# On the Design of Quantization Functions for Uplink Massive MIMO with Low-Resolution ADCs

Lifu Liu, Songyan Xue, Yi Ma, Na Yi and Rahim Tafazolli
Institute for Communication Systems (ICS), University of Surrey, Guildford, England, GU2 7XH
Email: (lifu.liu, songyan.xue, y.ma, n.yi, r.tafazolli)@surrey.ac.uk

***Abstract*—Quantization is the characterization of analogue-to-digital converters (ADC) in massive MIMO systems. The design of quantization function or quantization thresholds is found to relate to quantization step, which is the factor that adapts with the changing of transmit power and noise variance. With the objective of utilizing low-resolution ADC is reducing the cost of massive MIMO, we propose an idea as if it is necessary to have adaptive-threshold quantization function. It is found that when maximum-likelihood (ML) is employed as the detection method, having quantization thresholds fixed for low-resolution ADCs will not cause significant performance loss. Moreover, such fixed-threshold quantization function does not require any information of signal power which can reduce the hardware cost of ADCs. Simulations have been carried out in this paper to make comparisons between fixed-threshold and adaptive-threshold quantization regarding various factors.**

***Index Terms*—Quantization, low-resolution analogue-to-digital converter (ADC), multiple-input multiple-output (MIMO), maximum-Likelihood (ML).**

## I. Introduction

Massive multiple-input multiple-output (MIMO) is one of the most promising technologies for 5G communications; by using tens of antennas at the base station, throughput and reliability will be largely enhanced in communication system [1]. But this technology still faces a lot of technical challenges, for example, the huge demand of the antennas will lead to the requirement of a large number of RF chains, and this will result in the increase of the hardware complexity and hardware cost [2]. It is known that analogue-to-digital converter (ADC) contributes the most to the RF chain power consumption with the consumption increases exponentially with resolutions [3]; based on this background, employing low-resolution (1, 2, 3 bit) ADCs at the receiver side is recognized as one of the cost-effective schemes for future wireless communications. This technology has already attracted many and still increasing research efforts, mainly towards signal detection, channel estimation, as well as performance evaluation [4]–[7].

Considering the non-linearity low-resolution ADC brings to massive MIMO systems [8], signal detection is obviously more challenging than systems with higher-resolution ADCs. However, there are still numerous works have been done in this research domain: for detection, maximum likelihood (ML) detection has been well studied in [9], different from the minimum Euclidean distance approach considered in the conventional linear model, ML detection for receivers with low-resolution ADC is related to the conditioned probability; zero forcing (ZF) detector was also introduced in [4], while a modified minimum mean square error (MMSE) detector has been proposed in [10] which approximates the low-resolution quantization model into a linear one, and this detection method is specifically optimized for systems with low-resolution ADCs.

Besides numerous researches that have been done in signal detection for massive MIMO with low-resolution ADCs, quantization function for such systems has not attracted too much attention. The history of researches on quantization functions could be traced back to several decades ago while the first step of the design for quantization functions was about the fixed-rate scalar quantization; followed by the design and optimizations for several types of adaptive quantization functions, and these functions are widely used in recent ADC researches [11]–[13]. The original objective for doing optimizations for quantization functions is to reduce the mean square error (MSE) between the analogue signal and the quantized signal; and the adaptive quantization function has a significant characteristic is that it requires the knowledge of the received signal power which in another word is that it requires power estimation at the receiver side to do specific quantization optimization for each ADC. In this paper, the concern arises from the key feature of low-resolution ADC: the low-cost. Therefore, the practicality and complexity of the quantization function come to essential factors to be concerned; and it is useful and technologically important to do extra analytical works and extensive studies on this topic.

To facilitate our study, we model the multiuser multi-antenna signal reception into the problem of detection high-resolution signals with low-resolution ADCs. The resolution of signals is due to the employment of high-order modulations (e.g., 16-QAM) at transmitter, the presence of multiple transmitters, or both of them. Our key contribution in this paper is on the analysis of quantization functions for low-resolution ADCs in massive MIMO systems. ML detection method has been utilised in this paper for signal reconstructions considering its ability to deal with the non-linear signals. By means of computer simulations, our work reveals that from the error probability aspect, the fixed-threshold approach has a small difference from the widely used conventional quantization function. Based on the fixed-threshold quantization function, a ML detector has been derived to specifically conduct detection for low-resolution quantized MIMO systems; such detector considers a sign refinement for the quantized signals. Since the adaptive-threshold quantization function requires the knowl-

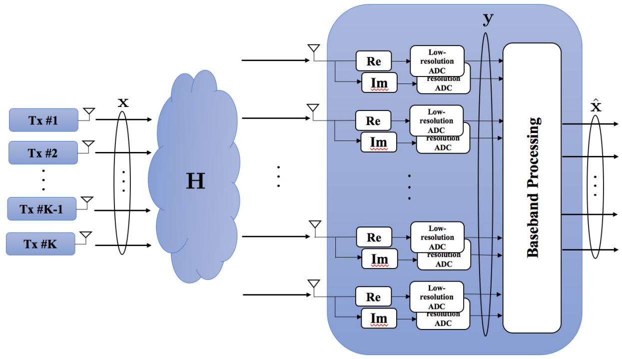


Fig. 1. System model of massive MIMO with low-resolution ADCs

edge of the received signal power so that a power estimation which is hardware-cost demanding is needed. It is trivial to conclude that the fixed-threshold quantization function can be a considerable solution for massive MIMO with low-resolution ADCs when employing ML detector. This conclusion conforms with the original low-cost objective of implementing low-resolution ADCs in massive MIMO systems. Simulation performance evaluation will be carried out in this paper to compare the performance of both fixed-threshold quantization function and adaptive-threshold quantization function for massive MIMO with low-resolution ADCs while error probability will be treated as the metric.

## II. System Model and Problem Formulation

### A. Signal Model for MIMO with Low-Resolution ADCs

The signal model of interest in its general form is depicted in Fig. 1. Basically, there are $K$ single-antenna transmitters, with each sending a symbol $x_k$, $k = 0, ..., K-1$, to the receiver. The receiver has $N$ ($N \gg K$) receive-antennas, with each associated by two low-cost RF chains (low-resolution ADCs). Each symbol $x_k$ goes through a single-input multiple-output (SIMO) channel $\mathbf{h}_k = [h_{0,k}, ..., h_{N-1,k}]^T$, where $h_{n,k}$ stands for the channel coefficient between the $k^{th}$ transmitter and the $n^{th}$ receive-antenna, and the superscript $[\cdot]^T$ for the matrix/vector transpose. Denote $\mathbf{H} = [\mathbf{h}_0, ..., \mathbf{h}_{K-1}]$ to be the MIMO channel matrix, $\mathbf{x} = [x_0, ..., x_{K-1}]^T$ the multiuser symbol block, $\mathbf{v} \sim \mathcal{CN}(0, \sigma^2)$ denotes the additive white Gaussian noise (AWGN), and the digital form of the unquantized signal $\mathbf{r} = \mathbf{Hx} + \mathbf{v}$. The output of low-resolution ADCs is a $N \times 1$ sequence $\mathbf{y}$ given by

$$\mathbf{y} = Q(\mathbf{r}) \tag{1}$$

and this complex signal model (1) can be represented by a real-form equivalent signal model [14], i.e.,

$$\underbrace{\begin{bmatrix} \Re(\mathbf{y}) \\ \Im(\mathbf{y}) \end{bmatrix}}_{=\bar{\mathbf{y}}} = Q\Big( \underbrace{\begin{bmatrix} \Re(\mathbf{H}) & -\Im(\mathbf{H}) \\ \Im(\mathbf{H}) & \Re(\mathbf{H}) \end{bmatrix}}_{=\overline{\mathbf{H}}} \underbrace{\begin{bmatrix} \Re(\mathbf{x}) \\ \Im(\mathbf{x}) \end{bmatrix}}_{=\bar{\mathbf{x}}} + \underbrace{\begin{bmatrix} \Re(\mathbf{v}) \\ \Im(\mathbf{v}) \end{bmatrix}}_{=\bar{\mathbf{v}}} \Big) \tag{2}$$

where $Q(\cdot)$ denotes the midrise $b$-bit uniform quantizer as

$$y_n = Q(r_n) = \begin{cases} \text{sign}(r_n) \cdot \left( \lfloor \frac{|r_n|}{\Delta} \rfloor \Delta + \frac{\Delta}{2} \right), & |r_n| < S + \frac{\Delta}{2} \\ \text{sign}(r_n) \cdot S, & \text{otherwise} \end{cases} \tag{3}$$

where $y_n$ and $r_n$ denote the $n_{th}, n \in [1, 2N]$ entry of $\bar{\mathbf{y}}$ and $\bar{\mathbf{r}}$ (the non-quantized received signal $\mathbf{r}$ in the real form ), respectively. $\Delta$ is the quantization step, and $S = (2^{b-1} - 1/2)\Delta$ is the saturation level determined by resolution bit $b$.

The quantization step $\Delta$ is chosen to minimize the distortion between the quantized and unquantized signals so that MSE $E(|y_n - r_n|^2)$ becomes a measurement standard. The setup of $\Delta$ has been analyzed in [12]; the distribution of the unquantized signals is the key determining factor for $\Delta$. For standard Gaussian signal with standard noise ($\sim \boldsymbol{\mathcal{N}}(0,1)$), there is a parameter called standard quantization step $\Delta^*$, whose values can be found in [13], for example, $\Delta^* = 0.9957$ for 2-bit ADC, and 0.5860 for 3-bit ADC. In most of the state-of-the-art researches, quantization step has been set as follow

$$\Delta = \sqrt{(P_t + \sigma_v^2)/2}\Delta^* \tag{4}$$

where $P_t$ is the transmitter power, $\sigma_v^2$ is the noise variance.

### B. ML Detector for Conventional Adaptive Quantization Function

Given the channel knowledge $\mathbf{H}$, the optimum decision about $\mathbf{x}$ can be formed by means of maximizing the conditional probability for a given received quantized signal $\mathbf{y}$

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \mathcal{P}(\mathbf{x}|\mathbf{H}, \mathbf{y}) \tag{5}$$

The solution of (5) leads to the ML detector algorithm. To analyze this in the real form, it can be rewritten by maximizing the conditioned probability $\mathcal{P}(\bar{\mathbf{x}}|\bar{\mathbf{y}})$ for a given received quantized signal $\bar{\mathbf{y}}$

$$\begin{aligned} \max_{\mathbf{x}} \mathcal{P}(\bar{\mathbf{x}}|\bar{\mathbf{y}}) &= \max_{\mathbf{x}} \mathcal{P}(\bar{\mathbf{x}}|\bar{\mathbf{r}} \in R(\bar{\mathbf{y}})) \\ &= \max_{\mathbf{x}} \int_{R(\bar{\mathbf{y}})} p(\bar{\mathbf{x}}|\bar{\mathbf{r}}) d\bar{\mathbf{r}} \end{aligned} \tag{6}$$

and $R(\bar{\mathbf{y}})$ represents the quantized hyper-rectangular region [7] in the (2N)-dimensional real space corresponding to the construction vector $\bar{\mathbf{r}}$, while

$$R(y_n) = \{t_n^{low} \leq r_n \leq t_n^{up}\}; \tag{7}$$

where the lower and upper boundaries of the quantization region read as

$$t_n^{low} = \begin{cases} y_n - \frac{\Delta}{2} & \text{for} \quad r_n \geq -\frac{\Delta}{2}(2^b - 2) \\ -\infty & \text{otherwise,} \end{cases} \tag{8}$$

and

$$t_n^{up} = \begin{cases} y_n + \frac{\Delta}{2} & \text{for} \quad r_n \leq \frac{\Delta}{2}(2^b - 2) \\ +\infty & \text{otherwise,} \end{cases} \tag{9}$$

$$
\begin{aligned}
L(\mathbf{z},\bar{\mathbf{x}}) &= \mathcal{P}\Big(\mathbf{h}_n^T\bar{\mathbf{x}}+\bar{v}_n < -(L-1)\Lambda, \forall n\in \mathbf{m}_L^{(1)}\Big)\times ... \times \mathcal{P}\Big(-\Lambda \le \mathbf{h}_n^T\bar{\mathbf{x}}+\bar{v}_n < 0, \forall n\in \mathbf{m}_1^{(1)}\Big)\\
&\quad \times \mathcal{P}\Big(0 \le \mathbf{h}_n^T\bar{\mathbf{x}}+\bar{v}_n < \Lambda, \forall n\in \mathbf{m}_1^{(2)}\Big)\times ... \times \mathcal{P}\Big((L-1)\Lambda \le \mathbf{h}_n^T\bar{\mathbf{x}}+\bar{v}_n, \forall n\in \mathbf{m}_L^{(2)})\\
&\overset{(a)}{=} \mathcal{P}\Big((L-1)\Lambda+\bar{v}_n \le \frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}}, \forall n\in \mathbf{m}_L^{(1)}\Big)\times ... \times \mathcal{P}\Big(-\bar{v}_n \le \frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}} < \Lambda-\bar{v}_n, \forall n\in \mathbf{m}_1^{(1)}\Big)\\
&\quad \times \mathcal{P}\Big(\bar{v}_n \le \frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}} < \Lambda+\bar{v}_n, \forall n\in \mathbf{m}_1^{(2)}\Big)\times ... \times \mathcal{P}\Big((L-1)\Lambda-\bar{v}_n \le \frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}}, \forall n\in \mathbf{m}_L^{(2)}\Big)\\
&\overset{(b)}{=} \mathcal{P}\Big((L-1)\Lambda+\bar{v}_n \le \frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}}, \forall n\in \mathbf{m}_L^{(1)}\Big)\times ... \times \mathcal{P}\Big(\bar{v}_n \le \frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}} < \Lambda+\bar{v}_n, \forall n\in \mathbf{m}_1^{(1)}\Big)\\
&\quad \times \mathcal{P}\Big(\bar{v}_n \le \frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}} < \Lambda+\bar{v}_n, \forall n\in \mathbf{m}_1^{(2)}\Big)\times ... \times \mathcal{P}\Big((L-1)\Lambda+\bar{v}_n \le \frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}}, \forall n\in \mathbf{m}_L^{(2)}\Big)\\
&\overset{(c)}{=} \mathcal{P}\Big(\bar{v}_n \le \frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}} < \Lambda+\bar{v}_n, \forall n\in \mathbf{m}_1^{(1)+(2)}\Big)\times ... \times \mathcal{P}\Big((l-1)\Lambda+\bar{v}_n \le \frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}} < l\Lambda+\bar{v}_n, \forall n\in \mathbf{m}_l^{(1)+(2)}\Big)\times\\
&\quad ... \times \mathcal{P}\Big((L-1)\Lambda+\bar{v}_n \le \frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}}, \forall n\in \mathbf{m}_L^{(1)+(2)}\Big)
\end{aligned}
\tag{10}
$$

According to [9], the conditional probability in (6) can be expressed as

$$
\begin{aligned}
\mathcal{P}(\bar{\mathbf{x}}|\bar{\mathbf{y}}) &= \prod_{n=1}^{2N}\int_{t_n^{low}}^{t_n^{up}} \frac{1}{\sqrt{\pi}\sigma}\exp\Big(-\big(\frac{r_n-\mathbf{h}_n^T\bar{\mathbf{x}}}{\sigma}\big)^2\Big)dr_n\\
&= \prod_{n=1}^{2N}\Big\{\Phi\Big(\frac{t_n^{up}-\mathbf{h}_n^T\bar{\mathbf{x}}}{\sigma/\sqrt{2}}\Big)-\Phi\Big(\frac{t_n^{low}-\mathbf{h}_n^T\bar{\mathbf{x}}}{\sigma/\sqrt{2}}\Big)\Big\}
\end{aligned}
\tag{11}
$$

where $\Phi(\cdot)$ represents the cumulative Gaussian distribution and reads as

$$
\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp(-t^2)dt \tag{12}
$$

Based on the setup for the quantization step $\Delta$ and the design for above ML detector, it can be found that the design of adaptive quantization function assume the knowledge of the signal power at the receiver side. This motivates us to rethink if adaptive quantization function necessary for low-resolution ADCs equipped massive MIMO systems as having the knowledge of received signal power requires power estimation and such would result in higher complexity. Moreover, the main objectives of using low-resolution ADCs in massive MIMO systems are reducing the hardware and energy cost, but doing such kind of estimation would not contribute to the objectives. Motivated by this background, an analysis of the fixed-threshold quantization function will be carried in the following part; and a new ML detector will be proposed based on fixed-threshold quantization function will be proposed.

## III. Maximum Likelihood Detector for Fixed-Threshold Quantization Function

Different from the setup for low-resolution ADCs with adaptive quantization thresholds in (8) and (9), the fixed-threshold quantization function has thresholds uniformly fixed in the range of [-1, 1] with $\Lambda = 1/2^{b-1}$ as the quantization step. Indicate $L=2^{b-1}$ overall thresholds, and the expression of the set $\boldsymbol{\mathcal{B}}$ for all thresholds should be

$$
\begin{aligned}
\boldsymbol{\mathcal{B}} &= \{-b_L,...,-b_1,b_0,b_1,...,b_L\}\\
&= \{-1,...,-1/2^{b-1},0,1/2^{b-1},...,1\}
\end{aligned}
\tag{13}
$$

In the fixed-threshold quantized massive MIMO system, the quantization function can be written as $Q^f(\cdot)$ with the quantized signal $y_n^f$ being expressed as

$$
y_n^f = Q^f(r_n) = \begin{cases}\mathrm{sign}(r_n)\cdot\lceil\frac{|r_n|}{\Lambda}\rceil\Lambda, & |r_n|<1\\ \mathrm{sign}(r_n), & \text{otherwise}\end{cases} \tag{14}
$$

Based on the definition of the fixed-threshold based quantization function in (14), our analysis will start from the following proposition.

**Proposition 1.** *Given the quantized received vector $\bar{\mathbf{y}}^f$, and $\mathbf{z}=\bar{\mathbf{y}}^f\odot|\bar{\mathbf{y}}^f|^{-1}\odot\bar{\mathbf{y}}^f$, then the following equation will be fulfilled*

$$
\begin{aligned}
\mathbf{z} &= \bar{\mathbf{y}}^f\odot|\bar{\mathbf{y}}^f|^{-1}\odot Q^f(\bar{\mathbf{H}}\bar{\mathbf{x}}+\bar{\mathbf{v}})\\
&= Q^f(\bar{\mathbf{y}}^f\odot|\bar{\mathbf{y}}^f|^{-1}\odot\bar{\mathbf{H}}\bar{\mathbf{x}}+\bar{\mathbf{y}}^f\odot|\bar{\mathbf{y}}^f|^{-1}\odot\bar{\mathbf{v}})
\end{aligned}
\tag{15}
$$

*where $\odot$ is the element-wise product and $|\bar{\mathbf{y}}^f|$ denotes the absolute vector based on $\bar{\mathbf{y}}^f$.*

Such proposition is because the product $\bar{\mathbf{y}}^f\odot|\bar{\mathbf{y}}^f|^{-1}$ is a binary vector and it will only change the sign of unquantized signal $\mathbf{r}$.

Let us define an index set

$$
\mathcal{M}=[\mathbf{m}_L^{(1)},...,\mathbf{m}_1^{(1)},\mathbf{m}_1^{(2)},...,\mathbf{m}_L^{(2)}] \tag{16}
$$

where $\mathbf{m}_l^{(1)}$ denotes a vector that contains all index $n$ that make $y_n^f=-l\Lambda$; and $\mathbf{m}_l^{(2)}$ refers to the index vector that has all $n$ result in $y_n^f=l\Lambda$; moreover, $l\in[1,L]$. Therefore, the likelihood function for a b-bit quantized massive MIMO system can be expressed as (10).

In (10), (a) is based on the sign refinement in **Proposition** 1; (b) is because $v_n$ and $-v_n$ have the same distribution method

$$\mathcal{P}^f(\bar{\mathbf{x}}|\bar{\mathbf{y}}) = \prod_{l=1}^{L-1} \prod_{n}^{n\in\mathbf{m}_l^{(1)+(2)}} \Big(\Phi\big(\frac{\frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}} - (l-1)\Lambda}{\sigma/\sqrt{2}}\big) - \Phi\big(\frac{\frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}} - l\Lambda}{\sigma/\sqrt{2}}\big)\Big) \times \prod_{n}^{n\in\mathbf{m}_L^{(1)+(2)}} \Phi\big(\frac{\frac{y_n^f}{|y_n^f|}\mathbf{h}_n^T\bar{\mathbf{x}} - (L-1)\Lambda}{\sigma/\sqrt{2}}\big) \tag{17}$$

(or the same probability density function (PDF)) so that $\mathcal{P}(c \geq v_n) = \mathcal{P}(c \geq -v_n)$ for arbitrary constant $c$; while (c) is just the combination of index sets.

Having $v_n$ independent for all $n$, we can build the conditional probability based on the proposed fixed-threshold quantization function as (17). It can be found in (10) and (17) that different from the unquantized signal $r_n$ that has been mapped into $[-b_{L-1}b_{L-1}]$; the upper and lower bound for the fixed-threshold system are $\pm 1$, however, there is probability that $r_n$ be larger than 1 or smaller than $-1$, this is the saturation that has been discussed in II-A, and we set $\pm 1$ to be the saturation levels.

Based on the same assumption that the channel state information (CSI) is known at the receiver side for both adaptive and fixed-threshold model, the most significant advantage the fixed-threshold approach has is that it does not require any extra beforehand signal processing where the adaption for the quantization step $\Delta$ has been removed; no signal power knowledge is needed for the fixed-threshold approach.

## IV. Simulation Results and Discussion

TABLE I
FIXED/ADAPTIVE-THRESHOLD QUANTIZATION FUNCTION SETUP FOR UPLINK MASSIVE MIMO

| Parameter | Configuration |
|---|---|
| UT ($K$) | $1, 2$ |
| Rx Ant. ($N$) | $20, 40$ |
| Modulation | 16QAM |
| ADC resolution | 2-bit, 3-bit |
| Quantization Function | fixed-threshold, adaptive-threshold |
| Detection method | Maximum-likelihood detection |

Computer simulations are carried out to do comparisons between different quantization functions for massive MIMO with low-resolution ADCs, with specific to the performance evaluation for different ML detectors. We strive to achieve our goals through intensive studies on several meaningful configurations: as specified in Table I. The wireless MIMO channel is independent flat-Rayleigh (complex Gaussian) block fading. The signal-to-noise ratio (SNR) is defined as the average received bit-energy to noise ratio (i.e. Eb/N0 in dB). It is worthwhile to note: 1) we skip the configuration of $K = 4$ since the complexity of ML detector is exponentially increased along with the increasing of the number of transmitters; limited by the hardware condition, it would cost a lot of time to get a single line; 2) we choose 16-QAM as the showcase modulation because it obeys our requirements of the high-resolution signals and it's general enough as our main concern of this paper is not the modulation scheme but the quantization function; 3) there are at least four states for the transmitted signals in the real form which is enough for these two kinds

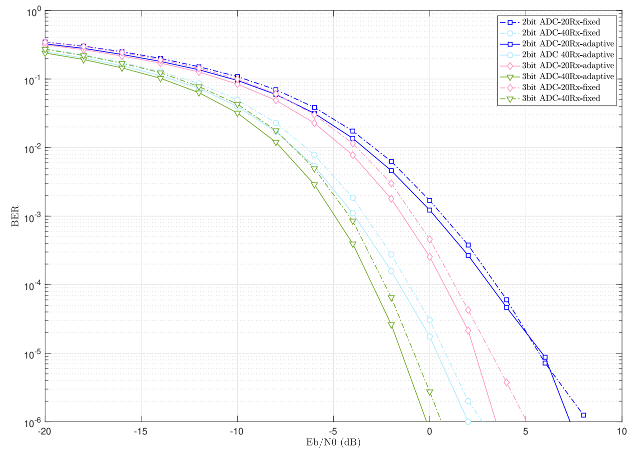


Fig. 2. Comparisons for single-user system.

of ADCs with different resolutions to build up Rx-Tx one to one bijection and this will avoid the stochastic resonance (SR) phenomenon that has been found and discussed in the 1-bit ADC model in [8].

Simulation results, in terms of the bit-error-rate (BER), are plotted in Fig. 2 for single-user system and Fig. 3 for two-users system with the number of receive antennas $N$ and ADC resolutions variable. The following phenomenons can be observed:

1. In massive MIMO system with low-resolution ADC, it is found that the signal detectability is monotonically increasing with the SNR in both Fig. 2 and Fig. 3. The performance can be improved by rising up the number of receive antennas ($N$) and bit resolutions ($b$) of ADC as well. Moreover, an interesting phenomenon can be found that double the number of receive antennas is more efficient than double the number of quantization levels for high-resolution signals to increase the signal detectability. all light blue curves with diamond symbols (2-bit ADC, 40Rx) in two figures outperform those pink ones with circle symbols (3-bit ADC, 20Rx) while comparing both curves with the blue one with square symbols (2-bit ADC, 20Rx). This can be explained as while the ADC resolution is enough to build up the Tx-Rx signal one-to-one bijection, signal detectability is more strictly limited by the ADC resolutions rather than number of receive antennas.
2. Both Fig. 2 and Fig. 3 exhibit clearly that there is a small difference between the fixed and adaptive quantization functions when configurations are the same. Concerning the typical massive MIMO setup, where the receive-antenna to transmit-antenna ratio fulfills: $N/K \gg 10$,

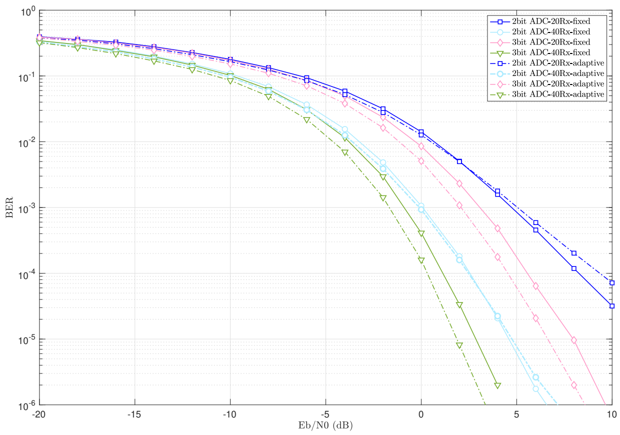


Fig. 3. Comaprisons for two-users system.

these two types of quantizations share the same spatial diversity-gain [15], thus the only factor that would cause difference is the quantization error. It is found that the fixed-threshold and adaptive-threshold approaches have narrow gap in terms of BER. Furthermore, the fixed-threshold can even outperform those with adaptive quantization function with 2-bit ADCs in Fig. 3. This phenomenon is because we use 16-QAM as the modulation scheme, the 2-bit ADC is not sufficient to form the bijection between the input and output signals for the two-user system. In addition to the non-sufficient bijection of 2-bit ADCs, the stochastic resonance of low-resolution ADC also makes the noise constructive for signal detection in a specific SNR range [8]. Therefore, the fixed-threshold outperforms the adaptive approach because the noise effect has been considered in the design of the adaptive quantization function.

Lots of previous researches that focuses on low-resolution quantized massive MIMO systems ignore the design for quantization function but use the adaptive approach. In conventional communication systems, with the knowledge of signal power and noise variance, it is the gain controller who determines how the threshold is adapted. Considering the relatively lower signal processing ability of the low-resolution quantized massive MIMO system; using the gain controller to adapt the quantization function comes to a burden. This motivates us to consider the fixed-threshold quantization for low-resolution quantized massive MIMO systems. Based on our simulation works, within the negligible small difference between the fixed-threshold quantization function and adaptive-threshold quantization function, doing adaptive quantization for the low-resolution ADCs is not necessary, and the gain this technology can bring to the low-cost massive MIMO system is limited.

## V. Conclusion

In this paper, we have investigated the problem of detecting low-resolution quantized signals in massive MIMO systems using different types of quantization functions. To better achieve the low-cost intention of utilizing low-resolution ADCs in massive MIMO systems, a fixed-threshold quantization function has been analytically revealed; and experimental comparisons have been made between the adaptive-threshold quantization function and the fixed-threshold quantization function with respect to different variables. Compared with the adaptive-threshold quantization function, it was found that the fixed-threshold quantization function could achieve comparable detection performance without the need for gain controller when using the ML detection algorithm to do the boundary research. Work in this paper was different from those concluded in conventional low-cost massive MIMO systems; thus it could be interesting and encouraging for future development in this research domain.

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