Stopping Condition for Greedy Block Sparse Signal Recovery

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***Abstract*—For greedy block sparse recovery where the sparsity level is unknown, we derive a stopping condition to stop the iteration process. Focused on the block orthogonal matching pursuit (BOMP) algorithm, we model the energy of residual signals at each iteration from a probabilistic perspective. At the iteration when the last supporting block is detected, the resulting energy of residual signals is supposed to suffer an obvious decrease. Based on this, we stop the iteration process when the energy of residual signals is below a given threshold. Compared with other approaches, our derived condition works well for the BOMP recovery. What is more, we promote our approach to the interference cancellation based BOMP (ICBOMP) recovery in paper [1]. Simulation results show that our derived condition can save many unnecessary iterations and at the same time guarantees a favorable recovery accuracy, both for the BOMP and ICBOMP recoveries.**

I. Introduction

In the last few years, compressed sensing (CS) [2] has drawn increased interest in many areas such as signal processing and multi-user communications [3], [4], [5]. The CS theory claims that when the signals of interest are sparse with many elements being zero, even sampling the signals using a rate less than the Nyquist rate, it can be recovered from the down-sampled measurements almost without losing the information. The early work on CS assumes that each of the nonzero signals is just randomly located among all possible positions of a vector, i.e., random-sparsity case. However, as stated in papers such as [6], the nonzero signals are usually clustered, exhibiting the structure of block-sparsity. The block-sparsity indicates that, when partitioning the sequential signals into blocks, only some blocks contain nonzero components and all other blocks are zero.

Suppose $\mathbf{s}$ is an $Nd \times 1$ signal vector given as $\mathbf{s} = \left[\mathbf{s}_1^T \cdots, \mathbf{s}_i^T \cdots, \mathbf{s}_N^T\right]^T$ where superscript T stands for the transpose and $\mathbf{s}_i$ is a $d \times 1$ sub-vector, $1 \le i \le N$. Suppose only N_a out of N sub-vectors are nonzero, usually with $N_a \ll N$. The sparsity level is thus N_a. When $d = 1$, $\mathbf{s}$ exhibits the property of random-sparsity, and when $d > 1$, $\mathbf{s}$ exhibits the structure of block-sparsity. The CS measures $\mathbf{s}$ using an $M \times Nd$ measurement matrix $\mathbf{B} = [\mathbf{B}_1, \mathbf{B}_2, \cdots, \mathbf{B}_N]$, given as $\mathbf{y} = \mathbf{Bs}$, with $M < Nd$, where M stands for the measurement number. If the measurement is performed in a noisy environment, it has $\mathbf{y} = \mathbf{Bs} + \mathbf{z}$ where $\mathbf{z}$ represents the noise vector.

For sparse signal recovery, many algorithms such as [7], [8] are proposed. Among all the algorithms, greedy algorithms [7], [8] are important since they are simple for practical use. All the greedy recovery algorithms of random-sparsity can be transplanted to the block-sparsity case. For example, the block OMP (BOMP) is developed from the OMP algorithm for the block-sparse recovery [6]. Existing results demonstrate that, compared with the random-sparsity, exploiting the block structure provides better signal reconstruction performance.

Sparsity level N_a is an important parameter for the sparse recovery, especially for the greedy recovery. Many works, such as [7], [8] assume that the N_a is a priori known to control the iteration number. Unfortunately in reality, N_a is usually unknown at the signal recovery side and its estimation is therefore necessary. It should be noted that, if the estimated sparsity level is smaller than N_a, some nonzero signals will certainly be missed to detect; if the estimated value is larger than N_a, unnecessary iterations will harm the recovery performance [9], including degradation in accuracy and increase in complexity. To address this problem, work in [10] proposes automatic double overrelaxation (ADORE) thresholding method to estimate the sparsity level and reconstruct the signal simultaneously. Other works such as [9], [11] also adopt some stop criterions to stop the iterations process of the greedy recovery. However, all the above works are for the random-sparse recovery.

In this paper, we focus on the BOMP recovery of the block-sparsity situation where the sparsity level is unknown. Rather than giving the stopping condition by experience, or setting a maximum iteration number as in [12], we theoretically derive the stopping condition. We model the energy of the residual signal vector from a probabilistic perspective and we use its distribution to derive a threshold to stop the greedy process. When the energy of residual signal is smaller than that threshold, all the supporting blocks are supposed to have been detected and the BOMP algorithm will finish its iteration process. This approach works well for the BOMP, as demonstrated by the simulation. This gives us the confidence to promote the method. Specially, we use the same method to derive the stopping condition for the iterations of interference cancellation based BOMP (ICBOMP) algorithm in [1]. The ICBOMP is developed from the BOMP algorithm for the small packet recovery.

The rest of the paper is organized as follows. In Section II, we derive the iteration stopping condition for the BOMP recovery. In Section III, we transplant the method to the ICBOMP recovery in [1]. In section IV, some related works are cited. Finally, numerical studies are shown in Section V, followed by the conclusion in Section VI.

Notation: Vectors and matrices are denoted by boldface lowercase and uppercase letters, respectively. The identity matrix of size $n \times n$ is denoted as $\mathbf{I}_n$. For a subset $I \subset [N] := \{1, 2, \cdots, N\}$ and a matrix $\mathbf{B} := [\mathbf{B}_1, \mathbf{B}_2, \cdots, \mathbf{B}_N]$ consisting of N sub-matrices (blocks) of equal size, $\mathbf{B}_I$ stands for a sub-matrix of $\mathbf{B}$ whose block indices are in set I; for a vector $\mathbf{s} := [\mathbf{s}_1^T, \mathbf{s}_2^T, \cdots, \mathbf{s}_N^T]^T$, $\mathbf{s}_I$ is similarly defined. Value $|I|$ stands for the cardinality of set I. Given two sets I_1 and I_2, $I_1 \backslash I_2 = I_1 \cap I_2^c$. $\otimes$ stands for the Kronecker product.

II. Stopping Condition for the BOMP Recovery

In this part, we give a more detailed description for the block-sparsity recovery problem, and we take the BOMP algorithm as an example to derive the stopping condition from the probabilistic perspective.

A. Block-sparsity Recovery Problem

As mentioned earlier, the measurement of block-sparse signal vector $\mathbf{s}$ in a noisy environment is given as $\mathbf{y} = \mathbf{B}\mathbf{s} + \mathbf{z}$. In this paper, all the parameters are assumed in complex field. Besides, for the later derivation convenience, we assume that: 1) matrix $\mathbf{B}$ is randomly generated, and all its entries are i.i.d. Gaussian variables with a mean zero and a variance $\frac{1}{M}$; 2) nonzero signals in $\mathbf{s}$ are i.i.d. variables of zero mean and unit variance; 3) noise $\mathbf{z} \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2 \mathbf{I}_M\right)$. Gaussian approach makes the $\mathbf{B}$ almost surely satisfy the restricted isometry property (RIP) which is necessary for the sparse recovery [13].

Let I be the set containing the unknown indices of N_a nonzero blocks, with cardinal number $|I| = N_a$. Then the measurement can be rewritten as

$$\mathbf{y} = \sum_{i \in I} \mathbf{B}_i \mathbf{s}_i + \mathbf{z} = \mathbf{B}_I \mathbf{s}_I + \mathbf{z}. \tag{1}$$

B. BOMP Recovery

Also for the later derivation convenience, the iteration process of the BOMP is summarized here. At the k-th iteration, $k \in \{1, 2, \cdots\}$, let $\mathbf{r}_k$ denote the residual signal and Λ_k denote the set containing the indices of already detected blocks, their initializations are respectively given as $\mathbf{r}_0 = \mathbf{y}$ and $\Lambda_0 = \emptyset$. Then, the BOMP iteration is performed as follows:

1) For $j \in \{[N] \backslash \Lambda_{k-1}\}$, select the block that has the maximum correlation with the residual signals:

$$j_k = \arg\max_j \left\| \mathbf{B}_j^H \mathbf{r}_{k-1} \right\|_2^2$$

2) update the index set:

$$\Lambda_k = \Lambda_{k-1} \cup \{j_k\}$$

3) update the signal by the least-square (LS) algorithm:

$$\bar{\mathbf{s}}_{\Lambda_k} = \arg\min_{\mathbf{s}_0} \|\mathbf{y} - \mathbf{B}_{\Lambda_k} \mathbf{s}_0\|_2$$

4) update the residual signals:

$$\mathbf{r}_k = \mathbf{y} - \mathbf{B}_{\Lambda_k} \bar{\mathbf{s}}_{\Lambda_k}$$

The above BOMP iterations are terminated when certain condition is satisfied, either it reaches to the maximum allowed iteration number as in [12], [1], or the energy of the residual signals is below an empirical value as in [14]. The later approach is based on the common fact that the energy of the residual signals will usually suffer and obvious decrease when the last supporting block is selected. Different from these two kinds of approaches, in the following, by viewing the energy of the residual signals to be a random variable, we theoretically derive the iteration stopping condition from a probabilistic perspective.

C. Energy Evaluation of Residual Signal

At the $k\text{-}th$ BOMP iteration, it has $|\Lambda_k| = k$. The signal update is given as

$$\bar{\mathbf{s}}_{\Lambda_k} = (\mathbf{B}_{\Lambda_k}^H \mathbf{B}_{\Lambda_k})^{-1} \mathbf{B}_{\Lambda_k}^H \mathbf{y}. \tag{2}$$

The energy of residual signal is a random variable and is defined as $E_k = \|\mathbf{r}_k\|_2^2 = \left(\mathbf{y} - \mathbf{B}_{\Lambda_k}\bar{\mathbf{s}}_{\Lambda_k}\right)^H \left(\mathbf{y} - \mathbf{B}_{\Lambda_k}\bar{\mathbf{s}}_{\Lambda_k}\right)$. Assume that there are n_a $(0 \le n_a \le N_a)$ supporting blocks remained to detect, i.e., $|I \backslash \Lambda_k| = n_a$, then the E_k has a mean value given as follows

$$\begin{aligned}
\mu_k =& \mathbf{E}\left[\mathbf{s}_{I\backslash\Lambda_k}^H \mathbf{B}_{I\backslash\Lambda_k}^H \mathbf{B}_{I\backslash\Lambda_k} \mathbf{s}_{I\backslash\Lambda_k}\right] \\
&- \mathbf{E}\left[\mathbf{s}_{I\backslash\Lambda_k}^H \mathbf{B}_{I\backslash\Lambda_k}^H \mathbf{B}_{\Lambda_k} \left(\mathbf{B}_{\Lambda_k}^H \mathbf{B}_{\Lambda_k}\right)^{-1} \mathbf{B}_{\Lambda_k}^H \mathbf{B}_{I\backslash\Lambda_k} \mathbf{s}_{I\backslash\Lambda_k}\right] \\
&+ \mathbf{E}\left[\mathbf{z}^H \left(\mathbf{I}_M - \mathbf{B}_{\Lambda_k} \left(\mathbf{B}_{\Lambda_k}^H \mathbf{B}_{\Lambda_k}\right)^{-1} \mathbf{B}_{\Lambda_k}^H\right) \mathbf{z}\right] \\
=& n_a d - n_a \frac{kd^2}{M} + (M - kd)\,\sigma^2 \\
=& (M - kd)\left(\sigma^2 + \frac{n_a d}{M}\right)
\end{aligned} \tag{3}$$

where the property of the mathematical trace operation is used. And it should be noted that a more exact mean value should consider the order statistics of signal blocks, but the expressions will be complicated. For deriving a usable mean value, the above derivations omit the order statistics.

Since each component of $\mathbf{r}_k$ is a superposition of many independent variables, it can be approximated as a Gaussian variable. We further assume that components of $\mathbf{r}_k$ are i.i.d. Gaussian variables and each of them has a mean of zero and a variance of $\tilde{\sigma}^2$, with $\tilde{\sigma}^2 = \frac{\mu_k}{M} = \frac{M - kd}{M}\left(\sigma^2 + \frac{n_a d}{M}\right)$. Then E_k follows a chi-squared distribution with $2M$ degrees of freedom, and its variance is given as

$$\begin{aligned}
\sigma_k^2 =& M(M+1)\left(\tilde{\sigma}^2\right)^2 - \mu_k^2 \\
=& \frac{(M - kd)^2}{M}\left(\sigma^2 + \frac{n_a d}{M}\right)^2
\end{aligned} \tag{4}$$

Usually, M is large. In this case, it's reasonable to treat E_k as a Gaussian variable, satisfying $E_k \sim \mathcal{N}\left(\mu_k, \sigma_k^2\right)$.

D. Stopping Condition

As above stated, when the last supporting block is selected at the k-th iteration of BOMP algorithm, a sharp decrease will happen to the energy of the residual signals. This gives us the idea to derive a threshold, to stop the BOMP iterations. That is if E_k is smaller than the set threshold, the last supporting block is supposed to have been selected and then the iterations can be terminated.

The E_k is a random variable, and its distribution is decided by the following two cases:

C1: $|I\backslash\Lambda_k| = n_a \geq 1$.

C0: $|I\backslash\Lambda_k| = n_a = 0$.

The mean and variance of the E_k are respectively given as (3) and (4), for both of the above two cases. When performing energy detection by a threshold $\eta_{k,1}$, a missed detection probability, say p_m, will happen by deciding the **C1** to be the **C0**. Applying Gaussian variable to approximate E_k, it has that

$$P(E_k \leq \eta_{k,1}) = \Phi\left(\frac{\eta_{k,1} - \mu_k}{\sigma_k}\right) = p_m \tag{5}$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\frac{t^2}{2}\right)dt$. By substituting (3) and (4) into (5), it gives that

$$\eta_{k,1} = (M - kd)\left(\sigma^2 + \frac{n_a d}{M}\right)\left(1 + \frac{\Phi^{-1}(p_m)}{\sqrt{M}}\right) \tag{6}$$

where $\Phi^{-1}(p_m)$ is the inverse function of $\Phi(x)$. $\eta_{k,1}$ can be regarded as the maximum threshold for a maximum allowed missed detection probability p_m.

On the other hand, if a maximum false detection probability, say p_f, is allowed for deciding the **C0** to be the **C1**, it has that

$$P(E_k \geq \eta_{k,0}) = 1 - \Phi\left(\frac{\eta_{k,0} - \mu_k}{\sigma_k}\right) = p_f \tag{7}$$

which gives that

$$\eta_{k,0} = (M - kd)\left(1 - \frac{\Phi^{-1}(p_f)}{\sqrt{M}}\right)\sigma^2 \tag{8}$$

Undoubtedly, if the set threshold, say η_k, is required to take both the missed detection probability and false detection probability into account, a tradeoff should be made between the two probabilities. Note that, if the false detection happens under the **C0**, the iteration continues and some non-supporting blocks will be selected for signal update. This will degrade the recovery accuracy and at the same time increase the recovery complexity; However, when missed detection happens to the **C1**, some supporting blocks will be identified to be non-supporting, which will severely have an adverse impact on the sparse recovery performance. Therefore, a more accuracy performance cares more about the missed detection probability. Suppose p_m and p_f are respectively the allowed missed and false alarm probabilities, then the reasonable η_k is given as follows

$$\eta_k = \min(\eta_{k,1}, \eta_{k,0}) \tag{9}$$

Remark 1: Since iteration is processed at the recovery side, parameter k can be exactly known at the recovery side. Therefore, threshold η_k will be adjusted with iteration k.

Remark 2: In practice, we set $n_a = 1$ to derive $\eta_{k,1}$, because: 1) n_a is unknown at the recovery side, which can not be directly used; 2) for the threshold derived from $n_a = 1$, conditional probability $P(E_k \leq \eta_k | n_a \geq 2)$ is smaller than conditional probability $P(E_k \leq \eta_k | n_a = 1)$, this means the derived threshold η_k is also applicable for the k-th iteration when two or more supporting blocks are remained to detect.

III. Stopping Condition for the ICBOMP Recovery

In the communication scenario of [1], an uplink system of N mobile users and a base station (BS) with M_{ant} antennas is considered. By exploiting the sparse block transmission that only N_a out of the total N users are actively and simultaneously transmitting data, the work also establishes the block-sparsity model as follows

$$\mathbf{y} = \sqrt{\rho_0}\sum_{n=1}^{N}\mathbf{B}_n\mathbf{s}_n + \mathbf{z} = \sqrt{\rho_0}\mathbf{B}\mathbf{s} + \mathbf{z} \tag{10}$$

where ρ_0 is the signal to noise ratio (SNR). As a block of $\mathbf{B} \in \mathbb{C}^{M_{\text{ant}}T \times Nd}$, $\mathbf{B}_n = \mathbf{P}_n \otimes \mathbf{h}_n \in \mathbb{C}^{M_{\text{ant}}T \times d}$ where $\mathbf{P}_n \in \mathbb{C}^{T \times d}$ is a kind of precoding matrix and $\mathbf{h}_n \in \mathbb{C}^{M_{\text{ant}} \times 1}$ is the channel gain from the n-th user to the BS, $1 \leq n \leq N$. $\mathbf{s}$ is the block-sparse signal to be recovered, with length d for each block $\mathbf{s}_n$. $\mathbf{z}$ is the complex Gaussian noise vector.

To improve the recovery performance, the authors in [1] propose the interference cancellation based BOMP (ICBOMP) algorithm, which improves from the BOMP algorithm by taking advantage of the error correction and detection code in the communication, to perform the recovery of $\mathbf{s}$. The ICBOMP behaves the same as the BOMP in block detection, signal update and residual update. Their main difference is that for the ICBOMP, some blocks of signals may have been correctly recovered before finishing all the iterations and need no further update. However, in [1] the problem of when to stop the ICBOMP iterations is not specially studied, the authors only set a maximum iteration number. In this part, we derive the stopping condition for the ICBOMP algorithm. For detailed process of the ICBOMP algorithm, please refer to [1].

As the performance analysis in [1], entries of $\mathbf{P}_n$, $\mathbf{h}_n$ and $\mathbf{z}$ are all assumed to be i.i.d. complex Gaussian variables, respectively in $\mathcal{CN}(0, \frac{1}{T})$, $\mathcal{CN}(0, 1)$ and $\mathcal{CN}(0, 1)$. Nonzero entries of $\mathbf{s}$ are i.i.d. Quadrature Phase Shift Keying (QPSK) symbols, each of which has unit energy. Besides, it should be noted that, by the ICBOMP algorithm, it has $1 \leq |\Lambda_k| = l \leq k$. Suppose n_a active users are remained to detect when the k-th iteration is finished, then similar to the derivations of (3) and (4), the mean and variance of the residual energy of ICBOMP are respectively given as

$$\begin{aligned}\mu_k &= (M_{\text{ant}}T - ld)\left(1 + \frac{n_a\rho_0 d}{T}\right)\\ \sigma_k^2 &= \frac{(M_{\text{ant}}T - ld)^2}{M_{\text{ant}}T}\left(1 + \frac{n_a\rho_0 d}{T}\right)^2\end{aligned} \tag{11}$$

and the final energy threshold is given by $\eta_k = \min(\eta_{k,1}, \eta_{k,0})$, where the $\eta_{k,1}$ and $\eta_{k,0}$ are respectively given by

$$\begin{aligned}\eta_{k,1} &= (M_{\text{ant}}T - ld)\left(1 + \frac{n_a \rho_0 d}{T}\right)\left(1 + \frac{\Phi^{-1}(p_m)}{\sqrt{M_{\text{ant}}T}}\right)\\ \eta_{k,0} &= (M_{\text{ant}}T - ld)\left(1 - \frac{\Phi^{-1}(p_f)}{\sqrt{M_{\text{ant}}T}}\right)\end{aligned} \tag{12}$$

for certain allowed missed alarm probability p_m and false alarm probability p_f. As the previous Section II-D, $n_a = 1$ is used to derive $\eta_{k,1}$.

IV. Related works

In sparse signal recovery literature, many earlier works have considered the stopping condition for greedy algorithms. As a conclusion, three common stopping conditions are

$$\text{Condition 1}: \frac{\|\bar{\mathbf{s}}_{k+1} - \bar{\mathbf{s}}_k\|_2}{\|\bar{\mathbf{s}}_k\|_2} < \epsilon_1 \tag{13}$$

$$\text{Condition 2}: \|\mathbf{r}_k\|_2^2 < \epsilon_2 \tag{14}$$

$$\text{Condition 3}: \text{setting a maximum iteration number.} \tag{15}$$

Condition 1 indicates that the algorithm will stop when the relative change of the reconstructed signals between two consecutive iterations is smaller than a certain value. This kind of approach is mentioned in [9], but no specific ϵ_1 is given in the paper. In [15], empirical values like 10^{-6} is set for ϵ_1. Condition 2 shows that the algorithm will stop when the energy of the residual signals is smaller than a certain threshold. In [9], threshold ϵ_2 is set to be the energy of noise vector. And in [11], such a stopping condition is also theoretically discussed. Other works like [12] sets a maximum iteration number, and [7] assumes that N_a is known and iteration number is exactly set N_a. However, such kinds of approaches are not feasible for practical use, especially when N_a cannot be a priori known. It should also be noted that all the above works are for random sparsity case.

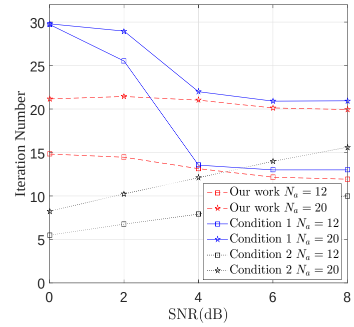
In our later numerical studies for the BOMP recovery, Condition 1 and Condition 2 will be simulated for the block sparsity case for comparison. And for the ICBOMP recovery, Condition 3 will be simulated for comparison.

V. Numerical Studies

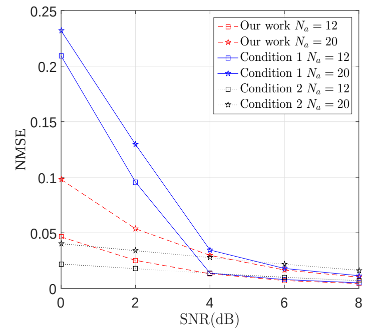
This section presents the numerical studies. To our derived thresholds for the BOMP and ICBOMP algorithms, probabilities p_m and p_f are respectively set to be 0.1% and 0.5%. The followings are some cited simulation results.

A. on the BOMP Recovery

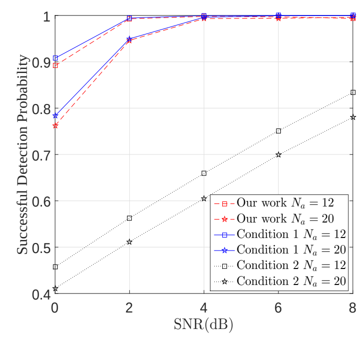
The system size for the BOMP recovery is set as: $d = 50$, $N = 640$ and $M = 2000$. The N_a supporting blocks are chosen uniformly at random among all N blocks. Entries of the measurement matrix and the nonzero signal blocks are generated as i.i.d. complex Gaussian variables, following $\mathcal{CN}(0, \frac{1}{M})$ and $\mathcal{CN}(0, 1)$, respectively. As comparisons, the thresholds in (13) and (14) are respectively given as $\epsilon_1 = 0.25$ and $\epsilon_2 = M\sigma^2$, where 0.25 is a reasonable value concluded from training simulations and $M\sigma^2$ is the energy of noise vector. The simulation results are presented as required iteration number vs. SNR, normalized mean square error (NMSE, calculated by $\frac{\|\mathbf{s}-\bar{\mathbf{s}}\|_2^2}{\|\mathbf{s}\|_2^2}$) vs. SNR and successful detection probability vs. SNR, respectively in Figure 1(a), Figure 1(b) and Figure 1(c). The SNR here is defined as $\frac{1}{\sigma^2}$. To accelerate the process, the maximum iteration number of the BOMP is set 30 to deal with case where the thresholds cannot stop the BOMP timely.



(a) Iteration number



(b) NMSE



(c) Successful detection probability for supporting blocks

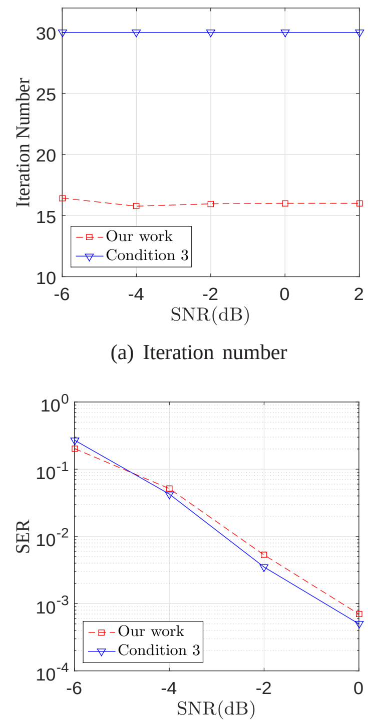
Fig. 1. Performance for BOMP recovery

Figure 1(a) tells us that our derived threshold can stop the iterations timely. As the SNR increases, the required iteration number nearly equals to the number of supporting blocks. However, the threshold given by (13) produces many false detections in low SNR regime, and threshold given by (14) will make certain number of supporting blocks missed to detect. Figure 1(b) shows that, the NMSE achieved by the derived threshold is a little higher than that of ϵ_2 in low SNR regime, it is because some false detections degrade the recovery performance. However it is always better than that of set ϵ_1. As the SNR increases, the output NMSE gradually becomes the smallest among the stopping conditions. Figure 1(c) demonstrates that, the derived threshold still guarantees a very high successful detection probability.

B. on the ICBOMP Recovery

For the communication scenario in [1] stated, system parameters are set: $d = 200$, $N = 640$, $M_{\text{ant}} = 8$, $T = 5d$ and $N_a = 16$, ρ_0 is the SNR. Entries of the precoding matrices and channel vectors are generated as i.i.d. complex Gaussian variables, following $\mathcal{CN}(0, \frac{1}{T})$ and $\mathcal{CN}(0, 1)$, respectively. QPSK is applied for signal modulation. Convolutional code is used as the error correction code and 24 bits cyclic redundancy check (CRC) code is used as the error detection code. Soft-decision Viterbi decoding of 16 quantization levels is used as the channel decoder. As a reference, the ICBOMP recovery will perform 30 iterations, which is exactly the case in [1]. The results required iteration number vs. SNR and symbol error rate (SER) vs. SNR are shown in Figure 2.

The results show that, in the given SNR regime from -6dB to 2dB, our derived threshold always makes the iteration number near the real sparsity level 16, which saves nearly 14 unnecessary iterations to greatly reduce the computational cost. In the accuracy performance, a slightly higher SER is observed for the threshold. This comes from the fact that compared with 30 iterations, more supporting blocks will be missed to detect when much less iterations are performed.



(a) Iteration number

(b) SER

Fig. 2. Performance for ICBOMP recovery

VI. Conclusions

In this paper, a theoretical stopping condition is derived for greedy block sparse recovery when the sparsity level is unknown. By studying the energy of the residual signals at each iteration, a condition is derived for stopping the iteration process of the BOMP algorithm. The approach works well for the BOMP recovery. And then we promote the work to derive the stopping condition for the ICBOMP recovery in [1]. The work contributes to saving many unnecessary iterations.

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