lceil R_k/R_{\max}\rceil, \forall k = 1,2,\ldots,K$

while $\sum_k S_k \geq N$ **do**

  $\hat{k} = \arg\min_k S_k$

  $S_{\hat{k}} = 0$

end while

$\Delta P_k = \frac{1}{\overline{\alpha_k^2}}\left[S_k f_{k,p_e}(\frac{R_k}{S_k}) - (S_k+1) f_{k,p_e}(\frac{R_k}{S_k+1})\right], \ \forall k$

while $\sum_k S_K < N$ **do**

  $\mathcal{A} = \left\{b_k \in \{1,2,\ldots,K\} | \Delta P_{b_k} \geq \Delta P_{b_{k+1}}, \forall k\right\}$

  $k^* = b_1$

  $S_{k^*} = S_{k^*} + 1$

  update ΔP_{k^*}, update $\mathcal{A}$

end while

The BAIQ algorithm can easily be shown to optimally solve the bandwidth allocation problem raised in (3) by mathematical induction when a feasible solution exists and the power-rate function $f_{k,p_e}(\cdot)$ is convex and monotonically increasing, which is true for most modulation techniques. Compared the BABS algorithm introduced in [10], this algorithm requires a much lower execution time in general as discussed later in the complexity section.

2) Subchannel Assignment: Since a flat fading assumption is used to reduce complexity in first stage, all the subchannels are essentially the same to each link; exact subchannel assignment information is not available. We now employ another greedy algorithm to allocate subchannels to users.

Although multiuser water-filling [7] achieves best power-rate efficiency, those users at the cell boundary whose channel conditions are normally disadvantaged would experience a very high outage probability. Hence, we perform a "Subchannel-Oriented Search" (SOS) [11] which only allows user to access its best S_k subchannels. A "best user" list $\mathcal{B}$ is also introduced to further simplify the two-dimensional search and consequently reduce the complexity. Let $\rho_{k,n}=1$ indicate user k getting subchannel n. Once $\sum_n \rho_{k,n} = S_k$, no more subchannels will be given.

$\rho_{k,n} = 0, \ \forall k,n$

$\mathcal{B} = \{(k,n) | (k,n) = \arg\max_k \alpha_{k,n}^2, \forall n\}$

while $\mathcal{B} \neq \phi$ **do**

  $(k^*, n^*) = \arg\max_{(k,n)\in\mathcal{B}} \alpha_{k,n}^2$

  while $\sum_n \rho_{k^*,n} = S_k^*$ **do**

    $\alpha_{k^*,n}^2 = 0, \ \forall n = 1,2,\ldots,N$

    $\mathcal{B} = \{(k,n) | (k,n) = \arg\max_k \alpha_{k,n}^2, \forall n\}$

    $(k^*, n^*) = \arg\max_{(k,n)\in\mathcal{B}} \alpha_{k,n}^2$

  end while

  $\rho_{k^*,n^*} = 1$

  $S_{k^*} = S_{k^*} + 1$

  $\mathcal{B} = \mathcal{B} \backslash (k^*, n^*)$

end while

After this simple algorithm has finished, subchannels assigned to a given user should be less variant than the entire bandwidth of subchannels, which better fits the initial flat-PSD assumption. This approach differs from the ACG approach in

[3]For the first iteration, flat fading is over the whole frequency band.

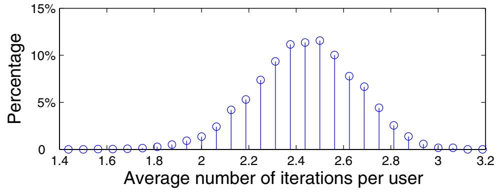

Fig. 2. Iteration number distribution for SADS.

[10] as the ordering of subchannels and/or users does not affect the resulting allocation. Furthermore, the SOS gives much better performance in terms of the total power consumption compared to the ACG approach.

B. Iterative Refinement

The theoretical idea behind the decomposition procedure is the *branch-and-bound* searching method [12]. The term *branch* refers to the partition process where groups of solutions with the same $\{S_k\}$ numbers, but not the same set of subchannels, $\{\rho_{k,n}\}$, are combined into a branch. The term *bound* refers to the availability of an efficient algorithm for calculating a lower bound inside a branch.

In the initial allocation stage, BAIQ helps us to focus on one branch that is most likely to be optimal, while the SOS algorithm provides a fast solution in finding the suboptimum on that branch. A fast allocation is now readily available after the first stage, but we can also further improve performance if the complexity limit allows. The S_k's are not the best choice in general due to the (invalid) flat fading assumption we made earlier. Even if the numbers are right, the $\{\rho_{k,n}\}$ may not be the best choice. Furthermore, the average SNR is calculated over the entire bandwidth rather than those subchannels assigned to that user, which also leaves room for improvement. Therefore, an iterative algorithm is proposed to search for better S_k and subchannel assignments, i.e., to move from one branch to another branch in order to alleviate the problem brought in by the flat fading assumption and further reduce the power consumption.

We first relax the fairness constraint to keep the power per bit for all users as low as possible. Average SNR is then updated to be $\overline{\alpha_k^2} = \frac{1}{S_k}\sum_n \alpha_{k,n}^2\rho_{k,n}$. We start with the worst user i with highest power per bit and search subchannel n belonging to the best user j with the lowest power per bit. By using the average-SNR approximation, the power change $\Delta P_{i,n}$ is defined as

$$\Delta P_{i,n} = P_k' - P_k \approx \frac{S_k'}{\overline{\alpha_k^2}} f_{k,p_e}\left(\frac{R_k}{S_k'}\right) - \frac{S_k}{\overline{\alpha_k^2}} f_{k,p_e}\left(\frac{R_k}{S_k}\right) \quad (4)$$

If the total power increment, $\Delta P_{i,j,n} = \Delta P_{i,n} + \Delta P_{j,n}$, is less then 0 when n is given from j to i, subchannel n will be dropped from j and given to i. The proposed "Steepest Decent Subchannel Adding/ dropping" (SDSA) repeats this procedure until any change will eventually increase total power.

$\mathcal{K} = \{1, 2, \ldots, K\}$
$k^* = \arg\max_{k\in\mathcal{K}} \frac{P_k}{R_k}$
while $\mathcal{K} \neq \phi$ **do**
  $\mathcal{C} = \{k' | P_{k'}/R_{k'} < P_{k^*}/R_{k^*}\}$
  $k' = \arg\min_{k\in\mathcal{C}} \frac{P_k}{R_k}$
  while $\mathcal{C} \neq \phi$ **do**
    $\mathcal{N} = \{n | \rho_{k',n} = 1 \quad \texttt{for } n = 1, 2, \ldots, N\}$
    $n^* = \arg\max_{n\in\mathcal{N}} \alpha_{k^*,n}^2$
    while $\Delta P_{k^*,k',n^*} \le 0$ **do**
      if $S_{k'} > \lceil \frac{R_k}{R_{\max}} \rceil$ **do**
        $S_{k^*} = S_{k^*} + 1$; $S_{k'} = S_{k'} - 1$
        $\rho_{k^*,n'} = 1$; $\rho_{k',n'} = 0$
        $\mathcal{N} = \mathcal{N} \backslash n^*$
        $n^* = \arg\max_{n\in\mathcal{C}} \alpha_{k^*,n}^2$
      elseif
        $\Delta P_{k^*,k',n^*} > 0$
      end if
    end while
    $\mathcal{C} = \mathcal{C} \backslash k'$
    $k' = \arg\min_{k\in\mathcal{C}} \frac{P_k}{R_k}$
  end while
  $\mathcal{K} = \mathcal{K} \backslash k^*$
end while

This algorithm is guaranteed converge as every time we add/drop a subchannel, the average $\sum P_k / \sum S_k$ and highest (worst user) P_k/S_k decreases. These two values are also bounded by the lowest (best user) P_k/S_k, which is non-decreasing, and thus, the algorithm will coverage.

C. Algorithm Complexity

For a real-time application, a critical constraint is computational complexity. In this section, computational complexities of some algorithms are briefly reviewed as a function of the number of subchannels N and number of users K.

1) Lagrangian Relaxation: Each iteration requires $N - S_k$ inversions and $S_k + 1$ evaluations of $f_{k,p_e}(\cdot)$ [10], where S_k is the number of subchannels assigned to the user with smallest rate who is being evaluated in that iteration. As the convergence speed is related to $\Delta\lambda$, the total number of iterations is quite unpredictable. Usually, it is computationally hazardous.

2) BABS+ACG: To calculate arithmetic means for K users, NK additions and K multiplications are needed. To decide the number of subchannels each user should have, the algorithm requires N iterations, with K evaluations and K comparisons of the function $f_{k,p_e}(\cdot)$ at each iteration. To decide which subchannel goes to which user, the algorithm requires N iterations as well, with K comparisons in each iteration. The total complexity is then $\mathcal{O}(NK)$.

3) BAIQ: Calculating the averaged SNRs requires NK additions and K multiplications. Initial sorting of power savings requires $K \log K$ comparisons. Allocating the N subchannels, the algorithm requires at most N iteration, with at most $\log K$

comparisons for reordering at each iteration. Therefore, the complexity for the BAIQ algorithm is $\mathcal{O}(N \log K)$, which is smaller than the BABS algorithm.

4) SOS: Finding best users requires a NK-comparison search over all $\alpha_{k,n}^2$'s. Among these N-best user-subchannel pairs, the user with the best overall subchannel gain will get that subchannel. After removing the best pair, SOS then searches for the best remaining, and so on. The complexity of this operation is $\mathcal{O}(N \log N)$, but since each user only requires S_k subchannels, once user k' has been satisfied, all of the (k', n)'s will be removed from further consideration. Thus, if user k' is the best user on some remaining subchannels, the best user then needs to be updated. Updating best users requires at most NK^2 comparisons, and the resulting total complexity for initial allocation would be $\mathcal{O}(\max(NK \log N, NK^2))$. It should be noted that this worst-case complexity order is extremely unlikely. In practice, it will be closer to the NK complexity order.

5) SDSA: For each "Add/Drop" procedure, it requires $(N - S_k)$ evaluations and four $f_{k,p_e}(\cdot)$ evaluations. Fig 2 shows simulation results of distribution of "Add/Drop" iterations numbers per user by using the proposed algorithm over 1000 different channels with 128 subchannels and a 16 user OFDM system. The total number of iterations varies from one realization to another, but it seems to remain a small number for moderate N and K. Hence, complexity would be along the order of $\mathcal{O}(NK^2)$ in this case.

IV. Simulation and Performance Comparison

We simulated 1,000 sets of a five-tap frequency-selective Rayleigh fading channels and an exponentially-decaying power profile. The multiuser OFDM system has 256 subchannels and guarantees 20 bits per symbol time for each user. Fig. 3 shows the normalized total transmission powers for different algorithms in various user numbers with BER all equal to 10^{-4}. Notice that power for the proposed algorithm is 4.3-12.3 dB less than predetermined OFDM and 1.5-4 dB less than BABS+ACG. The first stage of proposed algorithm only, without iteration (BAIQ+SOS), also gives 3.6-11.2 dB reduction over predetermined OFDM and additional 1.4-4.4 dB gain over BABS+ACG. This gain increases with the number of users, which is mainly because the larger the user number is, in general and given $N \gg K$, the more combinatorial subchannel allocation solutions exist and hence the proposed algorithm will perform more optimally than a randomly chosen predetermined allocation.

V. Conclusion

We proposed a computationally-efficient, adaptive and iterative suboptimal resource allocation scheme for multiuser OFDM systems. The algorithm uses an average SNR approximation and heuristic decomposition to reduce total complexity and make greedy decisions. The BAIQ algorithm optimally solved the bandwidth allocation problem under the flat fading assumption with a lower complexity. The SOS and SDSA offers a fast and beneficial solution to the subchannel assignment problem that we have considered. The proposed algorithms can be used in different channel environments and guarantee improvement throughout every iteration.

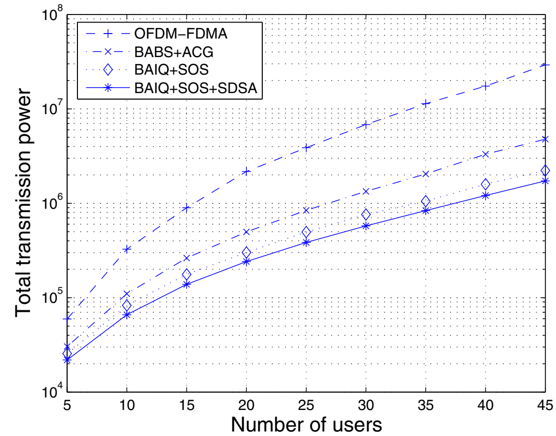

Fig. 3. Total transmission power versus number of users for a 256-subchannel OFDMA system with BER=10^{-4}

References

[1] R. W. Chang, "Synthesis of band-limited orthogonal signals for multichannel data transmission," *Bell Systems Technical Journal*, vol. 45, pp. 1775–1796, Dec. 1966.

[2] S. B. Weinstein and P. M. Ebert, "Data transmission by frequency-division multiplexing using the discrete Fourier transform," *IEEE Transactions on Communications*, vol. 19, no. 5, pp. 628–634, Oct. 1971.

[3] A. Peled and A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithm," *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 5, pp. 964–967, Apr. 1980.

[4] T. Keller and L. Hanzo, "Adaptive modulation techniques for duplex OFDM transmission," *IEEE Transactions on Vehicular Technology*, vol. 49, pp. 1893–1906, Sept. 2000.

[5] M. Wahlqvist, H. Olofsson, M. Ericson, C. Ostberg, and R. Larsson, "Capacity comparison of an OFDM based multiple access system using different dynamic resource allocation," *Proceedings of IEEE Vehicular Technology Conference*, vol. 3, pp. 1664–1668, May 1997.

[6] B. S. Krongold, K. Ramchandran, and D. L. Jones, "Computationally efficient optimal power allocation algorithm for multicarrier communication systems," *IEEE Transactions on Communications*, vol. 48, no. 1, pp. 23–27, Jan. 2000.

[7] D. N. C. Tse and S. V. Hanly, "Multiaccess fading channels. i. polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Transactions on Information Theory*, vol. 44, no. 7, pp. 2796–2815, Nov. 1998.

[8] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 10, pp. 1747–1758, Oct. 1999.

[9] I. Kim, H. L. Lee, B. Kim, and Y. H. Lee, "On the use of linear programming for dynamic subchannel and bit allocation in multiuser OFDM," *Proceedings of IEEE Global Telecommunications Conference*, vol. 6, pp. 3648–3652, Nov. 2001.

[10] D. Kivanc, G. Li, and H. Liu, "Computationally efficient bandwidth allocation and power control for OFDMA," *IEEE Transactions on Wireless Communications*, vol. 2, no. 6, pp. 1150–1158, Nov. 2003.

[11] L. Chen, B. Krongold, and J. Evans, "A computationally-efficient adaptive resource allocation algorithm for multiuser ofdm," *Proceedings of 12th European Wireless Conference*, April 2006.

[12] C. H. Papadimitriou and K. Steiglitz, *Combinatorial Optimization: Algorithms and Complexity*, 2nd ed. Dover Publications, Inc., 1998.