Joint Scheduling and Beamforming via Cloud-Radio Access Networks Coordination

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***Abstract*—Cloud radio access network (CRAN) emerges as a promising architecture for large-scale interference management. This paper addresses the benefit of one particular type of coordinated resource allocation in CRANs through the combined effect of joint scheduling and beamforming. Consider the downlink of a CRAN where the cloud is connected to several remote radio heads (RRHs), each equipped with multiple antennas. The transmit frame of every RRH is formed by several radio resource blocks (RRBs), each capable of serving multiple single-antenna users via spatial multiplexing using beamforming. The paper focuses on the problem of maximizing the network-wide weighted sum-rate by jointly determining the set of scheduled users at each RRB, and their corresponding beamforming vectors. The main contribution of the paper is to solve such a mixed discrete-continuous optimization problem using a graph-theoretical based approach. The paper introduces the joint scheduling and beamforming graph, wherein each independent set accounts for a feasible schedule and feasible beamforming vectors. Afterward, the joint scheduling and beamforming problem is shown to be equivalent to a maximum independent set problem in the proposed graph. Simulation results suggest that the proposed joint solution provides appreciable performance improvements as compared to the classical iterative approach.**

***Index Terms*—Cloud radio access networks, joint coordinated scheduling and beamforming, conflict graph, independent set.**

I. Introduction

The enormous demand for high data rates pushes wireless networks to enlarge both the spatial and spectral efficiency of next generation systems [1]. The spatial efficiency is enhanced by base-antenna densification via aggressively deploying multi-antenna base-stations with various cell sizes, e.g., macrocell, microcell, picocell. The spectral efficiency is enhanced through intelligent spectrum reuse via optimized resource allocation schemes. The combined effect of both techniques requires sophisticated platforms for interference management and inter-base-station coordination, so as to achieve the high data metrics of next generation wireless systems (5G) [1]. Cloud-radio access networks (CRANs) offer such a practical paradigm for large-scale interference management [2], as it connects all base-stations to the centralized cloud which allows for coordinated joint resource allocation.

Consider the downlink of a CRAN system, where the centralized processor (cloud) is connected to several multiple antennas remote-radio heads (RRH) and is responsible for allocating the available resources among the network entities. The transmit frame of every RRH is formed by several radio resource blocks (RRBs), each capable of simultaneously serving multiple users via spatial multiplexing using beamforming. Unlike the signal-level coordination which requires sharing the received waveforms among all base-stations with a substantial amount of backhaul/fronthaul communications [3], the focus of this paper is on maximizing the network-wide weighted sum-rate by means of coordination at scheduling and beamforming levels, which is easier to implement in practice [2].

The maximization problem addressed in this paper jointly optimizes both scheduling and beamforming strategies, which are considered both jointly and separately in the recent literature. The classical approach to solve the user scheduling problem utilizes the proportionally fair scheduling [4]; however, such approach assumes no inter-BS coordination. To best account for the cloud computing capabilities, recent references [5], [6] focus on coordinating the scheduling decisions across the multiple RRHs, so as to maximize a network-wide utility function. References [5], [6], however, do not account for the multi-antenna/beamforming case. The beamforming problem, on the other hand, is extensively studied in the literature of wireless networks, both under fixed scheduling [3], [7], and jointly with scheduling [8]–[10]. References [8]–[10], however, adopt an iterative, modular approach in which each parameter is optimized separately, i.e., the schedule is determined assuming the beamforming vectors are fixed, then the beamforming strategy is determined assuming that the schedules are fixed. Our paper shows that such a modular approach is, in fact, inferior to its joint counterpart.

The paper considers the downlink of a CRAN where the cloud is responsible for scheduling users to the available RRBs of the RRHs, determining the users beamforming vectors, and synchronizing the transmissions. Under such scheduling-beamforming coordination scheme, each user can only be connected to a single RRH since, otherwise, joint signal processing would be required between the RRHs. A single user, however, can be served simultaneously by several RRBs of one RRH, once it is associated with that particular RRH. The paper focuses on the problem of maximizing the weighted sum-rate subject to per-resource block power and user connectivity constraints, so as to jointly determine the set of scheduled users at each resource block and their corresponding beamforming vectors, in a coordinated manner via the cloud.

To the best of the authors' knowledge, while the modular approach is widely used, this is the first paper that provides a solution to the joint scheduling users and determining the beamforming vectors problem in CRANs. The main contribution of the paper is to solve such a mixed discrete-continuous optimization problem using graph theoretical techniques. The paper introduces the joint scheduling and beamforming graph wherein each independent set accounts for a feasible schedule and feasible beamforming vectors. The paper then shows that the problem can be reformulated as a maximum independent set problem in the proposed graph, which can be efficiently solved, e.g., [11], [12]. Simulation results suggest that the joint scheduling

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and beamforming approach provides appreciable performance improvements as compared to the classical iterative approach.

II. System Model and Problem Formulation

A. System Model

Consider the downlink of a CRAN where the cloud is connected to B RRHs, each equipped with N antennas. The network serves U users in total. Denote by $\mathcal{B}$, $\mathcal{U}$, and $\mathcal{N}$ the set of RRHs, the set of users, and the set of antennas per RRH, respectively. The transmit frame of every RRH is formed by a set $\mathcal{R}$ of R radio resource blocks, which are all synchronized via the cloud, i.e., interference at the r-th RRB of the b-th RRH only affects the same r-th RRB of the b'-th RRH, where $b' \neq b$. Figure 1 shows an example of the considered network and frame structure. The nominal maximum power level of the r-th RRB in the b-th RRH is denoted by P_{br}. In the context of this paper, the cloud is responsible for jointly determining the scheduling and beamforming polices, and for synchronizing the transmit frames across the connected RRHs.

Let $\mathbf{h}_{br}^u \in \mathbb{C}^N$ be the channel vector between the u-th user and the r-th RRB of the b-th RRH. Let $\mathbf{w}_{br}^u \in \mathbb{C}^N$ be the beamforming vector associated with the u-th user when it is scheduled at the r-th RRH of the b-th RRB. Under the scheduling-beamforming coordination scheme adopted in this paper, each user can only be connected to a single RRH, since otherwise, joint signal processing would be required between the RRHs. The signal-to-interference plus noise-ratio (SINR) achieved by the u-th user when served by the r-th RRB of the b-th RRH can be written as:

$$\text{SINR}_{br}^u(W) = \frac{|(\mathbf{w}_{br}^u)^H \mathbf{h}_{br}^u|^2}{\sigma^2 + \sum_{(u',b') \neq (u,b)} |(\mathbf{w}_{b'r}^{u'})^H \mathbf{h}_{b'r}^u|^2} \quad (1)$$

where $\mathbf{x}^H$ denotes the Hermitian of vector $\mathbf{x}$ and σ^2 is the Gaussian noise variance. Note that the interference term in (1), i.e., $\sum_{(u',b') \neq (u,b)} |(\mathbf{w}_{b'r}^{u'})^H \mathbf{h}_{b'r}^u|^2)$ can be separated into both intra-cell interference $\sum_{u' \neq u} |(\mathbf{w}_{br}^{u'})^H \mathbf{h}_{br}^u|^2$ and inter-cell interference $\sum_{b' \neq b} \sum_{u' \neq u} |(\mathbf{w}_{b'r}^{u'})^H \mathbf{h}_{b'r}^u|^2$.

B. Joint Scheduling and Beamforming Problem Formulation

In this paper context, the joint scheduling and beamforming problem focuses on jointly assigning users to RRHs, scheduling users to each RRB of each RRH, and choosing the beamforming vector of each user under the following connectivity constraints:

- C1: Each user can be assigned at most to a single RRH.
- C2: Each RRB serves between 1 and a maximum of K users where $1 \leq K \leq N$.

The above constraint C2 allows each RRB to simultaneously serve up to N users, so as to preserve high system multiplexing gain. The paper, in fact, quantifies both the computational complexity and the performance of the proposed solution versus the maximum number of users per RRB in sections Section III and Section IV, respectively.

The problem considered in this paper consists of maximizing the network-wide weighted sum-rate, subject to per RRB power constraints, in addition to constraints C1 and C2. Define π_{br}^u as the user' load balancing factors associated with the rate of the u-th user assigned to the r-th RRB of the b-th RRH. Such load balancing factors are typically adjusted at the cloud in an outer loop for fairness purposes. But such adjustment falls outside the

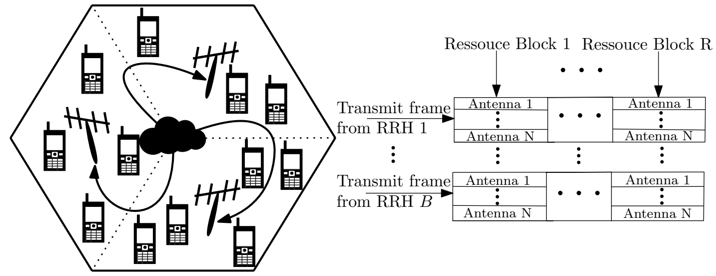

Fig. 1. A Cloud-enabled network connecting 3 remote radio head each with 4 antennas and serving a total of 12 users with the frame structure of the connected RRHs. The available radio ressource blocks are synchronized.

scope of the current paper, which assumes fixed π_{br}^u. Let X_{br}^u be a binary assignment variable which is 1 if the u-th user is served by the r-th RRB of the b-th RRH, and 0 otherwise. Similarly, let Y_b^u a binary assignment variable which is 1 if the u-th user is assigned to the b-th RRH, and 0 otherwise. The joint scheduling and beamforming problem considered in this paper can, therefore, be formulated as follows:

$$\max \sum_{u,b,r} \pi_{br}^u X_{br}^u \log\Big(1 + \frac{|(\mathbf{w}_{br}^u)^H \mathbf{h}_{br}^u|^2}{\sigma^2 + \sum\limits_{(u',b') \neq (u,b)} |(\mathbf{w}_{b'r}^{u'})^H \mathbf{h}_{b'r}^u|^2}\Big)$$

$$\text{s.t. } Y_b^u = \min\Big(\sum_{r \in \mathcal{R}} X_{br}^u, 1\Big), \forall\ (u,b) \in \mathcal{U} \times \mathcal{B}, \quad (2a)$$

$$\sum_{b \in \mathcal{B}} Y_b^u \leq 1, \qquad \forall\ u \in \mathcal{U}, \quad (2b)$$

$$1 \leq \sum_{u \in \mathcal{U}} X_{br}^u \leq K, \qquad \forall\ (b,r) \in \mathcal{B} \times \mathcal{R}, \quad (2c)$$

$$\sum_{u \in \mathcal{U}} X_{br}^u |\mathbf{w}_{br}^u|^2 \leq P_{br}, \quad \forall\ (b,r) \in \mathcal{B} \times \mathcal{R}, \quad (2d)$$

$$X_{br}^u, Y_b^u \in \{0,1\}, \qquad \forall\ (u,b,r) \in \mathcal{U} \times \mathcal{B} \times \mathcal{R}, \quad (2e)$$

$$\mathbf{w}_{br}^u \in \mathbb{C}^N, \qquad \forall\ (b,r,n) \in \mathcal{B} \times \mathcal{R} \times \mathcal{N}, \quad (2f)$$

where the optimization is over the discrete variables X_{br}^u and Y_b^u, and the continuous variables $\mathbf{w}_{br}^u$, where constraints (2a) and (2b) enforce that each user can be served at most by a single base-station, but possibly by many of its RRBs, where constraint (2c) enforces that each RRB is limited to serve K different users, and where (2d) corresponds to the per-RRB maximum power constraint.

The above optimization problem (2) is a mixed discrete and continuous optimization problem. Finding the optimal solution of such problem may require optimizing the beamforming vectors for each possible assignment of users to RRHs and RRBs, and then choosing the assignment with the maximum weighted sum-rate. Such approach is, however, computationally unfeasible for any reasonably sized network. The main contribution of this paper is to show that the above problem can be rather solved using techniques from graph theory with a relatively low computational complexity. The paper first sheds light on the classical modular approach which iteratively solves each subproblem by fixing the other subproblem. The subsequent section then presents our joint approach to solve the problem. The simulations results finally show that the modular approach is inferior to the joint solution proposed in our paper.

C. Iterative Modular Approach

The conventional modular approach to solve the scheduling and beamforming problem is to consider each problem on its own; see

[8]–[10] and references therein. Such approach optimizes each parameter separately, i.e., the schedule is determined assuming the beamforming vectors are fixed, then the beamforming strategy is determined assuming that the schedules are fixed. Such approach can be readily leveraged to the current paper system model. On one hand, for fixed beamforming vectors, the problem boils down to the following scheduling problem:

$$\max \sum_{u,b,r} \pi_{br}^{u} X_{br}^{u} \log\Big(1+\frac{|(\mathbf{w}_{br}^{u})^{H}\mathbf{h}_{br}^{u}|^2}{\sigma^2+\sum\limits_{(u',b')\neq(u,b)}|(\mathbf{w}_{b'r}^{u'})^{H}\mathbf{h}_{b'r}^{u}|^2}\Big)$$
$$\text{s.t. } (2a),(2b),(2c), \text{ and } (2e), \tag{3}$$

where the optimization is over the variables X_{br}^{u} and Y_{b}^{u}. Such problem can be solved using the approaches presented in [13]. On the other hand, for fixed assignment of users to the RRBs and RRHs, i.e., fixed X_{br}^{u} and Y_{b}^{u}, the problem boils down to the following beamforming problem:

$$\max \sum_{X_{br}^{u}=1} \pi_{br}^{u} \log\Big(1+\frac{|(\mathbf{w}_{br}^{u})^{H}\mathbf{h}_{br}^{u}|^2}{(\sigma^2+\sum\limits_{(u',b')\neq(u,b)}|(\mathbf{w}_{b'r}^{u'})^{H}\mathbf{h}_{b'r}^{u}|^2}\Big)$$
$$\text{s.t. } (2d), \text{ and } (2f), \tag{4}$$

where the optimization is over the variables $\mathbf{w}_{br}^{u}$. The above problem (4) is a well-known non-convex optimization problem. Despite its non-convexity, the problem can be solved locally using efficient techniques, e.g., using the weighted minimum mean-square error (WMMSE) approach proposed in [7]. In fact, this paper utilizes the WMMSE beamforming solution to compute the weights of the vertices while constructing the newly introduced joint scheduling and beamforming graph in the next section.

III. Joint Scheduling and Beamforming

This section presents the graph-theoretical based solution to problem (2). The section first introduces the local beamforming graph, in which each vertex represents a feasible schedule of a particular RRB. Solving the beamforming problem for each vertex allows getting the beamforming vectors. The joint scheduling and beamforming graph is then obtained by combining all local graphs, while preserving the feasibility of the solution. This section shows that the joint scheduling and beamforming solution is equivalent to the maximum independent set in the constructed graph, which can be globally solved using existing efficient solvers, e.g., [11], [12]. The section further presents the computational complexity analysis of the proposed solution.

A. Local Beamforming Graph

The local beamforming graph, denoted by $\mathcal{G}_r$, is the graph where each vertex represents a feasible schedule of a the rth RRB. In order to describe the graph construction steps, first introduce the set $\mathbb{U}$ of all possible multiplexing of users at a given RRB. From the restriction on the number of served users by each RRB, each element of the set $\mathbf{u} \in \mathbb{U}$ contains a maximum of K users. In other words, $\mathbb{U}$ can be mathematically written as $\mathbb{U} = \{\mathbf{u} \in \mathcal{P}(\mathcal{U}), \text{where } |\mathbf{u}| \leq K\}$, where $\mathcal{P}(\mathcal{X})$ is the power-set of the set $\mathcal{X}$, and $|\mathcal{X}|$ is the cardinality of $\mathcal{X}$.

Define the set of all associations between users, RRBs, and RRHs as $\mathcal{A}$ where $\mathcal{A}$ is the cartesian product $\mathcal{A} = \mathbb{U} \times \mathcal{B} \times \mathcal{R}$. In other words, every feasible association of users to a given RRB of a given RRH is represented by an element $\mathbf{a} \in \mathcal{A}$ ($\mathbf{a}$ is interchangeably called an association in this paper). In order to construct the set of vertices, define the mapping function φ_u from $\mathcal{A}$ to $\mathbb{U}$, which maps each association $\mathbf{a} = (\mathbf{u}, b, r)$ as follows: $\varphi_u(\mathbf{a}) = \mathbf{u}$. Similarly, let φ_b and φ_r be the mapping functions for the set of associations to the corresponding RRH and RRB, respectively, i.e., $\varphi_b(\mathbf{a}) = b$ and $\varphi_r(\mathbf{a}) = r$.

Let $\mathcal{A}_{rb} = \{\mathbf{a} \in \mathcal{A}, \text{where } \ \varphi_b(\mathbf{a}) = b \text{ and } \varphi_r(\mathbf{a}) = r\}$ be the subset of $\mathcal{A}$ that only includes the associations of the r-th RRB at the b-th RRH. From the system constraint C1, each user can only be scheduled to a single RRH. Therefore, the users scheduled at RRB r of each RRH need to be distinct from one RRH to another. The set of feasible associations of the r-th RRB, denoted by $\mathcal{V}_r$, can then be written as follows:

$$\mathcal{V}_r = \Big\{(\mathbf{a}_1, \ \cdots, \ \mathbf{a}_B) \in \prod_{b=1}^{B} \mathcal{A}_{rb} \ \Big| \ \bigcap_{b=1}^{B} \varphi_u(\mathbf{a}_b) = \varnothing\Big\}, \tag{5}$$

where $\prod_{b=1}^{B} \mathcal{A}_{rb}$ is the cartesian product of the sets $\mathcal{A}_{rb}$.

Proposition 1. *There is a one-to-one mapping between the elements of $\mathcal{V}_r$ and each feasible assignment of users to the r-th RRB across all connected RRHs.*

Proof. To show this proposition, it is sufficient to show that each feasible assignment of users to the r-th RRB across all connected RRHs is represented by a vertex. To prove the converse, it is sufficient to show that each vertex represents a feasible assignment. Let $\mathbf{a}_b = (\mathbf{u}_b, b, r)$ be a feasible association of a set of users $\mathbf{u}_b$ to the r-th RRB of the b-th RRH. From the limitation on the number of multiplexed users, we have $|\mathbf{u}_b| \leq K$. In other words, we have $\mathbf{u}_b \in \mathbb{U}$, and equivalently, $\mathbf{a}_b \in \mathcal{A}_{rb}$. Furthermore, since each user cannot be connected to multiple RRHs, then $\mathbf{u}_b \cap \mathbf{u}_{b'} = \varnothing, \ \forall \ b \neq b'$. The latter condition can be rewritten as $\bigcap_{b\in\mathcal{B}} \mathbf{u}_b = \varnothing$, which proves that the association is represented by a vertex in the graph. Conversely, one can note that each vertex represents a feasible assignment of users to the r-th RRB across the connected RRHs, since it satisfies the connectivity constraints. Therefore, there is a one-to-one mapping between each feasible assignment of users to the r-th RRB across all connected RRHs and all elements of $\mathcal{V}_r$. ■

Let $\mathcal{G}_r(\mathcal{V}_r, \mathcal{E}_r)$ be the local beamforming graph of the r-th RRB, where the set of vertices $\mathcal{V}_r$ is constructed as in (5). From Proposition 1, such set of vertices includes all possible assignments for the r-th RRB. As the original optimization problem (2) is a weighted sum-rate maximization problem, the weight of each vertex $\mathbf{v} = (\mathbf{a}_1, \ \cdots, \ \mathbf{a}_B)$ can be found by solving the following optimization problem:

$$\max \sum_{b=1}^{B} \pi_{br}^{u_b} \log\Big(1+\frac{|(\mathbf{w}_{br}^{u_b})^{H}\mathbf{h}_{br}|^2}{\sigma^2+\sum\limits_{(u',b')\neq(u_b,b)}|(\mathbf{w}_{b'r}^{u'})^{H}\mathbf{h}_{b'r}^{u_b}|^2}\Big)$$
$$\text{s.t. } (2d), \text{ and } (2f), \tag{6}$$

where the optimization is over the beamforming vectors and the notation $u_b = \varphi_u(\mathbf{a}_b)$. The above optimization problem has a similar structure to (4) (for the r-th RRB of interest), and so it can be solved efficiently using a WMMSE approach [7].

B. Joint Scheduling and Beamforming Graph

The local beamforming graph constructed above only focuses on a single RRB r. The paper now makes use of such construction to solve the *joint* scheduling and beamforming problem (2) by means of building the joint scheduling and beamforming graph, which represents the union of all local beamforming graphs, which preserve the feasibility of the solution of the original problem. Each local graph can be herein seen as a cluster, wherein each vertex represents the solution to a particular RRB. The connections between the clusters are then created in such a way that preserves the feasibility of the joint optimization problem. Finally, finding the solution of the original optimization problem requires determining the independent set with the maximum weights. Such construction process is described in details next.

Let $\mathcal{G}(\mathcal{V},\mathcal{E})$ be the joint scheduling and beamforming graph in which the set of vertices is $\mathcal{V} = \bigcup_{r \in \mathcal{R}} \mathcal{V}_r$. Two vertices $\mathbf{v}_r = (\mathbf{a}_1, \cdots, \mathbf{a}_B) \in \mathcal{V}_r$ and $\mathbf{v}_{r'} = (\mathbf{a}'_1, \cdots, \mathbf{a}'_B) \in \mathcal{V}_{r'}$ are connected if one of the following two *conflict connectivity conditions* (CC) are satisfied:

- CC1: $r = r'$.
- CC2: $r \neq r'$ and $\bigcup\limits_{i=1}^{B} \Big(\bigcup\limits_{j=1, j \neq i}^{B} \varphi_u(\mathbf{a}_i) \cap \varphi_u(\mathbf{a}'_j) \Big) \neq \varnothing$

Condition CC1 represents a conflict of scheduling, since same RRB r is scheduled more than once. Condition CC2, on the other hand, describes that each user is scheduled to possibly two or more RRHs, which makes the intersection $\varphi_u(\mathbf{a}_i) \cap \varphi_u(\mathbf{a}'_j)$ non-empty and generates an edge between the vertices $\mathbf{v}_r$ and $\mathbf{v}_{r'}$. Figure 2 shows the joint scheduling and beamforming graph for a network with 2 RRHs, each equipped with two antennas over one RRB in a network comprising 3 users.

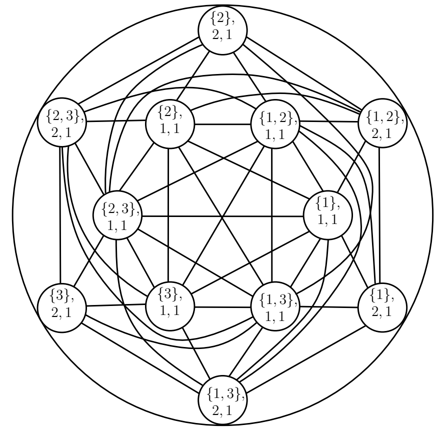

Fig. 2. Joint scheduling and beamforming graph for a network with 2 RRHs, 2 antennas per RRH, a single RRB and 3 users. Each vertex is labeled as $\{u_1, u_2\}, b, r$ where $\{u_1, u_2\}$ is the set of users associated with the r-th RRB of the b-th RRH.

C. Joint Scheduling and Beamforming Solution

To solve the joint scheduling and beamforming problem, the paper now shows that problem (2) can be reformulated as a maximum independent set problem in the joint scheduling and beamforming graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ constructed above. From a graph theoretical perspective [11], [12], an independent set in a graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ is a subset of vertices $\mathbf{V} \in \mathcal{V}$ where no connection exists between any two pairs, i.e., $\epsilon_{v,v'} = 0, \ \forall \ (v, v') \in \mathbf{V}^2$. Similarly, a maximal independent set is an independent set so that no other vertex can be added to the set without losing the independence property. The maximum weight independent set, on the other hand, is the independent set of maximum weight. Given such definitions, the following theorem describes the solution of the joint scheduling and beamforming beamforming problem (2).

Theorem 1. *A local optimal solution to the joint scheduling and beamforming problem (2) is the solution to the maximum weight independent set problem in the joint scheduling and beamforming graph. In other words, let $\mathcal{I}$ denotes the set of all maximal independent sets, the problem (2) can be formulated in the graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ as follows:*

$$\max_{I \in \mathcal{I}} \sum_{v \in I} w(v). \tag{7}$$

Proof. The proof of this theorem can be decomposed into the following parts. The first part of the proof shows that each feasible schedule can be represented by an independent set. The proof, afterward, shows that for a feasible scheduling, the beamforming vectors are the ones that are represented by the weights of the vertices as computed in (6). The proof then concludes with reformulating the problem as a search for the maximum weight independent set among the maximal ones.

First note that, as a consequence of Proposition 1, all feasible schedules are represented by a collection of vertices in the joint scheduling and beamforming graph. The first part of the proof then hinges upon a a contrapositive argument. In other words, assume first that the collection of vertices representing a certain feasible schedule is not an independent set, i.e., it contains connections between at least two vertices. The corresponding schedule resulting from such collection of vertices is, therefore, a non-feasible schedule. The reason behind such a conclusion is that the connection between the vertices of such a none-independent set can be due to one of the following conflict conditions:

1) CC1: Such connection means that the same resource block is scheduled more than once.
2) CC2: Such connection means that at least one user is scheduled to different RRHs.

Either connectivity constraint above (i.e., CC1 or CC2) makes the schedule unfeasible, which is a contradiction. We conclude then that that each feasible schedule can be represented by an independent set.

Moreover, for a feasible fixed scheduling, the locally optimal beamforming vectors are the ones which maximize the optimization problem (4). As shown in the previous subsection, the weights of the vertices are by design constructed in (6) (which is similar to (4)). The joint scheduling and beamforming problem (2) can, therefore, be solved by solving the maximization problem over the weights of independent vertices, i.e., the maximum weight independent set problem in the joint scheduling and beamforming graph $\mathcal{G}(\mathcal{V},\mathcal{E})$. ■

D. Complexity Analysis

This section describes the computational complexity of the proposed solution to problem (2). The maximum weight independent set (or equivalently the maximum weight clique) is an interesting problem in graph theory as it can be solved efficiently using both optimal, e.g., [11], and efficient heuristics, e.g., [12]. The complexity of all aformentionned algorithms depends on the number of vertices in the graph. The exact computation of the number of vertices $|\mathcal{V}|$ is challenging. However, one can show that the number of vertices in the joint scheduling and beamforming graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ is bounded by the following:

$$|\mathcal{V}| \leq R\Big(\frac{U!(U-K+1)}{(U-K)!K!(U-2K+1)}\Big)^B \tag{8}$$

IV. Simulation Results

This section evaluates the benefit of the proposed joint scheduling and beamforming solution as compared to state-of-art iterative

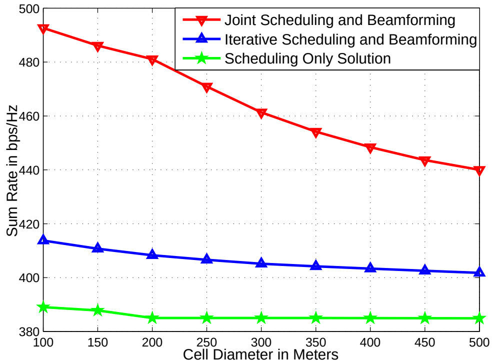

Fig. 3. Sum-rate in bps/Hz versus the cell diameter in meters. The network is composed of 3 RRHs, 5 RRBs, and 4 antennas per RRH. The maximum power is $P = -26.98dBm/Hz$, and the maximum number of multiplexed users is 4.

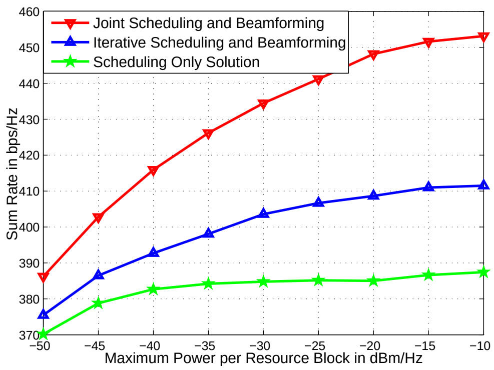

Fig. 4. Sum-rate in bps/Hz versus the maximum power per RRB. The network is composed of 3 RRHs, 5 RRBs, and 4 antennas. The cell diameter is 350m, and the maximum number of multiplexed users is 4.

methods. Consider a CRAN where the cloud is connected to three RRHs, and where each RRH frame has 5 distinct resources blocks. Users across the network are uniformly distributed, and the number of allowed users per RRB is varied in the simulations. The channel vectors are chosen according to SUI-3 Terrain type B model with log-normal shadowing and 10 MHz bandwidth. The background noise power is fixed to -168.60 dBm/Hz.

To illustrate the performance of the proposed algorithm as a function of the cell size, Figure 3 plots the sum-rate in bps/Hz versus the cell diameter, for a CRAN composed of 3 RRHs each having 5 RRBs with a maximum power $P = -26.98dBm/Hz$. The RRHs are equipped with 4 antennas, each serving a maximum of 4 users, and so the network can serve up to 60 users, i.e., 4 users per RRB $\times$ 5 RRBs per RRH $\times$ 3 RRHs). The figure shows that the proposed joint scheduling and beamforming approach outperforms the classical iterative approach for all values of cell size. The gain is particularly higher for small cell size, where there is a high level of interference. This is expected because it is specifically at high interference levels that the role of joint scheduling and beamforming becomes more pronounced. In fact, the proposed joint scheduling and beamforming solution can harvest about 20% gain as compared to the iterative solution. The figure further shows that both the joint approach and the iterative approach outperform the scheduling-only solution, i.e., the scheme which assumes that each antenna acts independently, which highlights the important role of beamforming in large-scale interference management.

Figure 4, on the other side, highlights the performance of the proposed algorithm for different system power capabilities. The figure plots the sum-rate in bps/Hz versus the maximum power per RRB for a network composed of 3 RRHs, each equipped with 4 antennas. As in Figure 3, the total number of available RRB is 5, and the maximum number of multiplexed users per RRB is 4. The cell size is set to 350m. Figure 4 shows that the proposed joint optimization solution outperforms the modular iterative approach, especially at high maximum transmit power levels, where the interference is high and where interference mitigation techniques are more vital. As expected, the figure shows once again that both the joint approach and the iterative approach have an appreciable performance improvement as compared to the scheduling-only solution for all values of the maximum power per RRB.

V. Conclusion

Sophisticated resource allocation schemes are expected to play a major role in the design of futuristic wireless systems. This paper evaluates the benefit of a particular type of coordinated resource allocation strategies in CRAN through joint scheduling and beamforming. The paper focuses on the problem of maximizing the network-wide weighted sum-rate by jointly determining the set of scheduled users at each resource block and their corresponding beamforming vectors. The paper presents a graph theoretical solution to the problem by introducing the joint scheduling and beamforming graph, in which each maximal independent set represents a feasible solution to the problem. Simulation results suggest that the joint scheduling and beamforming approach provides appreciable performance improvements as compared to the classical modular approach.

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