

# Time-Gap Based Traffic Model for Vehicular Traffic Flow

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**Abstract**—There have been many studies for modeling vehicular traffic flow using fluid models. However, these previous approaches do not accommodate realistic models for traffic density, flow, and velocity. The existing models also fail to uncover the relationships among energy efficiency, capacity, and safety. We investigate traffic networks from a system-level perspective. In result, we provide a time-gap based mathematical traffic model for vehicular traffic flow on highways. Our model explains the widely known triangular fundamental diagram, which represents vehicular traffic systems with the three primary parameters: maximum free-flow velocity, a typical safety length of vehicles, and a mean value of the time-gap of the traffic data during congested conditions. This result is also well validated with measured traffic data using least squares matching and with previous research outcomes about the propagation velocity. In addition, we suggest two distinct analysis techniques to estimate the time-gap from the traffic data measured on highways.

## I. INTRODUCTION

Many attempts to specify a relation among traffic flow, density, and speed have been made. A macroscopic traffic model to correlate them forms the so-called fundamental diagrams of traffic flow. Greenshields [1] derived a parabolic fundamental diagram between flow and density. Lighthill, Whitham, and Richards [2] [3] used Greenshields' hypothesis and a conservation law of vehicles to provide a concave fundamental diagram, which is called first order LWR model. Newell [4] proposed a triangular flow-density fundamental diagram as a simpler alternative to solve the LWR model. Only two velocities characterize this model: a maximum free-flow velocity in a free-flow regime and a propagation velocity for a congestion area. However, he did not explain the correlation between propagation velocity and driving patterns.

Banks [5] considered the time-gap which is the time required to travel the distance between the front end of a vehicle to the back end of its leading vehicle, and showed the relation between the time-gap and speed using real traffic data in the USA. He found that during congested times, the average time-gap was relatively constant, while it diverges with large deviation during free-flow periods. However, he did not derive a fundamental diagram from the time-gap nor an analysis method to estimate the time-gap from raw traffic data.

The remainder of this paper is organized as follows: In section II, we present a fundamental traffic pattern to describe

the Newell model in congested states, which we call a time-gap based traffic model. We propose two different methods to estimate the time-gap from real traffic data measured by a single-loop and double-loop system in section III. Section IV shows various analysis results about the relation between the time-gap and a triangular fundamental diagram, and validates our proposed time-gap based traffic model with empirical traffic data.

## II. TIME-GAP BASED TRAFFIC MODEL

In this section, we derive a time-gap based traffic model. The time-gap,  $\tau$ , is defined as the time required to travel to the bumper of the vehicle in front of a typical vehicle. We assume that highways can be divided into homogeneous sections with similar traffic characteristics and patterns at a local scale.

### A. Time-gap Based Triangular Fundamental Diagram

It has been empirically observed that drivers generally travel to maintain safe distance between themselves and the vehicles immediately in front of them in order to prevent collisions. Because of this distance, there exists a time-gap guaranteeing at least a driver's minimum reaction time, which has some average value with slight variation by driver or by region. We also assume that there is a maximum free-flow velocity,  $v_f$ , imposed either by physical limitations of the vehicles or by speed limits of the road.

Let  $L_i$  denote the safety length of the  $i$ th vehicle for  $i \in \mathbb{N}$  in feet. The safety length is a summed value of the actual physical length of a vehicle and the positive safety distance, say 0.1 feet, between any two consecutive vehicles on the same lane. In addition, let  $L$  denote the average safety length of vehicles in feet. Let  $\rho$ ,  $q$ , and  $v$  denote the density, flow, and velocity for the homogeneous section considered. Their units are vehicles per mile (vpm), vehicles per hour (vph), and miles per hour (mph), respectively. Since one mile is 5280 feet, the maximum density  $\rho_{max}$  per a lane is  $5280/L$  and thus  $\rho \leq 5280/L$  for all sections and for all times. For a value of density  $\rho \leq 5280/L$ , the average distance between the front ends of vehicles is  $5280\rho^{-1}$  and thus the average car spacing which is the distance from the front end of a vehicle to the back end of its leading vehicle is  $d = 5280\rho^{-1} - L$  in feet. At

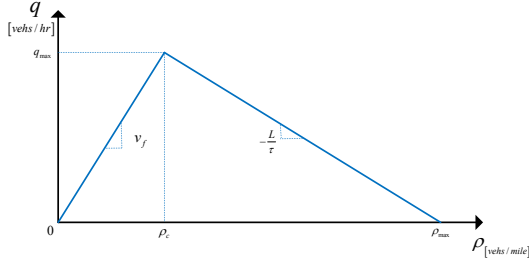


Fig. 1. Traffic Flow on a Road

a constant velocity of  $v$ , the time in seconds required to travel the car spacing  $d$  is  $d/(\bar{c}v)$ , where  $\bar{c}$  is 5280/3600 which is feet per second (fps). In order to follow the concept of the time-gap  $\tau$ , we must have  $d/(\bar{c}v) \geq \tau$ . We assume that the velocity has the limitation  $v \leq v_f$  and every driver travels as fast as possible. Hence,

$$v = \min \left[ v_f, \frac{5280\rho^{-1} - L}{\bar{c}\tau} \right]. \quad (1)$$

Since the relationship among flow, density, and velocity satisfies with  $q = \rho v$ , by (1),

$$q = \min \left[ \rho v_f, \frac{5280 - \rho L}{\bar{c}\tau} \right]. \quad (2)$$

This describes a relatively simple triangular fundamental diagram between the flow and density shown in Fig. 1. The vehicular traffic system for a considered homogeneous section has the maximum throughput  $q_{max}$  at a value of critical density  $\rho = \rho_c$  such that

$$\rho_c = \frac{5280}{\bar{c}\tau v_f + L}.$$

Our proposed time-gap based triangular fundamental diagram describes two distinct regions; a free-flow regime where the density is less than the critical density and a congestion regime where the density exceeds the critical density. The positive slope in a free-flow regime of a triangular fundamental diagram is the maximum free-flow velocity  $v_f$ , while the negative slope in a congestion regime is  $-L/(\bar{c}\tau)$  mainly consisting of the time-gap  $\tau$  by (2). Hence, we can explain how to derive a triangular fundamental diagram with both the maximum free-flow velocity and time-gap. We can easily imagine that the larger the maximum velocity is in a homogeneous section, the greater the throughput is. Additionally, a smaller time-gap value means that vehicles are tailgating closer to the vehicles immediately in front of them and thus allows for larger capacity on highways, as long as this tailgating behavior guarantees no crashes. This phenomenon can be also explained by our derivation (2); a smaller value of the time-gap shows that a negative slope in a congestion regime of our proposed triangular fundamental diagram is steeper, because we fix a value of the safety length  $L$  and thus a maximum density  $\rho_{max}$  is predetermined as  $5280/L$ . Hence, a steeper negative slope under a predetermined maximum density causes both larger critical density and higher maximum flow, i.e. larger capacity.

Note that our proposed model describes how the flow varies with the density at a local point. In fact, a triangular fundamental diagram was provided as an idealized traffic approximation by Newell [4], who recognized that this model provides simple algorithms for maximizing throughput using ramp metering. However, even though he presented a car-following model which is consistent with a triangular fundamental diagram, he has not explained it in terms of the traffic parameters  $v_f$ ,  $L$ , and  $\tau$ , as we have done here.

### III. COMPARATIVE ANALYSIS OF TIME-GAP FROM MEASUREMENT DATA

Both the USA and South Korea commonly embed an induction loop detector on highways to collect real traffic data. However, induction loop detectors can measure up to three parameters, so we are required to estimate the time-gap from these insufficient traffic observations. The USA uses a single-loop detector, whereas the South Korea employs a dual-loop detector constructed by two consecutive single-loop detectors feet apart. The type of measurement data collected from a single-loop detector is different from that by a dual-loop detector. Hence, we create the distinct methods to derive the time-gap from measurement data of these two different detector systems.

#### A. Time-gap Analysis from Single-loop Detectors

Single-loop detectors deployed in the USA report the number of vehicles passing over a detector and the occupancy counts occupied by passing vehicles per lane every 30 seconds. Because the sampling rate is at 30 Hz, the occupancy counts range from 0 to 900 over the 30 second window. Let  $n(kT)$  denote the number of passing vehicles per lane that is recorded during the  $k$ th time frame and reported at time  $kT$ , where  $T$  is the measurement interval (i.e., 30 seconds in this case). As another available raw data, the value of  $o(kT)$  is the total number of clock ticks, known as the occupancy counts when a vehicle is present over a detector in the  $k$ th sampling period. Let  $l_d$  denote the length of a detector to be 10 feet. Since  $l_d > 0$ , this causes the duration of on-time of each pulse to be larger, i.e. a fraction  $(L + l_d)/L$ , than if the detector diameter were the ideal length  $l_d = 0$ . We assume that all vehicles are uniformly distributed for the same measurement period. The ratio that a vehicle is not present at a certain point of the detector, i.e. at its leading side, for the  $k$ th measurement period can be estimated as

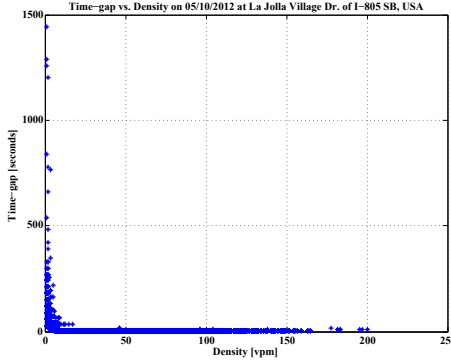
$$\left( \frac{900 - o(kT) \cdot \frac{L}{L+l_d}}{900} \right).$$

Hence, the average estimated time-gap for the  $k$ th measurement interval is

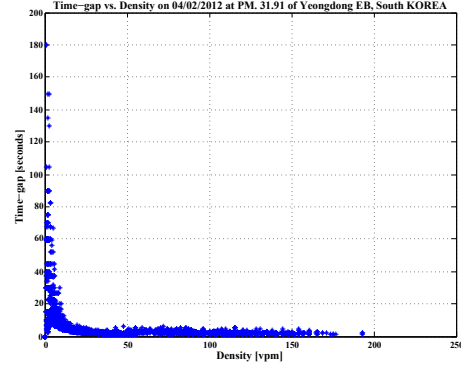
$$\tilde{\tau}(kT) = \frac{30 \left( 1 - \left( \frac{o(kT)}{900} \right) \cdot \left( \frac{L}{L+l_d} \right) \right)}{n(kT)} \quad (3)$$

seconds. The flow for the  $k$ th measurement period is estimated to be

$$\tilde{q}(kT) = \frac{n(kT)}{30} \cdot 3600 = 120n(kT) \quad (4)$$



(a) Measurement Data in the USA



(b) Measurement Data in the South Korea

Fig. 2. Estimated Time-Gap vs. Density

vph. In addition, we assume that all vehicles travel with the same velocity within the detector diameter. If we assume that the vehicles passing over a detector during the  $k$ th interval travel the distance  $n(kT)(L + l_d)$  feet for the time  $\frac{\rho(kT)}{900} 30$  seconds, then the average velocity for the  $k$ th interval is

$$\tilde{v}(kT) = n(kT)(L + l_d) \frac{30}{\rho(kT)} \quad (5)$$

mph. Now, by (4), (5), and the equation  $\rho(kT) = q(kT)v(kT)$  for all  $k \in \{0 \cup \mathbb{N}\}$ , we can draw a flow-density fundamental diagram and show the estimated time-gap value versus the density.

#### B. Time-gap Analysis from Double-loop Detectors

Since a double-loop detector can collect the travel time data of each vehicle from the first loop to the second loop, it can calculate the speed of an individual vehicle between the two known consecutive loops. However, the speed provided from a dual-loop detector system is not the velocity of an individual vehicle but the average speed of vehicles passing during the  $k$ th measurement interval. We suppose that the velocity of every vehicle belonging to the same measurement interval is equal to the average velocity provided by a dual-loop detector. Let  $v_i$  denote the velocity of the  $i$ th vehicle and  $v(kT)$  denote the average velocity for the  $k$ th measurement period. In addition,  $\tilde{t}_i$  is the estimated passage time at which the  $i$ th vehicle passes over the leading side of the first loop. Then,  $v_i = v(kT)$  for all  $i$  such that  $\tilde{t}_i \in ((k-1)T, kT]$ . Let  $\tilde{t}_{d(i,i-1)}$  denote the time difference of the estimated passage times between the  $i$ th vehicle and  $(i-1)$ th vehicle.  $x_{i-1}|_{(x_i=0)}$  is the position of the  $(i-1)$ th vehicle when its following  $i$ th vehicle just arrives at the leading side of the first loop. In addition, let  $\tilde{\tau}_i$  denote the estimated time-gap of the  $i$ th vehicle at that position. We then assume every vehicle travels with the same velocity after it passes over a detector under the existing steady-state. Then,

$$v_{i-1} \cdot \tilde{t}_{d(i,i-1)} = x_{i-1}|_{(x_i=0)} - L = v_i \cdot \tilde{\tau}_i$$

This gives the estimated time-gap for the  $i$ th vehicle

$$\tilde{\tau}_i = \frac{v_{i-1}}{v_i} \tilde{t}_{d(i,i-1)}. \quad (6)$$

Hence, since a dual-loop detector provides the number of passing vehicles  $n(kT)$  during the  $k$ th measurement interval, by arithmetic average and (6), the average estimated time-gap for the  $k$ th measurement period is

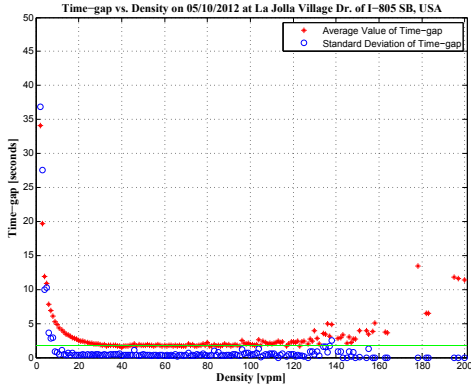
$$\tilde{\tau}(kT) = \frac{\sum_{i=1}^{n(kT)} \tilde{\tau}_i}{n(kT)} \quad (7)$$

for all  $i$  such that  $\tilde{t}_i \in ((k-1)T, kT]$ . Now, by using the measurement data with  $n(kT)$  and  $v(kT)$ , we can easily get the density for the  $k$ th measurement period and thus show a flow-density fundamental diagram as well as the estimated time-gap versus the density.

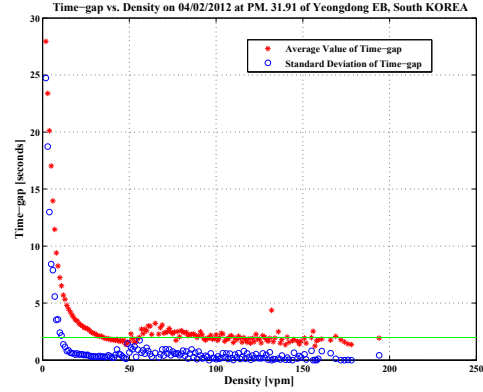
#### IV. ANALYSIS RESULTS AND DISCUSSIONS

We have measurements data collected from a single-loop detector in the USA and from a dual-loop detector in the South Korea. One data set was measured on May 10, 2012, at La Jolla Village Dr. of I-805 SB, USA and the other set was collected on Apr. 02, 2012, at PM. 31.91 of Yeongdong Express EB, South Korea. The raw traffic data presented here is processed to remove potentially erroneous observations; some examples of such measurements include the low number of passing vehicles during one measurement interval representing a low density value in spite of an extremely low velocity values. This case occurs when a vehicle temporarily stops over a detector or a loop detector system is malfunctioning.

The relationship between the traffic density and time-gap estimated by the analysis methods above is shown in Fig. 2. Fig. 2-(a) uses real traffic data measured in the USA, while Fig. 2-(b) is the result of real traffic data from the South Korea. Each point in the scatter plots gives the estimated density and time-gap using the analysis method above which are averaged over 30 seconds. Commonalities are apparent between the two different geographical data sets. It is seen that the time-gap varies widely when the density is low. However, it is nearly constant when the density is above a critical density. We show the average and standard deviation values of the estimated time-gap versus density in Fig. 3. These time-gap and density pairs are placed into bins according to their densities with

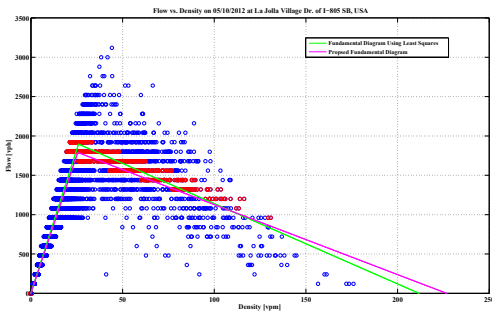


(a) Measurement Data in the USA

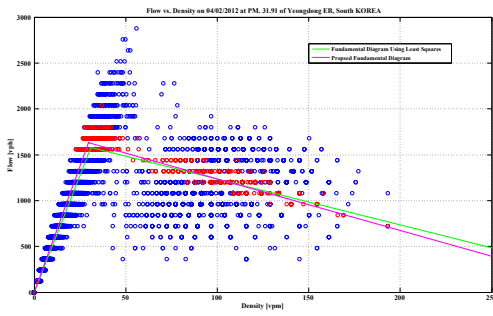


(b) Measurement Data in the South Korea

Fig. 3. Average and Standard Deviation of Estimated Time-Gap vs. Density



(a) Measurement Data in the USA



(b) Measurement Data in the South Korea

Fig. 4. Measurement Data of Flow vs. Density and A Time-Gap based Triangular Fundamental Diagram

a granularity of 1 vehicle per mile per lane per bin. The red starred points and blue-circled points represent the average and standard deviation of the estimated time-gap for each density bin, respectively. Both the average and standard deviation of the time-gap are large in a free-flow regime where the density is less than a critical density, i.e. 26 vpm for the USA data and 31 vpm for the South Korea. We will discuss later the method for how to get the critical density from real traffic data. As the density increases, both the average and standard deviation of the time-gap decrease until the density value approaches the critical density. Compared to a free-flow regime, the data shows that the average time-gap is fairly predictable as a constant value and that its standard deviation is relatively small in a congestion regime despite the existence of minor variation. The data shows several interesting points where average time-gap values are large even under high densities being larger than 170 vpm. The number of these interesting points is few, so this infrequent observation might be caused by erroneous instances when a vehicle stops just in front of a loop detector while maintaining a distance to its leading vehicle that does not adhere to the normal traffic patterns of following closely even under heavy congestion. A loop detector system error is also a possibility. The green line in the above plots shows the arithmetic mean value of the estimated time-gap for all measurement data points included in a congestion regime. The

mean time-gap in a congestion region are about  $\tau = 1.78$  seconds in the USA and  $\tau = 2.00$  seconds in the South Korea. It is obvious that drivers travel maintaining the positive time-gap to their leading vehicles to avoid collision, which is related to the brake reaction time. The brake reaction time is defined as the amount of time required to apply the brakes upon recognition of danger. Many research studies about a driver's reaction time provided that it ranges from 0.66 to 1.50 seconds [6] - [8]. Overall, we can observe that the mean value of the driver's reaction time is less than or almost equal to the average time-gap defined in a congestion regime. This observation provides the insight that drivers subconsciously drive with their own time-gap being larger than their reaction time so as to avoid a rear-end collision.

Fig. 4 shows the flow and estimated density pairs, and the corresponding fundamental diagram using the same data sets from the USA and South Korea. Each circle in the scatter plots provides the flow and estimated density averaged over 30 seconds. For reference, we draw a green line as a triangular flow-density fundamental diagram by regression analysis with least squares from all data points. We can derive this line by adjusting three important parameters: the critical density, and the positive and negative slopes of a triangular fundamental diagram so as to best fit these flow and estimated density sets. In the USA, the best fitted critical density, positive slope,

and negative slope are 26 vpm, 72.95 mph, and  $-10.21$  mph, respectively. The South Korea exhibits larger critical density and gentler slopes than the USA does. Since the speed limit of the measurement area in the South Korea is about 50 mph which is less than that in the USA with 65 mph, it is expected that the South Korea has a more gradual positive slope in a free-flow regime than the USA does.

A magenta line in Fig. 4 represents our proposed time-gap based traffic model drawn by (2). We need to use three parameters to draw it, which are the maximum velocity, typical safety length of a vehicle, and the mean value of a time-gap defined in a congestion regime. We added 5 mph to the pre-defined speed limit, because people normally travel with slightly higher speed than the regulated speed where there is no traffic. Therefore, we set a maximum velocity,  $v_f$ , as 70 and 55 mph for the USA and South Korea, respectively. We can get the best fit critical density by least squares from all data points. We then calculate the mean value of the time-gap  $\tau$  of the data points for which the density values are larger than the best fitted critical density, that is to say in a congestion regime. The calculated mean values of the time-gap are approximately  $\tau = 1.78$  seconds for the USA and  $\tau = 2.00$  seconds for the South Korea. Finally, we can draw our proposed time-gap based flow-density triangular fundamental diagram as shown in Fig. 4.

By comparing the green and magenta lines in Fig. 4, we can observe that our proposed time-gap based traffic model almost corresponds to the best fit triangular fundamental diagram by least squares. Therefore, not only does our proposed time-gap based traffic model explain well a triangular fundamental diagram provided by Newell [4] with both a maximum velocity and a mean value of the time-gap defined in a congestion regime by (2), but also it is a good and simple representative model for vehicular traffic flow.

There is additional indirect evidence of the validity of the time-gap based traffic model for the fundamental diagram. When the traffic density changes, a shock wave is launched and travels against the direction of the traffic flow at an almost constant speed, which is called the propagation velocity. This propagation velocity, which is independent of the density in a congestion regime, has been studied for many years [9] - [11]. These researches provided that the propagation velocity is approximately 9.32 to 12.43 mph. Thus, this previous research shows that the propagation velocity is consistent with our proposed time-gap based traffic model, and in fact by (2), this time-gap based traffic model predicts that the constant propagation velocity,  $v_p$ , is given by the simple formula  $v_p = L/(\bar{c}\tau)$ . Using  $L = 22$  feet and  $\tau = 1.78$  seconds estimated in the USA, the predicted propagation velocity is  $v_p = 10.21$  mph. We can observe that this value is in range of the propagation velocity provided by the above papers and thus the propagation velocity derived by our proposed time-gap based traffic model is consistent with earlier studies. Indeed, we believe that the characteristic properties in congested traffic can be largely explained using the model we provide in this paper.

## V. CONCLUSION

We provided a time-gap based traffic model which explains well a triangular flow-density fundamental diagram proposed by Newell with three principal parameters: maximum velocity, a typical safety length of vehicles, and a mean value of the time-gap of traffic data under congested conditions. Also, we proposed two different analysis methods to estimate the time-gap from real traffic data measured by a single-loop and a dual-loop detector system. We found that the average and standard deviation of the time-gap varies widely where the traffic density is low, while its average is nearly constant and its standard deviation is small in a congestion regime. This observation agrees with our time-gap based traffic model presented here, which shows that a mean value of the time-gap is a major factor to characterize vehicular traffic flow, especially where the traffic is congested. A further meaningful observation is that a mean value of the time-gap defined in a congestion area is larger than or almost equal to the average driver reaction time. This provides the insight that drivers unconsciously travel with their own time-gap being larger than their reaction time so as to avoid a real-end collision towards their leading vehicles.

In conclusion, we have shown the validity of our proposed time-gap based traffic model. The time-gap based traffic model represented with the three parameters above almost corresponds with the best fitting triangular fundamental diagram by least squares to the measured traffic. Also, the propagation velocity derived by a mean value of the time-gap is consistent with the propagation velocity studied for many years. Therefore, our proposed time-gap based traffic model is a good and simple representative model for vehicular traffic flow.

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