Optical Wireless Transmissions over Multi-layer Underwater Channels with Generalized Gamma Fading

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Abstract-Underwater optical communication (UWOC) is a potential solution for broadband connectivity in oceans and seas for underwater applications providing high data rate transmission with low latency and high reliability. Recent measurement campaigns suggest generalized Gamma distribution as a viable model for oceanic turbulence. In this paper, we analyze the performance of a UWOC system by modeling the vertical underwater link as a multi-layer cascaded channel, each distributed according to independent but not identically distributed (i.ni.d.) generalized Gamma random variables and considering the zero bore-sight model for pointing errors. We derive analytical expressions for probability density function (PDF) and cumulative distribution function (CDF) for the signal-to-noise ratios (SNR) of the combined channel and develop performance metrics of the considered UWOC system using outage probability, average bit error rate (BER), and ergodic capacity. We also derive the asymptotic expressions for outage probability and average BER to determine the diversity order of the proposed system for a better insight into the system performance. We use Monte-Carlo simulation results to validate our exact and asymptotic expressions and demonstrate the performance of the considered underwater UWOC system using measurement-based parametric data available for turbulent oceanic channels.

Index Terms-Cascaded channels, generalized Gamma, multilayer channels, performance analysis, UWOC, vertical link.

I. INTRODUCTION

Optical wireless communication (OWC) is a promising technology for underwater data transmission providing higher throughput with low latency and high reliability than radio frequency (RF) and acoustic wave communication systems. The underwater optical communication (UWOC) system transmits data in an unguided water environment using the wireless optical carrier for military, economic and scientific applications [1]. Despite several advantages of the UWOC, the underwater link suffers from signal attenuation, oceanic turbulence, and pointing errors. The signal attenuation occurs due to the molecular absorption and scattering effect of each photon propagating through water, generally modeled by the

extinction coefficient. Oceanic turbulence is the effect of random variations in the refractive index of the UWOC channel caused by random variations of water temperature, salinity, and air bubbles [2]. Pointing errors can also be detrimental to UWOC transmissions due to misalignment between the transmitter and detector apertures. Therefore, it is desirable to analyze the UWOC systems over various underwater channel impairments for an effective system design.

There has been tremendous research on the performance assessment of UWOC systems in recent years [3]-[10]. The authors in [3] provided an overview of various challenges associated with UWOC and proposed positioning, acquisition, and tracking scheme to mitigate the effect of pointing errors under turbulent channels. The average bit-errorrate (BER) performance under weak log-normal distributed turbulence channels was presented in [4]. The authors in [5] characterized a relay-assisted UWOC with optical code division multiple access (OCDMA) over log-normal turbulent channels. An analytic expression for the channel capacity of an orbital angular momentum (OAM) based free-space optical (FSO) communication in weak oceanic turbulence was developed in [6]. The effect of air bubbles on the UWOC was experimentally evaluated in [7]. The authors in [8], [11] analyzed the performance of multi-input and multi-output (MIMO) UWOC systems over log-normal turbulent channels. Further, a multihop UWOC system was investigated in [9]. The outage probability of a multiple decode-and-forward (DF) relay-assisted UWOC system with an on-off keying (OOK) modulation was studied in [10].

In those mentioned above and related research, a single layer of oceanic turbulence channel over the entire transmission range has been considered. However, experimental results reveal ocean stratification, i.e., the temperature gradient and salinity are depth-dependent (typically varying between a few meters to tens of meters), resulting in many non-mixing layers with different oceanic turbulence [12]. Thus, considering multiple oceanic layers for vertical transmissions may provide a more realistic performance assessment for UWOC systems. In [12]–[15], the author analyzed the performance

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Fig. 1. Schematic diagram for multi-layer UWOC system.

vertical UWOC links by cascading the end-to-end link as the concatenation of multiple layers considering both log-normal and Gamma-Gamma oceanic turbulent channels for each layer. In [2], the authors presented a holistic experimental view on the statistical characterization of oceanic turbulence in UWOC systems, considering the effect of the temperature gradient, salinity, and air bubbles. They used various statistical distributions such as log-normal, Gamma, Weibull, Exponentiated Weibull, Gamma-Gamma, and generalized Gamma to model underwater turbulence channels. Experimental investigation in [2] projected the generalized Gamma distribution as a more generic model and was valid for various underwater channel conditions. To the best of the authors' knowledge, there are no analyses available for the outage probability, average BER, and ergodic capacity of a multi-layer UWOC system over a generalized Gamma turbulent channel with pointing errors.

In this paper, we analyze the performance of a vertical UWOC system under the combined effect of cascaded underwater turbulence channels and pointing errors. The major contributions of the proposed work are summarized as follows:

- We use Mellin transform to derive the novel probability density function (PDF) and cumulative distribution function (CDF) of the signal-to-noise ratio (SNR) for vertical UWOC link in terms of a single-variate Fox H-function considering independent and not identically distributed (i.ni.d.) generalized Gamma distribution model for underwater turbulence channel and zero bore-sight model for pointing errors.
- We use the derived statistical results to develop analytical expressions for the outage probability, average BER, and ergodic capacity of the cascaded UWOC system.
- We also present an asymptotic analysis for both outage probability and average BER in the high SNR regime to derive the diversity order depicting the impact of system and channel parameters on the performance of the considered system.
- We use numerical and simulation analysis to validate our derived expressions and demonstrate the performance of the considered UWOC system for various parameters of interest.

II. SYSTEM MODEL

We consider a UWOC system by splitting the entire transmission channel in N distinct layers in succession, resulting in N vertical links, as shown in Fig 1. We use the non-coherent intensity modulation/direct detection (IM/DD) scheme, where the photodetector detects changes in the light intensity without employing a local oscillator. The heterodyne detection (HD) requires complex processing of mixing the received signal with a coherent signal produced by the local oscillator [16]. Thus, the received electrical signal for the transmitted signal *s* under additive white gaussian noise (AWGN) *w* with variance σ_w^2 can be expressed as

$$y = h_l h_c h_p s + w \tag{1}$$

where $h_l = e^{-\alpha l}$ is the atmospheric path gain with link distance l (in m) and extinction attenuation coefficient α , the term $h_c = \prod_{i=1}^{N} h_i$ (where $i = 1, 2, 3 \cdots N$) is the cascaded channel with h_i as the *i*-th layer of vertical link, and h_p models pointing errors. The fading coefficients h_i , $i = 1, 2, \cdots, N$ associated with the different layers are modeled using i.ni.d generalized Gamma random variables [2]:

$$f_{h_i}(h_i) = \frac{p_i}{a_i^{d_i} \Gamma\left(\frac{d_i}{p_i}\right)} h_i^{d_i - 1} \exp\left(-\left(\frac{h_i}{a_i}\right)^{p_i}\right), 0 \le h_i \le \infty$$
(2)

where $\Gamma(z) = \int_0^\infty x^{z-1} \exp(-x) dx$ denotes the Gamma function. Here, a_i , d_i , and p_i are distribution parameters for the *i*-th layer to model different oceanic turbulence scenarios, as given in [2] (see Table-I, Table-II, and Table-III). As such, $p_i = 1$ in (2) denotes a Gamma distribution representing a thermally uniform UWOC channel.

Assuming IM/DD technique and on-off keying (OOK) modulation with $x \in \{0, \sqrt{2}P_t\}$ and P_t as average transmitted optical, the instantaneous received electrical SNR is given by [17]

$$\gamma = \frac{P_t^2 h_l^2 h^2}{\sigma_w^2} = \gamma_0 h^2 \tag{3}$$

where $h = h_c h_p$ is the combined channel and $\gamma_0 = \frac{P_t^* h_l^2}{\sigma_w^2}$ is the average electrical SNR. Note that P_t^2 in (3) is attributed to the detection type IM/DD and becomes $\gamma = \frac{P_t h_l h}{\sigma_w^2}$ for the HD technique [16], [18].

The PDF of zero-boresight pointing errors fading h_p is given in [17]:

$$f_{h_p}(h_p) = \frac{\rho^2}{A_0^{\rho^2}} h_p^{\rho^2 - 1}, 0 \le h_p \le A_0$$
(4)

where $A_0 = \operatorname{erf}(\upsilon)^2$ with $\upsilon = \sqrt{\pi/2} r/\omega_z$, r is the aperture radius and ω_z is the beam width, and $\rho = \frac{\omega_{zeq}}{2\sigma_s}$ with ω_{zeq} as the equivalent beam width at the receiver and σ_s^2 as the variance of pointing errors displacement characterized by the horizontal sway and elevation.

III. PERFORMANCE ANALYSIS

In this section, we analyze the performance of a cascaded UWOC system by deriving exact expressions for the PDF and CDF of the SNR under the combined effect of oceanic turbulence and pointing errors and provide expressions for the outage probability, average BER, and ergodic capacity. The derived performance assessment can help network operators design efficient UWOC systems when the underwater transmission range is high.

A. PDF and CDF of SNR

Similar to [2], we assume that turbulence channels are independent for each layer to provide a tractable performance analysis. However, adjacent layers might become correlated if the layers are not sufficient apart, requiring rigorous analysis for the product of N correlated random variables.

Denoting $z_1 = x_1x_2$ as the product of two i.i.d. generalized Gamma distributions, we use [19, pp. 368, eq. 3.471.09] to get the PDF for two-layers UWOC system:

$$f_{z_1}(z) = \frac{2}{p} K_0\left(2\sqrt{\frac{z^p}{a^{2p}}}\right) \tag{5}$$

where K_0 denotes the Bessel function of the zeroth order. It can be seen that (5) does not represent a pattern comparing with (2), and thus cannot be generalized to derive the PDF for the product of N channels. In the following theorem, we use Mellin's inverse transform to provide PDF and CDF of the cascaded UWOC link under the combined effect of the oceanic turbulence and pointing errors under i.ni.d fading channels.

Theorem 1: The PDF and CDF of the end-to-end SNR for N cascaded channels distributed according to generalized Gamma channel combined with pointing errors are given as

$$f_{\gamma}(\gamma) = \frac{\rho^2}{2\gamma} \prod_{i=1}^{N} \frac{1}{\Gamma\left(\frac{d_i}{p_i}\right)} H_{1,N+1}^{N+1,0} \\ \left[\left\{ \left(\frac{d_{i_1}}{p_{i_1}}, \frac{1}{p_{i_1}}\right) \right\}_{i_1=1}^{N}, (\rho^2, 1) \right| \prod_{i_2=1}^{N} \left(\frac{1}{a_{i_2}A_0} \sqrt{\frac{\gamma}{\gamma_0}}\right) \right]$$
(6)

$$F_{\gamma}(\gamma) = \prod_{i=1}^{N} \frac{\rho^{2}}{\Gamma\left(\frac{d_{i}}{p_{i}}\right)} H_{2,N+2}^{N+1,1} \\ \left[\begin{pmatrix} (1,1), (1+\rho^{2},1) \\ \left\{ \left(\frac{d_{i_{1}}}{p_{i_{1}}}, \frac{1}{p_{i_{1}}}\right) \right\}_{i_{1}=1}^{N}, (\rho^{2},1), (0,1) \right| \prod_{i_{2}=1}^{N} \left(\frac{1}{a_{i_{2}}A_{0}} \sqrt{\frac{\gamma}{\gamma_{0}}} \right) \right]$$
(7)

Proof: See Appendix A.

In what follows, we use Theorem 1 to develop performance metrics of the UWOC system.

B. Outage Probability

Outage probability is a performance metric that demonstrates the effect of fading channels on the communication systems. It is defined as the probability that the instantaneous SNR falls below a certain threshold γ_{th} and is given as

$$P_{\text{out}}(\gamma_{\text{th}}, \gamma_0) = P(\gamma < \gamma_{\text{th}}) = F_{\gamma}(\gamma_{\text{th}})$$
(8)

Substituting (7) in (8) yields

$$P_{\text{out}} = \prod_{i=1}^{N} \frac{\rho^2}{\Gamma\left(\frac{d_i}{p_i}\right)} H_{2,N+2}^{N+1,1} \\ \left[\begin{pmatrix} (1,1), (1+\rho^2, 1) \\ \left\{ \left(\frac{d_{i_1}}{p_{i_1}}, \frac{1}{p_{i_1}}\right) \right\}_{i_1=1}^N, (\rho^2, 1), (0,1) \right| \prod_{i_2=1}^{N} \left(\frac{1}{a_{i_2}A_0} \sqrt{\frac{\gamma_{th}}{\gamma_0}}\right) \right]$$
(9)

We use [20, eq. 1.8.4] to develop an asymptotic expression for the outage probability in the high SNR regime $\gamma_0 \rightarrow \infty$:

$$P_{\text{out}}^{\infty} = \prod_{i=1}^{N} \frac{\rho^2}{\Gamma\left(\frac{d_i}{p_i}\right)} \sum_{k=1}^{N+1} \frac{\left(\frac{1}{a_i A_0} \sqrt{\frac{\gamma_{th}}{\gamma_0}}\right)^{\frac{b_k}{\beta_k}} \prod_{j=1, j \neq k}^{N+1} \Gamma\left(b_j - b_k \frac{\beta_j}{\beta_k}\right) \Gamma\left(\frac{b_k}{\beta_k}\right)}{\beta_k \Gamma\left(1 + \rho^2 - \frac{b_k}{\beta_k}\right) \Gamma\left(1 + \frac{b_k}{\beta_k}\right)}$$
(10)

where $b_j = b_k = \{\frac{d_i}{p_i}, \rho^2\}$ and $\beta_j = \beta_k = \{\frac{1}{p_i}, 1\}$. Combining the exponent of γ_0 in (10), the diversity order of the considered system can be expressed as $DO_{\text{out}} = \sum_{i=1}^N \min\{\frac{d_i}{2}, \frac{\rho^2}{2}\}$.

The diversity order reveals that the outage probability at a high SNR is dependent only on the parameter d of the oceanic turbulence channel and pointing errors. A sufficient higher beam-width can make the diversity order independent of pointing errors.

C. Average BER

In this subsection, we analyze the average BER performance of the cascaded UWOC system. The average BER can be obtained as [21]:

$$\bar{P}_e = \frac{\delta}{2\Gamma(\phi)} \sum_{n=1}^{M} q_n^{\phi} \int_0^{\infty} \gamma^{\phi-1} \exp(-q_n \gamma) F_{\gamma}(\gamma) d\gamma \qquad (11)$$

where the set $\{M, \delta, \phi, q_n\}$ can specify a variety of modulation schemes. Using (7) and substituting $\exp(-q_n\gamma) = G_{0,1}^{1,0} \begin{bmatrix} -\\ 0 \end{bmatrix} q_n \gamma$ in (11) and representing the Meijer G-function into Fox-H function, we express (11) as

$$\bar{P}_{e} = \frac{\delta\rho^{2}}{2\Gamma(\phi)} \sum_{n=1}^{M} q_{n}^{\phi} \prod_{i=1}^{N} \frac{1}{\Gamma\left(\frac{d_{i}}{p_{i}}\right)} \\ \times \int_{0}^{\infty} \gamma^{\phi-1} H_{0,1}^{1,0} \begin{bmatrix} -\\ (0,1) \end{bmatrix} q_{n} \gamma \end{bmatrix} H_{2,N+2}^{N+1,1} \\ \left[\begin{pmatrix} (1,1), (1+\rho^{2},1) \\ \left\{ \left(\frac{d_{i_{1}}}{p_{i_{1}}}, \frac{1}{p_{i_{1}}}\right) \right\}_{i_{1}=1}^{N}, (\rho^{2},1), (0,1) \end{bmatrix} \prod_{i_{2}=1}^{N} \left(\frac{1}{a_{i_{2}}A_{0}} \sqrt{\frac{\gamma}{\gamma_{0}}} \right) \right] d\gamma$$

$$(12)$$

We apply the definite integral of product of two Fox-H functions in [20, eq. 2.8.4] to express the average BER as

$$\bar{P}_{e} = \frac{\delta\rho^{2}}{2\Gamma(\phi)} \sum_{n=1}^{M} \prod_{i=1}^{N} \frac{1}{\Gamma\left(\frac{d_{i}}{p_{i}}\right)} H_{3,N+2}^{N+1,2} \\ \left[(1,1), (1-\phi, \frac{1}{2}), (1+\rho^{2}, 1) \\ \left\{ \left(\frac{d_{i_{1}}}{p_{i_{1}}}, \frac{1}{p_{i_{1}}}\right) \right\}_{i_{1}=1}^{N}, (\rho^{2}, 1), (0, 1) \right| \prod_{i_{2}=1}^{N} \frac{1}{a_{i_{2}}A_{0}\sqrt{q_{n}\gamma_{0}}} \right]$$

$$(13)$$

Transmitted optical power	P_t	-10 to 55 dBm
AWGN variance	σ_w^2	$10^{-14} \text{ A}^2/\text{GHz}$
Total link distance	l	50 m
Extinction coefficient	α	0.056 [2]
Pointing errors parameters	A_0, ρ^2	$0.0032, \{1, 6\}$
	$\{a_i\}_{i=1}^5$	$\{0.6302, 1.0750, 1.0173, 0.7598, 1.0990\}$
Generalized Gamma parameters [2]	${d_i}_{i=1}^5$	$\{1.1780, 3.2048, 1.6668, 2.3270, 4.5550\}$
	${p_i}_{i=1}^5$	$\{0.8444, 2.9222, 1.0380, 1.4353, 4.6208\}$
Modulation parameters	M, δ, ϕ, q_n	$1, 1, \frac{1}{2}, \frac{1}{2}$

TABLE I Simulation Parameters

We apply the series expansion in [20, eq. 1.8.4] to get an asymptotic expression for the average BER at a high SNR $\gamma_0 \rightarrow \infty$:

$$\bar{P}_{e}^{\infty} = \frac{\delta\rho^{2}}{2\Gamma(p)} \sum_{n=1}^{M} \prod_{i=1}^{N} \frac{1}{\Gamma\left(\frac{d_{i}}{p_{i}}\right)} \sum_{k=1}^{N+1} \left(\frac{1}{a_{i}A_{0}\sqrt{q_{n}\gamma_{0}}}\right)^{\frac{b_{k}}{\beta_{k}}} \prod_{j=1, j\neq k}^{N+1} \Gamma\left(b_{j}-b_{k}\frac{\beta_{j}}{\beta_{k}}\right) \Gamma\left(\frac{b_{k}}{\beta_{k}}\right) \Gamma\left(\phi+\frac{b_{k}}{2\beta_{k}}\right)}{\beta_{k}\Gamma\left(1+\rho^{2}-\frac{b_{k}}{\beta_{k}}\right) \Gamma\left(1+\frac{b_{k}}{\beta_{k}}\right)}$$
(14)

where $b_j = b_k = \{\frac{d_i}{p_i}, \rho^2\}$ and $\beta_j = \beta_k = \{\frac{1}{p_i}, 1\}$. Similar to the outage probability, the diversity order for the average BER can be obtained as $DO_{\bar{P}_e} = \sum_{i=1}^N \min\{\frac{d_i}{2}, \frac{\rho^2}{2}\}$. Thus turbulent channl parameter d_i and pointing error parameter ρ^2 determines the diversity order of the system. Thus, the effect of pointing errors can be mitigated using a sufficient higher beam-width.

D. Ergodic Capacity

The ergodic capacity C is an important performance metric for the design of communication systems, and it can be defined as [22]:

$$\bar{C} = \int_{0}^{\infty} \log_2(1 + \kappa \gamma) f_{\gamma}(\gamma) d\gamma$$
(15)

where $\kappa = \frac{e}{2\pi}$ for IM/DD and $\kappa = 1$ for the HD technique. Using (6) and substituting $\log_2(1 + \kappa\gamma) = 1.44G_{2,2}^{1,2} \begin{bmatrix} 1, 1\\ 1, 0 \end{bmatrix} \kappa\gamma$ in (15) and representing Meijer-G function into Fox-H function, we get

$$\bar{C} = 0.72\rho^2 \prod_{i=1}^{N} \frac{1}{\Gamma\left(\frac{d_i}{p_i}\right)} \int_0^\infty \gamma^{-1} H_{2,2}^{1,2} \begin{bmatrix} (1,1), (1,1) \\ (1,1), (0,1) \end{bmatrix} \kappa \gamma \\ H_{1,N+1}^{N+1,0} \left[\left\{ \begin{pmatrix} (1+\rho^2,1) \\ \left\{ \left(\frac{d_{i_1}}{p_{i_1}}, \frac{1}{p_{i_1}}\right) \right\}_{i_1=1}^N (\rho^2,1) \right| \prod_{i_2=1}^N \left(\frac{1}{a_{i_2}A_0} \sqrt{\frac{\gamma}{\gamma_0}} \right) \right] d\gamma$$
(16)

Thus, we apply the definite integral of product of two Fox-H functions in [20, eq. 2.8.4] to get an analytical expression for

the ergodic capacity of the considered UWOC system:

$$\bar{C} = 0.72\rho^2 \prod_{i=1}^{N} \frac{1}{\Gamma\left(\frac{d_i}{p_i}\right)} H_{3,N+3}^{N+3,1} \\ \left[\begin{pmatrix} (0,\frac{1}{2}), (1,\frac{1}{2}), (1+\rho^2,1) \\ \left\{ \left(\frac{d_{i_1}}{p_{i_1}}, \frac{1}{p_{i_1}}\right) \right\}_{i_1=1}^{N}, (\rho^2,1)(0,\frac{1}{2}), (0,\frac{1}{2}) \right| \prod_{i_2=1}^{N} \frac{(\kappa\gamma_0)^{-\frac{1}{2}}}{a_{i_2}A_0} \right]$$
(17)

IV. SIMULATION AND NUMERICAL ANALYSIS

In this section, we demonstrate the performance of vertical UWOC system over-generalized Gamma fading with pointing errors. We use Monte-Carlo (MC) simulation (averaged over 10^7 channel realizations) to validate the derived analytical expressions. We use standard inbuilt MATLAB and MATHE-MATICA libraries to calculate Meijer-G and Fox-H function, respectively. Since there is no measurement data to confirm the variation of distribution parameters with distance, we illustrate the performance by considering vertical underwater link length l = 50 m with N = 5 layers and the thickness of each layer is assumed to be 10 m. Other simulation parameters are listed in Table I.

First, we demonstrate the outage probability performance of the UWOC system in Fig. 2. It can be seen from the figure that the outage performance of the system improves with an increase in the values of generalized Gamma distribution parameters (a, d, and p) and a decrease in pointing errors (i.e., higher ρ^2). In the first plot of Fig. 2, we consider the pointing errors parameter $(\rho^2 = 1)$ and the generalized Gamma distribution parameters (a, d, and p), as given in Table I. The diversity order $DO_{\text{out}} = \sum_{i=1}^{N} \min\{\frac{d_i}{2}, \frac{\rho^2}{2}\}$ for the top and middle plots in Fig. 2 are given by $\min\{6.4658, 0.5\}$ and $\min\{7.1822, 0.5\}$, respectively. It can be clearly observed that the diversity order is dependent on the pointing error parameter (ρ^2) since the slope does not change with the oceanic channel parameter d_i . Further, in the third plot, the diversity order becomes $\min\{7.1822, 3\}$, demonstrating a change of slope with ρ^2 , thus confirming our diversity order analysis.

Next, we present the average BER performance of the vertical UWOC system in Fig. 3. The average BER of the system follows a similar trend as observed for the outage



Fig. 2. Outage probability performance for a 5-layer UWOC.



Fig. 3. Average BER performance of UWOC with $\rho^2 = 6$.

probability with turbulent channel and pointing error parameters. It is evident from the plots that the average BER of the system improves by almost ten times if the turbulent channel parameter d_i increases from 1.1780 to 2.6108 at an average SNR of 80 dB. Further, the diversity order follows a similar analysis as that of the outage probability, which can be confirmed by observing the slope change among the plots.

Finally, we demonstrate the effects of the different vertical link impairments on the UWOC system by plotting the ergodic capacity versus transmit power, as shown in Fig. 4. The figure shows that the ergodic capacity increases by almost 3 bits/sec/Hz if the effect of pointing error decreases by increasing the parameter ρ^2 from 1 to 6 for the given turbulent parameters. It can also been seen that the ergodic capacity increases by almost 1 bits/sec/Hz if we change the oceanic turbulence parameters ($a_3 = 0.3557$, $d_3 = 5.0965$, and $p_3 = 1.296$) for the both $\rho^2 = 1$ and $\rho^2 = 6$.

In all the above plots (Fig. 2, Fig. 3, and Fig. 4) we verify



Fig. 4. Ergodic capacity performance of UWOC system.

our derived results using MC simulation, demonstrating that our derived analytical expressions (depicted as 'Analysis') of outage probability, average BER, and ergodic capacity have an excellent match with MC simulations (depicted as 'Simulation'). Further, the asymptotic expression of outage probability and average BER can be see to converge with analysis and simulation results in the high SNR regime.

V. CONCLUSION

In this paper, we analyzed the performance of the UWOC system considering the vertical underwater link as a multilayer cascaded channel, each distributed according to i.ni.d. generalized Gamma random variables. We analyzed the system performance by deriving analytical expressions of the PDF and CDF of the end-to-end SNR, and developed outage probability, average BER, and ergodic capacity under the combined effect of cascaded oceanic turbulence and pointing errors in terms of Meijer's G and Fox's H functions. We also provided the asymptotic expressions using Gamma functions for the outage probability and average BER to determine the diversity order of the considered system. We validated our derived expressions using Monte-Carlo simulations and demonstrated the performance of UWOC under various channel conditions. The reported results may provide better performance assessment and design criteria of optical communication for higher underwater transmission links.

APPENDIX A

To derive (6) and (7), first, we need the PDF and CDF of N cascaded channels $h_c = \prod_{i=1}^N h_i$. We use the inverse Mellin transform to find the PDF of h_c . If $\mathbb{E}[X^n]$ denotes the *n*-th moment, where $\mathbb{E}[\cdot]$ denotes the expectation operator, then the inverse Mellin transform results the PDF of a random variable X as

$$f_X(x) = \frac{1}{2\pi i x} \int_{\zeta - i\infty}^{\zeta + i\infty} x^{-n} \mathbb{E}[X^n] dn$$
(18)

where $\zeta - i\infty$ to $\zeta + i\infty$ denotes the line integral. It should be mentioned that Mellin transform has been used to analyze the product of N random variable for different applications [18], [23]–[25]. The *n*-th order moment for h_c is derived as

$$\mathbb{E}[h_c^n] = \prod_{i=1}^N \int_0^\infty h_i^n f_{h_i}(h_i) dh_i$$
(19)

Substituting (2) in (19) and applying the identity $\int_0^\infty x^m e^{-\beta x^n} dx = \frac{\Gamma\left(\frac{m+1}{n}\right)}{n\beta^{\frac{m+1}{n}}}$ [19, pp. 347, eq. 3.381.10], we get the *n*-th moment of h_c as:

$$\mathbb{E}[h_c^n] = \prod_{i=1}^N \frac{\Gamma\left(\frac{n+d_i}{p_i}\right)}{a_i^{-n} \Gamma\left(\frac{d_i}{p_i}\right)}$$
(20)

Next, we use (20) in (18) and apply the definition of Fox H-function to get the PDF of h_c as

$$f_{h_c}(h_c) = \prod_{i=1}^{N} \frac{1}{h_c \Gamma\left(\frac{d_i}{p_i}\right)}$$
$$H_{0,N}^{N,0} \begin{bmatrix} - \\ \left\{ \left(\frac{d_{i_1}}{p_{i_1}}, \frac{1}{p_{i_1}}\right) \right\}_{i_1=1}^{N} \middle| \prod_{i_2=1}^{N} \left(\frac{h_c}{a_{i_2}}\right) \end{bmatrix}$$
(21)

Next, we use the theory of product distribution [26] to get the PDF of the combined channel $h = h_c h_p$ as

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$$f_{h}(h) = \int_{0}^{A_{0}} \frac{1}{h_{p}} f_{h_{p}}(h_{p}) f_{h_{c}}\left(\frac{h}{h_{p}}\right) dh_{p}$$
(22)

Using (4) and (21) in (22) and applying the definition of Fox H-function, we get

$$f_{h}(h) = \frac{\rho^{2}}{h} \prod_{i=1}^{N} \frac{1}{\Gamma\left(\frac{d_{i}}{p_{i}}\right)} H_{1,N+1}^{N+1,0}$$

$$\begin{bmatrix} (1+\rho^{2},1) \\ \left\{ \left(\frac{d_{i_{1}}}{p_{i_{1}}}, \frac{1}{p_{i_{1}}}\right) \right\}_{i_{1}=1}^{N}, (\rho^{2},1) \middle| \prod_{i_{2}=1}^{N} \left(\frac{h}{a_{i_{2}}A_{0}}\right) \end{bmatrix}$$
(23)

Finally, we use the transformation of random variable $\gamma = \gamma_0 h^2$ to get the PDF SNR in (6). To find the CDF of SNR under the combined channel, we use (6) in $F_{\gamma}(\gamma) = \int_0^{\gamma} f(\gamma) d\gamma$ and apply the definition of Fox H-function with inner integral $\int_0^{\gamma} \gamma^{-\frac{u}{2}-1} d\gamma = \frac{\gamma^{-\frac{u}{2}}}{-\frac{u}{2}} = \frac{2\gamma^{-\frac{u}{2}}\Gamma(-u)}{\Gamma(1-u)}$ to get the CDF of the SNR in (7), which concludes the proof of Theorem 1.

REFERENCES

- Z. Zeng, S. Fu, H. Zhang, Y. Dong, and J. Cheng, "A survey of underwater optical wireless communications," *IEEE Communications Surveys Tutorials*, vol. 19, no. 1, pp. 204–238, 2017.
- [2] M. V. Jamali, A. Mirani, A. Parsay, B. Abolhassani, P. Nabavi, A. Chizari, P. Khorramshahi, S. Abdollahramezani, and J. A. Salehi, "Statistical studies of fading in underwater wireless optical channels in the presence of air bubble, temperature, and salinity random variations," *IEEE Transactions on Communications*, vol. 66, no. 10, pp. 4706–4723, 2018.
- [3] X. Sun, C. H. Kang, M. Kong, O. Alkhazragi, Y. Guo, M. Ouhssain, Y. Weng, B. H. Jones, T. K. Ng, and B. S. Ooi, "A review on practical considerations and solutions in underwater wireless optical communication," *Journal of Lightwave Technology*, vol. 38, no. 2, pp. 421–431, 2020.
- [4] H. Gerçekcioğlu, "Bit error rate of focused gaussian beams in weak oceanic turbulence," *Journal of the Optical Society of America A*, vol. 31, no. 9, pp. 1963–1968, Sep 2014.

- [5] M. V. Jamali, F. Akhoundi, and J. A. Salehi, "Performance characterization of relay-assisted wireless optical CDMA networks in turbulent underwater channel," *IEEE Transactions on Wireless Communications*, vol. 15, no. 6, pp. 4104–4116, 2016.
- [6] M. Cheng, L. Guo, J. Li, and Y. Zhang, "Channel capacity of the OAMbased free-space optical communication links with bessel–gauss beams in turbulent ocean," *IEEE Photonics Journal*, vol. 8, no. 1, pp. 1–11, 2016.
- [7] M. Singh, H. S. Gill, and M. L. Singh, "Experimental evaluation of performance of underwater free space optical link in the presence of air bubbles," in 2018 6th Edition of International Conference on Wireless Networks Embedded Systems (WECON), 2018, pp. 58–62.
 [8] M. V. Jamali and J. A. Salehi, "On the BER of multiple-input multiple-
- [8] M. V. Jamali and J. A. Salehi, "On the BER of multiple-input multipleoutput underwater wireless optical communication systems," in 2015 4th International Workshop on Optical Wireless Communications (IWOW), 2015, pp. 26–30.
- [9] M. V. Jamali, A. Chizari, and J. A. Salehi, "Performance analysis of multi-hop underwater wireless optical communication systems," *IEEE Photonics Technology Letters*, vol. 29, no. 5, pp. 462–465, 2017.
- [10] A. Tabeshnezhad and M. A. Pourmina, "Outage analysis of relayassisted underwater wireless optical communication systems," *Optics Communications*, vol. 405, pp. 297–305, Dec. 2017.
- [11] M. V. Jamali, J. A. Salehi, and F. Akhoundi, "Performance studies of underwater wireless optical communication systems with spatial diversity: MIMO scheme," *IEEE Transactions on Communications*, vol. 65, no. 3, pp. 1176–1192, 2017.
- [12] M. Elamassie, F. Miramirkhani, and M. Uysal, "Performance characterization of underwater visible light communication," *IEEE Transactions* on Communications, vol. 67, no. 1, pp. 543–552, 2019.
- [13] M. Elamassie and M. Uysal, "Performance characterization of vertical underwater VLC links in the presence of turbulence," in 2018 11th International Symposium on Communication Systems, Networks Digital Signal Processing (CSNDSP), 2018, pp. 1–6.
- [14] M. Elamassie, S. M. Sait, and M. Uysal, "Underwater visible light communications in cascaded gamma-gamma turbulence," in 2018 IEEE Globecom Workshops (GC Wkshps), 2018, pp. 1–6.
- [15] M. Elamassie and M. Uysal, "Vertical underwater vlc links over cascaded gamma-gamma turbulence channels with pointing errors," in 2019 IEEE International Black Sea Conference on Communications and Networking (BlackSeaCom), 2019, pp. 1–5.
 [16] H. Melchior, M. Fisher, and F. Arams, "Photodetectors for optical manufacture and the sea conference on the second second
- [16] H. Melchior, M. Fisher, and F. Arams, "Photodetectors for optical communication systems," *Proceedings of the IEEE*, vol. 58, no. 10, pp. 1466–1486, 1970.
- [17] A. A. Farid and S. Hranilovic, "Outage capacity optimization for free-space optical links with pointing errors," *Journal of Lightwave Technology*, vol. 25, no. 7, pp. 1702–1710, July 2007.
- [18] V. K. Chapala and S. M. Zafaruddin, "Unified performance analysis of reconfigurable intelligent surface empowered free-space optical communications," *IEEE Transactions on Communications*, pp. 1–1, 2021.
- [19] D. Zwillinger, Table of integrals, series, and products. Elsevier, 2014.
 [20] A. Kilbas, Analytical methods and special functions, H-Transforms: Theory and Applications. New York, NY, USA: Taylor and Francis, 2004.
- [21] E. Zedini, H. Soury, and M.-S. Alouini, "Dual-hop FSO transmission systems over Gamma–Gamma turbulence with pointing errors," *IEEE Transactions on Wireless Communications*, vol. 16, no. 2, pp. 784–796, 2017.
- [22] H. E. Nistazakis, E. A. Karagianni, A. D. Tsigopoulos, M. E. Fafalios, and G. S. Tombras, "Average capacity of optical wireless communication systems over atmospheric turbulence channels," *Journal of Lightwave Technology*, vol. 27, no. 8, pp. 974–979, 2009.
- [23] L. Kong, G. Kaddoum, and D. Costa, "Cascaded α - μ fading channels: Reliability and security analysis," *IEEE Access*, vol. PP, pp. 41978 – 41992, 05 2018.
- [24] V. K. Chapala and S. M. Zafaruddin, "Exact analysis of RIS-Aided THz wireless systems over α-μ fading with pointing errors," *IEEE Communications Letters*, vol. 25, no. 11, pp. 3508–3512, 2021.
 [25] P. Bhardwaj and S. M. Zafaruddin, "On the performance of multihop
- [25] P. Bhardwaj and S. M. Zafaruddin, "On the performance of multihop THz wireless system over mixed channel fading with shadowing and antenna misalignment," *CoRR*, vol. abs/2110.15952, 2021. [Online]. Available: https://arxiv.org/abs/2110.15952
- [26] A. Papoulis and U. Pillai, Probability, random variables and stochastic processes, 4th ed. McGraw-Hill, Nov. 2001.