

COORDINATED RAMP-METERING CONTROL USING A TIME-GAP BASED TRAFFIC MODEL

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Abstract—Numerous studies have examined ramp-metering control to relieve highway congestion. Unlike previous research, this paper presents two different optimization problems for maximum system capacity over a highway corridor, using both a time-gap based traffic model describing traffic flow and the limited traffic data measured by existing field facilities. Our proposed algorithms are coordinated ramp-metering controls controlling the metered rates at system-wide entrance ramps. The origin-utilization relationship is taken into consideration in providing the mathematical derivation for the steady-state optimization problem. This scheme regulates on-ramp flows so as to keep traffic densities along the system below their critical densities. To prevent an increase in adjacent street traffic, which might be caused by this scheme, a time-variant linear programming problem is provided with on-ramp queue control and traffic flow estimation. Comparative simulation results for two optimization problems are presented.

I. INTRODUCTION

Ramp-metering control has been recognized as an effective method of relieving congestion on highways by regulating the inflow from on-ramps to the highway mainline. Chen [1] used Greenshields' model to investigate on-ramp control for travel-rate maximization over the traffic network. He posited that traffic origin-destination information is available from the vehicular traffic network. However, this is not based on real-time and empirical measurements from existing detectors in the traffic networks. Realistically, it is desirable to control on-ramp flow using the limited traffic observations measured from the existing traffic network such as the mainline flow, average speed on the mainline, on-ramp flow, demand flow, and off-ramp flow during each time interval. In addition, Chen calculated the mainline flow of each section of the traffic network by summing up the mainline flow at the upstream-most section and on-ramp flows of its upstream sections. Chen calculated this under the drastic condition that traffic on all sections of the traffic network is in a free-flow regime; this is accomplished by letting fewer vehicles enter highways, as a decrease in vehicle entry onto highways causes unexpected interference with neighboring arterial roads. In order to avoid such interference, the system designer has no choice but to permit more vehicles to enter the highway mainline in spite of the mainline congestion. Hence, it is necessary to estimate traffic density and flow using a traffic model that specifically takes into account congested traffic states.

The ALINEA proposed by Papageorgiou [2] - [4] is a traffic responsive ramp-metering control based on real-time measurements that seek to attain maximum capacity at the merge area of the on-ramp. This algorithm is simple and easily implemented, and it has been shown here to decrease the total time spent (TTS) in traffic and thus to relieve congestion at a local scale. However, a significant drawback of the ALINEA is that it is not a coordinated ramp-metering system but is instead a local feedback control. Hence, we need to provide optimal ramp-metering control to improve system-level capacity.

Macroscopic traffic models to specify the relationship among traffic flow, density, and velocity form the so-called fundamental diagrams. Greenshields [5] derived a parabolic fundamental diagram between traffic flow and density with the assumption of a linear velocity-density relationship under uninterrupted traffic flow conditions. Lighthill, Whitham, and Richards (LWR) [6] [7] used Greenshields' hypothesis and a non-linear conservation law of vehicles to provide a concave fundamental diagram, which is called the first-order LWR model. Newell [8] proposed a triangular flow-density fundamental diagram as a simpler alternative to the LWR model. As an evolved traffic model, a time-gap based traffic model verified by empirical traffic data explains the Newell model with both a mean value of the time-gap and a typical safety length of vehicles in a congestion regime [9].

In this paper, optimal coordinated ramp-metering control for maximizing system-level throughput using a time-gap based traffic model will be studied. In section II, we present vehicular traffic network assumptions and a time-gap based traffic model used for estimating traffic flow and achieving optimal coordinated ramp-metering control. In section III, we propose an on-ramp queue control and two methods of traffic estimation for use in free-flow and congestion regimes. We derive an optimization problem for achieving maximum system capacity in section IV. Section V shows the comparative simulation results of these optimization problems.

II. VEHICULAR TRAFFIC NETWORK

In this section, we describe a vehicular traffic network and specify the time-gap based fundamental diagram used for achieving the maximum average system capacity for vehicular traffic networks.

A. Vehicular Traffic Network Assumptions

Here, we first consider the vehicular traffic network shown in Fig. 1. We assume that there are I entrance ramps, J exit ramps, and M sections, where I , J , and M are bounded integer values. Let T be the time interval (e.g. 30 seconds) used for all controllers to measure traffic data and to apply the newly calculated metered rate along the highway corridor. Let $\rho(kT, x)$, $q(kT, x)$, and $v(kT, x)$ denote the average traffic density, flow, and velocity per lane that are reported at time kT and at point x with the traffic data measured during the k th time frame, where k is a non-negative integer. Their units are vehicles per mile (vpm), vehicles per hour (vph), and miles per hour (mph), respectively. We suppose that highways can be divided into homogeneous sections with similar traffic characteristics and patterns. Let $\rho_m(kT)$, $q_m(kT)$, and $v_m(kT)$ denote the average traffic density, flow, and velocity per lane reported at time kT over the m th section for $m \in [1, M]$. Hence, by the homogeneity of section, $\rho(kT, x) = \rho_m(kT)$, $q(kT, x) = q_m(kT)$, and $v(kT, x) = v_m(kT)$ for all points x belonging to the m th section and for every time kT .

Let $e_i(kT)$ denote the demand flow in vph entering the i th on-ramp queue detected during the k th time frame for $i \in [1, I]$. Let $w_i(kT)$ denote the number of vehicles queued on the i th on-ramp queue at time kT . That is, $w_i(kT)$ is the queue length of the i th on-ramp queue. Let $r_i(kT)$ denote the metered rate of the i th on-ramp queue in vph, which is applied for the k th time frame.

Let $f_j(kT)$ denote the off-ramp flow to the j th exit ramp measured during the k th time frame. Let $\beta_j(kT)$ denote the split ratio, which is the ratio of the number of vehicles exiting to the j th off-ramp over the number of vehicles traveling on the upstream section immediately before the j th off-ramp during the k th time frame, or which is the ratio of the j th off-ramp flow to the total summed flow of the j th off-ramp flow and the mainline flow of the downstream section just after the j th off-ramp. This gives $\beta_j(kT) \in [0, 1]$ for all $j \in [1, J]$ with $\beta_0(kT) = 0$. Let \tilde{m}^j denote the downstream section index immediately after the j th exit ramp. Then, $\beta_j(kT) = f_j(kT) / (f_j(kT) + q_{\tilde{m}^j}(kT))$ for all $j \in [1, J]$ and for all k .

B. Time-gap Based Traffic Model

The time-gap based traffic model shown in Fig. 2 explains a triangular fundamental diagram proposed by Newell [8] with three principal parameters: the maximum free-flow velocity in a free-flow region, a typical safety length of vehicles, and a mean value of the time-gap of traffic data under congested conditions [9]. Since this time-gap based traffic model is not only supported by empirical traffic measurements but is also simple, we will use this model for estimating traffic flow and providing optimal coordinated ramp-metering control for achieving maximum system throughput.

The traffic flow-density equation of a time-gap based traffic model at time kT and for the m th section is given by

$$q_m(kT) = \min \left\{ v_{f,m} \rho_m(kT), \frac{5280 - L \rho_m(kT)}{\bar{c} \tau_m} \right\}, \quad (1)$$

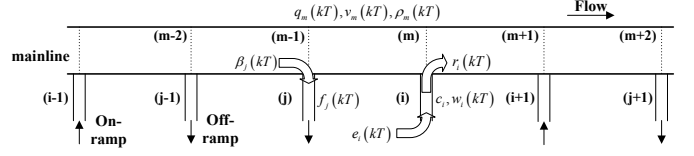


Fig. 1. Vehicular Traffic Network

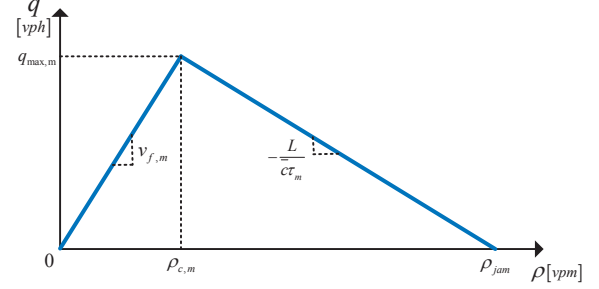


Fig. 2. Time-gap Based Traffic Model

where $v_{f,m}$ is the maximum free-flow velocity for the m th section in mph, L is a typical safety length of vehicles in feet, τ_m is the mean value of the time-gap defined in a congestion regime of the m th section in seconds, and \bar{c} is a constant with 5280/3600. The vehicular traffic network for the considered m th homogeneous section has the maximum capacity $q_{max,m}$ at a value of the critical density $\rho_m(kT) = \rho_{c,m}$ such that

$$v_{f,m} \rho_{c,m} = \frac{5280 - L \rho_{c,m}}{\bar{c} \tau_m}$$

which becomes

$$\rho_{c,m} = \frac{5280}{\bar{c} \tau_m v_{f,m} + L}. \quad (2)$$

The corresponding maximal throughput achieved at the critical density $\rho_{c,m}$ is given by

$$q_{max,m} = v_{f,m} \rho_{c,m} = \frac{5280 v_{f,m}}{\bar{c} \tau_m v_{f,m} + L} \quad (3)$$

and also $0 \leq q_m(kT) \leq q_{max,m}$ holds for all times kT and for all $m \in [1, M]$. We define the same value of the jam density for all $m \in [1, M]$ as $\rho_{jam} = 5280/L$. Note that $v_{f,m}$, τ_m , $\rho_{c,m}$, and $q_{max,m}$ are time-invariant.

III. STEADY-STATE OPTIMIZATION PROBLEM FOR MAXIMUM CAPACITY

In this section, we consider a method of achieving ramp-metering control with the steady-state optimization without on-ramp queue control. On-ramp queue management is used to prevent on-ramp queue spillover onto neighboring arterial streets by increasing ramp-metering rate [10] [11]. This strategy can have an adverse effect on highway mainline traffic and thus diminish the benefits of ramp-meter control. When the vehicular traffic network reaches the steady-state, traffic density of each homogeneous section is constant with respect to time kT . Likewise, traffic flow and velocity also become

time-invariant in the steady-state. Thus, by the steady-state conditions, $\rho_m(kT) = \rho_m$, $q_m(kT) = q_m$, $v_m(kT) = v_m$, $r_i(kT) = r_i$, $f_j(kT) = f_j$, and $\beta_j(kT) = \beta_j$ for all $m \in [1, M]$, $i \in [1, I]$, $j \in [1, J]$ and for all times kT .

A. Traffic Flow Estimation

We consider that the last indices of the entrance ramp and exit ramp located just before the m th section for some $m \in [1, M]$ are \bar{i}^m and \bar{j}^m , respectively. Then, $m = \bar{i}^m + \bar{j}^m$ for all $m \in [1, M]$. Let $\tilde{j}^{i,m}$ and $\bar{j}^{i,m}$ denote the indices of the upstream-most and downstream-most exit ramps located between the i th entrance ramp and m th homogeneous section, respectively. We assume that vehicles traveling on the section located just before each off-ramp exit from a highway with uniform distribution, no matter which upstream entrance ramps they entered. By this hypothesis, the origin-utilization equation for traffic flow of the m th section is given by

$$q_m = \frac{1}{n_m} \sum_{i=1}^{\bar{i}^m} \left[\prod_{j=\tilde{j}^{i,m}}^{\bar{j}^{i,m}} (1 - \beta_j) \right] r_i \quad (4)$$

where n_m is the number of lanes of the m th section. If a specific exit ramp cannot be defined for $\tilde{j}^{i,m}$, i.e. if there does not exist an exit ramp between the i th entrance ramp and m th section, then $\tilde{j}^{i,m} = 0$ and thus $\bar{j}^{i,m} = 0$.

B. Optimization Problem

The main purpose of a ramp-metering algorithm is to control the metering rate at all entrance ramps and thus to relieve congestion on highways. The most important evaluation criteria for a ramp-metering strategy are total travel time (TTT) on the mainline, total waiting time (TWT) at the entrance ramp, and total time spent (TTS), which is the sum of TTT and TWT. That is, in order to improve system-wide total time spent, it is desirable to utilize the limited throughput of all sections as close as possible to their maximum capacity, $q_{max,m}$, for all $m \in [1, M]$. Hence, we pursue the steady-state optimization problem to maximize average system throughput on highways. In the steady-state, the average system throughput per lane along the highway corridor, J_C , is given by

$$J_C = \frac{1}{D} \sum_{m=1}^M q_m d_m, \quad (5)$$

where D is the total length of the vehicular traffic network and d_m is the length of the m th section. Substituting (4) into (5),

$$J_C = \frac{1}{D} \sum_{m=1}^M \left\{ \sum_{i=1}^{\bar{i}^m} \left[\prod_{j=\tilde{j}^{i,m}}^{\bar{j}^{i,m}} (1 - \beta_j) \right] r_i \right\} \frac{d_m}{n_m} = \frac{1}{D} \bar{r}^\top \mathbf{B} \bar{a},$$

where \bar{a} is the $M \times 1$ vector whose element is the length over the number of lanes of the m th section (i.e. $a_m = d_m/n_m$ for all $m \in [1, M]$), \bar{r} is the $I \times 1$ vector whose element is the metered rate at the i th entrance ramp, \mathbf{B} is the $I \times M$ origin-utilization matrix, and $(\cdot)^\top$ is the transpose of the vector or matrix. The elements of the origin-utilization matrix \mathbf{B} are

$\beta_{(i,m)}$, which refer to the proportion of vehicles entering from the i th on-ramp and traveling on the m th section, where $0 \leq \beta_{(i,m)} \leq 1$ for all $i \in [1, I]$ and all $m \in [1, M]$. If there exists \hat{m}_i with $\hat{m}_i > i$ for each $i \in [1, I]$ such that

$$\begin{cases} \beta_{(i,m)} = 0 & \text{for all } m \in [1, \hat{m}_i), \\ \beta_{(i,m)} = 1 & \text{for } m = \hat{m}_i, \\ 0 \leq \beta_{(i,m)} \leq 1 & \text{for all } m \in (\hat{m}_i, M], \end{cases}$$

then the corresponding matrix \mathbf{B} is called an upper unit trapezoidal matrix. Note that the original-utilization matrix \mathbf{B} is an upper unit trapezoidal matrix, thus reducing the computational complexity needed to determine the average system throughput, J_C .

To utilize the limited throughput of the system fully, we need to prevent the sharp drop-off of traffic flow that occurs when traffic density is above critical density, which eventually causes serious congestion. Hence, it is desirable to control the metered rates at system-wide entrance ramps so as to keep traffic densities along the system below their critical densities; that is, $0 \leq \rho_m \leq \rho_{c,m}$ for all $m \in [1, M]$. By this optimization strategy and (2), for all $m \in [1, M]$,

$$0 \leq \rho_m \leq \frac{5280}{\bar{c}\tau_m v_{f,m} + L}. \quad (6)$$

By (1), for all $m \in [1, M]$,

$$\frac{q_m}{v_{f,m}} \leq \rho_m \leq \frac{5280 - \bar{c}\tau_m q_m}{L}. \quad (7)$$

Hence, by (6) and (7), the optimal traffic density range for the m th section is

$$\frac{q_m}{v_{f,m}} \leq \rho_m \leq \frac{5280}{\bar{c}\tau_m v_{f,m} + L}. \quad (8)$$

Therefore, by (3) and (8) the optimization problem for achieving maximum average system throughput, J_C , along the vehicular traffic network becomes

$$\begin{aligned} \max J_C &= \frac{1}{D} \sum_{i=1}^I \left\{ \sum_{m=\hat{m}^i}^M \left[\prod_{j=\tilde{j}^{i,m}}^{\bar{j}^{i,m}} (1 - \beta_j) \right] a_m \right\} r_i \\ &= \frac{1}{D} \bar{a}^\top \mathbf{B}^\top \bar{r}, \end{aligned} \quad (9)$$

subject to, for all $m \in [1, M]$,

$$\begin{cases} 0 \leq \frac{1}{n_m} \sum_{i=1}^{\bar{i}^m} \left[\prod_{j=\tilde{j}^{i,m}}^{\bar{j}^{i,m}} (1 - \beta_j) \right] r_i \leq \frac{5280 v_{f,m}}{\bar{c}\tau_m v_{f,m} + L}, \\ \frac{1}{n_m v_{f,m}} \sum_{i=1}^{\bar{i}^m} \left[\prod_{j=\tilde{j}^{i,m}}^{\bar{j}^{i,m}} (1 - \beta_j) \right] r_i \leq \rho_m \leq \frac{5280}{\bar{c}\tau_m v_{f,m} + L}, \end{cases}$$

where \tilde{k}^i is the downstream section index immediately after the i th entrance ramp.

IV. TIME-VARIANT OPTIMIZATION PROBLEM FOR MAXIMUM CAPACITY WITH ON-RAMP QUEUE CONTROL

The optimization problem (9) for achieving maximum average system throughput in the steady-state described in Section

III is valid as long as the traffic of the m th section is in a free-flow region. Although this scheme shows the maximum system capacity, it causes additional long-term congestion on entrance ramps and unexpected interference with adjacent street traffic. In order to prevent this undesirable effect of ramp-metering control, we need to consider an on-ramp queue management strategy. However, if the high demand flow arrives at on-ramps continually, the activation of on-ramp queue control unintentionally leads to the inevitable congestion on the highway mainline. In this case, if the traffic flow summed up with the inflows from upstream on-ramp queues of the m th section is larger than the corresponding maximum flow, $q_{max,m}$, then the equation (9) cannot be used. Hence, we introduce a traffic flow estimation method using both a fundamental diagram such as the time-gap based traffic model described in previous section and the vehicle conservation law. The conservation law of vehicles describes a physics law that the change in the number of vehicles on a highway section is equivalent to the net difference between the inflowing number of vehicles and the outflowing number of vehicles to/from the corresponding section.

A. On-ramp Queue Control

Let \hat{c}_i denote the maximum permissible queue length of the i th entrance ramp, which is strictly less than the i th entrance ramp's capacity, c_i , with $\hat{c}_i < c_i$. Let r_{min} denote the minimum on-ramp discharge rate, which is the same constant for all entrance ramps $i \in [1, I]$, i.e. 240 (vph). The minimum on-ramp discharge rate must be considered because drivers want to stay on the entrance ramp for as short a time as possible. Thus, for all $i \in [1, I]$ and for every time kT ,

$$r_{min} \leq r_i(kT). \quad (10)$$

Since the difference between the demand flow and metered rate should be less than or equal to the admissible on-ramp queue length, for all $i \in [1, I]$,

$$(e_i(kT) - r_i((k+1)T)) \leq \frac{(\hat{c}_i - w_i(kT))}{\hat{T}}, \quad (11)$$

where $\hat{T} = T/3600$. Also, since the metered rate should be less than or equal to the maximum possible inflow rate considering the demand flow and current on-ramp queue length, for all $i \in [1, I]$,

$$r_i((k+1)T) \leq e_i(kT) + \frac{w_i(kT)}{\hat{T}}. \quad (12)$$

By (10), (11), and (12), and by letting $\eta_i(kT) = e_i(kT) + \frac{w_i(kT)}{\hat{T}}$, on-ramp queue control is satisfied with

$$\begin{aligned} \max \left\{ \min \{r_{min}, \eta_i(kT)\}, \eta_i(kT) - \frac{\hat{c}_i}{\hat{T}} \right\} \\ \leq r_i((k+1)T) \leq \eta_i(kT) \end{aligned} \quad (13)$$

for all $i \in [1, I]$ and for all k .

B. Traffic Flow Estimation

Homogeneous sections comprising vehicular traffic networks are attached to either the entrance ramp or exit ramp. In the case of a homogeneous section with an entrance ramp, there exists disturbance by the vehicles entering from the on-ramp queue. Hence, based on the vehicle conservation law, traffic density of the m th section with an entrance ramp at the next time $(k+1)T$ can be estimated by

$$\begin{aligned} \rho_m(kT) = \rho_m((k-1)T) + \hat{T} [n_{m-1}q_{m-1}((k-1)T) \\ - n_m q_m((k-1)T) + r_{im}((k-1)T)] \frac{1}{n_m d_m}. \end{aligned} \quad (14)$$

Unlike a homogeneous section with an entrance ramp, since input disturbance does not exist in a section attached to an exit ramp, the estimated traffic density is linearly dependent on that of its upstream section as

$$\rho_m(kT) = (1 - \beta_{jm}((k-1)T)) \rho_{m-1}((k-1)T). \quad (15)$$

We can determine the estimated flow of the m th section at time kT by substituting (14) or (15) in (1).

C. Optimization Problem

The time-variant optimization problem to find the metered rate for each entrance ramp for achieving the maximum average system capacity per lane is a linear programming problem of choosing $r_i(kT)$ every time kT as, by (1), (3), and (13),

$$\max J_C(kT) = \frac{1}{D} \sum_{m=1}^M q_m(kT) d_m, \quad (16)$$

subjected to, for all $m \in [1, M]$,

$$\begin{cases} 0 \leq q_m(kT) \leq \frac{5280v_{f,m}}{\hat{e}\tau_m v_{f,m} + L}, \\ \max \left\{ \min \{r_{min}, \eta_{im}((k-1)T)\}, \eta_{im}((k-1)T) - \frac{\hat{c}_{im}}{\hat{T}} \right\} \leq r_{im}(kT) \leq \eta_{im}((k-1)T), \\ 0 \leq \rho_m(kT) \leq \rho_{jam}, \end{cases}$$

where $\eta_{im}((k-1)T) = e_{im}((k-1)T) + \frac{w_{im}((k-1)T)}{\hat{T}}$ and $q_m(kT)$ is given by either (14) or (15) depending on the type of the section.

V. ANALYSIS, RESULTS, AND DISCUSSION

In this section, we show the comparative simulation results of the average system capacity and velocity on a highway corridor with four different ramp-metering strategies: with no ramp-metering control, ALINEA, a steady-state optimization scheme, and a time-variant optimization method.

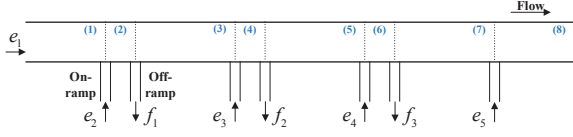


Fig. 3. Highway Corridor for Simulation Example

TABLE I
DEMAND FLOW

Entrance Ramp Number	Free-flow period [vph]	Rush-hour period [vph]
1	1000	1800
2, 3, 4, 5	200	1000

A. Simulation Scenario

Fig. 3 specifies a highway corridor that we consider for simulation; it consists of $M = 8$ sections, $I = 5$ entrance ramps including the first section that is the most upstream toward the mainline, and $J = 3$ exit ramps. The second section attaches to an entrance ramp and then each section contains, by turn, an entrance ramp and an exit ramp. Each section has $n_m = 4$ lanes for all $m \in [1, M]$. The length of section attached to an entrance ramp and an exit ramp is 1 and 0.1 miles, respectively. We set the average safety length of vehicles to $L = 22$ [feet], maximum free-flow velocity to $v_{f,m} = 70$ [mph], and mean value of the time-gap under congested status to $\tau_m = 1.78$ seconds for all $m \in [1, M]$ as typically measured in the United States [9]. From this, we can determine the critical density $\rho_{c,m}$, jam density ρ_{jam} , and maximum flow $q_{max,m}$ for each section using a time-gap based traffic model. We suppose that each on-ramp queue has the same permissible queue capacity of $\hat{c}_i = 200$ for all $i \in [1, I]$ and that the split ratio for each off-ramp is constant with $\beta_j(kT) = 0.2$ for all $j \in [1, J]$ and for all $k \geq 0$. In addition, we suppose that every section on the mainline has the same initial density of $\rho_m(0) = 10$ for all $m \in [1, M]$. Let the initial queue length on every on-ramp queue be $w_i(0) = 0$ for all $i \in [1, I]$. The demand flow entering to each on-ramp queue for each period is listed in Table I. The total simulation time is 5 hours and comprises three periods: we use the amount of demand flow vector defined in the free-flow period for the first hour, rush-hour period for the next 2 hours, and free-flow period again for the final 2 hours.

B. Simulation Results

Fig. 4 shows the average system capacity over 5 hours on a sample highway corridor using the four specified ramp-metering controls. In addition, Fig. 5 shows the average system velocity of all vehicles traveling over the highway corridor during the specified 5-hour time frame.

The average system flow is around 910 [vph] during the first free-flow period, which is the same without regard to the type of ramp-metering control and even under no ramp-metering operation. Once demand flow to all entrance

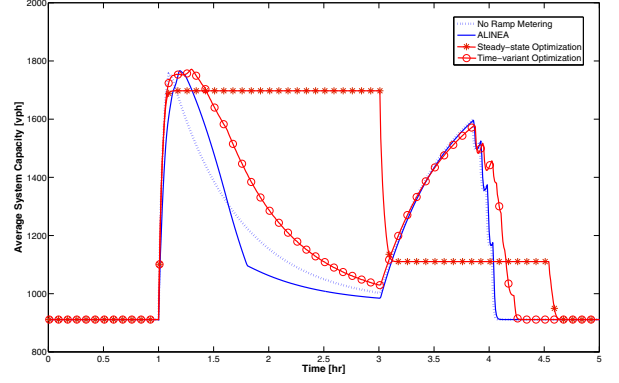


Fig. 4. Average System Capacity vs. Time

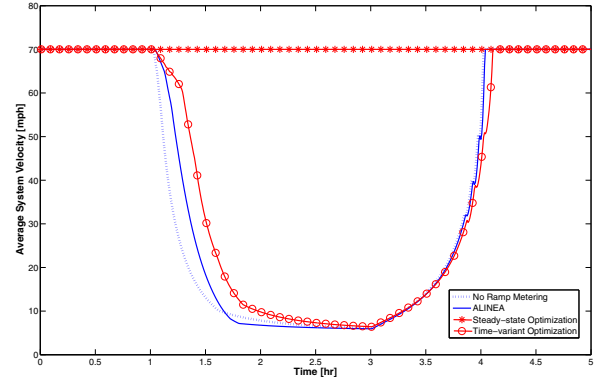


Fig. 5. Average System Velocity vs. Time

ramps increases dramatically as occurs during the rush-hour period, all four ramp-metering methods show that an instant throughput improvement approximate to the maximum system capacity around 1800 [vph] because of the sudden increase in traffic density. As high demand flow is injected into the vehicular traffic network continuously, no ramp-metering, ALINEA, and time-variant optimization controls lead to a gradual degradation of system throughput. However, the steady-state optimization control maintains a higher system flow by limiting the metered rates at system-wide entrance ramps so as to keep traffic densities along the system below their critical densities of about 25 [vpm], causing on-ramp queue spillover to adjacent local streets. In contrast to this observation, following the rush-hour period, since density along the system decreases, no ramp-metering, ALINEA, and time-variant optimization controls show an increase in the average system capacity. Note that the system capacity recovering from congestion does not reach capacity before flow breakdown. This observation describes traffic hysteresis phenomena [12] [13]. After vehicles of all on-ramp queues are discharged or traffic density on the mainline of the system decreases below the critical density, average system throughput of all ramp-

metering controls returns to the initial simulation state. In contrast to the other three ramp-metering approaches described here, the steady-state optimization algorithm requires more than an hour to clear congestion; this is because the steady-state scheme must disperse numerous vehicles waiting on both entrance ramp queues and adjacent arterial roads. Since ALINEA also does not support on-ramp queue control, it is observed that ALINEA similarly incurs neighboring street traffic. However, the time-variant ramp-metering algorithm not only keeps the queue length on entrance ramps short enough to prevent the build-up of vehicles at on-ramp queues, but it also generally improves the average system throughput, particularly in congestion conditions.

When we fulfill the requirements of the steady-state ramp-metering method, all vehicles on the mainline travel with a maximum free-flow velocity of 70 [mph] shown in Fig. 5, but it has the drawback of requiring longer congestion-clearance time for dispersing vehicles both on entrance ramps and adjacent roads. Compared to the steady-state method, other three schemes show that the average system speed decreases as the number of vehicles wanting to use the highway network increases during the rush-hour period, but the average speed of traveling vehicles on the mainline increases until traffic returns to the normal free-flow state following the rush-hour period. As with the average system capacity results, the time-variant ramp-metering scheme allows vehicles on the highway mainline to travel with a higher velocity than is permitted by either ALINEA or no ramp metering.

VI. CONCLUSION

The goal of this paper is to provide two optimal ramp-metering controls for achieving maximum average system throughput on highways. The steady-state optimization problem rigorously limits the inflow rate from entrance ramps into the mainline so as to keep traffic densities along a highway corridor below their critical densities, whereas the time-variant programming problem adopts on-ramp queue control to prevent additional congestion onto neighboring arterial roads. The time-gap based fundamental diagram is used for estimating traffic flow as well as for demonstrating the constraints of two optimization problems.

Use of the steady-state optimization scheme results in high average system capacity as well as increased average system velocity to the overall traffic network, but this method has the drawback of incurring significant on-ramp queue spillover on adjacent streets, thus requiring long congestion clearance time to return to the free-flow state. In contrast, the time-variant optimization problem for achieving the maximum average system capacity is proven as an effective coordinated ramp-metering algorithm to provide higher system throughput and to increase the average system velocity, particularly under congestion situations.

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