Approximation to the Sum of Two Correlated Gamma-Gamma Variates and its Applications in Free-Space Optical Communications

Guowei Yang[1], Mohammad-Ali Khalighi[1], Salah Bourennane[1], Zabih Ghassemlooy[2]
[1] Institut Fresnel, École Centrale Marseille, Marseille, France
[2] Optical Communications Research Group, Northumbria University, Newcastle Upon Tyne, UK
Email: {Guowei.Yang, Ali.Khalighi, Salah.Bourennane}@fresnel.fr , Z.Ghassemlooy@northumbria.ac.uk

***Abstract*—The Gamma-Gamma ($\Gamma\Gamma$) distribution is popularly accepted for modeling the received intensity fluctuations in the near-ground free-space optical (FSO) communication. We consider in this letter the case of dual space diversity FSO systems when the fading coefficients corresponding to the two sub-channels are correlated. In order to evaluate the receiver performance analytically, we propose to approximate the sum of correlated $\Gamma\Gamma$ random variables by an α-μ distribution. We show that there is a good match between the performance obtained by simulated data based on the $\Gamma\Gamma$ channel model and that obtained from analytical calculation based on the approximate α-μ model.**

***Index Terms*—Gamma-Gamma distribution, free-space optics (FSO), spatial diversity, correlated fading, α-μ distribution.**

I. Introduction

Free-space optical (FSO) communication has recently become very popular due to enabling cost efficient, highly secure, and very high rate data transmission. One of the problems that an FSO system is facing to is the atmospheric turbulence which induces signal fading at the receiver under clear weather conditions [1]. To mitigate the adverse effect of optical scintillation, one solution is aperture averaging which is efficient when the diameter of the receiver lens is larger than the coherence radius ρ_0 of the optical wave [2], [3]. However, at relatively large link distances, on the order of several kilometers, ρ_0 becomes too large and the required aperture size for efficient fading reduction necessitates too expensive optical devices and components. In such situations, spatial diversity arises to be a better solution. Usually, it consists of using several apertures at the receiver or several laser beams at the transmitter [1], [4].

When no spatial diversity technique is employed, for the statistical modeling of turbulence-induced channel fading at the receiver, the Gamma-Gamma ($\Gamma\Gamma$) distribution has widely been accepted thanks to its excellent agreement with the experimental data over a wide range of turbulence conditions [1]. For spatial diversity systems, this distribution can easily be modified if no spatial correlation is assumed between the corresponding subchannel fading coefficients. For instance, the sum of independent $\Gamma\Gamma$ random variables (RVs) is modeled by a $\Gamma\Gamma$ distribution in [5] and by an α-μ distribution in [6]. However, in practical systems, the correlation between the subchannels' fading coefficients is inevitable. As we have shown in a recent work [7], the correlation can be significant at large link distances and for relatively large aperture sizes. This fading correlation can substantially impair the system performance as it is reported in several previous works considering simplified statistical models for the turbulence [8], [9], [10]. However, obtaining a closed-form general formulation for the system performance for the case of correlated $\Gamma\Gamma$ channels is quite difficult. In [11], a multivariate $\Gamma\Gamma$ distribution is considered under the assumption of exponentially correlated large-scale and independent small-scale fluctuations. A similar study in [12] considers exponentially correlated fading envelopes using a multivariate K distribution. Exponential correlation is not usually valid in most FSO receiver configurations, however. We propose in this letter a general model for the correlated $\Gamma\Gamma$ channels by considering correlated large- and small-scale fluctuations. Our idea is to approximate the distribution of the sum of two correlated $\Gamma\Gamma$ RVs by an α-μ (also called *generalized Gamma*) distribution based on the moment-matching method. The α-μ distribution is a flexible distribution that can be reduced to several distributions such as Gamma, Nakagami-m, exponential, and Rayleigh [13]. We illustrate the accuracy of our proposed approximation by studying the corresponding probability density functions (PDFs) and also by using Monte-Carlo simulations for the cases of no-diversity and double receive diversity FSO systems, denoted by (1×1) and (1×2), respectively. The proposed method allows performance prediction of the FSO system at very low bit-error-rates (BERs) without resorting to highly time-consuming Monte Carlo simulations.

II. Case of no-diversity FSO system

A. *Single-variate $\Gamma\Gamma$ and α-μ distributions*

Let I denote the normalized received intensity. By the $\Gamma\Gamma$ model, we consider I as the product of two independent Gamma RVs, X and Y, which represent the irradiance fluctuations arising from large- and small-scale turbulence, respectively. The PDF of I is [1]:

$$f_I(i) = \frac{2(ab)^{(a+b)/2}}{\Gamma(a)\Gamma(b)}\, i^{\frac{(a+b)}{2}-1} K_{a-b}(2\sqrt{a\,b\,i}),\ i > 0. \quad (1)$$

Here, a and b denote the effective numbers of large- and small-scale turbulence eddies, respectively, $\Gamma(.)$ is the Gamma function and $K_\upsilon(.)$ the modified Bessel function of the second kind and order υ. The n^{th} moment of I is [5]:

$$\mathrm{E}\{I^n\} = \frac{\Gamma(a+n)\,\Gamma(b+n)}{\Gamma(a)\,\Gamma(b)}\,(ab)^{-n}\,, \tag{2}$$

where $\mathrm{E}\{.\}$ denotes the expected value. As stated previously, we approximate the $\Gamma\Gamma$ channel fading I by an α-μ RV that we denote by R. The PDF of R is given by:

$$f_R(r) = \frac{\alpha\,\mu^\mu\,r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\,\Gamma(\mu)}\exp\left(-\mu\,\frac{r^\alpha}{\hat{r}^\alpha}\right),\; r>0 \tag{3}$$

Here, $\alpha>0$, $\mu = \left(\mathrm{E}\left\{R^\alpha\right\}\right)^2/\mathrm{Var}\left\{R^\alpha\right\} > 0$ is the inverse of the normalized variance of R^α, $\hat{r} = \sqrt[\alpha]{\mathrm{E}\left\{R^\alpha\right\}}$, and $\mathrm{Var}(.)$ denotes variance. The m^{th} moment of R is given by:

$$\mathrm{E}\{R^m\} = \hat{r}^m\,\frac{\Gamma(\mu+m/\alpha)}{\mu^{m/\alpha}\,\Gamma(\mu)}. \tag{4}$$

B. Approximation of $\Gamma\Gamma$ by α-μ distribution

We use the moment-matching method for approximating a $\Gamma\Gamma$ by an α-μ distribution. After trying several different moments, we found that the best approximation is obtained when setting equal the first, second, and third moments of the two distributions to calculate the parameters α, μ and $\hat{r}$:

$$\begin{cases} \mathrm{E}\{R\} = \hat{r}\,\frac{\Gamma(\mu+1/\alpha)}{\mu^{1/\alpha}\,\Gamma(\mu)} = \frac{\Gamma(a+1)\Gamma(b+1)}{\Gamma(a)\,\Gamma(b)(ab)} \\ \mathrm{E}\{R^2\} = \hat{r}^2\,\frac{\Gamma(\mu+2/\alpha)}{\mu^{2/\alpha}\,\Gamma(\mu)} = \frac{\Gamma(a+2)\Gamma(b+2)}{\Gamma(a)\,\Gamma(b)(ab)^2} \\ \mathrm{E}\{R^3\} = \hat{r}^3\,\frac{\Gamma(\mu+3/\alpha)}{\mu^{3/\alpha}\,\Gamma(\mu)} = \frac{\Gamma(a+3)\Gamma(b+3)}{\Gamma(a)\,\Gamma(b)(ab)^3} \end{cases} \tag{5}$$

However, it is difficult to obtain a closed-form solution from (5) which contains nonlinear functions. Numerical methods can be used to calculate α, μ and $\hat{r}$ from these equations. For instance, we have used the *fsolve* function of MATLAB® to calculate these parameters. Note that the same approach is considered in [6] for the case of multiple independent $\Gamma\Gamma$ RVs, where the first, second, and fourth moments are considered. We have noticed that there is a slightly better match between the $\Gamma\Gamma$ and α-μ PDFs by using our approach.

III. Case of dual space diversity FSO system

Let us now consider the case of dual space diversity FSO systems. To the best of our knowledge, the case of correlated bivariate $\Gamma\Gamma$ distribution has not been treated in the literature so far. We propose here an approximation to the sum of two correlated $\Gamma\Gamma$ RVs based on the moment-matching method. In practical FSO systems, subchannels are naturally identical in terms of fading statistics. Therefore, it is quite reasonable to consider the same distribution parameters a and b for the corresponding $\Gamma\Gamma$ fading coefficients.

Let us denote by I_1 and I_2 the fading coefficients of the two sub-channels which are correlated and $\Gamma\Gamma$ distributed. We approximate their sum $I = I_1 + I_2$ by an α-μ RV R by setting equal the corresponding first three moments. Note that I corresponds to the received intensity for a dual-beam single aperture system, and to the received signal intensity after equal gain combining (EGC) [8] in a single-beam double-aperture system. The moment-matching method implies:

$$\begin{cases} \mathrm{E}\{R\} = \mathrm{E}\{I\} = \mathrm{E}\{I_1\} + \mathrm{E}\{I_2\}, \\ \mathrm{E}\{R^2\} = \mathrm{E}\{I^2\} = \mathrm{E}\{I_1^2\} + 2\,\mathrm{E}\{I_1 I_2\} + \mathrm{E}\{I_2^2\}, \\ \mathrm{E}\{R^3\} = \mathrm{E}\{I^3\} = \mathrm{E}\{I_1^3\} + 3\,\mathrm{E}\{I_1^2 I_2\} + 3\,\mathrm{E}\{I_1 I_2^2\} + \mathrm{E}\{I_2^3\}. \end{cases} \tag{6}$$

Considering normalized channel coefficients, we set $\mathrm{E}\{I_1\} = \mathrm{E}\{I_2\} = 1$. It can be shown that the $(m,n)^{\text{th}}$ joint moment of I_1 and I_2 is given by:

$$\begin{aligned} &\mathrm{E}\{I_1^m I_2^n\} = \mathrm{E}\{X_1^m X_2^n\}\,\mathrm{E}\{Y_1^m Y_2^n\} = \\ &\quad \frac{\Gamma\left(a+m\right)\Gamma\left(a+n\right){}_2F_1\left(-m,-n;a;\rho_X\right)}{a^{m+n}\left(\Gamma(a)\right)^2} \\ &\qquad \times \frac{\Gamma\left(b+m\right)\Gamma\left(b+n\right){}_2F_1\left(-m,-n;b;\rho_Y\right)}{b^{m+n}\left(\Gamma(b)\right)^2}. \end{aligned} \tag{7}$$

Here, ρ_X and ρ_Y stand for the correlation coefficients corresponding to the large- and small-scale turbulence of the subchannels, and ${}_2F_1$ is the Gauss hypergeometric function [14]. Note that, theoretically, separating ρ_X and ρ_Y is justified because according to the extended Rytov theory, the spatial cutoffs for large- and small-scale intensity fluctuations are effectively separated under the strong turbulence regime [1]. Notice that due to the independence of the large- and small-scale fading coefficients, ρ can be related to ρ_X and ρ_Y as follows [7]:

$$\rho = \frac{a\rho_Y + b\rho_X + \rho_X\,\rho_Y}{a+b+1}. \tag{8}$$

The correlation coefficient ρ can be obtained for a given system configuration through wave-optics simulations [7]. However, there is no simple way to determine ρ_X and ρ_Y even through wave-optics simulations. The calculation of these parameters will later be discussed in Subsection IV-B2.

Finally, note that this method can also be applied to the case of maximal ratio combining (MRC) at the receiver. For this case, moment matching should be done considering $\mathrm{E}\{R^2\}$, $\mathrm{E}\{R^4\}$, and $\mathrm{E}\{R^6\}$, for instance [15]. MRC is the optimal combining scheme regardless of fading statistics [16], [17], yet, EGC is simpler to implement and its performance is very close to that of MRC [10].

IV. Numerical results

We provide here some numerical results to study the accuracy of the proposed approximation method. We consider two case studies of a (1×1) system of aperture diameter D, and a (1×2) system where each aperture has the diameter D. Also, for the latter case, we consider the EGC scheme at the receiver. The numerical results we present are for the two link distances Z of 2 Km and 5 Km, where D is set to 50 mm and 100 mm, respectively. In the following, we will refer to them as cases (1) and (2), respectively. For these two cases, we consider a set of correlation coefficients $\rho = 0.2, 0.4, 0.6$ and $\rho = 0.3, 0.5, 0.7$, respectively, for the (1×2) system. Notice that here we would like just to see the accuracy of the α-μ approximation model and consider arbitrarily values for ρ. In particular, the relatively large ρ values may not happen in practice (it should be verified by wave-optics simulations [7]). Nonetheless, they can show how appropriate the α-μ approximation is for large ρ.

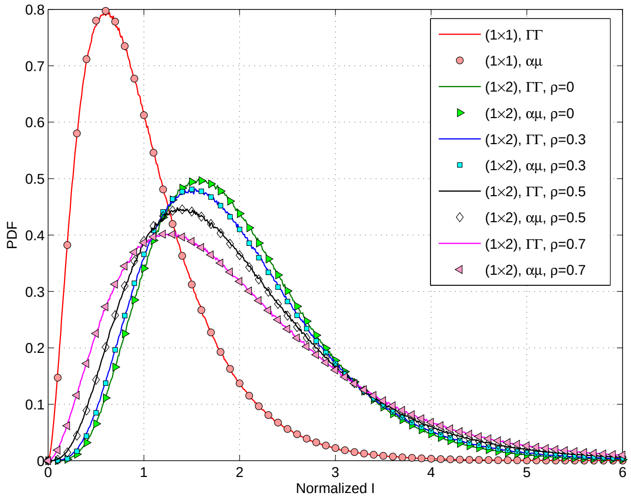

Fig. 1. Contrasting the PDFs of the received intensity I based on simulated data (using the $\Gamma\Gamma$ model) and α-μ approximation. $Z = 5\,\mathrm{Km}$, $D = 100\,\mathrm{mm}$ and different fading correlation coefficients ρ. $\mathrm{E}\{I\} = 1$ for (1×1) and $I = I_1 + I_2$ with $\mathrm{E}\{I_1\} = \mathrm{E}\{I_2\} = 1$ for (1×2) system.

TABLE I

KS TEST STATISTIC T FOR $\alpha = 5\%$ AND 10^4 SAMPLES.

T	(1×1)	$\rho = 0$	ρ_1*	ρ_2*	ρ_3*
Case (1)	0.0098	0.0090	0.0089	0.0086	0.0086
Case (2)	0.0097	0.0083	0.0084	0.0087	0.0086

* ρ_1, ρ_2, and ρ_3 are equal to 0.2, 0.4, 0.6, and 0.3, 0.5, 0.7, for the case studies (1) and (2), respectively.

We consider the strong turbulence regime with the turbulence strength parameter $C_n^2 = 6.5 \times 10^{-14}\,\mathrm{m}^{-2/3}$, the inner and outer scales of turbulence of $l_0 = 6.1\,\mathrm{mm}$ and $L_0 = 1.3\,\mathrm{m}$, respectively. Also, a diverging Gaussian beam is considered at the transmitter at $\lambda = 1550\,\mathrm{nm}$ with the beam waist $W_0 = 1.59\,\mathrm{cm}$ and the curvature radius of the phase front of $F_0 = -69.9\,\mathrm{m}$. These parameters correspond to the experimental works reported in [2].

A. *Comparison of $\Gamma\Gamma$ and α-μ distributions*

We have compared the PDF of $\Gamma\Gamma$ and α-μ distributions for the case studies explained above and noticed a very good match between them. Due to space limitations, we have only provided here the results for the case (2), i.e., $Z = 5\,\mathrm{Km}$ and $D = 100\,\mathrm{mm}$, in Fig. 1, where we notice an excellent match between the PDFs. We have also validated the accuracy of the proposed approximation by the Kolmogorov-Smirnov (KS) goodness-of-fit statistical test [5], [18]. We have calculated the KS test statistic T which represents the maximal difference between the cumulative distribution functions (CDFs) of I and R. Considering 10^4 random samples of I, the corresponding T values are given in Table I, where we have set the significance level to $\alpha = 5\%$, which results in the critical value $T_{\max} = 0.0136$ [18, (8-321)]. This means that the hypothesis that the random samples I belong to the approximate α-μ distribution is accepted with 95% significance when $T < T_{\max}$. The results of Table I show an excellent fit of the two distributions because all the T values are smaller than $T_{\max}$.

B. *BER analysis*

To appreciate the usefulness of the proposed α-μ approximation in predicting the BER performance, we consider a transmission system employing intensity modulation with direct detection using uncoded on-off keying (OOK) modulation. At the receiver, we perform optimal detection based on adaptive threshold setting [19], assuming perfect channel knowledge. Without loss of generality, we assume that the dominant noise source at the receiver is the background noise and model it by an additive white Gaussian noise [20]. We also set the transmit (on) signal intensity and the optical-to-electrical conversion efficient to one.

1) No-diversity system: Let us denote by i and σ_b^2 the signal intensity and the background noise variance at the receiver, respectively. Then, the average electrical signal-to-noise ratio (SNR) is defined as $\mathrm{E}\{i\}^2/4\sigma_b^2$ [8]. For this case, using the α-μ approximation, the BER is calculated as follows.

$$\mathrm{BER} \approx \frac{1}{2}\int_0^{\infty} f_R(r)\ \mathrm{erfc}\left(\frac{r}{2\sqrt{2}\,\sigma_b}\right)\, dr, \qquad (9)$$

where $\mathrm{erfc}(.)$ is the complementary error function defined as $\mathrm{erfc}(z) = 2/\sqrt{\pi}\int_z^{\infty} e^{-t^2} dt$.

2) Dual space diversity system: As mentioned previously, we perform EGC on the received signals after optical-to-electrical conversion. Let us denote by i_1 and i_2 the fading coefficients of the two subchannels. The received signal after EGC is $s = (i_1 + i_2) + n_{b1} + n_{b2}$, where n_{b1} and n_{b2} are the corresponding background noise components, which are independent and have the variance σ_b^2. Now, calculating the α-μ approximation model parameters from Section III, we can obtain the average BER from (9) by replacing σ_b by $\sqrt{2}\sigma_b$.

Given the correlation coefficient ρ, we should obtain ρ_X and ρ_Y, which are required for generating correlated channel coefficients in Monte Carlo simulations based on the $\Gamma\Gamma$ model, as well as for calculating the parameters of the α-μ approximation from (6) and (7). However, as we have recently shown in [7], given ρ, the BER performance has little dependence on the choice of ρ_X and ρ_Y. Here, for simplicity, we set $\rho_Y \approx 0$ which is a rather rational assumption. (Notice however that our model is general and can be used for any ρ_X and ρ_Y.) In fact, small-scale intensity fluctuations originate mostly from the turbulent cells smaller than the first Fresnel zone $F = \sqrt{L/k}$ (with $k = 2\pi/\lambda$ being the wave number) or the transverse coherence radius ρ_0, whichever smaller. Here, we have $F = 2.22\,\mathrm{cm}$ and $\rho_0 = 1.41\,\mathrm{cm}$ in case (1), and $F = 3.51\,\mathrm{cm}$ and $\rho_0 = 0.83\,\mathrm{cm}$ in case (2). As a result, if the two apertures are separated at least 1.41 cm and 0.83 cm apart (that can easily be attained in practice), in cases (1) and (2), respectively, we can effectively assume uncorrelated small-scale fluctuations and neglect ρ_Y. So, for the Monte Carlo simulations for the case of (1×2) system, we have generated independent Y, and correlated X RVs with the corresponding ρ_X using the method proposed in [7], [21], to obtain the channel fading coefficients I_1 and I_2.

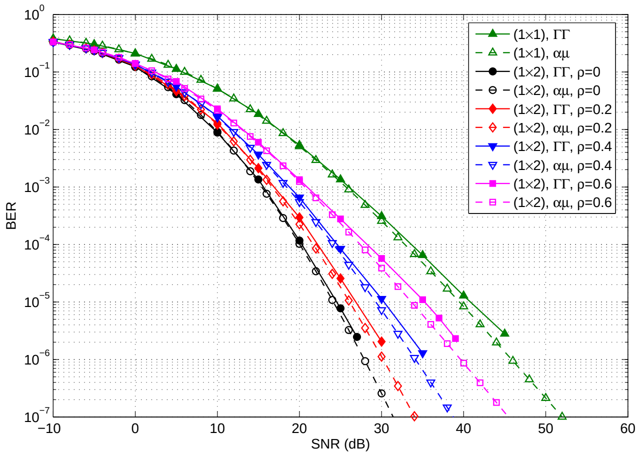

Fig. 2. Contrasting BER performance using ΓΓ and α-μ approximate models. $Z = 2\,$Km, $D = 50\,$mm. Uncoded OOK, background noise limited receiver.

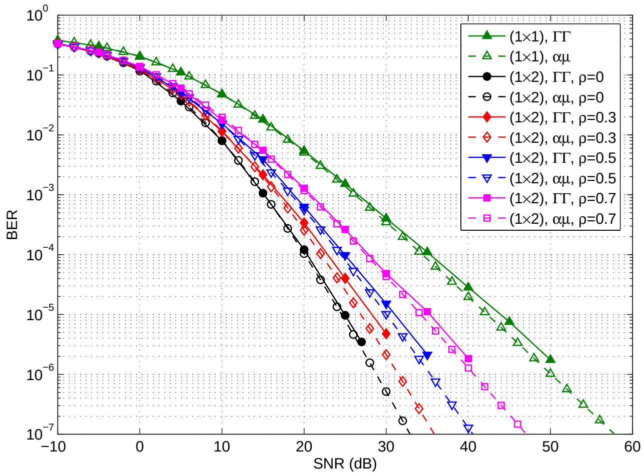

Fig. 3. Contrasting BER performance using ΓΓ and α-μ approximate models. $Z = 5\,$Km, $D = 100\,$mm. Uncoded OOK, background noise limited receiver.

3) BER comparison: We have shown in Figs. 2 and 3 the BER curves versus the electrical SNR for (1×1) and (1×2) systems, for cases (1) and (2), respectively. Solid lines correspond to simulated data using the ΓΓ model, and dashed lines to analytical BER based on the α-μ approximation model. We have compared the performance for different fading correlation coefficients ρ including the special case of uncorrelated fading. We notice for both cases that the predicted performance by the α-μ model is close to that based on the simulated ΓΓ channels. The best match is noticed for the (1×2) system with uncorrelated fading. Although the α-μ approximate model overestimates the receiver performance, it is quite useful to predict the BER with an SNR error of less than 1.6 dB, at BER $= 10^{-6}$, for instance.

V. Conclusions

We have proposed an accurate and useful approximation to the sum of two correlated ΓΓ RVs in order to evaluate the analytical BER performance of dual space diversity FSO systems. Numerical results showed a very good match between the performances obtained from the two models.

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