# Energy-Efficiency based Resource Allocation for the Orthogonal Multi-user Channel

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***Abstract*—Energy efficiency (EE) is emerging as a key design criterion for both power limited, i.e. mobile devices, and power-unlimited, i.e. cellular networks, applications. Whereas, resource allocation is a well-known technique for improving the performance of communication systems. In this paper, we design a simple and optimal EE-based resource allocation method for the orthogonal multi-user channel by adapting the transmit power and rate to the channel condition such that the energy-per-bit consumption is minimized. We present our EE framework, i.e. EE metric and node power consumption model, and utilize it for formulating our EE-based optimization problem with or without constraint. In both cases, we derive explicit formulations of the optimal energy-per-bit consumption as well as optimal power and rate for each user. Our results indicate that EE-based allocation can substantially reduce the consumed power and increase the EE in comparison with spectral efficiency-based allocation.**

***Index Terms*—Energy efficiency, resource allocation, orthogonal multi-user channel, realistic power model, single cell.**

## I. Introduction

In the current context of growing energy demand and increasing energy price, energy efficiency (EE) is emerging as a key design criterion for creating reliable and low-power consumption communication systems. In the recent past, EE has already been comprehensively investigated but mainly through the prism of power-limited applications such as battery-driven systems [1], e.g. mobile terminal, underwater acoustic telemetry [2], or wireless ad-hoc and sensor networks [3], [4]. This research topic is now being revisited for unlimited-power applications such as cellular networks [5], [6]. This shift of focus from power-limited to power-unlimited applications is driven by two factors; on the one hand, ICT as a whole aims at reducing its carbon footprint; on the other hand, network operators strive to curb their operational costs.

Resource allocation and link adaptation have been extensively utilized in the past for improving the peak rate or spectral efficiency (SE) performance of communication systems, but without any consideration about the energy consumption. With the emergence of the EE as a key system design criterion, EE-based resource allocation is becoming a popular research topic [7]–[9]. For instance, in [8], a EE-based link adaptation method has been developed for saving user equipment (UE) energy in the uplink of the orthogonal multi-user channel (OMC), i.e. an orthogonal frequency multiplexing (OFDM) transmission over a frequency-selective channel. This work assumed a linear power consumption model (PCM) that served as a basis for its EE-based objective function. After proving the convexity of this objective function, a gradient search method was used to solve an EE-based resource allocation problem subject to a rate or power constraint. Then, this method has been refined in [9] by considering a more realistic assumption on the circuit power and amplifier efficiency at the UE. Meanwhile, the authors in [10] have recently introduced a framework for optimizing the EE in the downlink of the OMC channel when considering an elastic traffic scenario.

In this paper, we revisit the work of [8] and design a simple algorithm that optimally allocates resources in terms of EE over the OMC channel when considering the total energy consumed within the cell. Our main contribution is the derivation of explicit expressions for the optimal energy-per-bit consumption as well as optimal power and rate for each user. In turn, these expressions have been used for solving the EE-based resource allocation problem subject to a power constraint over the OMC channel. Note that the simplicity of our algorithm is equivalent to that of the water-filling algorithm in SE-based optimization. Whereas, it was thought in [8] that this EE-based problem cannot be solved directly but only via an iterative method based on gradient search, which is clearly far more computationally demanding than our method. Furthermore, having an explicit formulation of the optimal power allocation allows us to prove that equal power allocation is both the most energy and spectral efficient strategy when the channel gain-to-noise ratio is high. The rest of the paper is organized as follows. Section II introduces the conventional OMC channel along with its per-user power and SE expressions. It also details our EE framework, i.e. PCMs for the base station (BS) and UE nodes of [6] and [8], respectively, as well as the Joule-per-bit metric that acts as an objective function for our EE-based optimization problem. This objective function is reformulated solely as a function of SE in Section III and its convexity is discussed. We then derive the explicit formulation of the users' optimal power and rate, which are utilized for solving analytically our EE-based optimization problem. In Section IV, we show the accuracy of our method by comparing it against simulation results and graphically validate our assertion that equal power allocation is EE-optimal at high channel gain-to-noise ratio. In addition, we compare our EE-based resource allocation method against the traditional SE-based method in a realistic scenario. Results show that our method reduces significantly the transmit power, which in turn increases EE, in comparison with the SE-based approach. Conclusions are finally drawn in Section V.

## II. System and Power Consumption Models

### A. System Model

We consider the OMC channel, where $K$ parallel subchannels are used for transmission and each of them has a different channel gain, i.e. equivalent to a closed-loop multi-input multi-output (MIMO) channel or an OFDM transmission over a frequency-selective channel. Moreover, we assume that each orthogonal subchannel is affected by block fading and that perfect channel state information is available at both the transmitter and receiver ends, such that the channel capacity per unit bandwidth of the $k$-th user can be expressed as [11]

$$\mathcal{C}_k = \log_2\left(1+\frac{g_k p_k}{N\Gamma}\right). \tag{1}$$

Conversely from (2), $p_k$ can be expressed as

$$p_k = \left(2^{\mathcal{C}_k}-1\right) g_k^{-1} N\Gamma, \tag{2}$$

where $g_k$ is the $k$-th user subchannel gain and $\Gamma$ denotes the SNR gap between the channel capacity and the performance of a practical coding and modulation scheme as in [9]. Consequently, the total transmit power over the OMC channel can be expressed as

$$P(\boldsymbol{\mathcal{C}}) = N\Gamma\sum_{k=1}^{K}\left(2^{\mathcal{C}_k}-1\right) g_k^{-1}, \tag{3}$$

where $\boldsymbol{\mathcal{C}} = [\mathcal{C}_1,\ldots,\mathcal{C}_K] \succeq 0$.

### B. Energy Efficiency Framework

In communication, the energy consumption is usually formulated in terms of the energy-per-bit metric, $E_b$. This metric indicates how much energy is consumed by the system for transmitting bits. In a single-cell system, it can simply be defined as the ratio of the total power consumption to the sum of all user rates within the cell, such that

$$\Sigma_{E_b}(\boldsymbol{\mathcal{C}}) = \frac{P_\Sigma(\boldsymbol{\mathcal{C}})}{W\sum_{k=1}^{K}\mathcal{C}_k}, \tag{4}$$

where $P_\Sigma(\boldsymbol{\mathcal{C}})$ is the cell total consumed power and $W$ is the bandwidth of each subchannel.

In order to model the cell total consumed power, one has to carefully model the power consumption of each node within the cell. In a classic cellular system, the two main types of nodes are the BS and UE, and the BS is clearly the more power demanding node. A BS itself is composed of various elements such as a transceiver, a power amplifier, a baseband interface, a signal processing unit, a power supply regulator, a cooling system, etc., and each of these elements consumes power in a different way, as it has been revealed by the comprehensive BS power consumption analysis of [6]. This work defines realistic BS PCMs, which take into account the non-linearity of the power amplifier, for five different types of BSs. However, it has also been shown in [6] that the relation between the relative radio frequency (RF) output power and BS power consumption is nearly linear, such that [6]

$$P_{\text{BS}} = \Delta_{P,\text{BS}}P + P_{0,\text{BS}}, \tag{5}$$

where $\Delta_{P,\text{BS}}$ and $P_{0,\text{BS}}$ are the slope and overhead power of the PCM, respectively. In addition, $P\in[0,P_{\max}]$ with $P_{\max}$ being the maximum RF output power, i.e. maximum transmit power. As far as the UE is concerned, it has been indicated in [8] that its total consumed power can be expressed as

$$P_{\text{UE}} = \Delta_{P,\text{UE}}P + P_{0,\text{UE}}, \tag{6}$$

which is a similar formulation as in (5). Consequently, the total consumed power for the downlink or uplink of a single-cell single-antenna multi-user system can be formulated as

$$P_\Sigma(\boldsymbol{\mathcal{C}}) = \Delta_P P(\boldsymbol{\mathcal{C}}) + P_{\text{c}}, \tag{7}$$

where $\Delta_P = \Delta_{P,\text{BS}}$, $P_{\text{c}} = P_{0,\text{BS}} + \varsigma K P_{0,\text{UE}}$ or $\Delta_P = \Delta_{P,\text{UE}}$, $P_{\text{c}} = KP_{0,\text{UE}} + \varsigma P_{0,\text{BS}}$ in the downlink or uplink scenario, respectively. Moreover, $\varsigma$ characterizes the ratio between transmission and reception overhead powers with $1 \geq \varsigma \geq 0$. Intuitively, less overhead power is necessary for receiving than for transmitting signals.

## III. Energy Consumption Minimization

Having defined $P_\Sigma$ in (7) as a function of the transmit power $P$ and having formulated $P$ in (3) as a function of each user SE, we can re-expressed $\Sigma_{E_b}$ in (4) solely as a function of the SE, as follows,

$$\Sigma_{E_b}(\mathbf{X}) = A\left(\sum_{k=1}^{K}\left(e^{X_k}-1\right)g_k^{-1}+B\right)\left(\sum_{k=1}^{K}X_k\right)^{-1}, \tag{8}$$

where $X_k = \ln(2)\mathcal{C}_k$ for $k\in\{1,\ldots,K\}$, $\mathbf{X} = [X_1,\ldots,X_K]\succeq 0$, $A = \ln(2)W^{-1}N\Gamma\Delta_P$ and $B = \frac{P_{\text{c}}}{N\Gamma\Delta_P}$. The function $\left(\sum_{k=1}^{K}X_k\right)^{-1}$ is clearly convex for $X_k \geq 0$ and, as long as $\sum_{k=1}^{K}X_k > 0$, then $\ln\left(\left(\sum_{k=1}^{K}X_k\right)^{-1}\right)$ is convex and, hence, $\left(\sum_{k=1}^{K}X_k\right)^{-1}$ is log-convex. Therefore, it implies that at least one $X_k$ variable must be strictly greater than zero or equivalently that there is always an active user in the system, i.e. the user with the largest channel gain. Similarly, it can easily be proved that $\left(\sum_{k=1}^{K}\left(e^{X_k}-1\right)g_k^{-1}+B\right)$ is also log-convex for $\mathbf{X}\succeq 0$. Since the product of two log-convex functions is a log-convex function, we can conclude that $\Sigma_{E_b}$ in (8) is a log-convex function and, hence, a convex function, as long as $\mathbf{X}\succeq 0$ and $X_i > 0$, where user $i$ is the user with the largest channel gain. Conversely, note that $1/\Sigma_{E_b}$ is log-concave and, hence, quasiconcave.

### A. Unconstrained Optimization

Knowing that $\Sigma_{E_b}$ is convex implies that there exits only one $\mathbf{X}$ value that minimizes $\Sigma_{E_b}(\mathbf{X})$ over its entire domain. Moreover, it is well-known that the $\mathbf{X}$ value, denoted $\mathbf{X}^\star$, minimizing $\Sigma_{E_b}$ satisfies $\nabla\Sigma_{E_b}(\mathbf{X}^\star) = \mathbf{0}$, which in turn implies after some simplifications that

$$\Sigma_{E_b}^\star = \Sigma_{E_b}(\mathbf{X}^\star) = Ae^{X_k^\star}g_k^{-1} \tag{9}$$

for any $k\in\mathcal{K}^\star$, where $\mathcal{K}^\star = \{j\in\mathcal{K}|X_j^\star > 0\}$ is the optimal set of active user indices, $\mathcal{K} = \{1,\ldots,K\}$ is the set of user indices and $\Sigma^\star_{E_b}$ is the optimal energy consumption per bit. For instance, in the 2-users case, (9) indicates that $\Sigma_{E_b}(\mathbf{X}^\star) = Ae^{X_1^\star} g_1^{-1}$ and $\Sigma_{E_b}(\mathbf{X}^\star) = Ae^{X_2^\star} g_2^{-1}$ such that $Ae^{X_1^\star} g_1^{-1} = Ae^{X_2^\star} g_2^{-1}$, or equivalently, $X_2^\star = X_1^\star + \ln(g_1^{-1} g_2)$ when assuming that $X_1^\star, X_2^\star > 0$. Similarly, we can obtain from (9) that

$$X_k^\star = X_1^\star + \ln(g_1^{-1} g_k) \tag{10}$$

in the $K$-users case for $k \in \mathcal{K}^\star \setminus \{1\}$ and when assuming that $X_1^\star > 0$. Inserting (10) into (8), we can re-express $\Sigma_{E_b}(\mathbf{X}^\star)$ solely as a function of $X_1^\star$ such that

$$\Sigma_{E_b}(X_1^\star) = A\left(K^\star e^{X_1^\star} g_1^{-1} - \alpha + B\right)\left(K^\star X_1^\star + \beta_1\right)^{-1}, \tag{11}$$

where $K^\star = |\mathcal{K}^\star|$ is the number of elements of $\mathcal{K}^\star$, $\alpha = \sum_{k\in\mathcal{K}^\star} g_k^{-1}$ and $\beta_k = \sum_{j\in\mathcal{K}^\star} \ln(g_k^{-1} g_j)$ for any $k \in \mathcal{K}^\star$. Then, by inserting (11) into (9), we can solve (9) solely as a function of $X_1^\star$ by means of the Lambert $W$ function such that

$$X_1^\star = W_0\left(\frac{(B-\alpha) g_1 e^{\frac{\beta_1}{K^\star}-1}}{K^\star}\right) + 1 - \frac{\beta_1}{K^\star}, \tag{12}$$

where $W_0$ denotes the real branch of the Lambert function [12]. In the general case, we obtain by using (10) that

$$X_k^\star = W_0\left(\frac{(B-\alpha)\left(\prod_{k\in\mathcal{K}^\star} g_k\right)^{\frac{1}{K^\star}} e^{-1}}{K^\star}\right) + 1 - \frac{\beta_k}{K^\star} \tag{13}$$

for $k \in \mathcal{K}^\star$. Note that $X_k^\star = 0$ for $k \in \mathcal{K} \setminus \mathcal{K}^\star$. Finally, we can obtain the value of $\Sigma^\star_{E_b}$ by inserting (13) into (9) and the values of the optimal power allocation per user, $p_k^\star$, by inserting (9) into (3) such that $p_k^\star = A^{-1} N\Gamma \Sigma^\star_{E_b} - g_k^{-1} N\Gamma$ for $k \in \mathcal{K}^\star$, or equivalently

$$p_k^\star = \left[A^{-1} N\Gamma \Sigma^\star_{E_b} - g_k^{-1} N\Gamma\right]_+ \tag{14}$$

for $k \in \mathcal{K}$, where $[x]_+ = \max\{0, x\}$.

In order to obtain the optimal $X_k^\star$ values in (13), one has first to obtain $\mathcal{K}^\star$. Let $\pi$ be the user index order, with $\pi = (\pi_1, \ldots, \pi_K)$ denotes a permutation of $\mathcal{K}$, such that user $\pi_1$ and $\pi_K$ are the users with the largest and smallest channel gains, respectively. Consequently, $X_{\pi_1} > 0$, which in turn implies that $X^\star_{\pi_1} > 0$ such that $\pi_1 \in \mathcal{K}^\star$ and $\mathcal{K}^\star$ has at least one element. Moreover, one can use the following inequality for obtaining the other elements

$$B > \sum_{k=1}^{U} g_{\pi_k}^{-1} - U\left(\prod_{k=1}^{U} g_{\pi_k}\right)^{-\frac{1}{U}}, \tag{15}$$

which is a direct consequence of the fact that the domain of $W_0$ is lower bounded by $-e^{-1}$ in (13). Starting from $U = K$ and decrementing $U$ by 1 as long as (15) does not hold, we obtain a trimmed set of indices $\mathcal{K} \setminus \{\pi_U, \ldots, \pi_K\}$, which will be further trimmed by removing the user indices for which the inequality $X_k^\star > 0$ does not hold in (13). Note that (15) always holds for $U = 1$.

As far as the optimization of the sum-rate over the OMC is concerned, it is well-known that the optimal SE-based power allocation strategy is obtained via water-filling [13] such that

$$p_k^\star = \left[(\nu^\star)^{-1} - g_k^{-1} N\Gamma\right]_+, \tag{16}$$

where $(\nu^\star)^{-1}$ is the water level. Hence, equal power allocation is the most spectral efficient power allocation when $g_k^{-1} N\Gamma \ll 1$, or equivalently, when the channel gain-to-noise ratio is high, such that $p_k^\star = \frac{P_{\max}}{K}$. Similarly, equation (14) provides a valuable insight on the optimal EE-based power allocation. It clearly shows that equal power allocation is also the most energy efficient power allocation when the channel gain-to-noise ratio is high, such that $p_k^\star = A^{-1} N\Gamma \Sigma^\star_{E_b}$, which reverts to $p_k^\star = \frac{P_{\max}}{K}$ when $\Sigma^\star_{E_b} \geq A P_{\max}(KN\Gamma)^{-1}$.

### B. Constrained Optimization

In the previous section, we have derived explicit expressions of the optimal users' rate and power that minimize the energy consumption per bit without constraint. Here, we generalize these expressions for the case of a total power constraint. Let us first define the EE-based optimization problem subject to a sum-power constraint as

$$\begin{aligned} \min_{\mathbf{X}} \quad & \Sigma_{E_b}(\mathbf{X}) \\ \text{s.t.} \quad & X_k \geq 0, \forall k \in \mathcal{K}; X_{\pi_1} > 0, \\ & P(\mathbf{X}) \leq P_{\max}. \end{aligned} \tag{17}$$

We know from (14) that if $N\Gamma(KA^{-1}\Sigma^\star_{E_b} - \alpha) < P_{\max}$ then $P(\mathbf{X}^\star) < P_{\max}$ and the optimal values for $\mathbf{X}$, $\mathbf{p} = [p_1, \ldots, p_K]$ and $\Sigma_{E_b}$ can be obtained from the previous section. However, if $P(\mathbf{X}^\star) \geq P_{\max}$, we can define the Lagrangian associated with the problem in (17) as

$$\mathcal{L}(\mathbf{X}, \lambda) = \Sigma_{E_b}(\mathbf{X}) + \lambda(P(\mathbf{X}) - P_{\max}). \tag{18}$$

Then, using the KKT conditions, i.e. solving $\nabla\mathcal{L}(\mathbf{X}^\star, \lambda^\star) = \mathbf{0}$, we obtain after some simplifications that

$$\Sigma^\star_{E_b} = e^{X_k^\star} g_k^{-1}\left[A + \lambda\left(\sum_{k=1}^{K} X_k\right) N\Gamma\right], \tag{19}$$

which in turn yields the same relation between $X_1^\star$ and $X_k^\star$ as in (10) for any $k \in \mathcal{K}^\star \setminus \{1\}$. Thus, we can obtain the optimal power allocation by inserting (10) into (4) such that

$$P_{\max} = P(\boldsymbol{\mathcal{C}}) = N\Gamma(K^\star e^{X_1^\star} g_1^{-1} - \alpha), \tag{20}$$

which in turn more generally implies that

$$X_k^\star = \ln\left(\frac{g_k}{K^\star}\left[\frac{P_{\max}}{N\Gamma} + \alpha\right]\right) \tag{21}$$

for $k \in \mathcal{K}^\star$ and $X_k^\star = 0$ for $k \in \mathcal{K} \setminus \mathcal{K}^\star$. Moreover, $p_k^\star$ can simply be obtained by inserting (21) in (2). The optimal energy consumption per bit value is then given by

$$\Sigma^\star_{E_b} = \ln(2)\left(\frac{\Delta_p P_{\max} + P_c}{W \sum_{k\in\mathcal{K}^\star} X_k^\star}\right), \tag{22}$$

with $X_k^\star$ is given in (21).

Overall, our simple procedure for optimizing the energy-per-bit metric subject to a sum-power constraint in (17) can be summarized as follows

**Algorithm 1** Fast algorithm for minimizing the total $E_b$ (FAME)

***Inputs:*** $P_{\max}, P_0, \Delta_p, W, N_0, \Gamma,$ $K$ and $g_k$ for $k \in \mathcal{K}$
Obtain $\pi$ by sorting $\mathbf{g} = [g_1, \ldots, g_K]$ in descending order;
$U = K$;
**While** (15) does not hold, $U = U - 1$;
Compute $\beta_k$ for $k \in \{\pi_1, \ldots, \pi_U\}$;
**While** $X^\star_{\pi_U} \leq 0$, $U = U - 1$, recompute $\beta_k$ for $k \in \{\pi_1, \ldots, \pi_U\}$;
Set $\mathcal{K}^\star = \{\pi_1, \ldots, \pi_U\}$ and obtain $X^\star_k$ in (13);
Using $X^\star_k$, compute $P(\mathbf{X}^\star)$ in (4);
**If** $P(\mathbf{X}^\star) \leq P_{\max}$ then obtain $\Sigma^\star_{E_b}$ via (9) and $p^\star_k$ via (14);
**Else** (constrained search)
**While** $\frac{g_{\pi_U}}{U}\left[\frac{P_{\max}}{N\Gamma} + \alpha\right] \leq 1$, $U = U - 1$;
Set $\mathcal{K}^\star = \{\pi_1, \ldots, \pi_U\}$ and obtain $X^\star_k$ in (21);
Obtain $\Sigma^\star_{E_b}$ via (22) and $p^\star_k$ via (2) knowing $X^\star_k$;
***Outputs:*** $p^\star_k$, $X^\star_k$ and $\Sigma^\star_{E_b}$.

## IV. Numerical Results and Discussions

In order to prove the reliability of our simple algorithm for solving the EE-based resource allocation problem over the OMC channel, i.e. FAME, we compare in Fig. 1 the optimal energy-per-bit values returned by FAME (line) and the Matlab function "fmincon" (dot), as well as their respective optimal power and rate allocations for each user. Note that since (17) is a convex optimization problem, it can be solved via "fmincon" or the algorithm in [8], which both rely on gradient search, but at the cost of extra complexity. Figure 1, which has been plotted for $P_{\mathrm{c}} = 130$ W, $\Delta_P = 4.7$, $N_0 = W = 1$, $\Gamma = 1$, $K = 4$ and the channel gain values $\mathbf{g} = [2.6, 0.3, 4.1, 0.9]$, indicates that our method is reliable since the power and rate allocations which have been obtained via our FAME algorithm and 'fmincon" are identical. In addition, both algorithms returned the same optimal energy-per-bit value. In this particular channel gain setting, the optimal unconstraint total transmit power is $P(\boldsymbol{X}^\star) = 17.57$ W. Thus, the EE-based resource allocation is constrained by $P_{\max}$ for $0.1 \leq P_{\max} \leq 17.57$ W, and power, rate as well as energy-per-bit consumption improves with $P_{\max}$ up to $P_{\max} = 17.57$ W. Then, the resource allocation becomes unconstrained and independent of $P_{\max}$, as it is shown in Fig. 1.

In Fig. 2, we depict the energy-per-bit and per-user transmit power for the same parameter values as in Fig. 1, except that $P_{\max} = 5$ W and $\mathbf{g} = \Delta[2.6, 0.3, 4.1, 0.9]$, where $\Delta$ varies from 1 to 1000. In the lower part of the graph, it can clearly be seen that as $\Delta$ increases, or equivalently as the channel gain-to-noise ratio increases (since $N$ is fixed), as the per-user optimal power allocation converges first to $p^\star_k = \frac{P_{\max}}{K}$ for $\Delta$ up to 425 and then to $p^\star_k = A^{-1}N\Gamma\Sigma^\star_{E_b}$ afterwards. The transition begins when $\Sigma^\star_{E_b}$ becomes lower than $\Sigma_{E_b} = AP_{\max}(KN\Gamma)^{-1}$ in the upper part of the graph. This graphically validates our assertion that equal power allocation is the most energy efficient power allocation when the the channel gain-to-noise ratio is high, such that $p^\star_k = \frac{P_{\max}}{K}$ if $\Sigma^\star_{E_b} \geq AP_{\max}(KN\Gamma)^{-1}$ and $p^\star_k = A^{-1}N\Gamma\Sigma^\star_{E_b}$ otherwise.

In order to show that our EE-based resource allocation method reduces the energy-per-bit consumption of the system and study the trade-off between energy and rate, we benchmark it against the sum-rate based resource allocation method subject to a total power constraint, which we denote as $\mathrm{RA}_{\Sigma_R}$ and is solved via water-filling by using (16). We consider a realistic downlink scenario where the channel gain is path-loss dependent such that the $k$-th user channel gain is expressed as

$$g_k = 10^{(G_{\mathrm{TxRx}} - \mathrm{PL}(d_k))/10}, \tag{23}$$

where $G_{\mathrm{TxRx}}$ is the antenna gain of the BS-UE transmission and $\mathrm{PL}(d_k) = \mathrm{Pb}_{\mathrm{LOS}}(d_k)\mathrm{PL}_{\mathrm{LOS}}(d_k) + (1 - \mathrm{Pb}_{\mathrm{LOS}})\mathrm{PL}_{\mathrm{NLOS}}(d_k)$ is the path-loss as a function of the distance $d_k$ between the BS and the $k$-th user. In addition, $\mathrm{Pb}_{\mathrm{LOS}}$ is the line-of-sight (LOS) probability, and $\mathrm{PL}_{\mathrm{LOS}}(d_k)$ and $\mathrm{PL}_{\mathrm{NLOS}}(d_k)$ are the LOS and non-LOS (NLOS) distance dependent path-loss functions. With regards to the PCM, values of $\Delta_{P,\mathrm{BS}}$, $P_{0,\mathrm{BS}}$ and $P_{\max}$ can be found in Table 2 of [6] for different types of

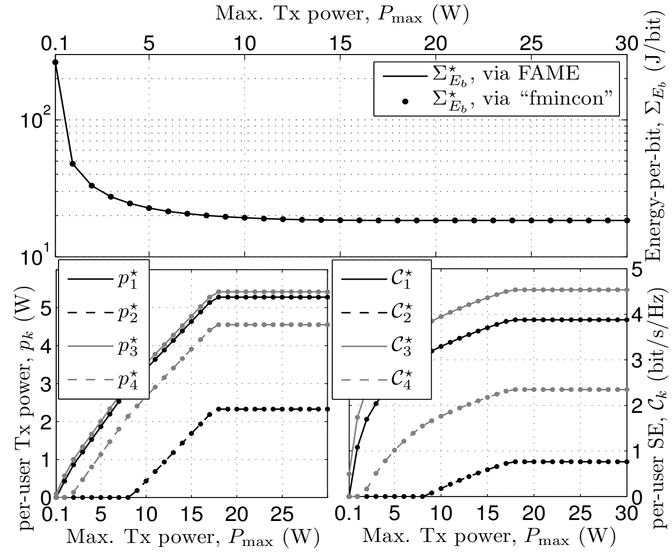


Fig. 1. Comparison of the optimal energy-per-bit consumption and related optimal per-user power and rate values obtained via FAME or "fmincon".

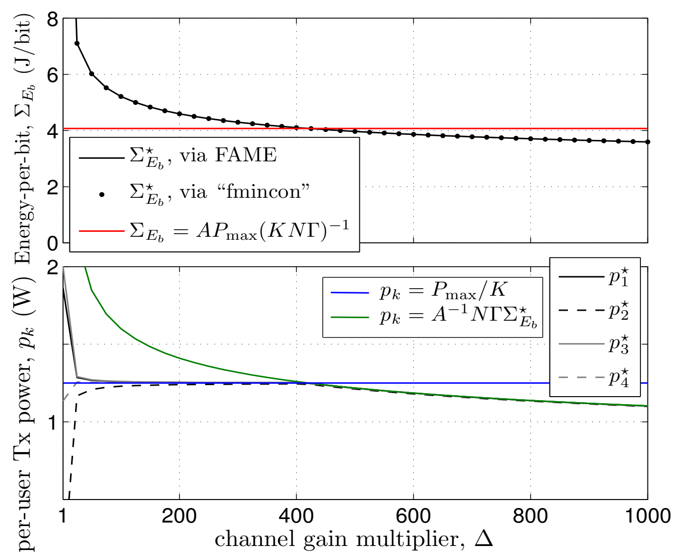


Fig. 2. Optimal energy-per-bit consumption and per-user power as a function of the channel gain multiplier $\Delta$.

TABLE I
SIMULATION PARAMETER VALUES

| | | Parameters | Values |
|---|---|---|---|
| PCM | BS [6] (1 sector) | $\Delta_{P,\text{BS}}$ | 4.7 |
| | | $P_{0,\text{BS}}$ | 130 W |
| | | $P_{\max}$ | 20 W |
| | UE [8] | $P_{0,\text{UE}}$ | 100 mW |
| System model [14] | | $f_c$ | 2.1 GHz |
| | | $W$ | 10 MHz |
| | | $N_0$ | $-165.2$ dBm/Hz |
| | | $G_{TxRx}$ | 14 dBi |
| | | $\text{PL}_{\text{LOS}}(d)$ | $24.8 + 20\log_{10}(f_c) + 24.2\log 10(d)$, |
| | | $\text{PL}_{\text{NLOS}}(d)$ | $-3.3 + 20\log_{10}(f_c) + 42.8\log 10(d)$, |
| | | | ($f_c$ in GHz and $d$ in m) |
| | | $\text{Pb}_{\text{LOS}}$ | $\max\{1, e^{(-(d-10)/200)}\}$ |

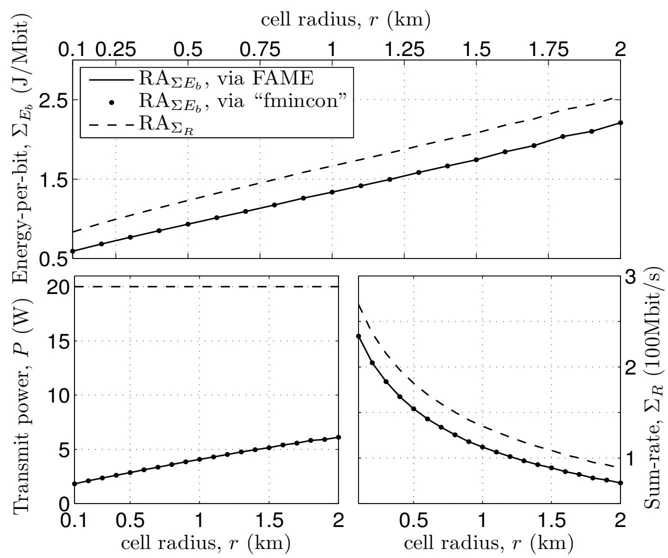


Fig. 3. Performance comparison of EE-based against SE-based resource allocation for various metrics vs. cell rate radius $r$, and for $K = 10$ users.

BSs and, here, we consider the macro BS values for one sector. Moreover, we use $P_{0,\text{UE}} = 100$ mW and consider that $\varsigma = 0.5$ for the UE reception and processing power. These PCM and system model values, which have been obtained from [14], are summarized in Table I. Using these values, we compare in Fig. 3 our EE-optimal resource allocation algorithm against the SE-optimal algorithm, i.e. $\text{RA}_{\Sigma E_b}$ vs. $\text{RA}_{\Sigma_R}$, in terms of the total BS transmit power $P$, cell sum-rate $\Sigma_R$ and cell energy-per-bit $\Sigma_{E_b}$. This graph is plotted for $K = 10$ users uniformly distributed within the cell. As it was expected, the results indicate that our EE-based resource allocation method $\text{RA}_{\Sigma E_b}$ provides the lowest energy-per-bit consumption, or equivalently the best EE. This improved EE is achieved by drastically reducing the total transmit power $P$ (by 70 to 85%) in comparison with $\text{RA}_{\Sigma_R}$. However, it comes at a cost of a lower sum-rate, about 30 Mbit/s on average.

## V. CONCLUSION

In this paper, a simple and optimal EE-based resource allocation algorithm has been proposed for the OMC channel when considering the total energy consumed within the cell. We have derived the explicit formulations of the optimal users' power and rate that minimize the energy consumption of the system for both the unconstrained and power constraint cases. In turn, our explicit formulation of the optimal power allocation has allowed us to prove that equal power allocation is both the most energy and spectral efficient strategy when the channel gain-to-noise ratio is high. Numerical results have confirmed the reliability of our algorithm that has then been utilized for comparing EE-based against SE-based resource allocation in a realistic single cell multi-user downlink scenario. Our results have showed that EE-based allocation can significantly reduce the consumed power and in turn increase the EE in comparison with the SE-based allocation.

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