Optimum Power Allocation for Beamforming-Based Regenerative Dual-Hop MISO Relay Channels

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Abstract – **We consider a power allocation strategy to maximize the end-to-end capacity of a regenerative dual-hop multiple-input single-output (MISO) relay channel with the use of transmit beamforming. In a dual-hop MISO relay channel, the optimum power allocation can be determined by means of a max-min technique with sum power constraints. The proposed scheme adaptively allocates the transmit power of the source and relay according to the achievable beamforming gain as well as the average channel gain to balance the capacity between the dual-hop MISO relay channels. Simulation results show that the proposed scheme can provide noticeable performance improvement over the average signal-to-noise ratio (SNR)-based power allocation scheme in a dual-hop MISO relay channel with the use of the transmit beamforming.**

I. INTRODUCTION

The use of wireless relays has actively been considered in wireless networks due to its potentials for the enhancement of cooperative diversity, capacity, and cell coverage [1]. Relay systems generally employ two types of operating modes, regenerative and non-regenerative relay [2]. Recent researches have focused on the use of regenerative relays, which is less affected from noise enhancement in low signal-to-noise ratio (SNR) environments than the use of non-regenerative relays by employing a decode-and-forward (DF) scheme [3].

In the regenerative relay mode, it is of important concern to allocate the resource (e.g., power and bandwidth) to maximize the end-to-end capacity of a dual-hop relay channel [4]–[7]. When the relays employ a single antenna, the end-to-end capacity can be maximized by balancing the capacity between the first and second hop channels based on the average SNRs of dual-hop channels [4]. Recently, resource allocation for multiple-input multiple-output (MIMO) relay channels have been under consideration due to the ability of MIMO channels, which enables to increase the data rate or decrease the error rate [5]–[7]. In MIMO relay channels, the resource also has been allocated according to the average SNRs of dual-hop MIMO relay channels with various transmission strategies (e.g., dirty-paper coding and distributed space-time coding) [5]. However, the resource allocation for the relays with the use of transmit beamforming has not been reported.

In this paper, we consider a power allocation for regenerative dual-hop multiple-input single-output (MISO) relay systems with the use of transmit beamforming in spatially-correlated channel environments. We optimize the transmit power to maximize the end-to-end capacity considering the achievable beamforming gain as well as the average channel gain between dual-hop MISO relay channels. We also consider the use of short-term channel state information (CSI) and long-term CSI for generating the beamforming vector at the source and relay [8], [9]. The performance of the proposed scheme is analyzed and verified by computer simulation. The analytic and numerical results show that the proposed scheme significantly outperforms the average SNR-based power allocation scheme in beamforming-based regenerative dual-hop relay channels.

The remainder of this paper is organized as follows. Section II describes a correlated dual-hop MISO relay channel with the use of the transmit beamforming in consideration. Section III proposes an optimum power allocation strategy that maximizes the end-to-end capacity of beamforming-based dual-hop MISO relay channels. Section IV verifies the performance of the proposed scheme by computer simulation. Finally, conclusions are given in Section V.

II. SYSTEM MODEL

We consider a regenerative dual-hop MISO relay system as shown in Fig. 1, where the source transmits the signal using M_1 antennas to the relay by means of transmit beamforming, the relay receives it using a single antenna and re-transmits it using M_2 antennas to the destination by means of transmit beamforming, the destination receives it using a single antenna. We assume that the first and second hop channel equally share the available channel bandwidth, and that the total sum power of the source and relay is p_0. We also assume that the direct link between the source and the destination is unavailable due to large path loss.

Let s_i, $\mathbf{w}_i$, and $\mathbf{h}_i$ be the transmit signal, $(M_i \times 1)$ transmit beamforming vector, and $(1 \times M_i)$ channel vector of the i-th hop channel, respectively. Then, the received signal through the i-th hop channel can be represented as

$$y_i = \sqrt{\gamma_i}\mathbf{h}_i\mathbf{w}_i s_i + z_i \qquad (1)$$

where z_i denotes additive noise of the i-th hop channel, which is a zero-mean complex Gaussian random variable with variance σ_i^2, and $\gamma_i \left(\triangleq p_i d_i^{-\tau}\right)$ denotes the large-scale fading

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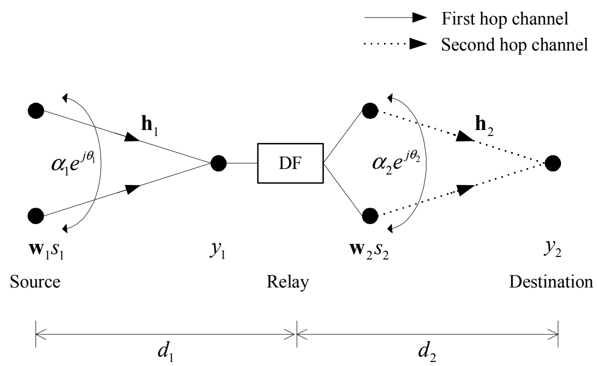

Fig. 1. Modeling of a dual-hop MISO relay systems.

coefficient of the i-th hop channel. Here, p_i is the transmit power, d_i is the propagation distance, and τ is the path loss exponent.

Assuming that each hop channel experiences spatially-correlated Rayleigh fading, the channel vector $\mathbf{h}_i$ can be generated using an independent and identically distributed (i.i.d) Rayleigh channel vector $\tilde{\mathbf{h}}_i$ by [12]

$$\mathbf{h}_i = \tilde{\mathbf{h}}_i \mathbf{R}_i^{1/2} \tag{2}$$

where $\mathbf{R}_i^{1/2}$ denotes the square root of the channel covariance matrix $\mathbf{R}_i$ defined by [13]

$$\begin{aligned} \mathbf{R}_i &= E\left\{\mathbf{h}_i^* \mathbf{h}_i\right\} \\ &= \begin{bmatrix} 1 & \rho_i & \cdots & \rho_i^{M_i-1} \\ \rho_i^* & 1 & \cdots & \rho_i^{M_i-2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_i^{*(M_i-1)} & \rho_i^{*(M_i-2)} & \cdots & 1 \end{bmatrix}. \end{aligned} \tag{3}$$

Here, the superscript $*$ denotes conjugate transpose, $E\{\mathbf{x}\}$ denotes the expectation of $\mathbf{x}$, and $\rho_i \left(\triangleq \alpha_i e^{j\theta_i}\right)$ denotes the transmit correlation coefficient between adjacent antennas, where α_i $\left(0 \le \alpha_i \le 1\right)$ and θ_i $\left(0 \le \theta_i \le 2\pi\right)$ denote its amplitude and phase, respectively. Since $\mathbf{R}_i$ is a positive definite Hermitian matrix, it can be decomposed as [13]

$$\mathbf{R}_i = \mathbf{U}_i \mathbf{\Sigma}_i \mathbf{U}_i^* \tag{4}$$

where $\mathbf{U}_i = \left[\mathbf{u}_{i,1} \ \cdots \ \mathbf{u}_{i,M_i}\right]$ is an $\left(M_i \times M_i\right)$-dimensional unitary matrix whose columns are the normalized eigenvectors of $\mathbf{R}_i$ and $\mathbf{\Sigma}_i$ is an $\left(M_i \times M_i\right)$-dimensional diagonal matrix whose diagonal elements $\left\{\lambda_{i,1},...,\lambda_{i,M_i}\right\}$ are descending ordered non-negative real values, i.e., $\lambda_{i,1} \ge ... \ge \lambda_{i,M_i} \ge 0$.

III. Proposed Power Allocation Scheme

In this section, we consider power allocation of a regenerative dual-hop MISO relay channel with the use of transmit beamforming. We first derive the capacity of each hop channel with the use of beamforming, and determine the transmit power to maximize the end-to-end capacity of a dual-hop MISO relay channel by means of a max-min technique [14]. Finally, the optimum strategy is applied to regenerative dual-hop MISO relay systems with the use of two beamforming schemes, coherent beamforming based on short-term CSI and eigen-beamforming based on long-term CSI.

A. Proposed Power Allocation Scheme

When the source and the relay transmit the signal with a beamforming vector $\mathbf{w}_i$ for $i = 1,2$, the capacity of the i-th hop channel can be represented as [13]

$$C_i\left(p_i\right) = E\left\{\log_2\left(1 + \frac{\gamma_i}{\sigma_i^2}\left|\mathbf{h}_i \mathbf{w}_i\right|^2\right)\right\} \tag{5}$$

where p_i is the transmit power of the i-th hop channel.

Since the end-to-end capacity of a regenerative dual-hop MISO relay channel is determined by the minimum capacity between dual-hop channels, i.e., $C = \min\left(C_1\left(p_1\right), C_2\left(p_2\right)\right)$ [1], it is desirable to distribute the total power p_0 to maximize the minimum capacity between dual-hop channels. Thus, the optimum power allocation problem for a regenerative dual-hop MISO relay channel can be formulated as

$$C = \max_{(p_1,p_2)} \min\left\{C_1\left(p_1\right), C_2\left(p_2\right)\right\} \text{ subject to } p_1 + p_2 = p_0. \tag{6}$$

By means of a max-min technique [14], it can be shown that the end-to-end capacity of a regenerative dual-hop MISO relay channel can be maximized by making

$$C_1\left(p_1^{\text{opt}}\right) = C_2\left(p_2^{\text{opt}}\right). \tag{7}$$

Utilizing the relationship (7) between the first and second hop channel, we can derive the optimum power allocation for a dual-hop MISO relay channel with the use of transmit beamforming. It can be shown from Jensen's inequality [13] that

$$\begin{aligned} C_i\left(p_i\right) &\le \log_2\left(1 + \frac{\gamma_i}{\sigma_i^2} E\left\{\left|\mathbf{h}_i \mathbf{w}_i\right|^2\right\}\right) \\ &= \log_2\left(1 + \frac{\gamma_i}{\sigma_i^2}\kappa_i\right) \end{aligned} \tag{8}$$

where $\kappa_i \left(\triangleq E\left\{\left|\mathbf{h}_i \mathbf{w}_i\right|^2\right\}\right)$ denotes the beamforming gain of the i-th hop channel. Thus, it can be seen that the condition satisfying (7) is given by

$$\frac{\gamma_1}{\sigma_1^2}\kappa_1 = \frac{\gamma_2}{\sigma_2^2}\kappa_2. \tag{9}$$

Since $\gamma_i = p_i d_i^{-\tau}$, (9) can be rewritten as

$$p_2 = \frac{\kappa_1}{\kappa_2}\left(\frac{d_1^{-\tau}/\sigma_1^2}{d_2^{-\tau}/\sigma_2^2}\right)p_1. \quad (10)$$

By inserting (10) into the sum power constraint in (6), it can be shown that the transmit power of a dual-hop MISO relay channel with the use of transmit beamforming can be optimally determined as

$$\left(p_1^{\text{opt}}, p_2^{\text{opt}}\right) = \left(\frac{1}{1+\frac{\kappa_1}{\kappa_2}\left(\frac{d_1^{-\tau}/\sigma_1^2}{d_2^{-\tau}/\sigma_2^2}\right)}p_0, \frac{1}{1+\frac{\kappa_2}{\kappa_1}\left(\frac{d_2^{-\tau}/\sigma_2^2}{d_1^{-\tau}/\sigma_1^2}\right)}p_0\right). \quad (11)$$

It can be seen that the transmit power $\left(p_1^{\text{opt}}, p_2^{\text{opt}}\right)$ should be determined by the achievable beamforming gain κ_i as well as the average channel gain $d_i^{-\tau}/\sigma_i^2$. In fact, p_i^{opt} is inversely proportional to the average channel gain and the beamforming gain of the i-th hop channel. Thus, when the average channel gain and beamforming gain of the first hop channel are larger than that of the second hop channel, it needs to decrease p_1^{opt} and increase p_2^{opt} to balance the capacity of the first and the second hop channel, and vice versa.

B. Application to Beamforming Schemes

We consider the application of the proposed power allocation scheme to two beamforming schemes.

1) Coherent Beamforming

When the signal is transmitted by means of coherent beamforming with beamforming weight [8]

$$\mathbf{w}_{i,\text{CB}} = \mathbf{h}_i^* / \|\mathbf{h}_i\| \text{ for } i = 1,2 \quad (12)$$

it can easily be shown that the end-to-end capacity can be maximized by allocating the transmit power as

$$\left(p_{1,\text{CB}}^{\text{opt}}, p_{2,\text{CB}}^{\text{opt}}\right) = \left(\frac{1}{1+\frac{M_1}{M_2}\left(\frac{d_1^{-\tau}/\sigma_1^2}{d_2^{-\tau}/\sigma_2^2}\right)}p_0, \frac{1}{1+\frac{M_2}{M_1}\left(\frac{d_2^{-\tau}/\sigma_2^2}{d_1^{-\tau}/\sigma_1^2}\right)}p_0\right). \quad (13)$$

It can be seen that the transmit power should be determined according to the number of transmit antennas M_i as well as the average channel gain $d_i^{-\tau}/\sigma_i^2$ since the coherent beamforming gain $\kappa_{i,\text{CB}} (\triangleq E\{|\mathbf{h}_i \mathbf{w}_{i,\text{CB}}|^2\})$ of the i-th hop channel equals to M_i.

2) Eigen Beamforming

When the signal is transmitted by means of eigen-beamforming, the beamforming weight can be determined by the principal eigenvector of the transmit correlation matrix of each hop MISO channel as [9]

$$\mathbf{w}_{i,\text{EB}} = \mathbf{u}_{i,\max} \text{ for } i = 1,2. \quad (14)$$

where $\mathbf{u}_{i,\max}$ denotes the principal eigenvector corresponding to $\lambda_{i,1}$. It can be shown that the eigen-beamforming gain $\kappa_{i,\text{EB}}$ of the i-th hop channel can be represented as

$$\begin{aligned}\kappa_{i,\text{EB}} &= E\left\{\left|\mathbf{h}_i \mathbf{u}_{i,\max}\right|^2\right\} \\ &= E\left\{\mathbf{h}_i \mathbf{u}_{i,\max} \mathbf{u}_{i,\max}^* \mathbf{h}_i^*\right\}.\end{aligned} \quad (15)$$

Since $\mathbf{h}_i = \tilde{\mathbf{h}}_i \mathbf{R}_i^{1/2}$ and $E\left\{\tilde{\mathbf{h}}_i \mathbf{A} \tilde{\mathbf{h}}_i^*\right\} = \text{tr}(\mathbf{A})$ [13], (15) can be represented as

$$\begin{aligned}\kappa_{i,\text{EB}} &= E\left\{\tilde{\mathbf{h}}_i \mathbf{R}_i^{1/2} \mathbf{u}_{i,\max} \mathbf{u}_{i,\max}^* \mathbf{R}_i^{*/2} \tilde{\mathbf{h}}_i^*\right\} \\ &= E\left\{\tilde{\mathbf{h}}_i \mathbf{R}_i \mathbf{u}_{i,\max} \mathbf{u}_{i,\max}^* \tilde{\mathbf{h}}_i^*\right\} \\ &= \text{tr}\left(\mathbf{R}_i \mathbf{u}_{i,\max} \mathbf{u}_{i,\max}^*\right) \\ &= \lambda_{i,\max}.\end{aligned} \quad (16)$$

Thus, the optimum power allocation that maximizes the end-to-end capacity can be determined as

$$\left(p_{1,\text{EB}}^{\text{opt}}, p_{2,\text{EB}}^{\text{opt}}\right) = \left(\frac{1}{1+\frac{\lambda_{1,\max}}{\lambda_{2,\max}}\left(\frac{d_1^{-\tau}/\sigma_1^2}{d_2^{-\tau}/\sigma_2^2}\right)}p_0, \frac{1}{1+\frac{\lambda_{2,\max}}{\lambda_{1,\max}}\left(\frac{d_2^{-\tau}/\sigma_2^2}{d_1^{-\tau}/\sigma_1^2}\right)}p_0\right). \quad (17)$$

It can be seen that when the eigen-beamforming is employed, the transmit power needs to be allocated according to the principal eigen-value $\lambda_{i,\max}$ of transmit correlation matrix and the average channel gain $d_i^{-\tau}/\sigma_i^2$. As a special case, when two transmit antennas are employed (i.e, $M_i = 2$), the optimum power allocation can be represented as

$$\left(p_{1,\text{EB}}^{\text{opt}}, p_{2,\text{EB}}^{\text{opt}}\right) = \left(\frac{1}{1+\frac{1+\alpha_1}{1+\alpha_2}\left(\frac{d_1^{-\tau}/\sigma_1^2}{d_2^{-\tau}/\sigma_2^2}\right)}p_0, \frac{1}{1+\frac{1+\alpha_2}{1+\alpha_1}\left(\frac{d_2^{-\tau}/\sigma_2^2}{d_1^{-\tau}/\sigma_1^2}\right)}p_0\right) \quad (18)$$

since $\lambda_{i,\max} = 1+\alpha_i$ for $M_i = 2$ [10]. It can be seen that the transmit power can simply be determined by amplitude α_i of the transmit correlation coefficient and the average channel gain $d_i^{-\tau}/\sigma_i^2$ between dual-hop channels.

IV. Simulation Results

The performance of the proposed power allocation scheme is verified by computer simulation. For comparison, the performance of the average SNR-based power allocation scheme [5] is also considered, where the total power is distributed according to only the average channel gain between dual-hop channels. The common simulation parameters are summarized in Table I.

Fig. 2 depicts the allocated power of the first hop channel with use of the coherent beamforming according to the number of source antennas M_1 when $d_1 = 0.7$ km, $d_2 = 0.3$ km, $\alpha_1 = 0.9$, $\alpha_1 = 0.5$, $M_2 = 2$, and $\gamma_0 = 10$ dB. It can be seen that the proposed scheme distributes less power to the first hop channel than the average SNR-based scheme. This is mainly because the beamforming gain of the first hop channel is larger than that of the second hop channel since $M_1 > M_2$. When $M_1 = 2$, the transmit power of the proposed scheme equals to that of the average SNR-based power allocation scheme since $\kappa_{i,\text{Prop}} > \kappa_{i,\text{SNR}}$.

Fig. 3 depicts the end-to-end capacity of the proposed scheme with use of the coherent beamforming according to γ_0 when $d_1 = 0.7$ km, $d_2 = 0.3$ km, $\alpha_1 = 0.9$, $\alpha_2 = 0.5$, $M_1 = 4$, and $M_2 = 2$. It can be seen that the proposed scheme noticeably outperforms the average SNR-based scheme. This is mainly because the proposed scheme adaptively distributes the transmit power according to the number of transmit antennas as well as the average channel gain between dual-hop MISO channels. It can be also seen that the analytic results are slightly larger than the simulation results due to Jensen's inequality in (8).

Fig. 4 depicts the allocated transmit power of the first hop channel according to $\Delta\alpha = |\alpha_1 - \alpha_2|$ when the proposed scheme is applied to use of the eigen-beamforming with $d_1 = 0.7$ km, $d_2 = 0.3$ km, $\alpha_1 = 0$, $M_1 = M_2 = 2$, and $\gamma_0 = 10$ dB. It can be seen that the proposed scheme distributes more power to the first hop channel than the average SNR-based scheme. This is mainly because the beamforming gain of the first hop channel is smaller than that of the second hop channel since $\alpha_1 < \alpha_2$.

Fig. 5 depicts the end-to-end capacity of the proposed scheme with the eigen-beamforming according to γ_0 for $d_1 = 0.7$ km, $d_2 = 0.3$ km, $\alpha_1 = 0.9$, $\alpha_2 = 0.5$, and $M_1 = M_2 = 2$. It can be seen that the power allocation based on the channel correlation and the average channel gain outperforms one based on the average channel gain.

V. Conclusion

We have considered a power allocation strategy that maximizes the end-to-end capacity of a regenerative dual-hop MISO relay channels with the use of transmit beamforming. The proposed scheme can adaptively allocate the transmit power according to the beamforming gain as well as the average channel gain between the dual-hop MISO relay channels to balance the capacity of each hop channel. The analytic and simulation results have shown that the proposed scheme can provide noticeable performance improvement over the average SNR-based scheme.

Table I. Simulation parameter

Parameters	Values
Relay protocol mode	Regenerative relay with DF scheme
Number of hops	2
Cell radius	1 km
Source antenna configuration	2 transmit antennas
Relay antenna configuration	2 transmit, 1 receive antenna
Destination antenna configuration	1 receive antenna
Sum power	1
Fading channel	Spatially-correlated Rayleigh fading
Path loss exponent	4
Link adaptation	Ideal (i.e., using the Shannon's capacity formula)

VI. Acknowledgement

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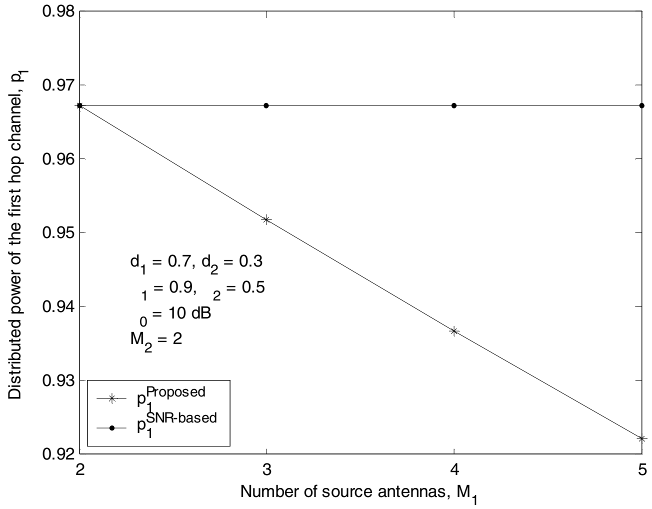

Fig. 2. Distributed power of the first hop channel with the use of coherent beamforming according to M_1 .

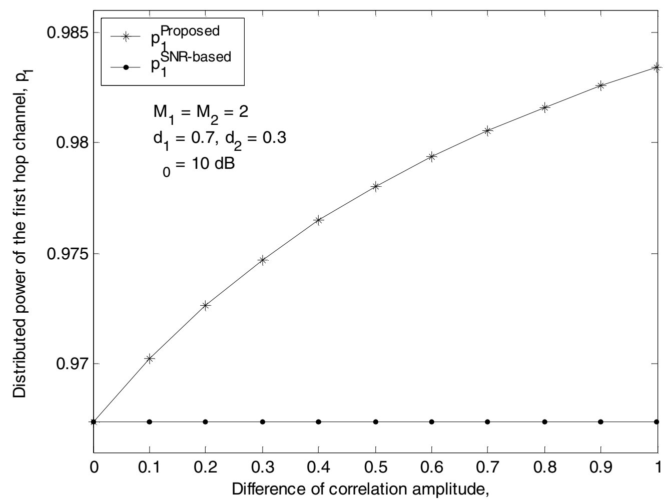

Fig. 4. Distributed power of the first hop channel with the use of eigen-beamforming according to $\Delta\alpha$.

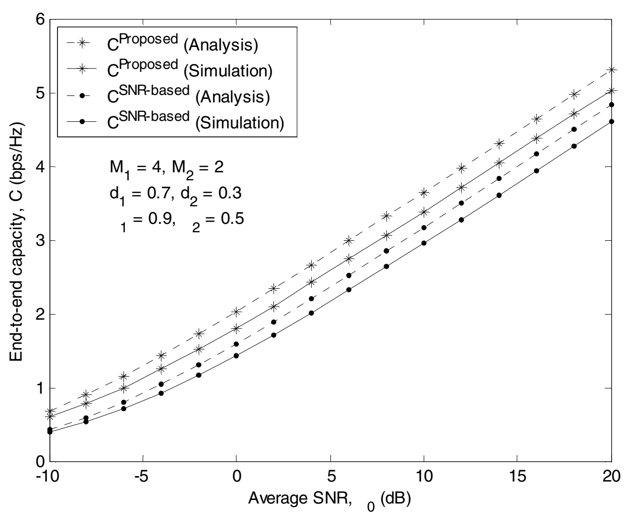

Fig. 3. End-to-end capacity of dual-hop relay channels with the use of coherent beamforming according to γ_0 .

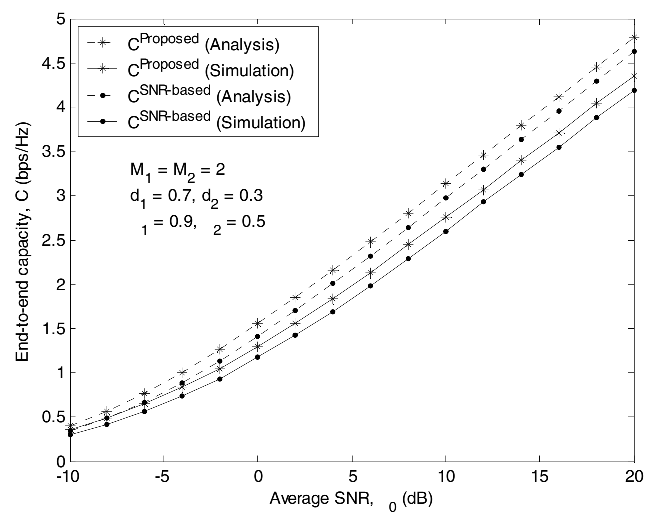

Fig. 5. End-to-end capacity of dual-hop relay channels with the use of eigen-beamforming according to γ_0 .