Practical Full Duplex Physical Layer Network Coding

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Abstract—We propose a practical network code for the wireless two-way relay channel where all nodes communicate in full duplex (FD) mode. The physical layer network coding (PNC) operation is applied with the FD operating nodes, reducing the transmission time to a single time slot, hence doubling the spectral efficiency when compared to classical PNC systems. In our system model, binary phase shift keying modulated signals are transmitted over Rayleigh fading channels. We derive the theoretical error rates at relay and end nodes according to the maximum likelihood detection rule, in case of non-ideal self-interference cancellation. Theoretical results are also verified via simulations.

Index Terms—Full duplex transmission, physical layer network coding, self-interference.

I. INTRODUCTION

FOLLOWING the seminal works [1] and [2] there has been a surge of research activities in the field of physical layer network coding (PNC). The main principle of the PNC relies on the use of the two-way relay channel (TWRC) shown in Fig. 1 in a two phase process. The first phase is the multiple access (MA) phase, where end nodes A and B transmit their symbols to the relay node, R. In the second phase, referred to as the broadcast (BC) phase, R transmits a function of the collided symbols from end nodes A and B. Having the *a priori* knowledge about their own transmitted symbols, A and B can detect the unknown symbol. Information exchange through PNC hence takes two time slots in TWRC.

In this paper, we propose a new physical layer network code for TWRC where the transmission takes a single time slot via combining the MA and BC phases by making use of full duplex (FD) transmission at all nodes. In the proposed network coding technique, named as full duplex physical layer network coding (FD-PNC), R uses detect-and-forward (DF) relaying technique as shown in Fig. 1. The maximum likelihood (ML) based joint detection is used at the relay in order to eliminate the multiple access interference, which occurs due to the reception of two symbols from distinct sources at the same time slot. We adopt the max-log approximation to the ML detection rule, which is frequently preferred in order to reduce the computational complexity of the receiver [3]. The proposed FD-PNC system can save 50% of the time needed by the classical PNC approaches. Note that the FD-PNC saves 66.7% of the time when compared to the classical network coding, where the MA phase takes two time slots (requiring 3 time slots in total [4]).

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Fig. 1. Full duplex physical layer network coding system. All nodes transmit in a single time slot. The relay transmits a function of the transmitted signals from end nodes via detect-and-forward relaying.

Despite the apparent advantages, making use of FD nodes has its challenges in terms of the generated self-interference (SI). In the literature, there are various SI cancellation techniques that are proposed in order to combat the associated performance degradation. Although the majority of the literature on FD transmission assume that the SI can ideally be eliminated [5]–[8], noting its deteriorating effects on the performance, throughout our analyses we take the residual SI into account. In this paper, after describing the details of the system model we derive the corresponding bit error rate (BER) expressions for the relay node and end nodes in the presence of Rayleigh fading channels for binary phase shift keying (BPSK) modulation. We verify our theoretical results via simulations.

A. Related Work

The use of FD nodes in TWRC is proposed for the first time in the pioneering work of Rankov and Wittneben, where authors investigate the information theoretical rate region without considering bit level transmission issues such as modulation and coding techniques [5]. A new lattice code is proposed for TWRC with FD nodes under non-fading channels, and the corresponding rate region analysis is performed in [6]. In [7], authors examine the effects of channel estimation errors on achievable rate region of the amplify-and-forward PNC system, again without including a bit level analysis. New linear relaying functions are proposed to maximize the mutual information between source nodes in [8], again without considering physical layer transmission issues. The studies of TWRC with FD nodes listed above explore the information-theoretic standpoints. In the literature there is one recently proposed practical transmission scheme for relay communications with FD transmission only at the relay node [9], where the single-way information transfer is completed in two time slots.

In the literature about FD transmission, there are several proposed SI cancellation techniques such as as active and

passive cancellation, operating at RF and baseband [10]–[12]. These works do not apply network coding.

II. SYSTEM MODEL

The proposed system model consists of two end nodes and one relay node indicated by A, B, and R, respectively. All nodes communicate in FD mode with separate transmit and receive antenna chains, as shown in Fig. 1. There is no communication between the end nodes. $d_i \in \{1, 0\}$ denotes the information bit of node i and $s_i \in \{1, -1\}$ represents the corresponding BPSK modulated symbol, $i = A, B$. In this TWRC, we have two types of channels; the communication channel between the nodes, and the SI channel observed due to FD transmission. Let the communication channels be denoted by h_{AR} and h_{BR} for $A - R$ and $B - R$ links, respectively. To simplify our notation we make use of the frequently utilized channel reciprocity assumption, and state that $h_{RA} = h_{AR}$ and $h_{RB} = h_{BR}$ [2], [5], [13]. The SI channels of A, B and R are represented by h_{AA}, h_{BB} and h_{RR}. Note that the time index for the channels are also omitted for notational simplicity. All channels are assumed to Rayleigh fading with unit energy. Block diagram of the system model is shown in Fig. 2.

In the proposed system, the received signal at R in the nth time slot can be modeled as

$$r_R[n] = h_{AR}s_A[n] + h_{BR}s_B[n] + \kappa_R h_{RR}s_R[n] + w_R[n], \quad (1)$$

where $w_R[n]$ denotes the additive white Gaussian noise (AWGN) with the distribution $\mathcal{N}(0, \sigma_R^2)$. Here, $s_R[n]$ is the symbol that is transmitted by R at the nth time slot. The term κ_R represents the residual SI coefficient associated with FD transmission, and it is used to quantify the associated performance degradation (due to practical implementation constraints, SI can not always be ideally canceled [10], [11]).

Representing the average bit energy of the relay node by E_R, the energy of the residual SI is equal to $\kappa_R^2 E_R$. Noting the independence of the noise process and the SI, the average signal to interference and noise ratio (SINR) of the signal transmitted by node i at relay node R is [14]

$$\gamma_i = \frac{E\left[|(h_{iR}s_i[n])|^2\right]}{E\left[|w_R[n] + (\kappa_R h_{RR}s_R[n])|^2\right]} = \frac{E_i}{\sigma_i^2 + \kappa_R^2 E_R} \quad (2)$$

where E_i represents the average bit energy of the ith node. The SI term acts as the Gaussian interference [15], [16].

In the proposed system, the symbol that is transmitted by R at the nth time slot, $s_R[n]$, is calculated according to

$$s_R[n] = -s_A[n-1] \times s_B[n-1]. \quad (3)$$

Here, $s_R[n]$ is determined based on the joint estimates of $s_A[n-1]$ and $s_B[n-1]$ obtained at R. Details of this joint detection process are given in the following subsection. Note that the relation in (3), corresponds to the modulated XOR of $d_A[n-1]$ and $d_B[n-1]$. In this system, the received baseband signal $r_R[n-1]$ is converted to the network coded symbol $s_R[n]$ without individually decoding $s_A[n-1]$ and $s_B[n-1]$.

As the relay node broadcasts a delayed version of the information symbols $s_R[n]$, the nodes A and B transmit $s_A[n]$

and $s_B[n]$, respectively. The received signal at the end node i is

$$r_i[n] = h_{iR}s_R[n] + \kappa_i h_{ii}s_i[n] + w_i[n] \text{ for } i = A, B, \quad (4)$$

where κ_i represents the residual SI coefficient and $w_i[n]$ is the AWGN component with the distribution $\mathcal{N}(0, \sigma_i^2)$. It can be observed from the system block diagram Fig. 2 that the MA and BC phases of TWRC are combined within a single time slot. At the nth time slot the end nodes detect the bit of $(n-1)^{th}$ time slot. Hence bits are detected with a fixed delay of one time slot.

A. Error Performance at the Relay Node

In the proposed FD-PNC, R needs to jointly detect $s_A[n]$ and $s_B[n]$. Hence, $s_A[n]$ and $s_B[n]$ signals cause interference in the MA phase, limiting the system's error performance. This problem can be addressed by the use of joint detection techniques as proposed in [15], [17]. Using joint detection will aid us to increase the robustness against bit errors in presence of MA interference. We use the max-log approximation which is an effective method for the application of the ML detection rule to reduce the computational complexity [3]. The joint detection process starts with expressing the approximated ML decision rule that can be stated as [2]

$$(\hat{s}_A, \hat{s}_B) = \arg \min_{s_A, s_B \in \mathcal{X}} |r_R[n] - h_{AR}s_A - h_{BR}s_B|^2, \quad (5)$$

where the alphabet is $\mathcal{X} = \pm 1$. We assume that channel coefficients are available at R. After using the ML decision rule in (5), we obtain the estimated values of $s_A[n-1]$ and $s_B[n-1]$, hence the $s_R[n]$ is determined through the equation given in (3). Noting that the Gaussian interference model for Rayleigh fading channels provides the exact bit error rate (BER) expression for BPSK modulated symbols (as proven in [16]), the exact instantaneous BER expressions of $s_A[n]$ and $s_B[n]$ are equal to

$$P_i^R(e|h_{iR}) = Q\left(\sqrt{2|h_{iR}|^2 \gamma_i}\right). \quad (6)$$

Since the absolute square of channel coefficients have exponential distribution, the average BER of the s_i signal at R can be shown to be

$$P_i^R(e) = E\left[P_i^R(e|h_{iR})\right] = \frac{1 - \sqrt{\alpha_i}}{2}, \quad (7)$$

where $\alpha_i = E_i/(E_i + \kappa_R^2 E_R + \sigma_i^2)$. Using this definitions, the average exact bit error expression at R can be calculated according to

$$\begin{aligned} P^R(e) &= P_A^R(e)(1 - P_B^R(e)) + P_B^R(e)(1 - P_A^R(e)), \\ &= \frac{1 - \sqrt{\alpha_A \alpha_B}}{2}. \end{aligned} \quad (8)$$

We present the theoretical end-to-end system BER performance in the next section by using (8).

B. End-to-End Error Performance

Here, we analyze the error performance of the node A and the analysis will be identical for node B. In order to determine

Fig. 2. The block diagram of proposed model of full duplex physical layer network coding system (SIC: Self-interference cancellation, ML: Maximum likelihood). While nodes A and B transmit their information bits, the relay node transmits the modulated XOR of detected bits from the end nodes. Hence two information bits are transmitted within a single time slot.

$s_R[n]$ at A, the ML detection rule is

$$\hat{s}_R[n] = \arg \min_{s_R \in \{\mathcal{X}\}} |r_A[n] - h_{AR}s_R|^2, \qquad (9)$$

where the estimate of $s_R[n]$ is denoted by $\hat{s}_R[n]$. Here we assume that h_{AR} is available at A. Following this step, the estimate of $s_B[n-1]$ (denoted by $\hat{s}_B[n-1]$), is calculated according to

$$\hat{s}_B[n-1] = -s_A[n-1] \times \hat{s}_R[n], \qquad (10)$$

since $s_A[n-1]$ is known *a priori* at A. If R correctly determines $s_R[n]$, the instantaneous error probability at A is equal to

$$P_B^A(e|h_{iR}) = Q\left(\sqrt{2|h_{AR}|^2 \gamma_R}\right), \qquad (11)$$

where $\gamma_R = E_R/(\sigma_A^2 + \kappa_A^2 E_A)$. The error rate at node A is a function of $P^R(e)$ and the average BER is equal to $P_B^A(e) = E[P_B^A(e|h_{iR})]$. Here two events ($P^R(e)$ and $P_B^A(e)$) are statistically independent due to physical separation of nodes. When $s_R[n]$ is transmitted in error by R, and A incorrectly detects the erroneously transmitted bit, $s_R[n]$ signal is obtained correctly. Hence the average error probability of the end-to-end system is equal to

$$\begin{aligned} P^A(e) &= P^R(e)(1 - P_B^A(e)) + (1 - P_E^R(e))P_B^A(e), \\ &= \frac{1 - \sqrt{\alpha_A \alpha_B \alpha_R}}{2} \end{aligned} \qquad (12)$$

where $\alpha_R = E_R/(E_R + \kappa_A^2 E_i + \sigma_A^2)$.

As the signal to noise ratio of communication channels increase, the effect of SI becomes more dominant, generating error-rate floors. The error-rate floor at A, Γ_A, can be obtained as

$$\begin{aligned} \Gamma_A &= \lim_{\sigma_R^2, \sigma_i^2 \to 0} \{P^A(e)\}, \\ &= \frac{1}{2} - \frac{1}{2(1 + \kappa_R^2)\sqrt{1 + \kappa_A^2}}, \end{aligned} \qquad (13)$$

for the case of unit energy transmissions.

III. NUMERICAL AND SIMULATION RESULTS

In this section, theoretical and matching simulation results are given for the FD-PNC system in presence of SI for all nodes. The results are presented for relay and end-to-end BER performances given in (8) and (12), respectively. We use BPSK modulated signals, distribution of all channels are Rayleigh fading and required channel coefficients are assumed to be known at all nodes. The analyses can be extended to higher order modulations through similar steps.

BER curves for the end-to-end system and the relay nodes are shown in Fig 3 for various residual SI coefficients for nodes R and A. Note that theoretical results match simulation results, confirming the validity of the Gaussian interference model [15], [16]. In this figure we can identify two operating regimes, the noise limited regime ($\sigma_i^2 > 2\kappa^2$ for unit energy transmissions) and the SI limited regime ($\sigma_i^2 \leq 2\kappa^2$). The error-rate floors of the system depending on the residual SI component given in (13) are also highlighted in the figure.

As noted throughout our analyses, the effects caused by the residual SI coefficients can not be ignored. Fig. 4 demonstrates the performance degradation due to imperfect implementation of transmit and receive antenna chains of the FD nodes. From this figure we can see that for the non-ideal case of SI cancellation $\kappa_A = \kappa_R = 2 \times 10^{-4}$ (which equivalently models the 74 dB SI suppression implemented with consumer of the shelf equipments in [12]) we can obtain a BER of 10^{-4} for $SNR \approx 38$ dB, verifying the practicality of the proposed FD-PNC.

Finally noting that the residual SI at the relay node has more deteriorating effects on the end-to-end BER, in Fig. 5 we quantify the impact of κ_A and κ_R on the BER performance for $SNR = 20$ dB and $SNR = 40$ dB. From this figure we can observe that the error rates can be improved if better

Fig. 3. Relay node and end-to-end bit error rate performances of the proposed FD-PNC system for various κ values ($\kappa_R = \kappa_A = \kappa$). The approximate boundary between the self-interference and noise dominated operating regimes are also shown.

Fig. 4. End-to-end bit error rate of the proposed FD-PNC system for different κ and SNR values where $\kappa_R = \kappa_A = \kappa$. Acceptable operating points can be achieved with high SNR values even when $\kappa \neq 0$.

performing FD nodes are selected as the relay nodes, rather than the end nodes.

IV. Conclusion

In this paper, we proposed a physical layer network code for TWRC with FD nodes, where each node is equipped with a single transmit and receive antenna. The proposed system decreased the total transmission time of two time slots of classical PNC systems to a single time slot, executing the MA and BA phases concurrently. The theoretical BER expressions in case of non-ideal SI cancellation over Rayleigh fading channels are derived for relay and end nodes. Monte Carlo simulations are used to verify the theoretical results.

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Fig. 5. End-to-end bit error rate performances of the proposed FD-PNC system for different κ_R and κ_A values. It can be observed that the residual self-interference in the relay node deteriorates the performance more that the residual self-interference at the end nodes.

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