Capacity Analysis of PLC Systems over Rayleigh Fading Channels with Nakagami-m Additive Noise

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***Abstract*—Power line communication (PLC) is an emerging technology for the realization of smart grid and home automation. It utilizes existing power line infrastructure for data communication in addition to the transmission of power. The PLC channel behaves significantly different from the wireless channel; and it is characterized by signal attenuation as well as by additive noise and multiplicative noise effects. The additive noise consists of background noise and impulsive noise; while the multiplicative noise results in fading of the received signal power. This paper investigates the impact of the channel characteristics on the capacity performance of a PLC system over Rayleigh fading channel with frequency-distance dependent attenuation and colored Nakagami-m distributed additive noise. We derive the exact closed-form expressions for the distribution of the instantaneous signal-to-noise ratio (SNR). Since closed-form expression of the capacity for channels with non-Gaussian noise is extremely difficult to obtain, we choose to use the lower limit of the PLC capacity to facilitate our analysis. Monte Carlo simulation results are used to verify the derived analytical expressions.**

I. Introduction

In recent years, power line communication (PLC) has gained increasing interests from both the industry and academia due to the vision of widespread information transmission through power lines. With the advantages of omnipresence of power line and no need to invest in new infrastructure, PLC is set to be a promising technology to meet the ever-growing demands of high speed and ubiquitous access to digital information [1]. However, the power line channel presents some constraints for reliable signal transmission such as fading, unpredictable fluctuation of noise levels and impedance, time varying signal attenuation along the transmission line, etc [2].

PLC channel is tremendously different from the wireless channel. Attenuation in PLC systems depends on the characteristics of the power cables, length of transmission, and the operating frequency. The additive noise in PLC can be classified into two broad categories, i.e., background noise and impulsive noise [2]. The impulsive noise is mostly modeled by the Gaussian-mixture distributions, e.g., Bernoulli-Gaussian or Middleton's class-A distributions [3], while studies show that background noise in PLC follows the Nakagami-m distribution [4]–[6]. Additionally, the background noise in the PLC channel is not white but colored. In this paper, we focus on the effects of background noise. The amplitude fading statistics in PLC environments are not well established compared to wireless communications. A vast number of measurement results show that distributions such as Rayleigh, Rician, and lognormal are recommended for defining the path amplitudes in PLC channels [7]. In our analysis, we will assume the amplitude following Rayleigh distribution, which was found to be the best fit for a wealth of PLC field measurements [8]–[12].

Due to the unfavorable characteristics of the PLC channel, performance analysis of PLC systems have been the focus of research. Most of existing works on the performance analysis of PLC systems have been focused on the bit error rate (BER) and outage probability. The PLC performance in terms of BER and outage probability for a binary phase shift keying (BPSK) modulated signal under Nakagami-m distributed additive noise is studied in [13]. A comparison of BER for an orthogonal frequency division multiplexing (OFDM) system with different pulse-shaping is studied in [14]. The BER of a PLC system under the combined effect of background and impulsive noises is analyzed in [15]. Some recent information-theoretic studies on PLC systems include [16]–[18]. In [16], the lower and upper bounds of the system capacity taken into consideration a Rayleigh fading channel and impulsive noise effect are derived. In [17] and [18], the capacity of PLC systems with different topologies is studied. However, only the attenuation effect was considered while fading (due to multiplicative noise) and additive noise were ignored.

In this paper, we study the system performance of a PLC over Rayleigh fading channel with Nakagami-m distributed additive noise. We choose the lower limit of the PLC capacity to facilitate our analysis. We derive expressions for the probability density function (PDF) of the instantaneous signal-to-noise ratio (SNR) taking into account the effects of attenuation, fading, and additive noise. Based on the above statistics, we investigate impact of the channel characteristics on the PLC system capacity.

The remainder of the paper is organized as follows. In Section II, we describe the considered system and PLC channel models. The PLC system is analyzed in Section III; and the impact of channel characteristics on the lower limit of the PLC capacity are derived. The analytical and simulation results are presented in Section IV. Section V concludes the paper.

II. System and Channel Model

The input/output model of a PLC system over Rayleigh fading channel with Nakagami-m noise can be expressed as

$$y_c = h_c \cdot x + w_c, \tag{1}$$

where x is the channel input with unit energy, i.e., $\mathrm{E}[|x|^2] = 1$;

and y_c is the channel output. The envelope h of the complex channel gain h_c is Rayleigh distributed with PDF given by

$$f_h(h) = \frac{h}{\sigma^2} \cdot \exp\Big(-\frac{h^2}{2\sigma^2}\Big), \tag{2}$$

where σ is the scale parameter of the distribution, which determines the statistical average and the variance of the random variable (RV) as $\mathrm{E}[h] = \sigma\sqrt{\pi/2}$ and $\mathrm{Var}[h] = (2 - 0.5\pi)\sigma^2$, respectively. In model (1), the average power of $h_c \cdot x$ depends on the transmit power P_t and the power attenuation $A(D, f)$ over transmission distance D at operating frequency f[1], i.e.,

$$\mathrm{E}[|h_c|^2 \cdot |x|^2] = \mathrm{E}[h^2] = P_t \cdot A(D, f). \tag{3}$$

Due to the nature of the cable propagation environment, the PLC attenuation model is significantly different from that of wireless channel and $A(D, f)$ can be expressed as [17]

$$A(D, f) = e^{-2(\alpha_1+\alpha_2 \cdot f^k)\cdot D}, \tag{4}$$

where α_1 and α_2 are constants with dependence on the system configurations; the exponent k is the attenuation factor with typical values between 0.5 and 1. It is obvious from (4) that the attenuation increases dramatically with higher frequency and larger transmission distance.

Utilizing (3), (4) and the equality $\mathrm{E}[h^2] = \mathrm{Var}[h] + (\mathrm{E}[h])^2$, the scale parameter σ in (2) can be represented as

$$\sigma = \sqrt{\frac{P_t}{2}} \cdot e^{-(\alpha_1+\alpha_2 \cdot f^k)\cdot D}. \tag{5}$$

In (1), the parameter w_c represents the complex background noise. The absolute value w of the RV w_c is Nakagami-m distributed and its PDF is given by

$$f_w(w) = \frac{2m^m}{\Gamma(m)\Omega^m} w^{2m-1} \cdot \exp\Big(-\frac{mw^2}{\Omega}\Big), \tag{6}$$

where m is the shape parameter of the distribution defined as $\mathrm{E}^2[w^2]/\mathrm{Var}[w^2]$ with $\mathrm{E}[\cdot]$ denoting the expectation operator and $\mathrm{Var}[\cdot]$ representing the variance of the random variable, and $\Gamma(\cdot)$ is the Gamma function. The parameter Ω is the average power defined as $\mathrm{E}[w^2]$. The widely used assumption of white noise for wireless channel does not hold for PLC channel. Instead, the background noise is colored and the average power per unit bandwidth, namely, the power spectral density (PSD), can be written as [17]

$$\Omega = \mathrm{E}[w^2] = 10^{0.1\cdot(\beta_1+\beta_2 \cdot e^{-f/\beta_3})} \qquad [\mathrm{mW/Hz}], \tag{7}$$

where β_1, β_2, and β_3 are some constants.

III. Capacity Performance Analysis

Closed-form expression for the capacity of non-Gaussian noise is extremely difficult to obtain. The shannon capacity provides a lower limit for arbitrary noise and is given by

$$C = \log_2(1+\gamma) \qquad [\mathrm{bit/s/Hz}], \tag{8}$$

where γ is the instantaneous SNR. Despite the capacity in (8) is only the lower limit for the PLC channel, the emphasis of this study is to investigate the impact of the PLC channel parameters on the capacity; so it suffices for our purpose.

[1]The frequency f is in MHz throughout the paper.

A. Statistics of the Instantaneous SNR

The instantaneous SNR γ of the PLC system in (1) is expressed as

$$\gamma = \frac{h^2}{w^2}. \tag{9}$$

For simplicity of notation, we replace the arguments h^2 and w^2 in (9) by h' and w', respectively. To obtain the statistics of the instantaneous SNR γ, we first derive the statistics of $h' = h^2$ and $w' = w^2$. The PDF $f_{h'}(h')$ of the RV h' is obtained by introducing a change of random variable in the expression for the PDF $f_h(h)$ of the RV h in (2), yielding

$$f_{h'}(h') = f_h(\sqrt{h'})\Big|\frac{dh}{dh'}\Big| = \frac{1}{2\sigma^2} \cdot \exp\Big(-\frac{h'}{2\sigma^2}\Big). \tag{10}$$

The PDF $f_{w'}(w')$ of the RV w' can be obtained in the same manner as above and is given by

$$f_{w'}(w') = \frac{m^m}{\Gamma(m)\Omega^m} w'^{(m-1)} \cdot \exp\Big(-\frac{mw'}{\Omega}\Big). \tag{11}$$

After obtaining the PDFs of the RVs h' and w', the PDF of a new RV defined as the quotient of the two RVs $\gamma = \frac{h'}{w'}$ can be obtained as

$$f_\gamma(\gamma) = \int_{-\infty}^{\infty} |w'| \cdot f_{h',w'}(w'\gamma, w')\, dw', \tag{12}$$

where $f_{h',w'}(\cdot,\cdot)$ is the joint PDF of the independent RVs h' and w'. Therefore, $f_{h',w'}(w'\gamma, w') = f_{h'}(w'\gamma) \cdot f_{w'}(w')$. Substituting this equality into (12) and after some manipulations, we obtain the distribution of the instantaneous SNR as

$$\begin{aligned} f_\gamma(\gamma) &= \int_0^{\infty} w' \cdot f_{h'}(w'\gamma) \cdot f_{w'}(w')\, dw' \\ &= \frac{m^m}{2\sigma^2\Omega^m\Gamma(m)} \int_0^{\infty} w'^m \cdot \exp\Big[-\Big(\frac{\gamma}{2\sigma^2} + \frac{m}{\Omega}\Big) \cdot w'\Big]\, dw' \\ &= \frac{m^m}{2\sigma^2\Omega^m \cdot \mathrm{B}(1,m)} \cdot \Big(\frac{\gamma}{2\sigma^2} + \frac{m}{\Omega}\Big)^{-(m+1)}, \end{aligned} \tag{13}$$

where $\mathrm{B}(\cdot,\cdot)$ is the Beta function [19, p. 258]; and the last equality is obtained by using [20, Eq. 3.478.1] and the functional relation between Beta function $\mathrm{B}(\cdot,\cdot)$ and Gamma function $\Gamma(\cdot)$ [20, Eq. 8.384].

The cumulative distribution function (CDF) $F_\gamma(\gamma)$ of the RV γ can be immediately obtained from its relationship with the PDF $f_\gamma(\cdot)$, i.e., $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(x)\, dx$, as follows:

$$F_\gamma(\gamma) = \frac{m^m}{2\sigma^2\Omega^m\mathrm{B}(1,m)} \cdot \int_0^\gamma \Big(\frac{x}{2\sigma^2} + \frac{m}{\Omega}\Big)^{-(m+1)} dx. \tag{14}$$

In (14), substituting $t = \frac{x}{\gamma}$ and with the appropriate change of the integration limits, the CDF $F_\gamma(\gamma)$ can be rewritten as

$$\begin{aligned} F_\gamma(\gamma) &= \frac{\Omega\gamma}{2\sigma^2 m \cdot \mathrm{B}(1,m)} \cdot \int_0^1 \Big(1 + \frac{\Omega\gamma}{2\sigma^2 m} t\Big)^{-(m+1)} dt \\ &= \frac{\Omega\gamma}{2\sigma^2 m \cdot \mathrm{B}(1,m)} \cdot {}_2F_1(m+1, 1; 2; -\frac{\Omega\gamma}{2\sigma^2 m}), \end{aligned} \tag{15}$$

where the last equality comes from the integral representation of Gauss hypergeometric function ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ [20, Eq. 9.111]. The Gauss hypergeometric function can be straightforwardly evaluated using mathematical softwares such as Matlab and Mathematica. The expression in (15) also serves as the outage probability of the PLC system, which is also an essential performance criterion quantity and is defined as the probability that the SNR γ falls below a predefined threshold γ_{th}.

B. Ergodic Capacity Analysis

The ergodic capacity is defined as the expectation of the information rate over all states of the fading channel, which is mathematically expressed by

$$C_{erg} = \mathrm{E}[\log_2(1+\gamma)] = \int_0^{\infty} \log_2(1+\gamma) \cdot f_\gamma(\gamma)\, d\gamma, \quad (16)$$

where the PDF $f_\gamma(\gamma)$ of the SNR γ is given in (13). The integral in (16) is difficult to solve directly, thus we evaluate it with the help of the Meijer's G-function. The term $\log_2(1+\gamma)$ can be expressed in the form of Meijer's G-function as

$$\log_2(1+\gamma) = \frac{1}{\ln 2} \cdot G_{2,2}^{1,2}\left(\gamma \middle|\, {}^{1,1}_{1,0}\right). \quad (17)$$

The Meijer's G-function representation of the PDF $f_\gamma(\gamma)$ is not obvious to obtain directly from (13). Alternatively, we first express the CDF in (15) in the form of Meijer's G-function using [20, Eq. 9.34.7] and then obtain the PDF using derivation, i.e.,

$$\begin{aligned} f_\gamma(\gamma) &= \frac{dF_\gamma(\gamma)}{d\gamma} = \frac{d}{d\gamma}\left[\frac{1}{\Gamma(m)} \cdot G_{2,2}^{1,2}\left(\frac{\Omega\gamma}{2\sigma^2 m} \middle|\, {}^{1-m,1}_{1,0}\right)\right] \\ &= \frac{1}{\Gamma(m)\gamma} \cdot G_{3,3}^{1,2}\left(\frac{\Omega\gamma}{2\sigma^2 m} \middle|\, {}^{0,1-m,1}_{1,0,1}\right). \end{aligned} \quad (18)$$

As a check, the expression (18) can reduce to (13) by utilizing the equalities associated with the Meijer's G-function [20, Eq. 9.31] and [21, Eq. (07.34.03.0271.01)]. Then, the ergodic capacity can be written as the integral of the product of two Meijer's G-functions as follows:

$$\begin{aligned} C_{erg} &= \frac{(\ln 2)^{-1}}{\Gamma(m)} \int_0^{\infty} \frac{1}{\gamma} G_{2,2}^{1,2}\left(\gamma \middle|\, {}^{1,1}_{1,0}\right) G_{3,3}^{1,2}\left(\frac{\Omega\gamma}{2\sigma^2 m} \middle|\, {}^{0,1-m,1}_{1,0,1}\right) d\gamma \\ &= \frac{1}{\ln 2 \cdot \Gamma(m)} G_{2,2}^{2,2}\left(\frac{\Omega}{2\sigma^2 m} \middle|\, {}^{1-m,0}_{0,0}\right), \end{aligned} \quad (19)$$

where the last equality is based on the convolution theorem of Meijer's G-function [22, Eq. (21)] and the functional relation [20, Eq. 9.31]. Finally, utilizing [21, Eq. (07.34.03.0873.01)] and after some manipulations, the ergodic capacity can be expressed in terms of Gauss hypergeometric function as shown in (20) at the bottom of this page. Substituting (5) and (7) into (20), we obtain the ergodic capacity of the communication system in (21) as a function of the parameters associated with the PLC attenuation, fading, and additive noise introduced in Section II. The validity of the analytical expressions will be verified using simulation in the next section.

C. Outage Capacity Analysis

The ε-outage capacity, denoted as C_ε, is defined as the long-term transmission rate, which is guaranteed for $(1-\varepsilon)$ of the channel realizations. The ε-outage capacity is given by

$$\Pr(C < C_\varepsilon) = F_\gamma(2^{C_\varepsilon} - 1) = \varepsilon. \quad (22)$$

Using the equalities ${}_2F_1(a, b; b+1; z) = bz^{-b}\mathrm{B}_z(b, 1-a)$ and $\mathrm{B}_z(1,c) \cdot \mathrm{B}(1,c) = [1-(1-z)^c]$ [19, Eq. 6.6], where $\mathrm{B}_z(\cdot,\cdot)$ is the incomplete Beta function [20, Eq. 8.39], the CDF $F_\gamma(\gamma)$ in (15) can be rewritten as

$$F_\gamma(\gamma) = \frac{1}{m \cdot \mathrm{B}(1,m)}\left[1 - \left(1 + \frac{\Omega\gamma}{2\sigma^2 m}\right)^{-m}\right]. \quad (23)$$

Substituting (23) into (22) and solving for C_ε, we obtain the ε-outage capacity in terms of the PLC channel model parameters defined in Section II as follows:

$$C_\varepsilon = \log_2\left[1 + \frac{2\sigma^2 m}{\Omega \cdot \sqrt[m]{1 - m\varepsilon \cdot \mathrm{B}(1,m)}} - \frac{2\sigma^2 m}{\Omega}\right] \quad (24)$$

$$= \log_2\left[1 + \frac{P_t m \cdot e^{-2(\alpha_1+\alpha_2\cdot f^k)\cdot D}}{10^{0.1\cdot(\beta_1+\beta_2\cdot e^{-f/\beta_3})} \cdot \sqrt[m]{1 - m\varepsilon \cdot \mathrm{B}(1,m)}} - \frac{P_t \cdot m \cdot e^{-2(\alpha_1+\alpha_2\cdot f^k)\cdot D}}{10^{0.1\cdot(\beta_1+\beta_2\cdot e^{-f/\beta_3})}}\right]. \quad (25)$$

D. Optimal Transmission Frequency

Different from its wireless counterpart, the transmission frequency of the PLC system influences both the attenuation and the noise that the system will experience. To find the optimal carrier frequency in terms of maximum ergodic capacity, we start with the average SNR $\overline{\gamma}$ of the PLC channel given by

$$\overline{\gamma} = \frac{\mathrm{E}[h^2]}{\mathrm{E}[w^2]} = \frac{P_t \cdot e^{-2(\alpha_1+\alpha_2\cdot f^k)\cdot D}}{10^{0.1\cdot(\beta_1+\beta_2\cdot e^{-f/\beta_3})}}. \quad (26)$$

Then, the optimal frequency f_{opt} can be obtained by taking the first derivative of the strictly monotonic increasing function $\log_{10}(\overline{\gamma})$ with respect to f and setting the derivative to 0, i.e.,

$$\begin{aligned} &\frac{d}{d\,f}\left[-2D \cdot (\alpha_1 + \alpha_2 \cdot f^k) \cdot \log_{10} e - 0.1(\beta_1 + \beta_2 \cdot e^{-f/\beta_3})\right] \\ &= -2\alpha_2 k D(\log_{10} e) f^{k-1} + \frac{0.1\beta_2}{\beta_3} e^{-f/\beta_3} = 0. \end{aligned} \quad (27)$$

The solution to the equation in (27) cannot be expressed with elementary functions. The usual way is to evaluate it numerically using the Euler or Newton's methods [19, p. 18].

$$C_{erg} = \frac{2\sigma^2}{\ln 2 \cdot \Omega} \cdot {}_2F_1\left(1, 1; 1+m; 1 - \frac{2m\sigma^2}{\Omega}\right) \quad (20)$$

$$= \frac{P_t \cdot e^{-2(\alpha_1+\alpha_2\cdot f^k)\cdot D}}{\ln 2 \cdot 10^{0.1\cdot(\beta_1+\beta_2\cdot e^{-f/\beta_3})}} \cdot {}_2F_1\left(1, 1; 1+m; 1 - \frac{P_t \cdot m \cdot e^{-2(\alpha_1+\alpha_2\cdot f^k)\cdot D}}{10^{0.1\cdot(\beta_1+\beta_2\cdot e^{-f/\beta_3})}}\right). \quad (21)$$

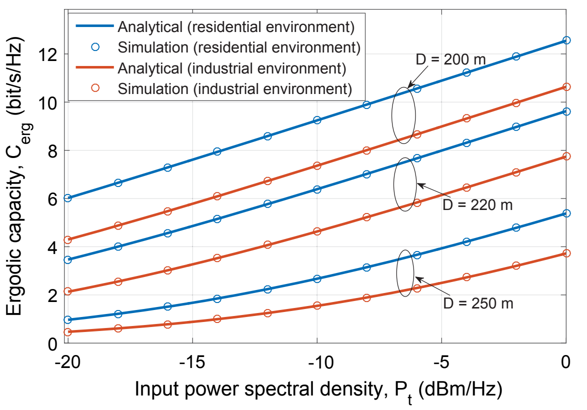

Fig. 1: Ergodic capacity versus input power per unit bandwidth with different transmission lengths, the carrier frequency is 20 MHz.

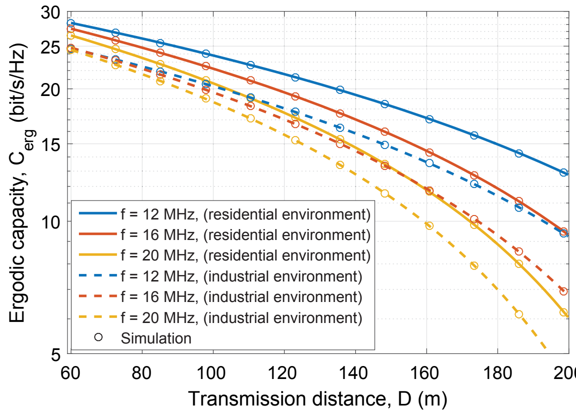

Fig. 2: Ergodic capacity versus transmission length at different transmission frequencies, the input PSD is -20 $\mathrm{dBm/Hz}$.

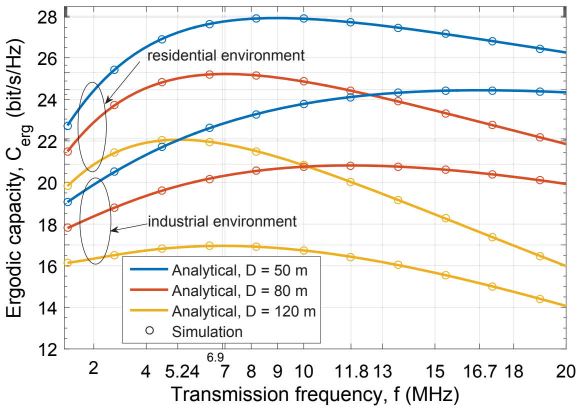

Fig. 3: Ergodic capacity versus transmission frequency with different transmission lengths, the input PSD is -25 $\mathrm{dBm/Hz}$.

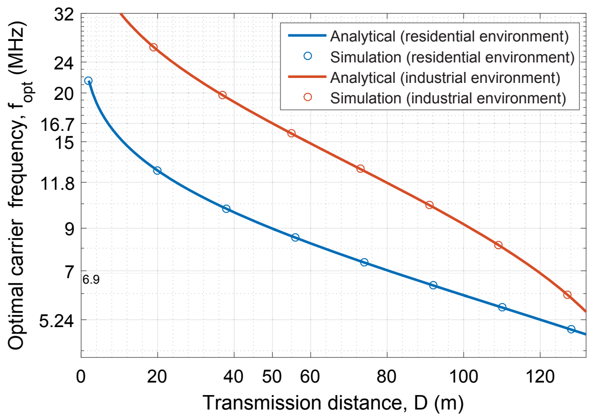

Fig. 4: Optimal transmission frequency versus transmission distance in the measured residential and industrial environments.

Here, we present a solution of the above equation in terms of Lambert W function from Appendix A:

$$f_{opt} = (k-1)\beta_3 \cdot W\left(\frac{\left(20(\log_{10} e)\alpha_2 k D \cdot \beta_3^k\right)^{\frac{1}{1-k}}}{(k-1)\cdot \beta_2^{\frac{1}{1-k}}}\right), \quad (28)$$

where $W(\cdot)$ is the Lambert W function (a.k.a., Omega function), which is defined as the multiple-valued inverse of the function $x \mapsto xe^x$ [23]. It can be effectively evaluated to arbitrary precision with Matlab. Obviously, the optimal carrier frequency is a function of the transmission distance D, which is quite different from that of wireless communications.

IV. Numerical Results

In this section, the analytical expressions presented in the previous sections are evaluated numerically and validated using simulations. We adopt the PLC channel parameter values shown in Table I, which are the experimental data from field measurements conducted in residential and industrial environments [24], [25]. Besides, the Nakagami-m parameter is set to 0.7 unless otherwise stated.

Figure 1 illustrates the ergodic channel capacity of the PLC system at transmission frequency 20 MHz with different transmission lengths in both environments. Figure 2 shows the capacity along the increasing power line length at different frequencies. It can be seen that the ergodic channel capacity depends highly on the transmission length. This dependance is two-fold. Firstly, longer transmission distance surely implies greater attenuation. Secondly, for a given frequency, it is more favorable for transmission at some distance than others, as will be shown in later analysis on the influence of transmission frequency on capacity. It can also be observed from Figs. 1 and 2 that PLC channel in the measured industrial environment exhibits much less ergodic capacity than that in the measured residential environment. This is due to stronger disturbance by large electrical loads in the industrial scenario, which leads to higher background noise level.

Figure 3 shows the ergodic channel capacity against carrier frequency with different transmission lengths in the measured residential and industrial environments. For a given wireless

TABLE I: Simulation Parameters

attenuation model parameters		
$\alpha_1 = 9.33 \times 10^{-3}\ \mathrm{m}^{-1}$	$\alpha_2 = 5.1 \times 10^{-3}\ \mathrm{s/m}$	$k = 0.7$
noise model parameters (residential environment)		
$\beta_1 = -125$	$\beta_2 = 35$	$\beta_3 = 3.6$
noise model parameters (industrial environment)		
$\beta_1 = -123$	$\beta_2 = 40$	$\beta_3 = 8.6$

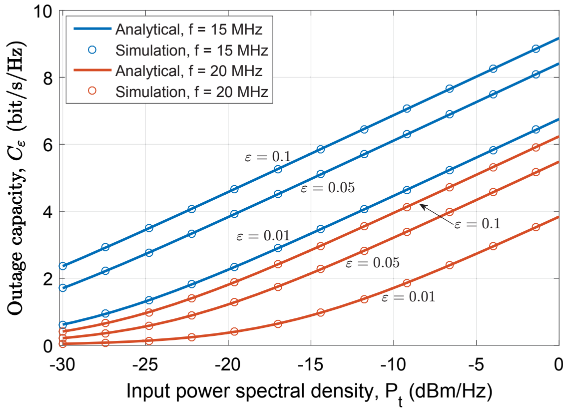

Fig. 5: Outage capacity versus input power per unit bandwidth in the measured residential scenario, the transmission distance is 200 m.

channel with fixed transmit power and propagation distance, larger carrier frequency generally indicates lower capacity per unit bandwidth. This is only partially true for the power line channel due to its different characteristics of the background noise. For a PLC link, the ergodic capacity first increases with larger carrier frequency before reaching the optimal frequency, then the capacity decreases with increasing frequency. Also, this optimal transmission frequency is lower for shorter transmission distance. This monotonic decreasing trend can also be seen from Fig. 4, which illustrates the relationship between the PLC transmission distance and the corresponding optimal carrier frequency. By comparing the frequencies corresponding to the largest capacities in Fig. 3 with the results of Fig. 4, the validity of the expression (28) is verified.

Figure 5 shows the outage capacity with different thresholds at two frequencies in the measured residential environment. It should be noted that the ε-outage capacity is not a monotone decreasing function against the carrier frequency.

V. Conclusion

In this paper, we studied the impact of PLC channel characteristics on the PLC system performance. The effects of the Rayleigh fading and the colored Nakagami-m background noise on the PLC system performance were evaluated from the information-theoretic perspective. Exact closed-form expressions for the SNR and the lower limit for the capacity of the PLC system were derived. The analytical expressions were validated using Monte Carlo simulation results.

Appendix A: Derivation of (28)

The relation (27) can be simplified as $x^a e^{-x} = b$ with following change of RVs: $a = 1-k$, $b = 20(\log_{10} e)\alpha_2 k D \beta_3^k/\beta_2$, $x = f/\beta_3$. Therefore, to obtain the optimal frequency $f_{opt} = \beta_3 x$, we just need to work out the solution of $x^a e^{-x} = b$.

According to the definition of Lambert W function, the solution of the equation $ye^y = X$ is given by $y = W(X)$, where $W(\cdot)$ is the Lambert W function. Substituting y and X with $-\frac{x}{a}$ and $-\frac{b^{\frac{1}{a}}}{a}$ respectively and with some manipulations, the solution of $x^a e^{-x} = b$ is written as $x = -a \cdot W(-\frac{b^{\frac{1}{a}}}{a})$. Finally, we can obtain the result shown in (28).

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References

[1] S. Galli *et al.*, “For the grid and through the grid: The role of power line communications in the smart grid,” *Proc. IEEE*, vol. 99, no. 6, pp. 998–1027, 2011.

[2] N. Pavlidou *et al.*, “Power line communications: State of the art and future trends,” *IEEE Commun. Mag.*, vol. 41, no. 4, pp. 34–40, 2003.

[3] T. Shongwey *et al.*, “On impulse noise and its models,” in *Proc. of IEEE Int. Symp. Power Line Commun. and its Applic. (ISPLC)*, 2014.

[4] H. Meng *et al.*, “Modeling and analysis of noise effects on broadband power-line communications,” *IEEE Trans. Power Del.*, vol. 20, no. 2, pp. 630–637, 2005.

[5] X. Yang *et al.*, “Investigation of transmission properties on 10-kV medium voltage power lines–part I: General properties,” *IEEE Trans. Power Del.*, vol. 22, no. 3, pp. 1446–1454, 2007.

[6] Y. Kim *et al.*, “Closed-form expression of Nakagami-like background noise in power-line channel,” *IEEE Trans. Power Del.*, vol. 23, no. 3, pp. 1410–1412, 2008.

[7] S. Güzelgöz *et al.*, “A review of wireless and PLC propagation channel characteristics for smart grid environments,” *J. Electr. Comput. Eng.*, vol. 2011, pp. 1–12, 2011.

[8] P. Karols *et al.*, “Mass transit power traction networks as communication channels,” *IEEE J. Sel. Areas Commum.*, vol. 24, pp. 1339–1350, 2006.

[9] S.-C. Kim *et al.*, “Wideband channel measurements and modeling for in-house power line communication,” in *Proc. of Int. Symp. Power Line Commun. and its Applic. (ISPLC)*, 2002.

[10] M. Tlich *et al.*, “Indoor power-line communications channel characterization up to 100 MHz–part I: One-parameter deterministic model,” *IEEE Trans. Power Del.*, vol. 23, no. 3, pp. 1392–1401, 2008.

[11] K. Hoque *et al.*, “Performance analysis of MC-CDMA power line communication system,” in *Proc. of Int. Conf. Wireless and Opt. Commun. Netw. (WOCN)*. IEEE, 2007, pp. 1–5.

[12] P. Karols, *Nachrichtentechnische Modellierung von Fahrleitungsnetzen in der Bahntechnik*. Mensch-und-Buch-Verlag, 2004.

[13] A. Mathur and M. R. Bhatnagar, “PLC performance analysis assuming BPSK modulation over Nakagami-m additive noise,” *IEEE Commun. Lett.*, vol. 18, no. 6, pp. 909–912, 2014.

[14] J. J. Sánchez-Martínez *et al.*, “Performance analysis of OFDM modulation on indoor PLC channels in the frequency band up to 210 MHz,” in *Proc. of Int. Symp. Power Line Commun. and its Applic. (ISPLC)*, 2010.

[15] A. Mathur *et al.*, “Performance evaluation of PLC under the combined effect of background and impulsive noises,” *IEEE Commun. Lett.*, vol. 19, no. 7, pp. 1117–1120, 2015.

[16] H. V. Vu *et al.*, “On the capacity of Bernoulli-Gaussian impulsive noise channels in Rayleigh fading,” in *Proc. of IEEE Int. Symp. Personal, Indoor, and Mobile Radio Commun. (PIMRC)*, 2013, pp. 1281–1285.

[17] X. Cheng *et al.*, “Relay-aided amplify-and-forward powerline communications,” *IEEE Trans. Smart Grid*, vol. 4, no. 1, pp. 265–272, 2013.

[18] S. Ezzine *et al.*, “Capacity analysis of an OFDM-based two-hops relaying PLC systems,” in *Proc. of IEEE Veh. Technol. Conf. (VTC-Spring)*, 2015, pp. 1–5.

[19] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions: with Formulas, Graphs, and Mathematical Tables*. Courier Corporation, 1964, no. 55.

[20] A. Jeffrey and D. Zwillinger, *Table of Integrals, Series, and Products*, 7th ed. Elsevier, 2007.

[21] Wolfram Research. The Wolfram Functions Site. [Online]. Available: http://functions.wolfram.com/

[22] V. Adamchik and O. Marichev, “The algorithm for calculating integrals of hypergeometric type functions and its realization in REDUCE system,” in *Proc. of Int. Symp. on Symbolic and Algebraic Comput.* ACM, 1990, pp. 212–224.

[23] R. M. Corless *et al.*, “On the Lambert W function,” *Advances in Comput. Math.*, vol. 5, no. 1, pp. 329–359, 1996.

[24] M. Zimmermann and K. Dostert, “A multipath model for the powerline channel,” *IEEE Trans. Commun.*, vol. 50, no. 4, pp. 553–559, 2002.

[25] H. Philipps, “Development of a statistical model for powerline communication channels,” in *Proc. of Int. Symp. Power Line Commun. and its Applic. (ISPLC)*, 2000, pp. 5–7.