

Skolem Sequence Based Self-adaptive Broadcast Protocol in Cognitive Radio Networks

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Abstract

The base station (BS) in a multi-channel cognitive radio (CR) network has to broadcast to secondary (or unlicensed) receivers/users on more than one broadcast channels via channel hopping (CH), because a single broadcast channel can be reclaimed by the primary (or licensed) user, leading to *broadcast failures*. Meanwhile, a secondary receiver needs to synchronize its clock with the BS's clock to avoid broadcast failures caused by the possible clock drift between the CH sequences of the secondary receiver and the BS. In this paper, we propose a CH-based broadcast protocol called *SASS*, which enables a BS to successfully broadcast to secondary receivers over multiple broadcast channels via channel hopping. Specifically, the CH sequences are constructed on basis of a mathematical construct—the *Self-Adaptive Skolem sequence*. Moreover, each secondary receiver under SASS is able to adaptively synchronize its clock with that of the BS without any information exchanges, regardless of any amount of clock drift.

1 Introduction

In an *infrastructure-based (or cellular)* cognitive radio (CR) network, the base station (BS) has to broadcast to secondary receivers/users on more than one broadcast channels via a channel hopping (CH) process.

The *broadcast failure problem* can occur for a CH-based broadcast protocol. First, the primary users (PU, or licensed users) may reclaim the spectrum band where broadcast channels reside, and the secondary receivers have to vacate this channel according to the requirement for protection of PU. Second, there may exist a clock drift between the BS and the secondary receiver, which can lead to broadcast failures due to the non-overlapping of their CH sequences. Note

that any fast synchronization scheme requires necessary information exchange that incurs additional control overhead.

In order to address these problems, we expect the CH-based broadcast protocol to have the following properties.

1. *Multiple broadcast channels.* Ideally, broadcast deliveries can occur over all available broadcast channels, thus becoming invulnerable to broadcast failures caused by the PU on a single broadcast channel.
2. *Self-adaptive synchronization without information exchange.* The secondary receivers are supposed to synchronize with the BS autonomously via a self-adaptive synchronization process without any information exchange, in order to minimize both the broadcast latency and control overhead.

In this paper, we present a channel-hopping based multi-channel broadcast protocol, called *SASS*, where the CH sequences are constructed on basis of a mathematical construct—the *Self-Adaptive Skolem Sequence*. The SASS protocol has the following two noteworthy features.

- The BS can successfully broadcast to secondary receivers over multiple (or up to the maximum number of) available channels within a bounded latency, for increasing (or maximizing) the broadcast channel diversity.
- The broadcast latency can be minimized to be near-zero given any amount of clock drift, as each secondary receiver can adaptively synchronize its clock with the BS without any information exchange by leveraging the historical information of successful broadcast deliveries.

Our analytical and simulation results show that SASS incurs small broadcast latency and guarantees a high successful delivery rate under various network conditions.

The rest of this paper is organized as follows. We introduce the related work on channel hopping broadcast protocols and Skolem sequence in Section 2. We provide the system model and formulate the problem in Section 3. In Section 4, we describe the SASS broadcast protocol. We evaluate the performance of our proposed broadcast schemes in Section 5. We conclude the paper in Section 6.

2 Related Work

The purpose of most CH protocols in the literature is to achieve channel rendezvous between a sender and a receiver using jump-stay techniques [5], the array-based quorum systems [1], or modular arithmetics [8]. It has not been largely investigated as yet to devise multi-channel broadcast protocols in the context of an infrastructure-based CR network.

In [7], a fully-distributed broadcast protocol is proposed to provide very high successful delivery rate while achieving the shortest broadcast delay. However,

most of existing research on channel-hopping based broadcast protocols in CR networks [2, 7] did not leverage the availability of information gathered from past successful broadcast deliveries to synchronize clocks independent of information exchange and minimize broadcast latencies. Note that existing clock synchronization techniques [5] rely on necessary information exchange (e.g., exchange of clock time and/or relevant parameters) after a successful broadcast delivery is established.

3 System Model

Multi-channel broadcast via channel hopping. In a CR network, a broadcast channel may become unavailable at any time due to the primary user’s activities. Therefore, the secondary BS (or broadcast sender) has to broadcast the content over multiple channels to ensure successful delivery to the secondary receiver, which we call a *multi-channel broadcast* process.

Every secondary user (SU) is equipped with a single radio interface, and we use the channel hopping sequence to define the order in which a radio (or a SU) visits a set of broadcast channels. Suppose there are N broadcast channels, labeled as $0, 1, 2, \dots, N - 1$. We consider a time-slotted communication system, in which a timeslot means the minimum time unit within which a network node accesses a channel. Thus, the CH sequence u_i of radio i (or SU i) is represented as a sequence of channel indices:

$$u_i = \{u_i^0, u_i^1, u_i^2, \dots, u_i^t, \dots\},$$

where u_i can be an infinite sequence and $u_i^t \in [0, N - 1]$ represents the channel index of u_i in the t -th timeslot.

Clock drift. Suppose the local clock of radio/SU i is $\Delta_i \in \mathbb{Z}$ timeslots behind the global clock; Δ_i can be a negative integer, and this means that the local clock of radio/SU i is in fact $-\Delta_i$ timeslots ahead of the global clock. From the perspective of the global clock, the CH sequence of radio i is a sequence that starts from timeslot Δ_i :

$$G(u_i) = \{G(u_i)^{\Delta_i}, G(u_i)^{\Delta_i+1}, \dots, G(u_i)^{\Delta_i+t}, \dots\},$$

where $G(u_i)^{\Delta_i+t} = u_i^t, t \in \mathbb{N} \cup \{0\}$. It is likely that the clock drift is not a multiple of a timeslot. This is also termed that these two nodes’ timeslots are *not aligned*. However, we can extend the slot duration to be twice of the original slot duration. We will easily arrive at the conclusion that as long as two nodes have successful broadcast delivery on a certain channel, the delivery duration is at least an original slot duration.

Successful broadcast delivery. Given two CH sequences u_i and u_j , if there exists $t_g \in \mathbb{Z}$ such that $t_g \geq \max\{\Delta_i, \Delta_j\}$ and $G(u_i)^{t_g} = G(u_j)^{t_g} = h$, where $h \in [0, N - 1]$, we say that a *successful broadcast delivery* occurs between radios i and j in the t_g -th (global) timeslot on broadcast channel h . The t_g -th timeslot is called a *delivery slot* and channel h is called a *delivery channel* between SUs i and j .

4 An ESS-based Broadcast Protocol

4.1 Preliminaries

A *Skolem sequence (SS)* [6], $\{\zeta_i\}_{0 \leq i \leq 2n-1}$ of order n , is a permutation of the sequence of $2n$ integers $\{1, 1, 2, 2, 3, 3, \dots, n, n\}$, and it satisfies the *Skolem property*:

- If $\zeta_i = \zeta_j, 0 \leq i < j \leq 2n - 1$, then $j - i = \zeta_i + 1$.

For example, the sequence $l = \{3, 1, 2, 1, 3, 2\}$ is a Skolem sequence of order $n = 3$. Given $i = 0$ and $j = 4$, we have $\zeta_0 = \zeta_4 = 3$, and $j - i = 3 + 1$; given other combinations of i and j , the sequence l also satisfies the Skolem property. The following lemma holds [6].

Lemma 1. *A Skolem sequence of order n exists if and only if n is congruent to 0 or 3 modulo 4.*

In [6], Skolem proposed a very efficient and general construction method for Skolem sequences in his proof of Theorem 2.

Extended Skolem sequence. We define an *extended Skolem sequence (ESS)*, $\{\zeta'_i\}_{0 \leq i \leq 2(n+1)-1}$ of order n , as a permutation of the sequence of $2(n+1)$ integers: $\{0, 0, 1, 1, 2, 2, \dots, n, n\}$.

The sequence satisfies the Skolem property, i.e., if $\zeta'_i = \zeta'_j, 0 \leq i < j \leq 2(n+1) - 1$, then $j - i = \zeta'_i + 1$. For example, the sequence $\zeta' = \{0, 0, 3, 1, 2, 1, 3, 2\}$ is an extended Skolem sequence (ESS) of order $n = 3$. It follows immediately from Lemma 1 that an extended Skolem sequence of order n exists if n is congruent to 0 or 3 modulo 4.

Given a Skolem sequence $\{\zeta_i\}_{0 \leq i \leq 2n-1}$ of order n , we can construct an extended Skolem sequence $\{\zeta'_i\}_{0 \leq i \leq 2(n+1)-1}$ of the same order by inserting two integers $\zeta'_0 = \zeta'_1 = 0$ at the beginning of the original SS—i.e., by letting $\zeta'_i = 0$ when $i = 0, 1$; and $\zeta'_i = \zeta_{i-2}$ when $1 < i \leq 2(n+1) - 1$. In an extended Skolem sequence ζ' of order n , any integer $k \in [0, n]$ appears exactly twice in the ESS.

4.2 ESS-based CH Sequences

In this subsection, we use ESS to generate channel hopping sequences for the base station and secondary receivers.

When the channel number N is congruent to 0 or 1 modulo 4, then $N - 1$ is congruent to 0 or 3, and by Lemma 1, there exists an ESS $\{\zeta'_i\}_{0 \leq i \leq 2N-1}$ of order $N - 1$. For example, when $N = 4$, the ESS-based CH sequence is

$$\{0, 0, 3, 1, 2, 1, 3, 2\}.$$

When $N \not\equiv 0, 1 \pmod{4}$, we can easily use the *padding technique*¹ to transform it into the case with the channel number N' congruent to 0 or 1 modulo

¹We may as well use the *downsizing technique* as an alternative. The downsizing technique means that we discard some channels so that the new channel number $N' \leq N$ is congruent to 0 or 1 modulo 4. We only need to discard at most 2 channels.

4. According to the padding technique, we increase the channel number N to the minimum integer N' so that $N' \geq N$ and N' is congruent to 0 or 1 modulo 4. Obviously, $N \leq N' \leq N + 2$. We regard the newly added $(N' - N)$ channels as aliases of the original N channels². With the padding scheme, we can focus on the case where the channel number N' is congruent to 0 or 1 modulo 4.

- Let u and v be two CH sequences of the same length, say, T . We denote the set of delivery channels by

$$\mathcal{C}(u, v) \triangleq \{h \in [0, N - 1] : \exists t \in [0, T - 1], u^t = v^t = h\}.$$

- Let $\mathcal{D}(u, v)$ denote the set of delivery slots between u and v and

$$\mathcal{D}(u, v) \triangleq \{t \in [0, T - 1] : u^t = v^t\}.$$

- We define the notion of circular shift to represent the clock drift, i.e.,

$$\text{shift}(u, \alpha) = \{w^0, w^1, w^2, \dots, w^{T-1}\}$$

is a sequence of length T and $w^t \triangleq u^{(t+\alpha) \bmod T}$.

- We let

$$\prod_{k=1}^K \mu_k = \mu_1 \parallel \mu_2 \parallel \dots \parallel \mu_K$$

denote the concatenation of sequences μ_k 's.

Consider two ESS-based CH sequences with different amount of clock drift, $\text{shift}(u, \alpha)$ and $\text{shift}(u, \beta)$, where u is an ESS. Their relative clock drift is $\alpha - \beta$.

- Theorem 2 shows the relationship between the set of broadcast delivery channels $\mathcal{C}(\text{shift}(u, \alpha), \text{shift}(u, \beta))$ and the relative clock drift $(\alpha - \beta)$.
- Theorem 3 shows the relationship between the set of broadcast delivery slots $\mathcal{D}(\text{shift}(u, \alpha), \text{shift}(u, \beta))$ and the relative clock drift $(\alpha - \beta)$.

According to these two theorems, a secondary receiver is able to figure out the exact clock drift between the BS and the receiver itself by simply looking at the historical information—i.e., the set of broadcast delivery channels—such that the receiver can synchronize to the BS without exchanging any control messages.

Theorem 2. *Suppose that $u = \{\zeta_i\}_{0 \leq i \leq 2N'-1}$ is an ESS of order $N' - 1$.*

1. $\mathcal{C}(\text{shift}(u, \alpha), \text{shift}(u, \beta)) = \{0, 1, 2, \dots, N' - 1\}$ if and only if $\alpha - \beta \equiv 0 \pmod{2N'}$.
2. $\mathcal{C}(\text{shift}(u, \alpha), \text{shift}(u, \beta)) = \{|g| - 1\}$, where $|g| \leq N'$, if and only if $\alpha - \beta \equiv g \not\equiv 0 \pmod{2N'}$.

²For example, if the channel number is 3, we add a new channel, say, Channel 4, so that the new channel number amounts to 4. Channel 4 serves as an alias of Channel 1.

Proof. The first clause is obvious. Now it suffices to show that

$$\alpha - \beta \equiv g \not\equiv 0 \pmod{2N'}$$

will imply

$$\mathcal{C}(\text{shift}(u, \alpha), \text{shift}(u, \beta)) = \{|g| - 1\},$$

where $|g| \leq N'$. If $g > 0$, then $\exists 0 \leq i_0 < j_0 < 2N'$ s.t. $u^{i_0} = u^{j_0} = g - 1$. Thus $j_0 - i_0 = g$, $\text{shift}(u, g)^{i_0} = u^{i_0+g} = u^{i_0} = g - 1$, i.e., $g - 1 \in \mathcal{C}(\text{shift}(u, g), u)$. Suppose $x \in \mathcal{C}(\text{shift}(u, g), u)$, then $\exists 0 \leq i_0 < 2N'$ s.t. $\text{shift}(u, g)^{i_0} = u^{i_0+g} = u^{i_0} = x$. By the definition of ESS, $x + 1 = (i_0 + g) - i_0 = g$, $x = g - 1$. Therefore

$$\mathcal{C}(\text{shift}(u, \alpha), \text{shift}(u, \beta)) = \mathcal{C}(\text{shift}(u, g), u) = \{|g| - 1\}.$$

If $g < 0$, then $\beta - \alpha \equiv -g \not\equiv 0 \pmod{2N'}$. Thus $\mathcal{C}(\text{shift}(u, \beta), \text{shift}(u, \alpha)) = \{|-g| - 1\} = \{|g| - 1\}$. \square

Theorem 3. Suppose that $u = \{\zeta'_i\}_{0 \leq i \leq 2N'-1}$ is an ESS of order $N' - 1$.

1. $\mathcal{D}(\text{shift}(u, \alpha), \text{shift}(u, \beta)) = \{0, 1, 2, 3, \dots, 2N' - 1\}$ if and only if $\alpha - \beta \equiv 0 \pmod{2N'}$.

2. If $\alpha - \beta \equiv g \pmod{2N'}$, where $|g| \leq N'$, then

(a) $|\mathcal{D}(\text{shift}(u, \alpha), \text{shift}(u, \beta))| = 1$ if and only if $0 < |g| < N'$.

(b) $|\mathcal{D}(\text{shift}(u, \alpha), \text{shift}(u, \beta))| = 2$ if and only if $|g| = N'$.

Proof. The first clause is obvious. Now suppose $\alpha - \beta \equiv g \pmod{2N'}$ ($|g| \leq N'$). It suffices to show that $0 < |g| < N' \Rightarrow |\mathcal{D}(\text{shift}(u, \alpha), \text{shift}(u, \beta))| = 1$ and $|g| = N' \Rightarrow |\mathcal{D}(\text{shift}(u, \alpha), \text{shift}(u, \beta))| = 2$. By Theorem 2, $|\mathcal{D}(\text{shift}(u, \alpha), \text{shift}(u, \beta))| \geq 1$ and $\mathcal{C}(\text{shift}(u, \alpha), \text{shift}(u, \beta)) = \{|g| - 1\}$, i.e., the delivery channel $h = |g| - 1$ and $\exists 0 \leq i_0 < 2N'$ s.t. $\text{shift}(u, \alpha)^{i_0} = \text{shift}(u, \beta)^{i_0} = h$. There are only two h 's in $\text{shift}(u, \alpha)$ and $\text{shift}(u, \beta)$, respectively. Without loss of generality, the remaining h in $\text{shift}(u, \alpha)$ is $\text{shift}(u, \alpha)^{[i_0 - (h+1)] \bmod 2N'}$ and that in $\text{shift}(u, \beta)$ is $\text{shift}(u, \beta)^{[i_0 + (h+1)] \bmod 2N'}$. If $i_0 + (h+1) \equiv i_0 - (h+1) \pmod{2N'}$, we have $2(h+1) \equiv 2|g| \equiv 0 \pmod{2N'}$. Since $g \neq 0$, we conclude that $|g| = N'$. Therefore if $|g| < N'$, $i_0 + (h+1) \not\equiv i_0 - (h+1) \pmod{2N'}$ and $|\mathcal{D}(\text{shift}(u, \alpha), \text{shift}(u, \beta))| = 1$; if $|g| = N'$, then $i_0 + (h+1) \equiv i_0 - (h+1) \pmod{2N'}$ and $|\mathcal{D}(\text{shift}(u, \alpha), \text{shift}(u, \beta))| = 2$. \square

4.3 SASS: A Two-phase Broadcast Protocol

The broadcast CH sequence for the BS (sender) is $\prod_{n=0}^{\infty} \mu$, where μ is a pre-specified ESS of order $N' - 1$ and thus with a length of $2N'$.

Every secondary receiver has an initial CH sequence. Then, the CH sequence is dynamically updated by the receiver based on the historical information of whether successful broadcast delivery occurs in the past timeslots. Specifically, it calibrates the clock in a self-adaptive manner by leveraging the mathematical

Table 1: The cases of broadcast channels if the sender uses u and the receiver uses $\text{shift}(u, a)$ with a clock drift of a slots, where a varies from 0 to $2N' - 1 = 7$, given $N' = 4$. The “All” here means that if the sender and receiver are synchronized, they can have broadcast delivery on all channels. The “0” here means that if the clock drift between the sender and receiver is 1, they can have broadcast delivery on Channel 0, and similarly for “1”, “2” and “3” hereinafter.

Rx	Bdcast ch(s)	Rx	Bdcast ch(s)
$\text{shift}(u, 0)$	All	$\text{shift}(u, 1)$	0
$\text{shift}(u, 2)$	1	$\text{shift}(u, 3)$	2
$\text{shift}(u, 4)$	3	$\text{shift}(u, 5)$	2
$\text{shift}(u, 6)$	1	$\text{shift}(u, 7)$	0

properties of ESS, so as to synchronize to the BS (sender) for minimizing the broadcast latency.

Phase 1: ESS-based CH sequence generation. A secondary receiver initially uses the CH sequence

$$\prod_{n=0}^{\infty} \text{shift}(\mu, n)$$

until the first successful broadcast delivery occurs. Meanwhile, it counts/maintains the number of successful broadcast deliveries since its local clock's timeslot $\lfloor \frac{t}{2N'} \rfloor \cdot 2N'$, where t is the current timeslot according to its local clock.

The secondary receiver will not wait long for the first successful broadcast delivery to occur. If the PU signal is not present in all channels, the first successful broadcast delivery will occur within $4N'(N' - 1)$ timeslots after both the secondary sender and receiver start channel hopping, as shown by Theorem 4.

Theorem 4. *Under SASS, the first successful broadcast delivery occurs within $4N'(N' - 1)$ timeslots after both the secondary sender and receiver start channel hopping.*

Proof. Suppose the sender uses the ESS u of length $2N'$, and the receiver uses $u, \text{shift}(u, 1), \text{shift}(u, 2), \text{shift}(u, 3), \dots, \text{shift}(u, 2N' - 1)$, sequentially. Since the receiver exhausts all possible cyclic rotations of u , the first successful broadcast delivery will occur within $2N' \cdot 2N' = 4N'^2$ slots.

Then, we can further improve this upper bound to $4N'(N' - 1)$. Now we establish our argument for the case $N' = 4$ as an example. For a general N' , we can have a similar argument.

Suppose $N' = 4$. From the perspective of the sender's clock, the receiver uses $\text{shift}(u, a), \text{shift}(u, a + 1), \text{shift}(u, a + 2), \dots, \text{shift}(u, a + 7)$, where a is the clock drift between the sender and the receiver. Now we consider the consecutive $4N'^2 = 64$ slots.

Table 1 shows the cases of broadcast channels for all possible clock drifts. If the sender and the receiver are synchronized, the receiver starts from the second row (i.e., it starts from $\text{shift}(u, 0) = u$), and then uses $\text{shift}(u, 1), \text{shift}(u, 2)$,

shift($u, 3$), \dots , shift($u, 7$), and then back to shift($u, 0$) = u and so on. If they have a clock drift of a slots, the receiver uses shift(u, a), shift($u, a+1$), shift($u, a+2$), \dots , shift($u, a+7$), then back to shift(u, a) and so on. If $a = 0$, broadcast delivery occurs on all channels, which is the best case. The worst case is when the receiver starts from shift($u, 1$). In this case, when it arrives at shift($u, 6$), successful broadcast delivery must have occurred because from shift($u, 1$) to shift($u, 6$), they have tried all possible channels 0, 1, 2, and 3. Thus first successful delivery will occur within $2 \times 4 \times (2 \times 4 - 2)$ slots. For a general N' , the upper bound is $2N'(2N' - 2) = 4N'(N' - 1)$. \square

Phase 2: Self-adaptive clock calibration. Upon the first successful broadcast delivery, the secondary receiver enters the clock calibration phase: it knows the exact clock drift between the sender (BS) and itself, and then adaptively synchronize to the sender.

According to the local clock of the secondary receiver, we group every $2N'$ timeslots into a (time) frame, e.g. timeslots 0 to $2N' - 1$ of the receiver form the first frame. Suppose SU i is the sender, SU j is the receiver, and that the first broadcast delivery occurs in channel $\alpha \in [0, N' - 1]$ in timeslot τ_1 that lies in the ϕ -th frame according to SU j 's clock.

By the definition of ESS, there exists another timeslot $\tau_2 \neq \tau_1$ that lies in the ϕ -th frame and contains channel α . Let $u_j[\phi]$ denote the segment of SU j 's CH sequence in the ϕ -th frame according to SU j 's local clock, and we have $u_j[\phi] = \text{shift}(\mu, \phi - 1)$.

In the calibration phase, the receiver (i.e. SU j) continues using its original CH sequence $\prod_{n=0}^{\infty} \text{shift}(\mu, n)$ until the end of its ϕ -th frame; and in the meanwhile, SU j continues counting the number of successful broadcast deliveries since its local clock's timeslot $\lfloor \frac{t}{2N'} \rfloor \cdot 2N' = 2N'(\phi - 1)$, where t is the current timeslot according to its local clock.

When the ϕ -th frame ends, the receiver checks if successful broadcast delivery occurs in timeslot τ_2 and obtains the total number of successful broadcast deliveries in the ϕ -frame, which is denoted by $SB[\phi]$. The secondary receiver chooses different CH sequences in the following cases.

Case 1: Successful broadcast delivery occurs in timeslot τ_2 and $\alpha \neq N' - 1$. In this case, the receiver knows that the segment of SU i 's CH sequence in the ϕ -th frame according to SU j 's clock, denoted by $u_i[\phi]$, is exactly $u_j[\phi] = \text{shift}(\mu, \phi - 1)$. From the next frame (the $(\phi + 1)$ -th frame) on, the receiver uses

$$\prod_{n=0}^{\infty} \text{shift}(\mu, \phi - 1)$$

as its new CH sequence.

Fig. 1(a) shows an example of Case 1 with $N' = 4$. The PU occupies channels 0 and 3. The first successful delivery occurs on channel 1 (thus $\alpha = 1 \neq N' - 1$) and in the 4th slot of the ϕ -th frame. Slot τ_2 is the 6th slot of frame ϕ and successful delivery occurs in slot τ_2 . Thus the receiver (SU j) knows that it has

been synchronized with the sender (SU i). So it continues using $\text{shift}(\mu, \phi - 1)$ as its CH sequence.

Case 2: $\alpha = N' - 1$. This implies that successful broadcast delivery occurs in timeslot τ_2 ; and that $u_i[\phi]$ is either $u_j[\phi]$ or $\text{shift}(u_j[\phi], N')$.

- In the next frame (the $(\phi+1)$ -th frame), the receiver uses $\text{shift}(u_j[\phi], N')$ as its CH sequence and counts the number of successful broadcast deliveries in the $(\phi + 1)$ -th frame, denoted by $SB[\phi + 1]$.
- From the $(\phi + 2)$ -th frame on, the receiver knows the exact difference between its clock and the sender's, and it can choose the CH sequence synchronized with the sender. If $SB[\phi] \geq SB[\phi + 1]$, it chooses

$$\prod_{n=0}^{\infty} u_j[\phi] = \prod_{n=0}^{\infty} \text{shift}(\mu, \phi - 1);$$

otherwise, it uses

$$\prod_{n=0}^{\infty} u_j[\phi + 1] = \prod_{n=0}^{\infty} \text{shift}(\mu, \phi - 1 + N').$$

Fig. 1(b) shows an example of Case 2 with $N' = 4$. The PU occupies channels 1 and 2. The first successful delivery occurs on channel 3 (thus $\alpha = 3 = N' - 1$) and in the 3th slot of the ϕ -th frame. Slot τ_2 is the 7th slot of frame ϕ and successful delivery occurs in slot τ_2 . Thus the receiver (SU j) knows that the segment of the sender's CH sequence in frame ϕ is either $u_j[\phi] = \{2, 1, 3, 2, 0, 0, 3, 1\}$ or $\text{shift}(u_j[\phi], 4) = \{0, 0, 3, 1, 2, 1, 3, 2\}$. So it tries $\text{shift}(u_j[\phi], 4) = \{0, 0, 3, 1, 2, 1, 3, 2\}$ in frame $(\phi + 1)$ and finds that $SB[\phi] = 2$, $SB[\phi + 1] = 4$ and $SB[\phi] < SB[\phi + 1]$. Therefore it knows that the sender uses $\text{shift}(u_j[\phi], 4) = \{0, 0, 3, 1, 2, 1, 3, 2\}$. So it synchronizes its clock with the sender and uses $\{0, 0, 3, 1, 2, 1, 3, 2\}$ as its CH sequence from frame $(\phi + 2)$ on.

Case 3: Successful broadcast delivery does not occur in timeslot τ_2 . This implies that $\alpha \neq N' - 1$ and that either $u_i[\phi] = \text{shift}(u_j[\phi], \alpha + 1)$ or $u_i[\phi] = \text{shift}(u_j[\phi], -(\alpha + 1))$.

- The receiver uses $\text{shift}(u_j[\phi], \alpha + 1)$ and $\text{shift}(u_j[\phi], -(\alpha + 1))$ as its CH sequences in the $(\phi + 1)$ -th and $(\phi + 2)$ -th frames; it counts the numbers of successful broadcast deliveries in these two frames, denoted by $SB[\phi + 1]$ and $SB[\phi + 2]$, respectively.
- From the $(\phi + 3)$ -th frame on, if

$$SB[\phi + 1] \geq SB[\phi + 2],$$

the receiver chooses

$$\prod_{n=0}^{\infty} u_j[\phi + 1] = \prod_{n=0}^{\infty} \text{shift}(\mu, \phi - 1 + (\alpha - 1));$$

otherwise, it uses

$$\prod_{n=0}^{\infty} u_j[\phi + 2] = \prod_{n=0}^{\infty} \text{shift}(\mu, \phi - 1 - (\alpha - 1)).$$

Fig. 1(c) shows an example of Case 3 with $N' = 4$. The PU occupies channels 2 and 3. The first successful delivery occurs on channel 1 (thus $\alpha = 1 \neq N' - 1$) and in the 6th slot of the ϕ -th frame. Slot τ_2 is the last slot of frame ϕ and successful delivery does not occur in slot τ_2 . Thus the receiver (SU j) knows that the segment of the sender's CH sequence in frame ϕ is either $\text{shift}(u_j[\phi], 2) = \{0, 0, 3, 1, 2, 1, 3, 2\}$ or $\text{shift}(u_j[\phi], -2) = \{2, 1, 3, 2, 0, 0, 3, 1\}$. So it tries $\text{shift}(u_j[\phi], 2) = \{0, 0, 3, 1, 2, 1, 3, 2\}$ in frame $(\phi+1)$ and tries $\text{shift}(u_j[\phi], -2) = \{2, 1, 3, 2, 0, 0, 3, 1\}$ in frame $(\phi + 1)$. It finds that $SB[\phi + 1] = 4$, $SB[\phi + 2] = 0$ and $SB[\phi + 1] > SB[\phi + 2]$. Thus it knows that the sender uses $\text{shift}(u_j[\phi], 2) = \{0, 0, 3, 1, 2, 1, 3, 2\}$. So it synchronizes its clock with the sender and uses $\text{shift}(u_j[\phi], 2) = \{0, 0, 3, 1, 2, 1, 3, 2\}$ as its CH sequence from frame $(\phi + 3)$ on.

After completing the clock calibration, every secondary receiver is synchronized to the BS. Since the BS and secondary receivers use the same ESS-based CH sequence, the broadcast latency will be minimized to zero upon the successful clock calibration in Phase 2.

5 Performance Evaluation

5.1 Simulation Setup

In this section, we compare the performance of the proposed broadcast protocol SASS and existing CH based broadcast protocols via simulation results: the random channel hopping broadcast protocol (RCH), the canonical Skolem sequence based broadcast protocol without self-adaptivity (CSS) and the asynchronous channel hopping protocol (ACH) [1].

In each simulated broadcast pair (secondary sender/BS and secondary receiver), each node can access N broadcast channels (i.e., the number of broadcast channels available to the broadcast pair is N). And both of the two nodes generate their CH sequences using the agreed broadcast protocol (i.e., either the proposed SASS protocol or other existing broadcast protocols) and perform channel hopping in accordance with the sequences.

Primary user traffic. We simulated a number of X primary transmitters operating on X channels independently, and these channels were randomly chosen in each simulation run. In most existing work, it is assumed that a primary user transmitter follows a “busy/idle” transmission pattern on a licensed channel [3, 4], and we assume the same traffic pattern here — i.e., the busy period has a fixed length of b timeslots, and the idle period follows an exponential distribution with a mean of l timeslots. A channel is considered *unavailable* when PU signals are present in it. The intensity of primary user traffic can be characterized as $PU = \frac{X}{N} \cdot \frac{b}{l+b} \times 100\%$.

	ϕ -th frame				$(\phi+1)$ -th frame													
Sender:	...	0	0	3	1	2	1	3	2	0	0	3	1	2	1	3	2	...
Receiver:	...	0	0	3	1	2	1	3	2	0	0	3	1	2	1	3	2	...

(a) An example of Case 1 with $N' = 4$. The PU occupies channels 0 and 3. Therefore the grids with 0 and 3 inside are gray.

	ϕ -th frame				$(\phi+1)$ -th frame				$(\phi+2)$ -th frame																	
Sender:	...	0	0	3	1	2	1	3	2	0	0	3	1	2	1	3	2	0	0	3	1	2	1	3	2	...
Receiver:	...	2	1	3	2	0	0	3	1	0	0	3	1	2	1	3	2	0	0	3	1	2	1	3	2	...

(b) An example of Case 2 with $N' = 4$. The PU occupies channels 1 and 2. Therefore the grids with 1 and 2 inside are gray.

	ϕ -th frame				$(\phi+1)$ -th frame				$(\phi+2)$ -th frame				$(\phi+3)$ -th frame																					
Sender:	...	0	0	3	1	2	1	3	2	0	0	3	1	2	1	3	2	0	0	3	1	2	1	3	2	0	0	3	1	2	1	3	2	...
Receiver:	...	3	2	0	0	3	1	2	1	0	0	3	1	2	1	3	2	2	1	3	2	0	0	3	1	0	0	3	1	2	1	3	2	...

(c) An example of Case 3 with $N' = 4$. The PU occupies channels 2 and 3. Therefore the grids with 2 and 3 inside are gray.

Figure 1: Examples illustrating the three cases. Each grid denotes a timeslot. The number in a grid denotes the index of a channel that the node hops to in this timeslot. A gray grid represents that the PU occupies this channel in the timeslot, while a non-gray one represents no PU presence on this channel in the timeslot. A red grid indicates that successful broadcast delivery occurs in the timeslot and over this channel.

Random clock drift. In a CR network, the nodes may lose clock synchronization or even link connectivity at any time when they experience the broadcast failure problem due to primary user activities. Hence, the clock of the nodes are not necessarily synchronized. In each simulation run, each secondary node determines its clock time independently of other nodes.

5.2 Proportion of Successful Broadcast Slots

We define the proportion of successful delivery slots in the first t timeslots, $\rho(t)$, as the percentage of timeslots in the first t timeslots in which successful broadcast delivery occurs.

Figs. 2(a), 2(b), 2(c) and 2(d) illustrate the results given the PU traffic $PU = 0\%$, 25% , 50% and 75% , respectively. In the proposed SASS protocol, the proportion of successful broadcast slots progressively approximates to the theoretical maximum $1 - PU$ —the proportion values are 100% , 75% , 50% , and 25% respectively. However, the performance of other protocols is approximately stable at $\frac{1-PU}{N}$.

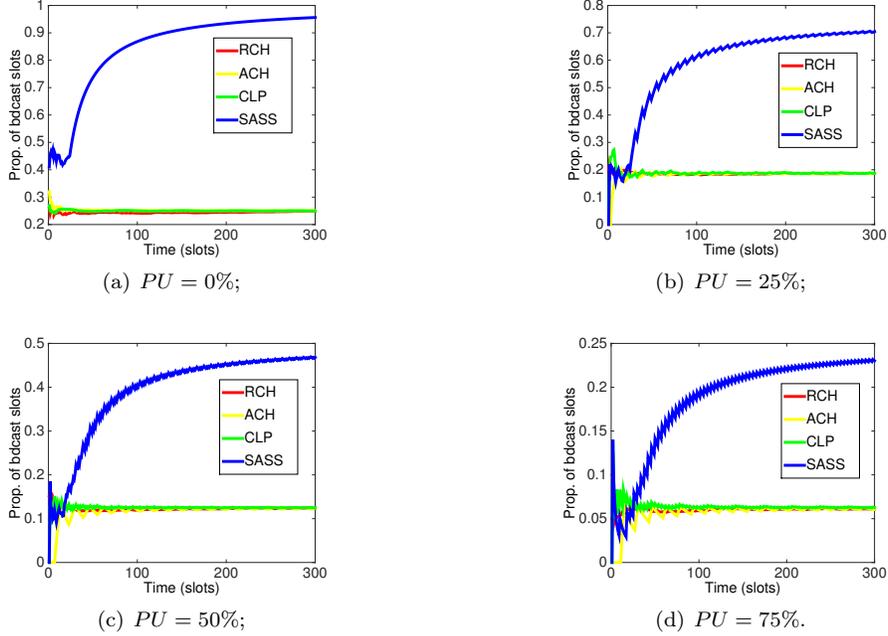


Figure 2: Proportion of successful broadcast slots vs. time.

5.3 Broadcast Latency

In this set of simulations, we simulate 1000 pairs of nodes, and investigate the broadcast latencies under the proposed SASS and other existing CH based broadcast protocols in the following five scenarios: (1) the latency until the first successful broadcast delivery occurs; and the average delivery latency in the first (2) 50, (3) 100, (4) 150, and (5) 200 timeslots. The results are showed in Fig. 3.

We observe that the latency under the SASS protocol progressively outperforms the other three protocols as the number of successful broadcast deliveries increases. Its delivery latency drops down to 7 and then 5 and finally decreases below 5, while the other three protocols' latency remains above 15. This can be attributed to the fact that the SASS protocol can synchronize all of the receivers with the broadcast sender, thus greatly reducing the delivery latency on average.

6 Conclusion

In this paper, we propose a channel hopping based multi-channel broadcast protocol, called SASS, where the CH sequences are constructed on basis of the self-adaptive extended Skolem sequence. SASS allows the network base station (broadcast sender) to broadcast over multiple channels such that the broadcasts

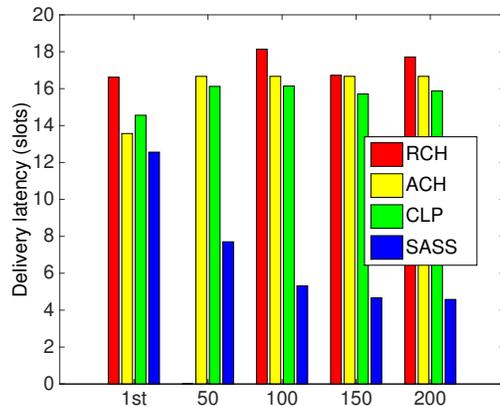


Figure 3: The average delivery latency in the following five scenarios: the latency until the first successful broadcast delivery occurs (see the leftmost group of bar labeled “1st”); and the average delivery latency in the first 50, 100, 150, and 200 timeslots (please see the groups of bars labeled “50, 100, 150, and 200”, respectively).

can be successfully delivered to secondary receivers. Meanwhile, each secondary receiver can infer the difference between its clock and the clock of the sender, and then adaptively synchronize with the sender to further reduce the broadcast latency. SASS is robust to the broadcast failure caused by primary user activities and the clock drift between two CH sequences.

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