Throughput Optimal Multi-user Scheduling via Hierarchical Modulation

Mehmet Karaca and Ozgur Ercetin

Abstract—We investigate the network stability problem when two users are scheduled simultaneously. The key idea is to simultaneously transmit to more than one users experiencing different channel conditions by employing hierarchical modulation. For two-user scheduling problem, we develop a throughput-optimal algorithm which can stabilize the network whenever this is possible. In addition, we analytically prove that the proposed algorithm achieves larger achievable rate region compared to the conventional Max-Weight algorithm which employs uniform modulation and transmits a single user. We demonstrate the efficacy of the algorithm on a realistic simulation environment using the parameters of High Data Rate protocol in a Code Division Multiple Access system. Simulation results show that with the proposed algorithm, the network can carry higher user traffic with lower delays.

Index Terms—Max-Weight scheduling, hierarchical modulation, power allocation, queue stability, stochastic control.

I. INTRODUCTION

THE scheduling is an essential problem for any shared resource. The problem becomes more challenging in a dynamic setting such as wireless networks where the channel capacity is time varying due to multiple superimposed random effects such as mobility and multipath fading. In a queueing system, the most important property of a scheduling algorithm is to keep the network stable (e.g., the queue sizes remain bounded over time). The seminal work by Tassiulas and Ephremides has shown that *Max-Weight* algorithm scheduling the user with the highest queue backlog and transmission rate product at every time slot can stabilize the network whenever this is possible [1].

It is well-known that Max-Weight algorithm is *throughput-optimal*, i.e., it stabilizes the network for all arrival rate vectors that are *strictly* within the achievable rate region. The performance of Max-Weight algorithm has been investigated in depth for the single user scheduling case. However, determining a throughput-optimal algorithm when more than one users are scheduled simultaneously has not received much attention. In this paper, we propose a modulation-assisted throughput optimal scheduling algorithm scheme where we employ hierarchical modulation (HM).

The authors in [2] are the first to show the advantage of HM in broadcast systems. In [3], the authors proposed a multiuser scheduling algorithm and showed that HM offers lower queueing delay at the transmission buffer. However, in neither of these works, the authors considered the stability of the network. Network utility maximization problem with HM was investigated in [4]. Meanwhile, [5] proposed scheduling and flow control algorithms but did not take into account the effect of modulation on the performance of the algorithm.

Our contribution can summarized as follows: i) we propose a throughput optimal algorithm, namely Max-Weight with Hierarchical Modulation (MWHM) when two users are scheduled simultaneously;i) we give the conditions under HM that should be employed by considering both analytical and implementation issues. ii) we prove that the proposed algorithm achieves larger rate region compared to the conventional Max-Weight algorithm; iv) we develop a lower complexity version of MWHM algorithm; v) we demonstrate via realistic simulations that our algorithm not only keeps the network stable with higher arrival rate but also reduces the average delay.

II. SYSTEM MODEL

We consider a cellular system with a single base station (BS) transmitting to N users. Let \mathcal{N} denote the set of users in the cell. Time is slotted, $t \in \{0, 1, 2, \ldots\}$. Let T_s denote the length of the time slot in seconds. Let $h_n(t)$ represent channel gain of user n at time $t, n \in \{1, 2, \ldots, N\}$. The gain of the channel is constant over the duration of a time slot but varies between slots.

HM is one of the techniques for multiplexing and modulating multiple data streams into one single symbol stream, where those multiple symbols are superimposed together before transmission. In this paper, for the sake of ease of exposition we assume that only two layers of hierarchical modulation is used to serve two users simultaneously. Specifically, we assume that QPSK/16-QAM HM is implemented. Let BS transmit to two users, i.e., user n and user m, at time slot t by employing two layers of HM. Assume that user j has a better channel than user i, i.e., $h_n \leq h_m$. Then, user n is assigned to QPSK constellation which we refer to as base layer. User m is assigned to 16-QAM constellation which we refer to as incremental layer. More information about hierarchical modulation can be found in [3] and references there in. Since two modulated signals are mixed before being transmitted, they interfere with each other at the receiver side. However, in [6], the authors propose a decoding technique to cancel the interference seen at the incremental layer. Specifically, when mixed signal reaches to the receivers, the data at the base layer is first decoded and removed. Hence, the data at the incremental layer does not suffer from the transmission at the base layer. In [6], the achievable rates for user n and user mare given respectively as follows:

$$\mu_n^b(t) = T_s \times BW \times \log\left(1 + \frac{h_n(t)P_{n,b}(t)}{h_n(t)P_{m,i}(t) + \sigma}\right), \quad (1)$$

$$\mu_m^i(t) = T_s \times BW \times \log\left(1 + \frac{h_m(t)P_{m,i}(t)}{\sigma}\right),\tag{2}$$

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where $P_{n,b}(t)$ and $P_{m,i}(t)$ are the transmission powers for user n and m at the base and incremental layers, respectively. σ is the noise power and BW is the bandwidth of the channel. We assume that the BS transmits at fixed power and the total power consumption is equal to P, i.e., $P_{n,b}(t) + P_{m,i}(t) =$ $P \quad \forall t. \ \mu_n^k(t)$ is upper-bounded such that $\mu_n^k(t) \leq \mu_{max} \quad \forall t,$ $k \in \{b, i\}$. Note that when UM is applied at the physical layer, full power is assigned to a single user and the amount of data that can be transmitted in that case is given by,

$$\mu_n^{um}(t) = T_s \times BW \times \log\left(1 + \frac{h_n(t)P}{\sigma}\right).$$
 (3)

where $n \in \{1, 2, ..., N\}$. Let $a_n(t)$ be the amount of data (bits or packets) arriving into the queue of user n at time slot t and $a_n(t) \leq a_{max} \forall t$, and assume that $a_n(t)$ is a time and user independent stationary process. We denote the arrival rate vector as $\lambda = (\lambda_1, \lambda_2, ..., \lambda_N)$, where $\lambda_n = \mathbb{E}[a_n(t)]$. Let $\mathbf{q}(t) = (q_1(t), q_2(t), ..., q_N(t))$ denote the vector of queue sizes, where $q_n(t)$ is the queue length of user n at time slot t. The dynamics of the queue of user n is given as,

$$q_n(t+1) = [q_n(t) + a_n(t) - \mu_n^k(t)]^+.$$
 (4)

where $(x)^+ = \max(x, 0)$ and $k \in \{b, i\}$. Let Λ denote the *achievable rate region* (or rate region) defined as the closure of the set of all arrival rate vectors for which there exists an appropriate scheduling policy stabilizing the network.

III. THROUGHPUT OPTIMAL SCHEDULING

In this section, we give a throughput optimal scheduling algorithm when HM is employed. However, we start with the conventional Max-Weight algorithm employing UM to schedule a single user at every time slot.

Max-Weight with UM (MWUM): At time t, given $h_n(t)$ and $q_n(t)$ for all $n \in \mathcal{N}$, schedule user n^* which has the maximum queue length and service rate product, i.e., [1]:

$$w_n^{um}(t) = q_n(t)\mu_n^{um}(t) \tag{5}$$

$$n^* = \operatorname*{argmax}_{n \in \mathcal{N}} w_n^{um}(t). \tag{6}$$

We define $W_u(t) \triangleq w_{n^*}^{um}(t)$. Let Λ_u denote the rate region achieved by MWUM.

Max-Weight with HM (MWHM): At time t, given $h_n(t)$ and $q_n(t)$ for all $n \in \mathcal{N}$ schedule two users (n^*, m^*) such that $h_{n^*}(t) \leq h_{m^*}(t)$ to maximize the sum of queue length and service rate products, i.e.,:

$$w_{n,m}^{hm}(t) = q_n(t)\mu_n^b(t) + q_m(t)\mu_m^i(t)$$
(7)

$$(n^*, m^*) = \underset{\substack{(n,m) \in \mathcal{N}, n \neq m \\ h_n(t) \le h_m(t)}}{\operatorname{argmax}} w_{n,m}^{hm}(t).$$
(8)

We define $W_h(t) \triangleq w_{n^*,m^*}^{hm}(t)$. Let Λ_h denote the rate region achieved by MWHM.

Since BS has limited power budget, power allocation must be performed to determine $\mu_n^b(t)$ and $\mu_m^i(t)$.

Power Allocation with HM: Recall that Max-Weight type scheduling algorithms aim to maximize the weight $w_{n,m}^{hm}(t)$ (or $w_n^{um}(t)$) at each time slot. It is easy to determine the maximum weight achieved by UM, $w_{n^*}^{um}(t)$. However, the

maximum weight under HM depends on power allocations $P_{n,b}^*(t)$ and $P_{m,i}^*(t)$. Without loss of generality, for a given pair of users, e.g., user n and user m such that $h_n(t) \leq h_m(t)$, the optimal power allocation maximizing the weight $w_{n,m}^{hm}(t)$ is obtained by solving the following optimization problem:

$$\max_{P_{n,b}(t), P_{m,i}(t)} q_n(t) \mu_n^b(t) + q_m(t) \mu_m^i(t)$$
(9)

s.t.
$$P_{n,b}(t) + P_{m,i}(t) = P$$
 (10)

Note that $P_{n,b}^{*}(t)$ and $P_{m,i}^{*}(t)$ both have a non-zero value when (9)-(10) is a convex problem. Now, we give a Lemma which states the necessary conditions for this to hold. For notational convenience, we drop the time index. Let us define,

$$A \triangleq \frac{h_n^2}{(h_n P_{m,i} + \sigma)^2}$$
 and $B \triangleq \frac{h_m^2}{(h_m P_{m,i} + \sigma)^2}$

where $0 \leq P_{m,i}(t) \leq P$.

Lemma 1: The problem (9)-(10) is a convex optimization problem when the following inequality is satisfied

$$q_n A \le q_m B \tag{11}$$

Proof: We show that the objective function in (9) is concave under the given condition. The objective function can be rewritten by noting that $P_{n,b} = P - P_{m,i}$. Since the parameters T_s and BW do not effect the concavity, we have the following objective function,

$$f = q_n \log \left(1 + \frac{h_n (P - P_{m,i})}{h_n P_{m,i} + \sigma} \right) + q_m \log \left(1 + \frac{h_m P_{m,i}}{\sigma} \right).$$

Taking the second derivative of f with respect to $P_{m,i}$ yields,

$$\frac{d^2 f}{dP_{m,i}^2} = q_n A - q_m B.$$
 (12)

For concavity, $\frac{d^2 f}{dP_{m,i}^2}$ must be less than or equal zero, i.e., $\frac{d^2 f}{dP_{m,i}^2} \leq 0$. Thus, $q_n A \leq q_m B$.

As long as the condition in Lemma 2 is satisfied, the optimal power allocation can be found by taking the first derivative of f and setting it to zero. The first derivative of f with respect to $P_{m,i}$ is given by,

$$\frac{df}{dP_{m,i}} = -q_n\sqrt{A} + q_m\sqrt{B} = 0.$$
 (13)

Thus, we have,

$$P_{m,i}^{*} = \frac{\sigma(q_m h_m - q_n h_n)}{h_m h_m (q_m - q_n)},$$
(14)

$$P_{n,b}^* = P - P_{m,i}^*.$$
 (15)

Lemma 2: If $\lambda \in \Lambda_h$ (i.e., λ is feasible), then MWHM algorithm stabilizes the network and it is throughput optimal.

Proof: The proof is given in Appendix A.

IV. MAX-WEIGHT ALGORITHM WITH DYNAMIC MODULATION (MWDM)

Note that MWHM can only be used when there is an inner point solution to the problem (9)-(10), i.e., $0 < P_{m,i}^* < P$. If the solution is on the boundary, i.e., $P_{m,i}^* = 0$ or $P_{m,i}^* = P$, then full power is assigned to a single user and HM is no longer employed, i.e., transmission to a single user is optimal.

Now, we propose Max-Weight algorithm with dynamic modulation (MWDM) that dynamically decides which modulation (HM or UM) must be employed at every time slot. Let $W_d(t)$ and Λ_d be the maximum weight at time t and the rate region achieved by MWDM, respectively. MWDM is implemented as follows:

- Step 1: The scheduler applies MWUM and finds the maximum weight $W_u(t)$ by using (5) and (6).
- Step 2: The scheduler applies MWHM as follows: For every pair of users (n, m), find $w_{n,m}^{hm}(t)$:
 - if $h_n \leq h_m$, then user n is embedded at the base layer whereas user m is the incremental layer or vice versa.
 - Check whether the condition in Lemma 2 is satisfied.
 - If not, $w_{n,m}^{hm}(t) = \max\{w_n^{um}, w_m^{um}\}$ and UM is employed.
 - Otherwise, determine the optimal power allocation $P_{m,i}^*$ and $P_{n,b}^*$ according to (14) and (15), respectively.
 - Then determine $w_{n,m}^{hm}(t)$ according to (7).
- Step 3: After finding w^{hm}_{n,m} for all pairs, determine W_h(t).
 Step 4: If W_h(t) > W_u(t), then W_d(t) = W_h(t) and HM is employed. Otherwise, $W_d(t) = W_u(t)$ and UM is employed.

Let us define the expected weights achieved by MWDM and MWUM as $\mathbb{E}[W_d(t)]$ and $\mathbb{E}[W_u(t)]$, respectively.

Theorem 3: The achievable rate region of MWUM algorithm is a subset of the achievable rate region of MWDM, i.e., $\Lambda_u \subseteq \Lambda_d$.

Proof: We use the theorem given in [7] to prove the lemma. According to the theorem, if $\mathbb{E}[W_d(t) > \mathbb{E}[W_u(t)]]$, then MWDM can at least achieve the rate region of MWUM. The average weight achieved by MWDM is always greater than or equal to the weight achieved by MWUM since MWDM can either apply HM or UM according the maximum weight. Thus, the following inequality holds at every time slot,

$$W_d(t) \ge W_u(t) \tag{16}$$

Taking the expectation of both sides of (16) yields that $\mathbb{E}[W_d(t)] \ge \mathbb{E}[W_u(t)].$

Hence, MWDM can be used to increase the total network throughput.

A. A low complexity algorithm

Note that the implementation of MWDM algorithm requires the calculation of the optimal power allocation and the weight of every pair of users. This requires a computational complexity of $O(N^2)$. Now, we propose a low complexity algorithm (L-MWDM) that has a computational complexity of O(N).

L-MWDM algorithm is implemented as follows:

- Step 1: Determine user n^* according to (5) and (6) which is the optimal user selected under UM. Determine $W_{\mu}(t)$ by using (5).
- Step 2: For every user $m \neq n^*$, $m \in \mathcal{N}$ do:
 - if $h_{n^*} \leq h_m$, then user n^* is embedded at the base layer whereas user m is embedded at the incremental layer or vice versa.
 - Check the condition in Lemma 2 is satisfied.
 - If it is not, $w_{n^*,m}^{hm}(t) = \max\{w_{n^*}^{um}, w_m^{um}\}$ and UM is employed.
 - Otherwise, determine the optimal power allocation $P_{n^*,b}^*$ and $P_{m,i}^*$ according to (14) and (15), respectively. Then, determine $w_{n^*,m}^{hm}(t)$ according to (7).
- Step 3: After finding $w^{hm}_{n^*,m}(t)$ for every user $m \neq n^*$ determine $W_h(t)$.
- Step 4: If $W_h(t) > W_u(t)$, then $W_d(t) = W_h(t)$ and HM is employed. Otherwise, $W_d(t) = W_u(t)$ and UM is employed.

Note that the difference between MWDM and L-MWDM is that MWDM checks the weights achieved by every pair of users. Hence, its complexity increases quadratically with the number of users. On the other hand, L-MWDM calculates the weights assuming that user n^* is always scheduled. Thus, its complexity is linear with N. However, the maximum weight obtained with MWDM is always greater or equal to that of L-MWDM.

V. SIMULATION RESULTS

In our simulations, we model a single cell downlink transmission utilizing high data rate (HDR) [8]. The base station serves 20 users and keeps a separate queue for each user. Time is slotted with length $T_s = 1.67$ ms as defined in HDR specifications. We set BW = 1.25 MHz, P = 10 Watts and $\sigma = 10^{-6}$ Watts. Packets arrive at each slot according to Bernoulli distribution. The size of a packet is set to 128 bytes which corresponds to the size of an HDR packet. The wireless channel between the BS and each user is modeled as a correlated Rayleigh fading according to Jakes' model with different Doppler frequencies varying randomly between 5 Hz and 15 Hz.

Figure 1 depicts the maximum arrival rate that can be supported by MWDM, L-MWDM and MWUM. Clearly, as the overall arrival rate exceeds 30 packets/slot queue sizes suddenly increase with MWUM and the network becomes unstable. However, MWDM and L-MWDM improves over MWUM by supporting the overall arrival rate of up to 32 packets/slot. Therefore, MWDM can achieve a larger rate region than MWUM as verified analytically in Theorem 3. Figure 1 also shows the sum of the queue lengths vs. mean of overall arrival rate (packets/slot), i.e., the increase is about 1.2 Mbps. As the average arrival rate increases the average queue backlogs increase as well in all algorithms. Following Littles' Law, larger queue backlogs yield longer network delays. However, due to the possibility of serving more than one queue at a time, MWDM and L-MWDM outperform MWUM in terms of the average delay. This result indicates that MWDM and L-MWDM are better techniques for delay



Fig. 1. Average total queue sizes vs. overall mean arrival rate.

sensitive applications. In addition, the delay performance of MWDM and L-MWDM are very close. Recall that L-MWDM always schedules user n^* which is the optimal user in MWUM algorithm. Similar to L-MWDM, MWDM schedules user n^* most of the time. Figure 2 shows the sum of the queue lengths vs. transmit power, P in Watts when the overall mean arrival rate is 28 packets/slot. Clearly, as P increases, the average queue size with both MWDM and MWUM decreases as well. However, the steady state queue length is achieved with MWUM at P = 4 whereas MWDM reaches to the steady state at P = 3. As a conclusion, HM based scheduling requires less power than the uniform constellation based scheduling to stabilize the network.

VI. CONCLUSIONS

In this paper, we investigate the advantages of transmissions of more than one data streams simultaneously in a network stability problem. We propose to use hierarchical modulation with Max-Weight algorithm when two user are scheduled simultaneously. First, we give the optimal power allocation among users. Then, we show that the proposed algorithm can support higher user traffic compared to the conventional Max-Weight Algorithm. In addition, we demonstrate that with the proposed algorithm the average delay reduces dramatically. HM is a good technique for scheduling problem especially when the number of users in the system is large and BS transmits high transmission powers.

APPENDIX A Proof of Lemma 2

We can write the following inequality by using the fact $([a]^+)^2 \leq (a)^2, \quad \forall a$:

$$q_n^2(t+1) \le q_n^2(t) + (\mu_{max})^2 + (a_{max})^2 - 2q_n(t)[\mu_n(t) - a_n(t)]$$
(17)

Define the following Lyapunov function and conditional Lyapunov drift:

$$L(\mathbf{q}(t)) \triangleq \sum_{n=1}^{N} q_n^2(t), \tag{18}$$

$$\Delta(t) \triangleq \mathbb{E}\left[L(\mathbf{q}(t+1)) - L(\mathbf{q}(t))|\mathbf{q}(t)\right].$$
 (19)



Fig. 2. Average total queue sizes vs. transmit power, P

By using (17) and (18), one can show that the Lyapunov drift of the system satisfies the following inequality at every time slot,

$$\Delta(t) \le B - \sum_{n} \mathbb{E} \left\{ q_n(t) \mu_n(t) | \mathbf{q}(t) \right\} - \sum_{n} q_n(t) \lambda_n \quad (20)$$

where $B = \frac{N}{2}((\mu_{max})^2 + (a_{max})^2)$. Note that the second term in the right hand side of (20) can be rewritten as follows when two users are scheduled:

$$\sum_{n} \mathbb{E} \left\{ q_n(t)\mu_n(t) | \mathbf{q}(t) \right\} = \sum_{\substack{(i,j)\\h_i \le h_j}} \mathbb{E} \left\{ q_i(t)\mu_i(t) + q_j(t)\mu_j(t) | \mathbf{q}(t) \right\}$$

Now, it is easy to see that MWHM minimizes the right hand side of (20) at every time slot. Therefore, according to Lyapunov-Foster criteria, $\mathbf{q}(t)$ process is positive recurrent Markov chain and MWHM can stabilize the network whenever this is possible [9].

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