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Optimum HDAF Relay-Assisted Combining Scheme with Relay Decision Information

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Abstract—For single-input multiple-output (SIMO) systems, Maximum Ratio Combining (MRC), employed at the receiver, achieves the best performance compared to other combining schemes in the literature, such as Selection Combining (SC) and Equal Gain Combining (EGC). However, for cooperative relay-based systems, MRC has limited performance due to the lack of relay decision information awareness at the destination combiner. To overcome this limitation, this paper proposes a new optimum combining scheme, which is being demonstrated for Hybrid-Decode-Amplify-Forward (HDAF) cooperative system. This scheme utilizes relay decision information in the form of the number of errors per received packet over the source-relay link. The derivation of the optimum combining scheme is based on a mathematical model that utilizes conditional error probability. The improved performance of the proposed optimum combining scheme is demonstrated through analytical results and Monte Carlo simulations.

Index Terms—Conditional error probability, MRC, Optimum combining, HDAF, Cooperative relay-based system.

I. INTRODUCTION

In cooperative relay-based systems, spatial diversity is of primary importance due to its ability of combating fading in wireless propagation channels. The relay node processes the received information using different signaling protocol, such as amplify-forward (AF) or decode-forward (DF). In AF protocol, the relay forwards the noisy version from the source without detection or any further processing. On the contrary, in DF protocol, the relay decodes the received information before sending it to the destination. The DF mode is further classified as: 1) Fixed Decode-Forward (FDF): always forward relay data, 2) Adaptive Decode-Forward (ADF): forward correctly received packets only and drop wrong packets. For the purpose of this study, and in order to avoid error propagation to the destination, ADF is considered. A hybrid relaying protocol called Hybrid-Decode-Amplify-Forward (HDAF) has been investigated in [1]. HDAF combines the metrics of both DF and AF relaying [2]. It has been shown in [2] that HDAF, with proper relay location, outperforms each of AF and DF.

The performance of cooperative relay-based systems with spatial diversity is primarily dependent on the choice of the employed combining scheme at the receiver. For single-input multiple-output (SIMO) systems, it has been demonstrated that Maximum Ratio Combining (MRC) is the optimum combining scheme [3]. However, in the case of cooperative relay-based systems, the performance of MRC is not always optimum since

the Signal-to-Noise Ratio (SNR) at the receiver is determined by both, the direct link and the relayed links. In [4], it was shown that MRC achieved full diversity when AF is being employed at the relay. However, when DF is used at the relay, MRC can not achieve full diversity. Since HDAF only uses DF when the relay packet is correct, MRC achieves full diversity when DF is used in HDAF systems.

In the literature, it was shown that combining at the destination can be further improved through the use of relay information. In [5], we proposed a new combining scheme for DF cooperative systems which uses Relay Decision information (i.e., whether or not the packet is received correctly at the relay) in order to further improve the performance. The authors in [6] derived a combiner capable of collecting full diversity with DF through the use of the instantaneous Bit Error Probability (BEP) of the source-relay link at the destination. Moreover, the authors in [7] derived a Maximum-Likelihood (ML) decoder for DF cooperative systems in which the decoder uses the average BER of the source-relay link at the destination. Such combiners or decoders optimize the combining process and improves the performance of DF cooperative systems. To the best of our knowledge, no optimum scheme, which uses relay information at the destination, is available for HDAF cooperative systems.

In this paper, we propose a new optimum combining scheme which utilizes relay decision information in the combining process at the receiver. The proposed combining scheme, which is being demonstrated for HDAF cooperative system, aims to improve the diversity gain, in comparison to existing combining scheme such as MRC. Relay decision information is defined as the number of errors in the received source-relay packet. Then, relay decision information assists the combiner at the receiver to optimize the detection of the combined packet. The formulated mathematical model of the proposed optimum combining scheme is based on minimizing a conditional error probability function. Numerical results of the analytical model as well as Monte Carlo simulations are provided to validate the benefits of the proposed scheme. Results emphasize that the proposed scheme achieves considerable performance improvement compared to MRC.

The rest of the paper is organized as follows: The system

model is presented in section II. The conditional error probability expressions are derived in Section III. The optimum combining coefficient is obtained in section IV. Numerical and system level simulations are demonstrated in section V. Finally, conclusions are provided in section VI.

Notation: $(\cdot)^*$ denotes conjugation; $\mathcal{CN}(0, \sigma^2)$ denotes the circular symmetric complex Gaussian distribution with zero mean and variance σ^2 ; $P(\alpha)N$ denotes the probability of the event α with a packet length of N symbols. For simplicity, when $N = 1$, we remove the subscript 1 in $P(\alpha)$ and hence, will be expressed as $P(\alpha)$. $\Re(x)$, $\Im(x)$, and $|x|$ are real part, imaginary part, and absolute value of complex number x , respectively. The event \bar{F} is the complement of the event F . Bold symbols are used to indicate complex variables.

II. SYSTEM MODEL

Consider a Source (S) node that cooperates with a Relay (R) node to send information, denoted by x , to a destination (D) node, at the receiver, using HDAF system. For the purpose of this study, time division duplexing is considered with all nodes having a single antenna. Moreover, half-duplex relaying is assumed, where R can not simultaneously transmit and receive. For the considered HDAF system, S broadcasts N -bits packet to both R and D at the first time slot (T_1). R detects the packet and identifies the number of errors per packet (e_n). If R receives the packet with errors (i.e., $e_n \neq 0$), it retransmits the packet using AF at the second time slot (T_2). On the other hand, if R receives the packet correctly, it retransmits the packet using DF and sets $e_n = 0$ at T_2 .

Our model further assumes the following: D can track the variations in the channel coefficient \mathbf{h} , implying that the combining is done with the knowledge of the fading parameters. The packets are transmitted using Binary Phase Shift Keying (BPSK) modulation. The cooperative links are assumed to be mutually independent. We also assume that the proposed system is equipped with CRC codes. Hence, R is capable of detecting packet errors. Moreover, we assume that the number e_n is obtained using channel coding at R . It should be pointed out that e_n is transmitted at T_2 to D over a reliable control channel (i.e., e_n will always be received correctly at D node).

Next, in order to set up the paper discussions, the mathematical formulation and underlying assumptions of our model are described. The signal \mathbf{y}_{SR} received by R over the Source-Relay (S - R) link at T_1 is given by

$$\mathbf{y}_{SR} = \sqrt{P_s} \mathbf{h}_{sr} x + \mathbf{n}_r, \quad (1)$$

where P_s and \mathbf{h}_{sr} are the power and the channel coefficient over S - R link, respectively. \mathbf{n}_r is the AWGN at R with $\mathbf{n}_r \sim \mathcal{CN}(0, \sigma_r^2)$. We point out that all terms are complex representing in-phase and quadrature components. For the direct path link, the signal \mathbf{y}_{SD} received by D over the Source-Destination (S - D) link at T_1 is given by

$$\mathbf{y}_{SD} = \sqrt{P_s} \mathbf{h}_{sd} x + \mathbf{n}_d^{(1)}, \quad (2)$$

where \mathbf{h}_{sd} is the channel coefficient over S - D link. $\mathbf{n}_d^{(1)}$ is the AWGN at D in T_1 with $\mathbf{n}_d^{(1)} \sim \mathcal{CN}(0, \sigma_d^2)$.

A. DF Relaying Mode

When R uses DF protocol to forward the packet to D , the signal $\mathbf{y}_{RD(DF)}$ received by D over the Relay-Destination (R - D) link at T_2 is given by

$$\mathbf{y}_{RD(DF)} = \sqrt{P_r} \mathbf{h}_{rd} x + \mathbf{n}_d^{(2)}, \quad (3)$$

where P_r and \mathbf{h}_{rd} are the power and the channel coefficient over R - D link, respectively. $\mathbf{n}_d^{(2)}$ is the AWGN at D at T_2 with $\mathbf{n}_d^{(2)} \sim \mathcal{CN}(0, \sigma_d^2)$. The combining coefficient at D , denoted as \mathbf{k}_{DF} , represents the combining ratio between \mathbf{y}_{SD} and $\mathbf{y}_{RD(DF)}$ [8]. Hence, the combined signal $\mathbf{y}_{c(DF)}$ at D can be given by

$$\mathbf{y}_{c(DF)} = \frac{\mathbf{k}_{DF} \mathbf{y}_{RD(DF)} + \mathbf{y}_{SD}}{\mathbf{k}_{DF} \sqrt{P_r} \mathbf{h}_{rd} + \sqrt{P_s} \mathbf{h}_{sd}}. \quad (4)$$

B. AF Relaying Mode

When R uses AF protocol to forward the packet to D , $\mathbf{y}_{RD(AF)}$ received by D over R - D link at T_2 is given by

$$\mathbf{y}_{RD(AF)} = \sqrt{P_r} \beta \mathbf{h}_{rd} \mathbf{y}_{SR} + \mathbf{n}_d^{(2)}, \quad (5)$$

where β is the amplification gain for AF protocol which is given in [9] by

$$\beta = \frac{1}{\sqrt{P_s} |\mathbf{h}_{sr}|^2 + \sigma_r^2}. \quad (6)$$

The combining coefficient at D , denoted as \mathbf{k}_{AF} , represents the combining ratio between \mathbf{y}_{SD} and $\mathbf{y}_{RD(AF)}$ [8]. Hence, the combined signal $\mathbf{y}_{c(AF)}$ at D can be given by

$$\mathbf{y}_{c(AF)} = \frac{\mathbf{k}_{AF} \mathbf{y}_{RD(AF)} + \mathbf{y}_{SD}}{\mathbf{k}_{AF} \sqrt{P_r} \sqrt{P_s} \beta \mathbf{h}_{rd} \mathbf{h}_{sr} + \sqrt{P_s} \mathbf{h}_{sd}}. \quad (7)$$

After substituting (2) and (5) into (7), $\mathbf{y}_{c(AF)}$ is given by

$$\mathbf{y}_{c(AF)} = x + \mathbf{n}_{c(AF)}, \quad (8)$$

where $\mathbf{n}_{c(AF)}$ is given as

$$\mathbf{n}_{c(AF)} = \frac{\mathbf{k}_{AF} \sqrt{P_r} \beta \mathbf{h}_{rd} \mathbf{n}_r + \mathbf{k}_{AF} \mathbf{n}_d^{(2)} + \mathbf{n}_d^{(1)}}{\mathbf{k}_{AF} \sqrt{P_r} \sqrt{P_s} \beta \mathbf{h}_{rd} \mathbf{h}_{sr} + \sqrt{P_s} \mathbf{h}_{sd}}. \quad (9)$$

Since user cooperation is most useful when channels are varying slowly, the channel coefficients \mathbf{h}_i , where $i \in \{sr, sd, rd\}$, remain constant over at least one symbol period, resulting in flat fading. The instantaneous SNR of the signal transmitted over i link is defined as $\gamma_i = \frac{P_j |\mathbf{h}_i|^2}{\sigma_u^2}$ where $j \in \{s, r\}$ and $u \in \{r, d\}$.

Given the above model, the problem lies in finding the best way to utilize Relay information e_n in order to optimize the combining process at D . It is important to note that the optimum value for \mathbf{k}_{DF} , when it is used in HDAF systems, will always be the MRC coefficient $\mathbf{k}_{MRC} = \sqrt{P_r} \mathbf{h}_{rd}^* / \sqrt{P_s} \mathbf{h}_{sd}^*$ [5]. In the upcoming sections, in order to achieve this goal, the combining coefficient \mathbf{k}_{AF} is redesigned so that the error probability of the combined signal is minimized.

III. CONDITIONAL ERROR PROBABILITY

For the purpose of optimizing the combining coefficient k_{AF} , we propose the conditional error probability as a mathematical model to be used in order to model the proposed optimum combining scheme. The conditional error probability model was first proposed to analyze HARQ retransmissions in [10] and [8]. In [11], we derived in details the conditional error probability for HDAF system when MRC is used at the receiver. Ultimately, later in this paper, we will show that the combining coefficient which minimizes the conditional error probability is the optimum combining coefficient.

In order to proceed with the conditional error probability model, we introduce the following definitions. Let F_e defines the event that the transmitted packet over S - R link is received with $e_n \neq 0$ (i.e., at least one bit in error). Similarly, $F_{e_n=i}$ defines the event that the number of errors in the received packet at R is i errors. Moreover, F_c defines the event that the combined packets at D have errors.

We begin by deriving the exact error probability of the combined packets conditioned on the events F_e and $F_{e_n=i}$. The conditional error probability $P(F_c|F_e, F_{e_n=i})_N$ can be computed using **Proposition 1**.

Proposition 1. *The error probability of the event that the detected combined packet at D has errors conditioned on the events that the detected S - R packet is received with errors (F_e) and the number of errors is equal to i ($F_{e_n=i}$), can be given by*

$$P(F_c|F_e, F_{e_n=i})_N = \frac{P(F_e, F_{e_n=i})_N - P(\bar{F}_c, F_e, F_{e_n=i})_N}{P(F_e, F_{e_n=i})_N}, \quad (10)$$

where

$$P(F_e, F_{e_n=i})_N = \binom{N}{i} P(F_e)^i P(\bar{F}_e)^{N-i}, \quad (11)$$

$$P(\bar{F}_c, F_e, F_{e_n=i})_N = \binom{N}{i} P(\bar{F}_c, F_e)^i P(\bar{F}_c, \bar{F}_e)^{N-i}, \quad (12)$$

where $\binom{N}{i}$ is the number of subsets of size i from a set of N elements.

Proof: See Appendix A \blacksquare

For that purpose, $P(F_e, F_{e_n=i})_N$ and $P(\bar{F}_c, F_e, F_{e_n=i})_N$ in (11) and (12), respectively, will be derived next. In (11), F_e is readily computed as

$$P(F_e) = Q\left(\frac{\sqrt{P_s}|\mathbf{h}_{sr}|}{\sigma_r}\right). \quad (13)$$

Propositions 2 and **3** are used to derive $P(\bar{F}_c, F_e)$ and $P(\bar{F}_c, \bar{F}_e)$, respectively, which are needed in (12).

Proposition 2. *The probability of the event that the detected combined bit at D is error-free and the event that the detected S - R bit is in error, can be given by*

$$P(\bar{F}_c, F_e) = \begin{cases} P\left(\frac{\Re(\mathbf{n}_r)}{\sqrt{P_s}|\mathbf{h}_{sr}|} > 1, \Re(\mathbf{n}_{c(AF)}) < 1\right) & , x = -1 \\ P\left(\frac{\Re(\mathbf{n}_r)}{\sqrt{P_s}|\mathbf{h}_{sr}|} < -1, \Re(\mathbf{n}_{c(AF)}) > -1\right) & , x = 1 \end{cases} \quad (14)$$

Proposition 3. *The probability of the event that the detected combined bit at D is error-free and the event that the detected S - R bit is error-free, can be given by*

$$P(\bar{F}_c, \bar{F}_e) = \begin{cases} P\left(\frac{\Re(\mathbf{n}_r)}{\sqrt{P_s}|\mathbf{h}_{sr}|} < 1, \Re(\mathbf{n}_{c(AF)}) < 1\right) & , x = -1 \\ P\left(\frac{\Re(\mathbf{n}_r)}{\sqrt{P_s}|\mathbf{h}_{sr}|} > -1, \Re(\mathbf{n}_{c(AF)}) > -1\right) & , x = 1 \end{cases} \quad (15)$$

Using **Proposition 2**, $P(\bar{F}_c, F_e)$ can be simplified to the following:

$$P(\bar{F}_c, F_e) = \frac{1}{\sqrt{2\pi}\sigma_r^2} \int_{\sqrt{P_s}|\mathbf{h}_{sr}|}^{\infty} (1 - Q\left(\frac{\mathcal{U}(t, \mathbf{k}_{AF})}{\sigma_{eq(AF)}}\right)) e^{-\frac{t^2}{2\sigma_r^2}} dt, \quad (16)$$

where \mathcal{U} is given by (17) and $\sigma_{eq(AF)}$ is given by (18). Similarly, using **Proposition 3**, $P(\bar{F}_c, \bar{F}_e)$ can be simplified to the following:

$$P(\bar{F}_c, \bar{F}_e) = \frac{1}{\sqrt{2\pi}\sigma_r^2} \int_{-\infty}^{\sqrt{P_s}|\mathbf{h}_{sr}|} (1 - Q\left(\frac{\mathcal{U}(t, \mathbf{k}_{AF})}{\sigma_{eq(AF)}}\right)) e^{-\frac{t^2}{2\sigma_r^2}} dt, \quad (19)$$

Remark 1. S is an equiprobable source, hence, the derived $P(\bar{F}_c, F_e)$ in (16) and (19) for $x = -1$ is equivalent to that for $x = 1$.

The probability expressions in (16) and (19) do not have closed-form expressions which renders the analysis to be possible only through numerical simulations¹. Finally, $P(F_c|F_e, F_{e_n=i})_N$ can be obtained by substituting (13), (16) and (19) in **Proposition 1**, where P_{Q_1} and P_{Q_2} are defined as follows

$$P_{Q_1}(\sigma, \sigma_{eq}, \phi, \mathcal{U}) = \frac{1}{\sqrt{2\pi}\sigma^2} \int_{\phi}^{\infty} (1 - Q\left(\frac{\mathcal{U}(t, \mathbf{k}_{AF})}{\sigma_{eq}}\right)) e^{-\frac{t^2}{2\sigma^2}} dt, \quad (20)$$

¹There is a way around this predicament which is to find alternative approximated expressions of (16) and (19) using one of the Q-function approximations. However, we skip this step due to space limitation.

$$\mathcal{U}(t, \mathbf{k}_{AF}) = \sqrt{P_r}\sqrt{P_s}\beta(\Re(\mathbf{k}_{AF})\Re(\mathbf{h}_{rd})\Re(\mathbf{h}_{sr}) - \Im(\mathbf{k}_{AF})\Im(\mathbf{h}_{rd})\Re(\mathbf{h}_{sr}) - \Re(\mathbf{k}_{AF})\Im(\mathbf{h}_{rd})\Im(\mathbf{h}_{sr}) - \Im(\mathbf{k}_{AF})\Re(\mathbf{h}_{rd})\Im(\mathbf{h}_{sr})) + \sqrt{P_s}\Re(\mathbf{h}_{sd}) + (\sqrt{P_r}\beta\Im(\mathbf{k}_{AF})\Im(\mathbf{h}_{rd}) - \sqrt{P_r}\beta\Re(\mathbf{k}_{AF})\Re(\mathbf{h}_{rd}))t \quad (16)$$

$$\sigma_{eq(AF)}(\mathbf{k}_{AF}) = \sqrt{P_{rd}\beta^2(I(\mathbf{h}_{rd})\Re(\mathbf{k}_{AF}) + I(\mathbf{k}_{AF})\Re(\mathbf{h}_{rd}))^2\sigma_r + (\Re(\mathbf{k}_{AF}) + 1)^2\sigma_d + I(\mathbf{k}_{AF})^2\sigma_d} \quad (18)$$

$$P(F_c|F_e, F_{e_n=i})_N(\mathbf{k}_{AF}) = \frac{1}{\binom{N}{i} (Q(\frac{\sqrt{P_s}|\mathbf{h}_{sr}|}{\sigma_r}))^i (1 - Q(\frac{\sqrt{P_s}|\mathbf{h}_{sr}|}{\sigma_r}))^{N-i}} \left(\binom{N}{i} (Q(\frac{\sqrt{P_s}|\mathbf{h}_{sr}|}{\sigma_r}))^i \right. \\ \left. (1 - Q(\frac{\sqrt{P_s}|\mathbf{h}_{sr}|}{\sigma_r}))^{N-i} - \binom{N}{i} (P_{Q_1}(\sigma, \sigma_{eq(AF)}, \sqrt{P_s}|\mathbf{h}_{sr}|, \mathcal{U}))^i (P_{Q_2}(\sigma, \sigma_{eq(AF)}, \sqrt{P_s}|\mathbf{h}_{sr}|, \mathcal{U}))^{N-i} \right) \quad (22)$$

$$P_{Q_2}(\sigma, \sigma_{eq}, \phi, \mathcal{U}) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\phi} (1 - Q(\frac{\mathcal{U}(t, \mathbf{k}_{AF})}{\sigma_{eq}})) e^{-\frac{t^2}{2\sigma^2}} dt. \quad (21)$$

Finally, the conditional error probability $P(F_c|F_e, F_{e_n=i})_N(\mathbf{k}_{AF})$ is given by (22).

IV. HDAF OPTIMUM COMBINING SCHEME

At the receiver, the objective of the proposed optimum combining scheme is to use relay decision information in order to obtain the *combining coefficient* which minimizes the error probability. For the purpose of achieving this objective, the conditional error probability expression, presented in section III, will be utilized in the formulation of \mathbf{k}_{AF}^{opt} problem as follows:

P 1.

$$\begin{aligned} & \underset{\mathbf{k}_{AF}}{\operatorname{argmin}} P(F_c|F_e, F_{e_n=i})_N \\ & \text{subject to} \quad |\mathbf{k}_{AF}| \leq |\mathbf{k}_{MRC}| \end{aligned}$$

P 1 is a two-dimensional optimization problem [12]. The proposed optimum combining algorithm is summarized in **Algorithm 1**.

Algorithm 1 : Proposed Optimum Combining

- 1: R detects the transmitted packet and identifies i which is the number of errors in received S - R packet.
 - 2: Using control channel, R sends i to D .
 - 3: D uses i to compute $P(F_c|F_e, F_{e_n=i})_N$ as function of \mathbf{k}_{AF} .
 - 4: D solves **P 1** for \mathbf{k}_{AF}^{opt} which minimizes $P(F_c|F_e, F_{e_n=i})_N$.
 - 5: D uses the combining coefficient value \mathbf{k}_{AF}^{opt} to combine and optimally detect the packet.
-

V. RESULTS AND DISCUSSIONS

A. Numerical Results

In order to verify the validity of the derived conditional error probability function as a minimization problem, Fig.1 plots $P(F_c|F_e, F_{e_n=i})_7$ as a function of both, the number of errors e_n and the combining coefficient \mathbf{k}_{AF} . Numerical results in Fig.2 explicitly show that $P(F_c|F_e, F_{e_n=i})_7$ is minimized using \mathbf{k}_{AF}^{opt} , compared to the MRC combining coefficient \mathbf{k}_{MRC} (intersection of the MRC plane and $P(F_c|F_e, F_{e_n=i})_7$). It is worth noting that there is an inverse relation between \mathbf{k}_{AF}^{opt} and the number of errors in S - R packet (i.e., e_n). This indicates that when the number of errors is large, the proposed scheme assigns a smaller value for the optimum combining coefficient. This leads to the conclusion that the proposed

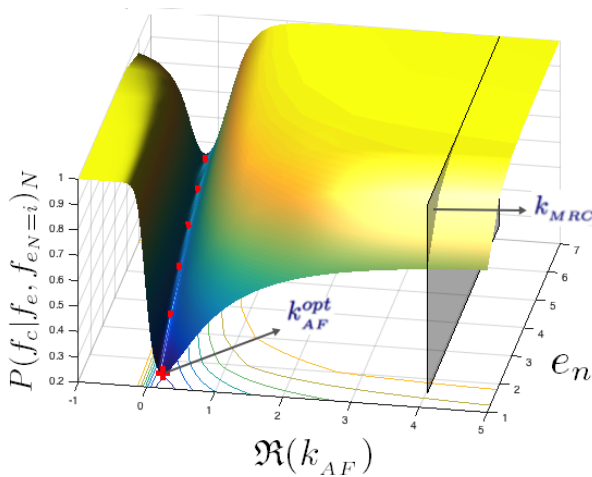


Fig. 1. $P(F_c|F_e, F_{e_n=i})_7$ vs. $\Re(\mathbf{k}_{AF})$ vs. e_n for $N=7$ and for the simulation scenario $\gamma_{SR} \cdot 10 \cdot \log(3) = \gamma_{SD} \cdot 10 \cdot \log(3) = \gamma_{RD} = 6.8\text{dB}$.

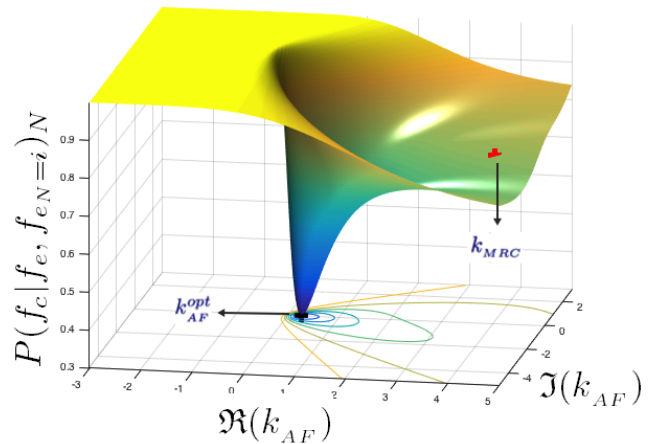


Fig. 2. $P(F_c|F_e, F_{e_n=i})_7$ vs. $\Re(\mathbf{k}_{AF})$ vs. $\Im(\mathbf{k}_{AF})$ for $N=7$, $e_n = 1$ and for the simulation scenario $\gamma_{SR} \cdot 10 \cdot \log(3) = \gamma_{SD} \cdot 10 \cdot \log(3) = \gamma_{RD} = 6.8\text{dB}$.

schemes minimizes the effect of errors in S - R packet.

B. Simulation Results

System level simulations have been conducted in order to validate the benefits of the proposed HDAF optimum combining scheme. Table I lists the simulation parameters and their respective values. In order to validate the effectiveness of the proposed optimum combining scheme, we compare its performance with MRC. In Fig.3, we compare the instantaneous Block Error Rate (BLER) of HDAF cooperative relaying system when MRC and optimum combining is utilized at the destination receiver for various values of SNR. Moreover, we compare between HDAF, DF and AF cooperative relaying systems, when MRC is employed at the receiver, in terms of BLER. Its worth noting that the combining coefficient are calculated through optimizing the expressions in (22). On the other hand, the BLER curves for the optimum coefficient scheme and the benchmark schemes are plotted using Monte Carlo Simulations. As shown in Fig. 3, the performance of HDAF is better than AF or DF alone. Moreover, we observe that that HDAF when the proposed optimum combining scheme is used outperforms HDAF with MRC receiver. Another important thing to note is that the improvement achieved by the proposed optimum combining scheme depends on my factors including: the number of errors in the packet , the fading channels condition and the packet size N .

C. Impact of the number of errors e_n on the conditional error probability $P(F_c|F_e, F_{e_n=i})_N$ and the improvement Δ_{CEP}

For the purpose of studying the improvement achieved by the proposed optimum combining over MRC, we introduce the Improvement (i.e., reduction) percentage in conditional error probability Δ_{CEP} as follows:

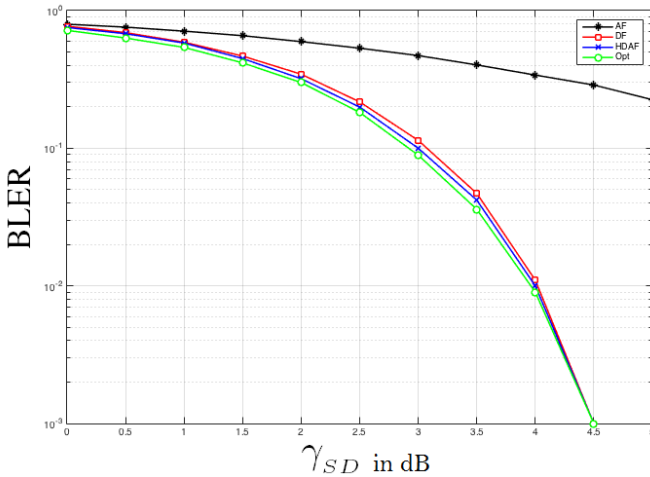


Fig. 3. Comparison between the proposed optimum combining scheme and the MRC in terms of BLER with varying SNR and the simulation parameters given in Table I.

TABLE I
SIMULATION PARAMETERS AND VALUES

Parameter	Value
$\sigma_R^2 = \sigma_D^2$	1
Modulation	BPSK
Slot duration	0.5 ms
Number of symbols per slot	10 symbols
Packet length	10 symbols
Power allocation	$P_S = P_R = 1$
Simulation Scenario	$4\gamma_{SR}=2\gamma_{SD}=\gamma_{RD}$

$$\frac{P(F_c|F_e, F_{e_n=i})_N(\mathbf{k}_{AF}^{opt}) - P(F_c|F_e, F_{e_n=i})_N(\mathbf{k}_{MRC})}{P(F_c|F_e, F_{e_n=i})_N(\mathbf{k}_{MRC})} \times 100\% \quad (23)$$

where $P(F_c|F_e, F_{e_n=i})_N(\mathbf{k}_{AF}^{opt})$ and $P(F_c|F_e, F_{e_n=i})_N(\mathbf{k}_{MRC})$ are the conditional error probabilities when \mathbf{k}_{AF} and \mathbf{k}_{MRC} are used, respectively. It should be noted that the reduction in conditional error probability is proportional to the increase in throughput and the reduction in error probability.

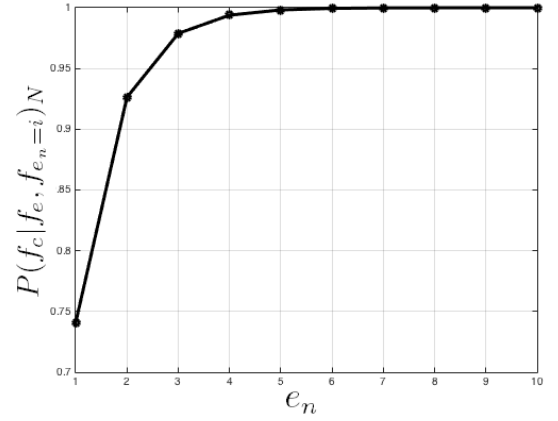


Fig. 4. $P(F_c|F_e, F_{e_n=i})_N$ vs. e_n for $N=10$, and for the simulation scenario $\gamma_{SR}=\gamma_{SD}=\gamma_{RD}=4\text{dB}$.

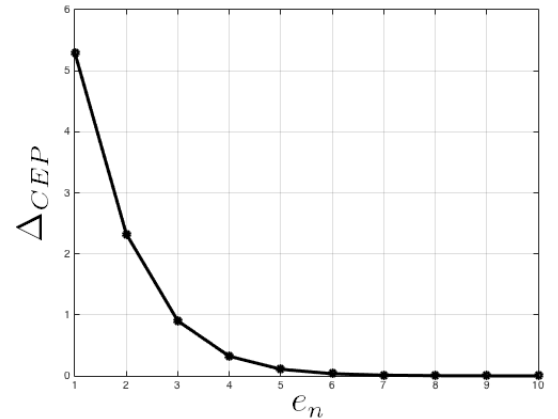


Fig. 5. Δ_{CEP} vs. e_n for $N=10$, and for the simulation scenario $\gamma_{SR}=\gamma_{SD}=\gamma_{RD}=4\text{dB}$.

In Fig.4, we plot the conditional error probability $P(F_c|F_e, F_{e_n=i})_N$ vs. the number of errors in the relay packet e_n . The first thing to note is the conditional error probability increases until e_n is half the packet. This indicate that the information that can be extracted from the packet when $e_n > N/2$ is zero. Hence the optimum combining coefficient will be zero which is equivalent to dropping the relay packet. Similarly, in Fig.5, we plot Δ_{CEP} vs. the number of errors in the relay packet e_n . It is clear that the improvement Δ_{CEP} decrease with the increase of e_n .

VI. CONCLUSION

In this paper, we proposed a new optimum combining scheme for HDFAF cooperative relay-based system. The proposed scheme uses relay decision information in order to optimize the combining coefficient and therefore minimizing the error probability at the destination. The relay decision information is defined as the number of errors in the received packet at the relay node. Simulation results demonstrated that the developed optimum combining scheme improves the overall system performance in the form of increased throughput. Moreover, it was shown that the achieved improvement depends on the number of detected errors at the relay node as well as the channel fading conditions of both direct and relayed links.

APPENDIX A

PROOF OF PROPOSITION 1.

Using the second axiom of probability, the sample space probability of the received combined signal can be given by

$$P(F_c)_N + P(\bar{F}_c)_N = 1 \quad (A.1)$$

Using Bayes' formula, (A.1) can be given by

$$P(F_c, F_e)_N + P(F_c, \bar{F}_e)_N + P(\bar{F}_c, F_e)_N + P(\bar{F}_c, \bar{F}_e)_N = 1 \quad (A.2)$$

Since the event $F_{e_n=i}$ occurs only when F_e happens, $F_{e_n=i} \subset F_e$. Therefore, using Bayes' formula, (A.2) can be rewritten as

$$\begin{aligned} &P(F_c, F_e, F_{e_n=i})_N + \underbrace{P(F_c, F_e, \bar{F}_{e_n=i})_N + P(\bar{F}_c, F_e, \bar{F}_{e_n=i})_N}_{=P(F_e, \bar{F}_{e_n=i})_N} \\ &+ P(\bar{F}_c, F_e, F_{e_n=i})_N + \underbrace{P(F_c, \bar{F}_e)_N + P(\bar{F}_c, \bar{F}_e)_N}_{=P(\bar{F}_e)_N} = 1 \end{aligned} \quad (A.3)$$

Using Bayes' formula again, $P(F_e, \bar{F}_{e_n=i})_N$ can be given by

$$P(F_e, \bar{F}_{e_n=i})_N = P(F_e)_N - P(F_e, F_{e_n=i})_N \quad (A.4)$$

After substituting (A.4) into (A.3), $P(F_c, F_e, F_{e_n=i})_N$ can be given by

$$P(F_c, F_e, F_{e_n=i})_N = P(F_e, F_{e_n=i})_N - P(\bar{F}_c, F_e, F_{e_n=i})_N \quad (A.5)$$

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