Achievable Degrees of Freedom of the K-user MISO Broadcast Channel with Alternating CSIT via Interference Creation-Resurrection

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***Abstract*—Channel state information at the transmitter affects the degrees of freedom of the wireless networks. In this paper, we analyze the DoF for the K-user multiple-input single-output (MISO) broadcast channel (BC) with synergistic alternating channel state information at the transmitter (CSIT). Specifically, the CSIT of each user alternates between three states, namely, perfect CSIT (P), delayed CSIT (D) and no CSIT (N) among different time slots. For the K-user MISO BC, we show that the total achievable degrees of freedom (DoF) are given by $\frac{K^2}{2K-1}$ through utilizing the synergistic benefits of CSIT patterns. We compare the achievable DoF with results reported previously in the literature in the case of delayed CSIT and hybrid CSIT models.**

Index Terms: Broadcast channel, degrees of freedom, interference alignment, alternating CSIT, interference creation-resurrection.

I. Introduction

Due to the rapid growth in wireless traffic, interference management is essential to provide the required quality of service (QoS) for future wireless networks. Traditional prior work focused on reducing the interference power at the receivers. Recently, interference alignment (IA) has been proposed and studied on various networks such as the interference, broadcast and X channels. IA is an elegant technique to decrease the impact of interference through reducing the dimension of the interference subspace thanks to the seminal work of [1], [2].

An important performance measure for a communication network is its degrees of freedom (DoF) which determines the behavior of the sum capacity in the high signal-to-noise ratio (SNR) regime. In particular, the network capacity under a transmission power P is given by [3]

$$C(P) = \text{DoF}\log(P) + o(\log(P)) \tag{1}$$

where $\lim_{P\to\infty} \frac{o(\log(P))}{\log(P)} = 0$.

In capacity characterization work, it is a common assumption that receivers know the channel state information (CSI) perfectly and instantaneously, while the CSI knowledge at the transmitter(s) (CSIT) is usually subject to some limitations. At one extreme, it is assumed that the transmitters know the CSI instantaneously and perfectly (full CSIT assumption). Under this condition, the capacity region and, hence the DoF region, of the multiple-input multiple-output (MIMO) broadcast channel was characterized in [4]. The DoF of the K-user single-input single-output (SISO) interference channel was shown to be $\frac{K}{2}$ with full CSIT [2]. Also, it was shown in [5] that the $M \times K$ SISO X channel with full CSIT has $\frac{MK}{M+K-1}$ DoF. In [6], it was proved that channel output feedback does not provide any DoF benefit in interference and X channels under the full CSIT assumption. At the other extreme, the transmitter(s) are assumed to have no knowledge about CSI. In this case, the K-user multiple-input single-output (MISO) broadcast channel was studied in [7]. Other works include [8] which characterized the DoF regions of the K-user MIMO broadcast channel, interference channel and X channel. Also, [9]–[11] studied the DoF region of the two-user MIMO broadcast and interference channels with no CSIT by developing upper and lower bounds on the DoF. It was shown in [8] that the MISO broadcast, SISO interference and SISO X channels under isotropic i.i.d. fading can achieve no more than one DoF.

Maddah Ali and Tse investigated a delayed CSIT model, which is an intermediate assumption between the two extremes; full CSIT and no CSIT. This model was introduced in [12] for the K-user Gaussian MISO broadcast channel (BC). They showed that the K-user BC under delayed CSIT can achieve at most $K/(1 + \frac{1}{2} + \cdots + \frac{1}{K})$ DoF which is strictly greater than one DoF. Also, in [13], Maleki et al. applied the delayed CSIT model to the X-channel and showed that the 2 user SISO X channel under delayed CSIT assumption can achieve $\frac{8}{7}$ DoF. A variety of work concerning CSIT availability models have been studied such as: quantized CSIT [14], [15], compound CSIT [16]–[18] and mixed CSIT [19].

Related Work

Another interesting model is the alternating CSIT model that was first introduced by Tandon et. al. in [20]. The authors of the pre-mentioned paper studied the synergistic benefits of alternating CSIT for the 2-user MISO broadcast channel and defined the DoF region $\mathcal{D}$ for different patterns of alteration. Also, the same authors in [21] studied the K-user case and identified the minimum CSIT pattern to achieve the upper bound on the total DoF, which is given by $\min(M, K)$, for the MISO broadcast channel with an M antenna transmitter and K single antenna users. The achievable DoF under this

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model is upper bounded by

$$D_{\Sigma}(K) \leq \frac{K(M + (\min(M,K) - 1)\lambda)}{M + K - 1} \quad (2)$$

where $\lambda = \frac{\min(M,K)}{K}$ is the fraction time that CSIT is perfect per user.

In [22], the authors considered the hybrid CSIT model for the BC; in which the CSIT pattern is fixed during the channel uses. In their framework, there is a perfect CSIT for a subset of receivers and delayed CSIT for the remaining receivers. For the 3-user case, they showed that for a 2-antenna transmitter with perfect CSIT for one user and delayed CSIT for the other two users, the BC can achieve at most $\frac{5}{3}$ DoF. Also, they studied the system with 3 antennas at the transmitter and showed that for the previous hybrid CSIT pattern a total DoF of $\frac{9}{5}$ is achievable. For the same number of antennas, i.e. three, but with a higher CSIT setting; in which perfect CSIT is available for two users while delayed CSIT for the third user, $\frac{9}{4}$ total DoF can be achieved.

The authors of [23] studied the SISO X channel with synergistic alternating CSIT. They proposed schemes based on interference creation-resurrection (ICR) that achieve the upper bound on the DoF of the 2-user network which is $\frac{4}{3}$ DoF. Also, they characterized the DoF region $\mathcal{D}$ as a function of the distribution of CSIT states, that are basically; perfect (P), delayed (D) and no CSIT (N).

In this paper, we propose a scheme based on ICR under alternating CSIT for the K-user BC. The ICR scheme is partitioned into two phases: phase one is associated with the delayed CSIT and no CSIT states. In this phase, information terms are delivered to receivers with no CSIT availability and interference terms (to be resurrected in phase two) are received by receivers with delayed CSIT. In phase two, we deliver useful linear combinations of past interference terms to the receivers in order to decode their desired messages. We show that the achievable DoF for this network is given by

$$D_{\Sigma}(K) = \frac{K^2}{2K - 1} \quad (3)$$

and the distribution of fraction of time of the different states $\{P, D, N\}$ required for our proposed scheme is

$$\lambda_P = \frac{(K-1)^2}{2K^2 - K}, \lambda_D = \frac{K-1}{2K-1}, \lambda_N = \frac{1}{K}. \quad (4)$$

The rest of the paper is organized as follows. Section II describes the system model. The proposed scheme is discussed in Section III. Section IV provides numerical evaluation of the attained DoF expression and shows the performance gains for our proposed system compared to previous work. Finally, we conclude the paper in Section V.

II. System Model

We consider a MISO broadcast channel with K transmit antennas and K single antenna receivers. The received signal at the ith receiver is given by

$$Y_i(t) = H_i(t)X(t) + N_i(t), \quad i = 1, \ldots, K \quad (5)$$

where $X(t)$ is the $K \times 1$ transmitted signal at time t with a power constraint $E\{|X(t)|^2\} \leq P$. The additive noise $N_i(t) \sim \mathcal{CN}(0,1)$ at time t generated at receiver R_i is circularly symmetric white Gaussian noise with zero mean and unit variance. $H_i(t)$ is the $1 \times K$ channel vector from the transmitter to receiver R_i at time t which is sampled from a continuous distribution whose elements are complex Gaussian. The channel coefficients are assumed to be i.i.d. across the receivers. Let $r_i(P)$ denote the achievable rate of message W_i for a given transmission power P defined as $r_i(P) = \frac{\log_2(|W_i|)}{n}$ where $|W_i|$ is the cardinality of the message set and n is the number of channel uses. The DoF region $\mathcal{D}$ is defined as the set of all achievable tuples $(d_1, d_2, \ldots, d_K) \in \mathbb{R}_+^K$ where $d_i = \lim_{P\to\infty} \frac{r_i(P)}{\log_2(P)}$ is the DoF for message W_i. The total DoF of the network is defined as

$$D_{\Sigma}(K) = \max_{(d_1,d_2,\ldots,d_K)\in\mathcal{D}} d_1 + d_2 + \cdots + d_K. \quad (6)$$

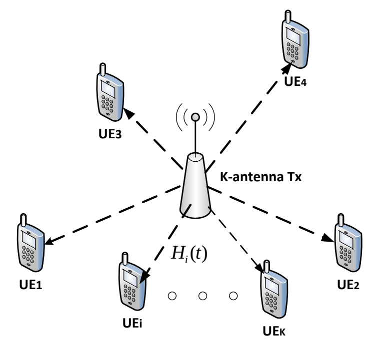

Fig. 1. Network Model: A MISO BC with a K-antenna transmitter and K single antenna users.

We assume that the receivers have perfect and global channel state information. Furthermore, we consider three different states of the availability of CSIT

1) Perfect CSIT (P): identifies the state of CSIT in which CSIT is available to the transmitter instantaneously and without error.
2) Delayed CSIT (D): identifies the state of CSIT in which CSIT is available to the transmitter with some delay greater than or equal one time slot duration and without error.
3) No CSIT (N): identifies the state of CSIT in which CSIT is not available to transmitter at all.

The state of CSIT availability of the channel to the ith receiver at time instant t is denoted by $S_i(t)$; where, $S_i(t) \in \{P, D, N\}$. For instance, $S_2(t) = P$ indicates that the transmitter has perfect and instantaneous knowledge of H_2 at time instant t. In addition, let $S_{12\ldots K}(t)$ denote the collection of the states of CSIT availability of the channels to the receivers $\{1, 2, \ldots, K\}$ at time slot t, respectively. Therefore, $S_{12\ldots K}(t) \in \{PP\ldots P, PP\ldots D, \ldots, NN\ldots N\}$. For example, $S_{123}(t) = PDN$, refers to the case where the transmitter has perfect knowledge to H_1, delayed information about H_2 and no information about H_3. We denote the CSIT availability of the channels to the ith receiver over n time slots by S_i^n. For instance, the CSIT availability over three time slots for receiver R_i is given by $S_i^3 = (x, y, z)$ where $x, y, z \in S_i$ and x, y and z denote the availability of CSIT in the first, second and third time slots, respectively. Similarly, we denote the availability of CSIT for the channels to the first and second receivers in three time slots "CSIT pattern" by $S_{12}^3 = (X, Y, Z)$ where $X, Y, Z \in S_{12}$.

The fraction of time associated with the availability of CSIT state S for the network, denoted by λ_S where $S \in \{P, D, N\}$, is given by

$$\lambda_S = \frac{\sum_{t=1}^{n} \sum_{i=1}^{K} \mathbb{I}(S_i(t) = S)}{nK} \tag{7}$$

where

$$\mathbb{I}(S_i(t) = S) = \begin{cases} 1, & \text{if } S_i(t) = S \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

and n is the number of channel uses, and hence,

$$\sum_{S \in \{P, D, N\}} \lambda_S = 1. \tag{9}$$

Furthermore, we use $\Lambda(\lambda_P, \lambda_D, \lambda_N)$ to denote the distribution of fraction of time of the different states $\{P, D, N\}$ of CSIT availability.

III. Proposed Interference Creation-Resurrection Scheme

Motivated by the previous work of [23] for the X channel, we extend this work to the BC. In this section, we propose a precoding scheme for the BC under alternating CSIT. The scheme is divided into two phases. The first phase is associated with the delayed and no CSIT states where the transmitter sends its messages. As a result, the receivers get linear combinations of their desired messages in addition to interference terms during this phase. This phase is called "interference creation." On the other hand, the second phase is associated with the perfect CSIT state and is called "interference resurrection" phase. In this phase, the transmitter reconstructs the old interference by exploiting the delayed CSIT in phase one in order to deliver new linear combinations to the receivers free from the interference and enable the receivers to extract their desired messages via physical network coding.

As an illustrative example of the K-user case: first, we consider a 3-user MISO BC with alternating CSIT pattern given by $S_{123}^5 = (NDD, DND, DDN, PPN, PNP)$ over five time slots. Let u_1, u_2 and u_3 be three independent messages intended to receiver R_1, v_1, v_2 and v_3 be three independent messages intended to receiver R_2, and p_1, p_2 and p_3 be three independent messages intended to receiver R_3. Consequently, the proposed scheme is performed over two phases as follows in the next subsections.

A. Phase 1: ***Interference Creation***

This phase consists of three time slots, each time slot is intended to deliver an interference-free linear combination of the messages intended for one receiver. Therefore, at the ith time slot, R_i receives a linear combination of its desired symbols while the two other receivers $R_j, j \in \{1, 2, 3\} \backslash \{i\}$ receive interference terms.

At $t = 1$:
The transmitter sends all data symbols for R_1, i.e.,

$$X(1) = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}. \tag{10}$$

As a result, the received signals are given as:

$$\begin{aligned} Y_1(1) &= H_1(1)X(1) = L_1^1(u_1, u_2, u_3) && (11) \\ Y_2(1) &= H_2(1)X(1) = I_2^1(u_1, u_2, u_3) && (12) \\ Y_3(1) &= H_3(1)X(1) = I_3^1(u_1, u_2, u_3) && (13) \end{aligned}$$

where $L_i^j(x_1, x_2, x_3)$ denotes the jth linear combination of the messages x_1, x_2 and x_3 that is intended for receiver R_i and $I_i^j(z_1, z_2, z_3)$ denotes the jth interference term for receiver R_i which is a function of the messages z_1, z_2 and z_3 overheard by receiver R_i.

At $t = 2$:
Similarly, the transmitter sends all data symbols for R_2 as follows:

$$X(2) = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}. \tag{14}$$

Then, the received signals are:

$$\begin{aligned} Y_1(2) &= H_1(2)X(2) = I_1^1(v_1, v_2, v_3) && (15) \\ Y_2(2) &= H_2(2)X(2) = L_2^1(v_1, v_2, v_3) && (16) \\ Y_3(2) &= H_3(2)X(2) = I_3^2(v_1, v_2, v_3) && (17) \end{aligned}$$

At $t = 3$:
Finally, the transmitter sends all data symbols for R_3:

$$X(3) = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}. \tag{18}$$

Then,

$$\begin{aligned} Y_1(3) &= H_1(3)X(3) = I_1^2(p_1, p_2, p_3) && (19) \\ Y_2(3) &= H_2(3)X(3) = I_2^2(p_1, p_2, p_3) && (20) \\ Y_3(3) &= H_3(3)X(3) = L_3^1(p_1, p_2, p_3). && (21) \end{aligned}$$

B. Phase 2: ***Interference Resurrection***

This phase consists of two time slots where in each time slot the transmitted signal is designed such that it provides two interference-free linear combinations of the messages intended to two receivers while the third receiver gets a linear combination of its desired messages corrupted by an interference term that can be removed using the received interference in previous time slots.

At $t = 4$:
In this time slot, the transmitter utilizes the perfect CSIT at R_1 and R_2. The transmitter delivers two interference-free terms to R_1 and R_2 while providing an interference-corrupted desired term for R_3. The transmitted signal is given by

$$\begin{aligned} X(4) &= h_1^{\perp}(4) \begin{bmatrix} I_3^2(v_1, v_2, v_3) \\ 0 \\ 0 \end{bmatrix} + h_2^{\perp}(4) \begin{bmatrix} I_3^1(u_1, u_2, u_3) \\ 0 \\ 0 \end{bmatrix} \\ &+ h_{(1,2)}^{\perp}(4) \begin{bmatrix} I_1^1(p_1, p_2, p_3) \\ 0 \\ 0 \end{bmatrix} \end{aligned} \tag{22}$$

where $h_i(t)^{\perp}$ and $h_{(i,j)}(t)^{\perp} \in \mathbb{C}^{3\times 3}$ are the orthogonal projection matrices on the null space of $H_i(t)$ and on the null space of the subspace spanned by both $H_i(t), H_j(t)$, respectively. Then,

$$\begin{aligned} Y_1(4) &= \left[H_1(4)h_2^{\perp}(4)\right]_1 I_3^1(u_1,u_2,u_3) && (23)\\ &= L_1^2(u_1,u_2,u_3) && (24)\\ Y_2(4) &= \left[H_2(4)h_1^{\perp}(4)\right]_1 I_3^2(v_1,v_2,v_3) && (25)\\ &= L_2^2(v_1,v_2,v_3) && (26)\\ Y_3(4) &= L_3^2(p_1,p_2,p_3) + \left[H_3(4)h_1^{\perp}(4)\right]_1 I_3^2(v_1,v_2,v_3)\\ &+ \left[H_3(4)h_2^{\perp}(4)\right]_1 I_3^1(u_1,u_2,u_3) && (27) \end{aligned}$$

where $[X]_1$ is the first element of a vector $X \in \mathbb{C}^{1\times 3}$. In spite of receiving an interference-corrupted signal, receiver R_3 can get a linear combination of its desired signals only and remove the interference by applying a simple physical network coding as follows:

$$\begin{aligned} L_3^2(p_1,p_2,p_3) &= Y_3(4) - \left[H_3(4)h_1^{\perp}(4)\right]_1 Y_3(2)\\ &- \left[H_3(4)h_2^{\perp}(4)\right]_1 Y_3(1) && (28) \end{aligned}$$

At $t = 5$:
In this time slot, we deliver two interference-free terms to R_1 and R_3 while providing a desired term for R_2 corrupted by removable interference, i.e.,

$$\begin{aligned} X(5) &= h_1^{\perp}(5)\begin{bmatrix} I_2^2(p_1,p_2,p_3)\\ 0\\ 0\end{bmatrix} + h_3^{\perp}(5)\begin{bmatrix} I_2^1(u_1,u_2,u_3)\\ 0\\ 0\end{bmatrix}\\ &+ h_{(1,3)}^{\perp}(5)\begin{bmatrix} I_1^1(v_1,v_2,v_3)\\ 0\\ 0\end{bmatrix} && (29) \end{aligned}$$

Then,

$$\begin{aligned} Y_1(5) &= \left[H_1(5)h_3^{\perp}(5)\right]_1 I_2^1(u_1,u_2,u_3) && (30)\\ &= L_1^3(u_1,u_2,u_3) && (31)\\ Y_2(5) &= L_2^3(v_1,v_2,v_3) + \left[H_2(5)h_1^{\perp}(5)\right]_1 I_2^2(p_1,p_2,p_3)\\ &+ \left[H_2(5)h_3^{\perp}(5)\right]_1 I_2^1(u_1,u_2,u_3) && (32)\\ Y_3(5) &= \left[H_3(5)h_1^{\perp}(5)\right]_1 I_2^2(p_1,p_2,p_3) && (33)\\ &= L_3^3(p_1,p_2,p_3) && (34) \end{aligned}$$

Receiver R_2 can also remove the interference signal using its received signal in previous time slots, i.e.,

$$\begin{aligned} L_2^3(v_1,v_2,v_3) &= Y_2(5) - \left[H_2(5)h_3^{\perp}(5)\right]_1 Y_2(1)\\ &- \left[H_2(5)h_1^{\perp}(5)\right]_1 Y_2(3) && (35) \end{aligned}$$

Hence, after five time slots, each receiver has three different linear combinations of its three desired messages and the total achieved DoF for the 3-user BC is given by $D_{\Sigma}(3) = \frac{9}{5}$.

Theorem 1. *The K-user broadcast channel with synergistic alternating CSIT with distribution* $\in \Lambda(\lambda_P = \frac{(K-1)^2}{2K^2-K}, \lambda_D = \frac{K-1}{2K-1}, \lambda_N = \frac{1}{K})$ *can achieve almost surely*

$$D_{\Sigma}(K) = \frac{K^2}{2K-1} \quad (36)$$

Proof: The transmission scheme starts with sending information symbols in phase one, i.e., interference creation phase, to provide each receiver with a linear combination of its intended data symbols while creating $K-1$ interference terms at each receiver. This phase consumes K time slots to deliver K different linear combinations of the data symbols to K different receivers while creating $K \times (K-1)$ interference terms that will be useful as a side information for the receivers in the subsequent time slots. This phase requires $K \times (K-1)$ delayed CSIT states and K no CSIT states.

In contrast, phase two, i.e., interference resurrection phase, consumes $(K-1)$ time slots to deliver $(K-1)$ messages of order-K, i.e., intended for the K receivers, in order to make each receiver decode K symbols successfully. This phase requires $(K-1)^2$ perfect CSIT states and $(K-1)$ no CSIT states. The fraction of CSIT states during the two phases is given by

$$\begin{aligned} \lambda_P &= \frac{(K-1)^2}{K\times(2K-1)} = \frac{(K-1)^2}{2K^2-K} && (37)\\ \lambda_D &= \frac{K\times(K-1)}{K\times(2K-1)} = \frac{K-1}{2K-1} && (38)\\ \lambda_N &= \frac{(2K-1)}{K\times(2K-1)} = \frac{1}{K} && (39) \end{aligned}$$

■

IV. Discussion

Remark 1: **Comparison with all delayed CSIT [12]**
For the K-user BC model, the achievable DoF under the CSIT alternation pattern with the distribution given in Theorem 1 is strictly greater than the best known upper bound for the all delayed CSIT pattern [12], i.e., with distribution $\Lambda(0,1,0)$, which is $K/(1+\frac{1}{2}+\cdots+\frac{1}{K})$ DoF. In order to send K^2 successfully decoded messages, the proposed scheme in [12] needs $K\times(1+\frac{1}{2}+\cdots+\frac{1}{K}) \approx K\times\ln(K)$ time slots while our proposed scheme needs only $2K-1$ time slots thanks to the alternating CSIT feature. Fig. 2 shows the synergistic benefits of CSIT alternation on the DoF versus the number of users K.

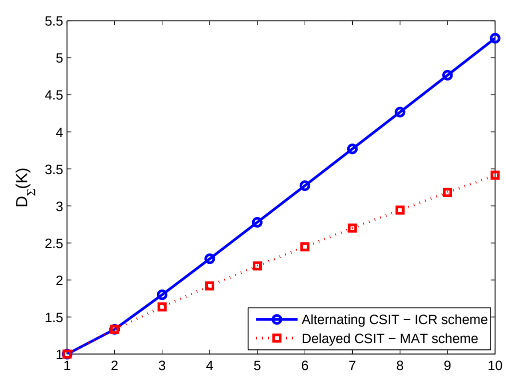

Fig. 2. DoF comparison for broadcast channel between all delayed and alternating CSIT models.

Remark 2: **Comparison with Hybrid CSIT [22]**
The system model is similar to ours but with hybrid CSIT, i.e., the link availability is constant over the channel uses. As a comparison, for the case of (P, D, D) the proposed

S_{123}^3	$S_{123}(4,5)$
(NDD, DND, DDN)	(PPN, PNP)
(NDD, DDN, DND)	(PNP, PPN)
(DND, DDN, DDN)	(PPN, NPP)
(DND, DDN, NDD)	(NPP, PPN)
(DDN, DND, NDD)	(NPP, PNP)
(DDN, NDD, DND)	(PNP, NPP)

TABLE I. ALL SYNERGISTIC CSIT PATTERNS FOR THE 3-USER BC WITH $\Lambda(\frac{4}{15}, \frac{6}{15}, \frac{5}{15})$.

scheme in [22] achieves $\frac{9}{5}$ DoF, which implies that the channel states availability pattern over the channel uses is $S_{123}^5 = (PDD, PDD, PDD, PDD, PDD)$. Note that this CSIT pattern has a distribution given by $\Lambda(\frac{5}{15}, \frac{10}{15}, \frac{0}{15})$. However, by harnessing the synergy benefits of CSIT alternation in our case, with less distribution of CSIT availability, i.e., $\Lambda(\frac{4}{15}, \frac{6}{15}, \frac{5}{15})$, our proposed scheme can achieve the same $\frac{9}{5}$ DoF. Also, the authors needed extensive channel extension to achieve this DoF by sending 18 symbols (10 symbols for R_1, 4 symbols for R_2 and 4 symbols for R_3) in 10 time slots. On the other hand, the proposed scheme requires only 5 time slots to send 3 messages to each user.

Remark 3: **Comparison with Tandon et.al. [21]**
For an M-antenna transmitter and K users, the proposed scheme in [21] assumes that at each time slot perfect CSIT is present to $\min(M, K)$ receivers and no CSIT to the remaining $K - \min(M, K)$ receivers. A total DoF of $\min(M, K)$ is achievable at each time slot and therefore a sum DoF of $\min(M, K)$ is also achievable for this scheme. The fraction of time λ that perfect CSIT obtained from any specific receiver is $\min(M, K)/K$. For $M = K$, the fraction of time for perfect CSIT $\lambda = 1$ which means perfect CSIT should be available about all receivers.

Remark 4: **Synergy benefits of CSIT pattern**
The synergy gain of delayed CSIT followed by perfect CSIT is useful to reconstruct the interference terms in prior time slots and constructing messages useful for the receivers in subsequent time slots. Note that the DoF for the 3-user BC with perfect CSIT is 3, with delayed CSIT is bounded by $\frac{18}{11}$, and with no CSIT is one DoF. The alternation of CSIT states S_{123} over five time slots works cooperatively to provide a DoF greater than the DoF of the sum of their individual DoF for the same network. As an example, consider the CSIT alternation pattern given by $S_{123}^5 = (NNN, DDD, DDD, DDD, PPP)$. If there is no interaction between the five time slots, the DoF that can be obtained are given by $1\times\frac{3}{15}+\frac{18}{11}\times\frac{9}{15}+3\times\frac{3}{15} = \frac{98}{55} < \frac{9}{5}$. However, harnessing the synergistic benefits of alternating CSIT, we can achieve more DoF ($\frac{9}{5}$ DoF) with less CSIT pattern $S_{123}^5 = (NDD, DND, DDN, PPN, PNP)$. Table 1 lists the beneficial synergistic CSIT alternation patterns with $\Lambda(\frac{4}{15}, \frac{6}{15}, \frac{5}{15})$ that can be utilized to achieve $\frac{9}{5}$ DoF for the 3-user BC channel. We can see from Table. 1 that there are only $|S_{123}^5| = |S_{123}^3| \times |S_{123}(4,5)|$ = 36 CSIT alternation patterns with synergistic benefits.

Remark 5: **Upper bound on the DoF**
An outer bound on the DoF region of the K-user BC under alternating CSIT was introduced in [21]. The achievable DoF, $\{d_i\}_{i=1}^K$, to the K receivers is bounded by

$$Kd_1 + d_2 + \cdots + d_K \le K + (K-1)\gamma_1 \quad (40)$$
$$d_1 + Kd_2 + \cdots + d_K \le K + (K-1)\gamma_2 \quad (41)$$
$$\vdots$$
$$d_1 + d_2 + \cdots + Kd_K \le K + (K-1)\gamma_K \quad (42)$$

where

$$\gamma_i = \frac{\sum_{t=1}^n \mathbb{I}(S_i(t) = P)}{n} \le \gamma, \forall i = 1, \ldots, K \quad (43)$$

is the fraction of time where perfect CSIT for receiver i is available. Adding the the previous K bounds, yields the following upper bound on the total DoF

$$D_\Sigma(K) = d_1+d_2+\cdots+d_K \le \frac{K^2 + (K-1)\sum_{i=1}^K \gamma_i}{2K-1} \quad (44)$$

Fig. 3 depicts the comparison between the achievable DoF with perfect CSIT fraction $(\gamma_1, \gamma_2, \ldots, \gamma_K)$ where $\gamma_i = \gamma = \frac{K-1}{2K-1}$ and $\gamma_{j\neq i} = \frac{K-2}{2K-1} < \gamma$, $\forall i, j \in \{1, \ldots, K\}$ with the upper bound on the achievable DoF with the same alternating CSIT fraction, and the upper bound when $\gamma = 1$ for the K-user BC.

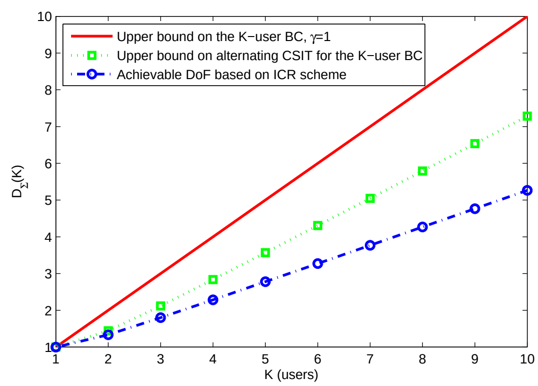

Fig. 3. DoF comparison for the K-user BC.

Remark 6: **DoF region characterization**

For the 3-user case, in order to find the optimal DoF for each receiver for a given perfect CSIT distribution $(\gamma_1,\gamma_2,\gamma_3)$, we solve the following linear program

$$\text{P1:} \max_{d_1,d_2,d_3} \quad d_1 + d_2 + d_3$$
$$\text{s.t.} \quad 3d_1 + d_2 + d_3 \le 3 + 2\gamma_1 \quad (45)$$
$$d_1 + 3d_2 + d_3 \le 3 + 2\gamma_2 \quad (46)$$
$$d_1 + d_2 + 3d_3 \le 3 + 2\gamma_3 \quad (47)$$
$$0 \le d_i \le 1, \quad \forall i = 1, 2, 3 \quad (48)$$

Since the constraints of the linear program are active, we can get a general closed form expression as a function of γ_i's by using the reduced echelon form method. Then, the solution will be as follows

$$d_i^* = \frac{3 + 4\gamma_i - \sum_{j=1,j\neq i}^3 \gamma_j}{5}, \quad \forall i = 1, 2, 3 \quad (49)$$

$(\gamma_1, \gamma_2, \gamma_3)$	(d_1, d_2, d_3)	Scheme
$(1, 0, 0)$	$(1, 0, 0)$	—
$(0, 1, 0)$	$(0, 1, 0)$	—
$(0, 0, 1)$	$(0, 0, 1)$	—
$(1/3, 1/3, 1/3)$	$(1/3, 1/3, 1/3)$	Time sharing
$(2/5, 1/5, 1/5)$	(3/5,3/5,3/5)	ICR
$(1/5, 2/5, 1/5)$	(3/5,3/5,3/5)	ICR
$(1/5, 1/5, 2/5)$	(3/5,3/5,3/5)	ICR
$(1, 1, 1)$	(1,1,1)	Conventional

TABLE II. PERFECT CSIT DISTRIBUTION AMONG THREE USERS AND ITS ACHIEVABLE DEGREES OF FREEDOM.

For a perfect CSIT distribution $(\gamma_1, \gamma_2, \gamma_3) = (\frac{2}{5}, \frac{1}{5}, \frac{1}{5})$ then the optimal DoF tuple is given by $d^* = (0.84, 0.64, 0.64)$ which is greater than the achievable DoF tuple $d = (0.6, 0.6, 0.6)$. Fig. 4 shows the achievable DoF region for the 3-user BC: the red point is the achievable DoF under perfect CSIT fraction with $(\gamma_1, \gamma_2, \gamma_3) = (2/5, 1/5, 1/5)$ (W.L.O.G we set $\gamma_1 = \gamma$ and $\gamma_{i \neq 1} < \gamma$), and the time sharing scheme is achieved by any convex combinations of the corner points.

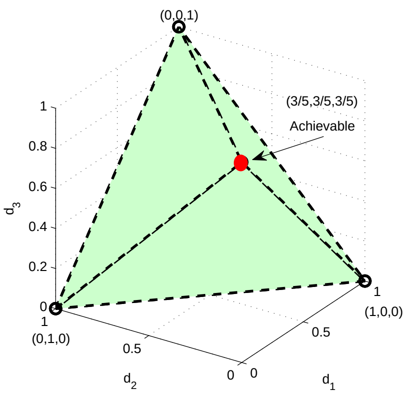

Fig. 4. Achievable DoF Region for the 3-user BC.

V. CONCLUSION

We have investigated the synergistic benefits of the alternation of CSIT for the K-user broadcast channel. The available CSIT alternates between three possible states of availability (P, D, N). We have showed that $\frac{K^2}{2K-1}$ DoF can be attained almost surely under CSIT distribution $\in \Lambda(\lambda_P = \frac{(K-1)^2}{K \times (2K-1)}, \lambda_D = \frac{K-1}{2K-1}, \lambda_N = \frac{1}{K})$. Also, we have compared our scheme with prior work and highlighted the advantages of having alternating CSIT to different receivers.

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