Constant-Envelope Multi-User Precoding for Frequency-Selective Massive MIMO Systems

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Abstract

We consider downlink precoding in a frequency-selective multi-user Massive MIMO system with highly efficient but non-linear power amplifiers at the base station (BS). A low-complexity precoding algorithm is proposed, which generates constant-envelope (CE) signals at each BS antenna. To achieve a desired per-user information rate, the extra total transmit power required under the per-antenna CE constraint when compared to the commonly used less stringent total average transmit power constraint, is small.

Index Terms

Massive MIMO, per-antenna constant-envelope.

I. INTRODUCTION AND SYSTEM MODEL

Massive MIMO refers to a communication system where a base station (BS) with a large number of antennas (N, a few hundreds) communicates with several user terminals (M, a few tens) on the same time-frequency resource [1]. There has been recent interest in massive MIMO systems due to their ability to increase the spectral and power efficiency even with very low-complexity multi-user detection and precoding [2], [3]. However, physically building cost-effective and power-efficient large arrays is a challenge. Specifically in the downlink, the power amplifiers (PAs) used in the BS should be highly power-efficient. Since there is a trade-off between the power-efficiency and linearity of the PA [4], highly power-efficient but non-linear PAs must be used. Since non-linear PAs could introduce distortion, it is desirable that the input signal has *constant envelope* (CE), that, the signal transmitted from each BS

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antenna has a constant amplitude irrespectively of the channel gains and the information symbols to be communicated.

With this motivation, in [5] we proposed a CE precoding algorithm for the frequency-flat multi-user MIMO broadcast channel, see Section II for a brief summary. In this paper, we make a non-trivial extension of the CE precoding idea to the case of frequency-selective channels, and propose a low-complexity perantenna CE precoding algorithm, see Section III-A. Since the channel has memory we propose to jointly optimize the CE signals transmitted at consecutive time instances. When $N \gg M$, numerical studies for the frequency-selective channel with i.i.d. Rayleigh fading taps reveal that, to achieve a desired per-user ergodic information rate, the proposed CE precoding algorithm needs less than 1.0 dB extra total transmit power compared to what is required under the less stringent and commonly used total average transmit power constraint. We also observe that even under a stringent per-antenna CE constraint, an O(N) array gain is achievable, i.e., with every doubling in the number of BS antennas the total transmit power can be reduced by 3 dB while maintaining a fixed information rate to each user (assuming the number of users is kept fixed).

The complex baseband constant envelope signal transmitted from the i-th BS antenna at time t is of the form

$$x_i[t] = \sqrt{\frac{P_T}{N}} e^{j\theta_i[t]}, \quad i = 1, 2, \cdots, N,$$
 (1)

where $j \triangleq \sqrt{-1}$, P_T is the total power transmitted from the N BS antennas and $\theta_i[t] \in [-\pi, \pi)$ is the phase of the CE signal transmitted from the *i*-th BS antenna at time *t*. The equivalent discrete-time complex baseband channel between the *i*-th BS antenna and the *k*-th user (having a single-antenna) is modeled as a finite impulse response filter with an impulse response of duration *L* samples and denoted by $(h_{k,i}[0], h_{k,i}[1], \dots, h_{k,i}[L-1])$. The signal received at the *k*-th user $(k = 1, 2, \dots, M)$ at time *t* is given by

$$y_k[t] = \sqrt{\frac{P_T}{N}} \sum_{i=1}^N \sum_{l=0}^{L-1} h_{k,i}[l] e^{j\theta_i[t-l]} + w_k[t], \qquad (2)$$

where $w_k[t] \sim C\mathcal{N}(0, \sigma^2)$ is the AWGN at the k-th user at time t (AWGN is i.i.d. across time and across the users).

II. PHILOSOPHY OF CONSTANT ENVELOPE PRECODING

Suppose that, at time t we are interested in communicating the information symbol $\sqrt{E_k}u_k[t] \in \mathcal{U}_k \subset \mathbb{C}$ to the k-th user. Let $\mathbb{E}[|u_k[t]|^2] = 1$, $k = 1, \dots, M$. Also, let $\mathbf{u}[t] = (\sqrt{E_1}u_1[t], \dots, \sqrt{E_M}u_M[t]) \in \mathcal{U}_1 \times \dots \times \mathcal{U}_M$ be the vector of information symbols to be communicated at time t. In [5], for frequencyflat channels we had proposed the idea of choosing the transmit phase angles $\theta_1[t], \theta_2[t], \dots, \theta_N[t]$ in such a way that the noise-free signal received at each user is very close to the desired information symbol, i.e., $\sqrt{P_T/N} \sum_{i=1}^N h_{k,i}[0]e^{j\theta_i[t]} \approx \sqrt{P_T}\sqrt{E_k}u_k[t]$ for all $k = 1, \dots, M$. In [5], the transmit phase angles corresponding to a desired information symbol vector $\mathbf{u}[t]$ are given by

$$(\theta_{1}^{u}[t], \cdots, \theta_{N}^{u}[t]) = \underset{\theta_{i}[t] \in [-\pi, \pi)}{\operatorname{argmin}} \sum_{k=1}^{M} \left| \frac{\sum_{i=1}^{N} h_{k,i}[0] e^{j\theta_{i}[t]}}{\sqrt{N}} - \sqrt{E_{k}} u_{k}[t] \right|^{2}$$
(3)

The idea is to minimize the energy of the difference between the received noise-free signal and the desired information symbol for each user (this difference was also referred to as multi-user interference (MUI) in [5]). Further, in [5] a low-complexity iterative algorithm was proposed for solving (3) near-optimally when N is sufficiently large. Here, near-optimality refers to achieving a per-user MUI energy level below the AWGN variance.

The main reason behind near-optimality of the precoder in (3) when $N \gg M$ is the availability of $(N - M) \gg 1$ excess degrees of freedom. Let $\mathbf{H} \in \mathbb{C}^{M \times N}$ be the channel gain matrix with $h_{k,i}[0]$ being its (k, i)-th entry. Consider a non-CE precoder like the ZF precoder, where $\mathbf{x}[t] = \mathbf{H}^H (\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{u}[t]$ is the transmit vector (with appropriate transmit power normalization). Note that adding any vector $\mathbf{v}[t]$ in the null space of \mathbf{H} to the transmit vector $\mathbf{x}[t]$ results in the same received vector (i.e., $\mathbf{H}\mathbf{x}[t] = \mathbf{H}(\mathbf{x}[t] + \mathbf{v}[t]) = \mathbf{u}[t]$). Since the null space dimensionality is very large when $N \gg M$, it is likely that there exists some $\mathbf{v}[t]$ such that $\mathbf{x}[t] + \mathbf{v}[t]$ is a component-wise CE vector, which then implies that the minimum value of the objective function in (3) is close to zero, and hence very little MUI is experienced by each user.

We had analytically shown that for a broad class of frequency-flat channels (including i.i.d. fading), for a fixed M and fixed symbol energy levels (E_1, \dots, E_M) , by having a sufficiently large $N \gg M$ it is always possible to choose the transmit phase angles so that the received signals at the users are arbitrarily

$$(\Theta^{u}[1], \cdots, \Theta^{u}[T]) = \arg \min_{\Theta[t] \in [-\pi, \pi)^{N}} \sum_{t=1}^{T} \sum_{k=1}^{M} \left| \frac{\sum_{i=1}^{N} \sum_{l=0}^{L-1} h_{k,i}[l] e^{j\theta_{i}[t-l]}}{\sqrt{N}} - \sqrt{E_{k}} u_{k}[t] \right|^{2}.$$
(4)

$$(\Theta^{u}[1], \cdots, \Theta^{u}[T]) = \arg \min_{\Theta[t] \in [-\pi,\pi)^{N}} \sum_{r=1}^{[T/\tau]} I_{r}$$

$$I_{r} \stackrel{\Delta}{=} f(\Theta[(r-1)\tau - L+2], \cdots, \Theta[\min(T, r\tau)], \mathbf{u}[(r-1)\tau + 1], \cdots, \mathbf{u}[\min(T, r\tau)])$$

$$= \sum_{t=(r-1)\tau+1}^{\min(T, r\tau)} \sum_{k=1}^{M} \left| \frac{\sum_{i=1}^{N} \sum_{l=0}^{L-1} h_{k,i}[l] e^{j\theta_{i}[t-l]}}{\sqrt{N}} - \sqrt{E_{k}} u_{k}[t] \right|^{2}.$$

$$(5)$$

close to the desired information symbols. In the next section, inspired by [5] we propose a CE precoding algorithm for frequency-selective channels.

III. PROPOSED CE PRECODING ALGORITHM

A. Algorithm

For frequency-selective channels, the CE precoder of [5] cannot be applied as it stands, because due to channel memory we cannot find the transmit phase angles for each time instance independently of the transmit phase angles at the previous time instances. We therefore propose to consider transmission in blocks of T channel uses (time instances), and jointly choose the transmit phase angles $\theta_i[t]$, i = $1, 2, \cdots, N, t = 1, \cdots, T$ in such a way that the sum of the energies of the difference between the desired and the received signals (over all users and over all the T channel uses) is minimized. Blockwise transmission is also motivated by the fact that real world channels have finite coherence time. More exactly, for a given desired information symbol vector sequence $\mathbf{u}[1]$, $\mathbf{u}[2]$, \cdots , $\mathbf{u}[T]$, the transmit phase angles are given by the solution to the optimization problem in (4) where $\Theta[t] \stackrel{\Delta}{=} (\theta_1[t], \theta_2[t], \cdots, \theta_N[t])$ is the vector of transmitted phase angles at time t. We assume that each block of T channel uses is preceded by a uplink training phase used for channel estimation (Time Division Duplexed operation with channel reciprocity is assumed). Since the duration of the training phase is proportional to L, we must have $T \gg L$ to avoid loss in overall information rate due to the training overhead. However, in (4) a large T is also associated with more optimization variables, and therefore a higher complexity. Hence, in the following we present a method where we split the optimization in (4) into several separate optimization problems with each problem having a much lesser number of optimization variables. This splitting of (4) is not necessarily the optimal way to solve it, but numerical studies confirm its near-optimality (see Section IV).

In (4) we group the MUI terms of τ consecutive time instances into one compound term, as shown in (5) (I_r is the r-th such compound term, $r = 1, 2 \cdots, \lceil T/\tau \rceil$). I_r is a function of $\Theta^r \triangleq (\Theta[(r - 1)\tau + 1]^T, \cdots, \Theta[\min(T, r\tau)]^T)^T$, which is the vector of phase angles to be transmitted at time instances $(r - 1)\tau + 1, \cdots, \min(T, r\tau)$. Note that I_r also depends on the phase angles transmitted during the last L -1 time instances of the previous compound term I_{r-1} (i.e., $\Theta[(r - 1)\tau - L + 2], \cdots, \Theta[(r - 1)\tau]$).

If the time duration τ is sufficiently larger than the channel memory L - 1, then I_r depends more strongly on Θ^r and less strongly on the phase angles transmitted in the last L -1 time instances of I_{r-1} . This intuition led us to develop a sequential CE precoding algorithm, where we start off with I_1 and optimize it as a function of only the transmit phase angles during the first τ time instances (i.e., $t = 1, 2, \dots, \tau$). In the *r*-th step of this sequential CE precoding algorithm, we minimize I_r as a function of only Θ^r while keeping the values of $\Theta[(r-1)\tau - L + 2], \dots, \Theta[(r-1)\tau]$ fixed to the value assigned to them after the optimization of I_{r-1} . The proposed sequential algorithm stops after the $\lceil T/\tau \rceil$ -th step.

Next, we propose a low-complexity iterative method to minimize I_r as a function of Θ^r (i.e., the *r*-th step of the sequential CE precoding algorithm). Let us use the notation $I_r = f_r(\Theta^r)$ for brevity. For any time instance $t \in [(r-1)\tau + 1, \cdots, \min(T, r\tau)]$, let us define

$$L_r(t) \stackrel{\Delta}{=} \min(L-1, \min(T, r\tau) - t) , \quad d_r \stackrel{\Delta}{=} \min(\tau, T - (r-1)\tau).$$
(6)

Excess degrees of freedom $(N \gg M)$ results in most local minimum of the function $f_r(\cdot)$ having a very small value (compared to the AWGN variance). Through numerical studies we also observe that a simple iterative algorithm which optimizes one phase angle at a time while keeping the others fixed, achieves very small values for the objective function $f_r(\cdot)$. Each iteration of the proposed iterative method is in turn split into Nd_r sub-iterations. In the (n,q)-th sub-iteration of any given iteration $(q \in [1, \dots, d_r], n \in [1, \dots, N])$, we minimize the current value of I_r as a function of only the phase angle to be transmitted from the *n*-th BS antenna at time $(r-1)\tau + q$ while keeping the other $Nd_r - 1$ phase angles fixed to their value from the previous sub-iteration. The phase angle $\theta_n[(r-1)\tau + q]$ is then updated to the optimal value resulting from this single variable optimization and the algorithm moves to the next sub-iteration. In the next sub-iteration, we repeat the same for the phase angle to be transmitted from the n + 1-th BS antenna at time $(r-1)\tau + q$. For the special case when n = N, the next sub-iteration minimizes I_r as a

$$\begin{aligned} \theta_n'[(r-1)\tau + q] &= \arg\min_{\phi \in [-\pi,\pi)} f_r \Big(\theta_1[(r-1)\tau + 1], \cdots, \theta_N[(r-1)\tau + 1], \cdots, \theta_1[(r-1)\tau + q], \cdots \\ &, \theta_{n-1}[(r-1)\tau + q], \phi, \theta_{n+1}[(r-1)\tau + q], \cdots \\ &, \theta_N[(r-1)\tau + q], \cdots, \theta_1[(r-1)\tau + d_r], \cdots, \theta_N[(r-1)\tau + d_r] \Big) \end{aligned}$$

$$= \pi + \arg \Big\{ \sum_{t=t_1}^{t_1 + L_r(t_1)} \sum_{k=1}^{M} h_{k,n}^*[t-t_1] S_{n,t_1}(k,t) \Big\} , \text{ where } t_1 \stackrel{\Delta}{=} (r-1)\tau + q \\ S_{n,t_1}(k,t) \stackrel{\Delta}{=} \frac{\sum_{i=1}^{N} \sum_{l=0, (i,l) \neq (n,(t-t_1))}^{L-1} h_{k,i}[l] e^{j\theta_i[t-l]}}{\sqrt{N}} - \sqrt{E_k} u_k[t] \end{aligned}$$

$$(7)$$

function of only the phase angle to be transmitted from the first BS antenna at time $(r-1)\tau + q + 1$, i.e., $\theta_1[(r-1)\tau + q + 1]$. One full iteration is completed once we have finished the sequence of one-dimensional optimizations for all the Nd_r transmit phase angles, after which we start over with the first sub-iteration (1, 1) of the next iteration.

The single variable optimization in the (n, q)-th sub-iteration is given by (7). At the end of the (n, q)-th sub-iteration the value of $\theta_n[(r-1)\tau + q]$ is modified to $\theta'_n[(r-1)\tau + q]$ (see (7)), whereas all the other Nd_r-1 angles remain unchanged. Note that the term $S_{n,t_1}(k,t)$ in (7) does not depend on $\theta_n[(r-1)\tau + q]$. Further, in the (n, q)-th sub-iteration

$$S_{n,t_{1}}(k,t) = S(k,t) - \frac{h_{k,n}[t-t_{1}]e^{j\theta_{n}[(r-1)\tau+q]}}{\sqrt{N}}$$

$$S(k,t) \triangleq \frac{\sum_{i=1}^{N} \sum_{l=0}^{L-1} h_{k,i}[l]e^{j\theta_{i}[t-l]}}{\sqrt{N}} - \sqrt{E_{k}}u_{k}[t]$$

$$k = 1, \cdots, M, t = (r-1)\tau + 1, \cdots, (r-1)\tau + d_{r}$$
(8)
(9)

From (7), (8) and (9), it is clear that for computing the optimal $\theta'_n[(r-1)\tau + q]$ we only need to know S(k,t), $k = 1, \dots, M$, $t = t_1, \dots, t_1 + L_r(t_1)$ ($t_1 \triangleq (r-1)\tau + q$). In fact, for each sub-iteration we need not compute S(k,t) as a double sum over (i,l) (as defined in (9)). We only need to compute S(k,t) for all $k = 1, \dots, M$, $t = (r-1)\tau + 1, \dots, (r-1)\tau + d_r$ only once during the start of the (1,1)-th subiteration of the first iteration. Thereafter, since we only update one phase angle per sub-iteration, the new updated value of S(k,t) at the end of the (n,q)-th sub-iteration is given by

$$S'(k,t) = S(k,t) + \frac{h_{k,n}[t-t_1]}{\sqrt{N}} \left(e^{j\theta'_n[t_1]} - e^{j\theta_n[t_1]} \right)$$

$$t = t_1, \cdots, t_1 + L_r(t_1) , \ k = 1, \cdots, M.$$
(10)

The value of S(k,t) for the remaining time instances is not changed. For $\tau \ge L$ the per-iteration complexity of computing all the NT phase angles is O(NMLT), i.e., a per-channel-use complexity of O(NML). Therefore the complexity increases only linearly in the length of the channel impulse response. Numerically, it has been observed that very few iterations (≤ 5) are required to converge to small values of $f_r(\cdot)$.

B. Performance Analysis

For a given set of information symbol vectors $\mathbf{u}[t]$, $t = 1, \dots, T$, let the output phase angles of the proposed CE precoding algorithm be denoted by $\tilde{\Theta}^u[1], \tilde{\Theta}^u[2], \dots, \tilde{\Theta}^u[T]$. The phase angle to be transmitted from the *n*-th antenna at time *t* is denoted by $\tilde{\theta}_n^u[t]$. The signal received at the *k*-th receiver at time *t* is then given by

$$y_{k}[t] = \sqrt{P_{T}}\sqrt{E_{k}}u_{k}[t] + \sqrt{P_{T}}I_{k}^{u}[t] + w_{k}[t]$$

$$I_{k}^{u}[t] \triangleq \left(\frac{\sum_{i=1}^{N}\sum_{l=0}^{L-1}h_{k,i}[l]e^{j\tilde{\theta}_{i}^{u}[t-l]}}{\sqrt{N}} - \sqrt{E_{k}}u_{k}[t]\right)$$
(11)

From (11) we observe that $I_k^u[t]$ essentially behaves like MUI. We also define the following vector notation for later use, $\mathbf{y}_k \stackrel{\Delta}{=} (y_k[1], \cdots, y_k[T])^T$, $\mathbf{u}_k \stackrel{\Delta}{=} (\sqrt{E_k}u_k[1], \cdots, \sqrt{E_k}u_k[T])^T$, $\mathbf{I}_k^u \stackrel{\Delta}{=} (I_k^u[1], \cdots, I_k^u[T])^T$ and $\mathbf{w}_k \stackrel{\Delta}{=} (w_k[1], \cdots, w_k[T])^T$. From (11) it follows that

$$\mathbf{z}_k \stackrel{\Delta}{=} \mathbf{u}_k - \mathbf{y}_k / \sqrt{P_T} = -\mathbf{I}_k^u - \mathbf{w}_k / \sqrt{P_T}$$
(12)

Let $\mathbf{H} = \{h_{k,i}[l]\} \in \mathbb{C}^{N \times M \times L}$ be the collection of all the channel impulse responses between the N BS antennas and the M users. For a given \mathbf{H} , an achievable rate for the k-th user is then given by the mutual information $I(\mathbf{y}_k; \mathbf{u}_k \mid \mathbf{H})/T$ [6],[7]. For any arbitrary input distribution on \mathbf{u}_k , $k = 1, \dots, M$, it is in general difficult to compute this mutual information in closed form and therefore in the following we derive a lower bound assuming $\sqrt{E_k u_k[t]}$, $t = 1, \dots, T$ to be i.i.d. complex Gaussian with mean 0 and variance E_k .

$$I(\mathbf{y}_{k}; \mathbf{u}_{k} | \mathbf{H}) = h(\mathbf{u}_{k}) - h(\mathbf{u}_{k} | \mathbf{y}_{k}, \mathbf{H})$$

$$= h(\mathbf{u}_{k}) - h(\mathbf{u}_{k} - \mathbf{y}_{k} / \sqrt{P_{T}} | \mathbf{y}_{k}, \mathbf{H})$$

$$= T \log_{2}(\pi e E_{k}) - h(\mathbf{z}_{k} | \mathbf{y}_{k}, \mathbf{H})$$

$$\geq T \log_{2}(\pi e E_{k}) - h(\mathbf{z}_{k} | \mathbf{H})$$
(13)

$$\frac{I(\mathbf{y}_k; \mathbf{u}_k \mid \mathbf{H})}{T} \ge \max\left(0, \log_2(E_k) - \frac{\log_2\left(\left|\mathbb{E}[\mathbf{I}_k^u \, \mathbf{I}_k^{u^H} \mid \mathbf{H}] + (\sigma^2/P_T)\mathbf{I}\right|\right)}{T}\right) \stackrel{\Delta}{=} R_k(\mathbf{H}, \mathbf{E}, \frac{P_T}{\sigma^2}) (14)$$

where \mathbf{z}_k is defined in (12), $h(\cdot)$ denotes the differential entropy function, and the last inequality follows from the fact that conditioning reduces entropy [7]. Further, for any random complex vector $\mathbf{z}_k \in \mathbb{C}^T$ with autocorrelation matrix $\mathbf{R}_z \stackrel{\Delta}{=} \mathbb{E}[\mathbf{z}_k \mathbf{z}_k^H]$, $h(\mathbf{z}_k)$ is maximized when \mathbf{z}_k is a zero mean proper complex Gaussian distributed vector and therefore $h(\mathbf{z}_k | \mathbf{H}) \leq \log_2((\pi e)^T |\mathbf{R}_z|)$ [8] (here $|\mathbf{R}_z|$ is the determinant of \mathbf{R}_z). From (12) it follows that $\mathbf{R}_z = \mathbb{E}[\mathbf{I}_k^u \mathbf{I}_k^{u^H} | \mathbf{H}] + (\sigma^2/P_T)\mathbf{I}$, where the expectation is over $\mathbf{u}_1, \dots, \mathbf{u}_M$. Using this upper bound (on $h(\mathbf{z}_k | \mathbf{H})$) in (13) we get an achievable rate for the k-th user (the lower bound $R_k(\mathbf{H}, \mathbf{E}, P_T/\sigma^2)$ in (14)). $\mathbf{E} \stackrel{\Delta}{=} (E_1, E_2, \dots, E_M)^T$ is the vector of the average information symbol energies of the M users. The ergodic mutual information lower bound for the k-th user is then given by $\mathbb{E}[R_k(\mathbf{H}, \mathbf{E}, P_T/\sigma^2)]$ (expectation is over \mathbf{H}).

IV. NUMERICAL RESULTS AND DISCUSSION

We consider a Rayleigh fading frequency selective channel with a uniform average power delay profile, i.e., $\mathbb{E}[|h_{k,i}[l]|^2] = 1/L$, $l = 0, 1, \dots, (L-1)$. The channel gains $h_{k,i}[l]$ are i.i.d. circularly symmetric complex Gaussian (mean 0, variance 1/L). We consider the ergodic sum rate $\sum_{k=1}^{M} \mathbb{E}[R_k(\mathbf{H}, \mathbf{E}, P_T/\sigma^2)]$ as the measure of performance. In order to further optimize the ergodic sum rate, we would need to maximize it as a function of E_1, \dots, E_M . This is however a difficult task. Since the users have identical channel statistics, it is likely that the optimal \mathbf{E} vector is such that $E_1 = E_2 = \dots = E_M = E^*$. Using numerical methods, for a given P_T/σ^2 we therefore find the optimal E^* which results in the largest ergodic sum rate. With $E_1 = E_2 = \dots = E_M = E^*$, we also numerically observe that $\mathbb{E}[R_1(\mathbf{H}, \mathbf{E}, P_T/\sigma^2)] =$ $\dots = \mathbb{E}[R_M(\mathbf{H}, \mathbf{E}, P_T/\sigma^2)]$, i.e., all users have the same ergodic rate. Let us denote this per-user ergodic achievable rate by $R(N, M, P_T/\sigma^2)$.

Fig. 1 shows the minimum required P_T/σ^2 to achieve a per-user ergodic rate of $R(N, M, P_T/\sigma^2) = 2$ bits-per-channel-use (bpcu), as a function of τ ($T \gg \tau$) for different values of the channel memory L and for fixed N = 80 and M = 10. For the sake of comparison we also consider the performance achieved under a average only total transmit power constraint (TAPC). Since the CE constraint is more stringent than the TAPC, it is expected that the performance under TAPC must be better. Under TAPC,



Fig. 1. Minimum required P_T/σ^2 to achieve a per-user ergodic rate of 2 bpcu, plotted as a function of τ . Fixed N = 80 BS antennas, M = 10 users.

we compute both a lower bound (ZF - Zero Forcing precoder) and an upper bound (cooperative users) on the downlink sum capacity. We observe that for a fixed L, increasing τ improves the performance. However when $\tau \gg L$, the improvement is not so significant and the performance appears to saturate as $\tau \to \infty$. For $L \ge 2$, we observe that when $\tau \gg L$, the minimum required P_T/σ^2 for the proposed CE precoding algorithm is within 1.0 dB of the minimum P_T/σ^2 required under TAPC. This shows the near-optimality of the proposed iterative algorithm for solving (4). Typically, a non-linear PA is 4 - 6times more power-efficient than a highly linear PA [4]. Therefore using non-linear PAs with CE input signals needs $10 \log_{10}(4) - 1.0 = 5.0$ dB less total transmit power compared to using highly linear PAs with a average only total transmit power constrained input. For the proposed CE precoding algorithm, it also appears that when $\tau \gg L$, the minimum required P_T/σ^2 is smaller for a larger L.

In Fig. 2, we plot the minimum required P_T/σ^2 to achieve 2 bpcu/user as a function of the number of BS antennas N for a fixed number of users M = 10. It can be seen that under both TAPC and CE, the minimum required P_T/σ^2 reduces by 3 dB with every doubling in the number of BS antennas (holds when $N \gg M$).



Fig. 2. Minimum required P_T/σ^2 to achieve a per-user ergodic rate of 2 bpcu, plotted as a function of N. Fixed M = 10 users, $\tau = 3L$ and $T \gg \tau$. Four iterations in the proposed CE precoding algorithm.

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