A New Multicarrier Transceiver Based on the Discrete Cosine Transform

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***Abstract*— We derive conditions on the impulse response and input signal of a frequency-selective FIR channel to be diagonalized by the DCT into parallel, decoupled, and memoryless subchannels. We show how these conditions can be satisfied in a practical multi-carrier transceiver through a novel design of the guard sequence and the front-end prefilter. This DCT-based design completely eliminates inter-block and inter-carrier interference at a lower complexity and without incurring any additional guard sequence overhead compared to DFT-based multicarrier transceivers. Extensions to multi-input multi-output frequency-selective channels are also described.**

I. Introduction

Multi-carrier Modulation (MCM) [1] based on the Discrete Fourier Transform (DFT) has been adopted as the modulation/demodulation scheme of choice in several digital communications standards. These include wireline scenarios such as digital subscriber lines (DSL) where it is commonly known as Discrete Multitone (DMT) and wireless scenarios such as digital audio broadcast, local area networks (IEEE 802.11a), and metropolitan area networks (IEEE802.16a) where it is commonly known as Orthogonal Frequency Division Multiplexing (OFDM). In this paper, we shall refer to it generically by DFT-MCM.

DFT-MCM divides the frequency response of a finite impulse response (FIR) frequency-selective channel into parallel, decoupled, and memoryless subchannels by adding a special guard sequence known as a *cyclic prefix* (CP) to each information block. This CP guard sequence (of length greater than or equal to the channel memory) is chosen as a *periodic* extension of the information sequence causing the linear convolution performed by the FIR channel to resemble a circular convolution. This renders the equivalent channel matrix *circulant* and hence diagonalizable by the DFT. Therefore, both inter-block interference (IBI) between successive transmitted blocks and inter-channel interference (ICI) between the frequency subchannels are eliminated. Additional attractive features of using the DFT orthogonal basis vectors for modulation/demodulation are efficient computation using the Fast Fourier Transform (FFT) algorithm and their independence of the channel characteristics. These two features are lost when using an all-zeros guard sequence (commonly known as *zero stuffing* [2]) since the optimum orthogonal modulation/demodulation basis vectors in this case are the eigenvectors of the channel matrix which are both channel-dependent and computationally-intense.

The only restrictions on the guard sequence are being redundant (i.e. it carries no new information) and of length greater than or equal to the channel memory. As far as channel throughput is concerned, it was shown in [3] that, in each transmitted block, the guard sequence can be assumed a *linear deterministic* function of the information sequence without loss of optimality. This leads us to the following fundamental question : *are there guard sequence design choices (other than CP) that result in an equivalent channel matrix perfectly diagonalizable (i.e. no IBI and ICI) by an orthogonal channel-independent set of finite-dimensional modulation/demodulation vectors which have a fast implementation algorithm comparable to the FFT* ?

In this paper, we develop another MCM design that satisfies these desirable properties based on the Discrete Cosine Transform (DCT) and a guard sequence that *symmetrically* extends the information sequence. In addition, our new design, referred to henceforth by DCT-MCM, has the following additional attractive features inherited from the DCT [4] :

- The DCT basis is well-known to have excellent spectral compaction and energy concentration properties. This, in turn, leads to improved performance with interpolation-based channel estimation [5] and in the presence of narrow-band interference and residual frequency offsets. In addition, it can result in improved adaptive filtering convergence (see e.g. [6] page 584).
- The DCT is widely adopted in image/video coding stan-

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dards (e.g. JPEG, MPEG, H.261). Using it for modulation/demodulation on frequency-selective channels results in a better integrated system design and a reduced overall implementation cost.

- The DCT uses only *real* arithmetic as opposed to the complex-valued DFT. This reduces the signal processing complexity especially for real pulse-amplitude modulation (PAM) signalling where DFT-based processing still uses complex arithmetic and suffers from in-phase/quadrature (I/Q) imbalance problems which can cause appreciable performance degradation.

II. BACKGROUND

A. Channel Model and Assumptions

We consider block-by-block transmission where the information symbols are divided into blocks of length N and assume the standard discrete-time representation of a linear time-invariant[1] frequency-selective channel. The received symbols are given by

$$y_k = \sum_{m=-\nu}^{\nu} h_m x_{k-m} + z_k \tag{1}$$

where h_m is the m^{th} coefficient of the channel impulse response (CIR) which has a memory of 2ν. The information symbols $\{x_k\}$ are assumed zero-mean, with an N-dimensional auto-correlation matrix $\mathbf{R}_{xx}$. For simplicity, we shall assume one-dimensional (PAM) signalling where both the input symbols and the CIR are real but the results can be easily extended to the complex case (see Remark 2 in Section 3). The additive white Gaussian noise (AWGN) symbols are denoted by $\{z_k\}$ and have variance σ_z^2. Additional 2ν *guard* symbols are added before (ν prefix symbols) and after (ν suffix symbols) each information block to eliminate inter-block interference (IBI) at the expense of a throughput loss factor of $\frac{2\nu}{N+2\nu}$. Furthermore, output symbols corresponding to the guard symbols are discarded at the receiver to eliminate their interfering effect. Over the remaining N output symbols, denoted by $\mathbf{y}_{k:k+N-1}$, Equation (1) can be expressed as follows[2]

$$\begin{aligned}\mathbf{y}_{k:k+N-1} &= \mathbf{H}\mathbf{x}_{k-\nu:k+N+\nu-1} + \mathbf{z}_{k:k+N-1} &(2)\\ &= \mathbf{H}_{pre}\mathbf{x}_{k-\nu:k-1} + \mathbf{H}_{info}\mathbf{x}_{k:k+N-1}\\ &+ \mathbf{H}_{suf}\mathbf{x}_{k+N:k+N+\nu-1} + \mathbf{z}_{k:k+N-1} &(3)\end{aligned}$$

[1]The channel is assumed time-invariant over each transmission block but can vary from block to block.

[2]The subscripts of a vector are used to denote the time indices of its first and last elements separated by a colon.

where $k = i(N+2\nu)+\nu$ and $i = 0, 1, \cdots$ is the block index. The length-$(N+2\nu)$ input vector was partitioned into 3 components corresponding to the *prefix*, *suffix*, and *information* symbols. The corresponding partitioning of the channel matrix $\mathbf{H}$ is defined by

$$\mathbf{H}_{pre} = \mathbf{H}\begin{bmatrix}\mathbf{I}_\nu\\ \mathbf{0}_{N\times\nu}\\ \mathbf{0}_{\nu\times\nu}\end{bmatrix} = \begin{bmatrix} h_\nu & \cdots & h_1\\ 0 & \ddots & \vdots\\ \vdots & \ddots & h_\nu\\ 0 & \cdots & 0\\ \vdots & & \vdots\\ 0 & \cdots & 0\end{bmatrix} \tag{4}$$

$$\mathbf{H}_{suf} = \mathbf{H}\begin{bmatrix}\mathbf{0}_{\nu\times\nu}\\ \mathbf{0}_{N\times\nu}\\ \mathbf{I}_\nu\end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0\\ \vdots & & \vdots\\ 0 & \cdots & 0\\ h_{-\nu} & \ddots & \vdots\\ \vdots & \ddots & 0\\ h_{-1} & \cdots & h_{-\nu}\end{bmatrix}$$

$$\mathbf{H}_{info} = \mathbf{H}\begin{bmatrix}\mathbf{0}_{\nu\times N}\\ \mathbf{I}_N\\ \mathbf{0}_{\nu\times N}\end{bmatrix} = \begin{bmatrix} h_0 & \cdots & h_{-\nu} & 0 & \\ \vdots & \ddots & & \ddots & 0\\ h_\nu & & h_0 & & h_{-\nu}\\ 0 & \ddots & & \ddots & \vdots\\ & 0 & h_\nu & \cdots & h_0\end{bmatrix} \tag{5}$$

where $\mathbf{I}_l$ denotes the identity matrix of size l and $\mathbf{0}_{m\times n}$ denotes the all-zeros matrix with m rows and n columns.

B. Guard Sequence

The two length-ν guard sequences do not carry new information, i.e., they are related by deterministic functions to the length-N information sequence $\mathbf{x}_{k:k+N-1}$. It was shown in [3] that assuming these functions to be *linear* does not limit the achievable throughput. Hence, we follow this assumption in this paper, i.e., we can write

$$\mathbf{x}_{k-\nu:k-1} = \mathbf{G}_{pre}\mathbf{x}_{k:k+N-1} \tag{6}$$

$$\mathbf{x}_{k+N:k+N+\nu-1} = \mathbf{G}_{suf}\mathbf{x}_{k:k+N-1} \tag{7}$$

where $\mathbf{G}_{pre}$ and $\mathbf{G}_{suf}$ are $\nu\times N$ fixed (independent of the channel) matrices that completely determine the guard sequences. Therefore, Equation (3) becomes

$$\begin{aligned}\mathbf{y}_{k:k+N-1} &= (\mathbf{H}_{info} + \mathbf{H}_{pre}\mathbf{G}_{pre} + \mathbf{H}_{suf}\mathbf{G}_{suf})\mathbf{x}_{k:k+N-1}\\ &+ \mathbf{z}_{k:k+N-1}\\ &= \mathbf{H}_{eqv}\mathbf{x}_{k:k+N-1} + \mathbf{z}_{k:k+N-1} &(8)\end{aligned}$$

where $\mathbf{H}_{eqv}$ is the $N \times N$ equivalent channel matrix which is a function of the original CIR, $\mathbf{G}_{pre}$, and $\mathbf{G}_{suf}$. Equation (8) clearly shows how different choices of the guard sequence result in a different overall channel matrix $\mathbf{H}_{eqv}$. For example, for DFT-MCM, the guard sequences are chosen as *cyclic extension* of the information sequence; i.e. $\mathbf{G}_{pre} = \begin{bmatrix} \mathbf{0}_{\nu\times(N-\nu)} & \mathbf{I}_\nu \end{bmatrix}$ and $\mathbf{G}_{post} = \begin{bmatrix} \mathbf{I}_\nu & \mathbf{0}_{\nu\times(N-\nu)} \end{bmatrix}$. It can be readily checked that in this case $\mathbf{H}_{eqv}$ becomes a *circulant* matrix which is diagonalizable by the DFT.

C. Discrete Cosine Transform

There are eight types of DCT [7]. In this paper, we only consider the type-II DCT because it is the first one discovered in [8] and the most popular in practice (used in JPEG, MPEG, and H.261 standards). The size-N type-II DCT is defined by the real orthogonal matrix whose (l, m) entry is given by (for $1 \le l, m \le N$)

$$\mathbf{C}(l,m) = \begin{cases} \sqrt{\frac{2}{N}}\cos(\frac{(l-1)(2m-1)\pi}{2N}) & : \quad l \neq 1 \\ \sqrt{\frac{1}{N}} & : \quad l = 1 \end{cases}$$

It can be readily checked that $\mathbf{C}^t\mathbf{C} = \mathbf{C}\mathbf{C}^t = \mathbf{I}_N$ where $(.)^t$ denotes the transpose.

We shall utilize the following key fact from ([7], page 2634) in designing our DCT-MCM transceiver.

Fact : All $N \times N$ matrices diagonalizable by the type-II DCT matrix can be written as the sum of an $N \times N$ *symmetric* Toeplitz matrix $\mathbf{T}$ and an $N \times N$ Hankel[3] matrix $\mathbf{L}$; i.e., $\mathbf{C}(\mathbf{T}+\mathbf{L})\mathbf{C}^t = \mathbf{D}$ where $\mathbf{D}$ is a diagonal matrix. Moreover, $\mathbf{L}$ is determined from $\mathbf{T}$ through the relations :[4]

$$\begin{aligned} \mathbf{L}\mathbf{e}_1 &= \mathbf{S}_N\mathbf{T}\mathbf{e}_1 \\ \mathbf{L}\mathbf{e}_N &= \mathbf{J}_N\mathbf{L}\mathbf{e}_1 \end{aligned} \qquad (9)$$

where $\mathbf{S}_k$ is the $k \times k$ upper-shift matrix (has ones on the first upper diagonal and zeroes everywhere else), $\mathbf{J}_k$ is the $k \times k$ reversal matrix (has ones on the main anti-diagonal and zeros everywhere else), and $\mathbf{e}_i$ is the i^{th} unit vector (has a one in the i^{th} position and zeros everywhere else). Note that since $\mathbf{L}$ is a Hankel matrix, it is completely determined by its first and last columns defined in (9). It is also important to note that this Fact does not imply that all Toeplitz plus Hankel matrices are diagonalized by the type-II DCT but only a subclass of them where the Toeplitz matrix is symmetric and the Hankel matrix is related to it by (9). Henceforth, we shall refer to the type-II DCT by DCT only for brevity.

[3]A matrix is called Toeplitz if it is constant along the main diagonals and called Hankel if it is constant along the main anti-diagonals.

[4]These mathematical relations are not stated explicitly in [7] but can be inferred from the development.

III. MAIN RESULT

In this section, we address the fundamental question of this paper : *Can we design the guard sequences $\mathbf{G}_{pre}$ and $\mathbf{G}_{suf}$ so that $\mathbf{H}_{eqv}$ is diagonalizable by the DCT ?*

A. Channel Block Throughput

For the equivalent N-dimensional frequency-selective channel with AWGN in (8), the channel block throughput, denoted by $I(\mathbf{X};\mathbf{Y})$, for PAM signalling is given by [5] [9]

$$\begin{aligned} I(\mathbf{X};\mathbf{Y}) &= \frac{1}{2}\frac{N}{N+2\nu}\log det(\mathbf{I}_N + \frac{1}{\sigma_z^2}\mathbf{H}_{eqv}^t\mathbf{H}_{eqv}\mathbf{R}_{xx}) \\ &= \frac{1}{2}\frac{N}{N+2\nu}\log det(\mathbf{I}_N + \frac{1}{\sigma_z^2}\mathbf{C}\mathbf{H}_{eqv}^t\mathbf{H}_{eqv}\mathbf{C}^t\mathbf{\Sigma}_x) \end{aligned} \qquad (10)$$

where $det(.)$ denotes the determinant of a matrix and we used the matrix identity $det(\mathbf{I}+\mathbf{AB}) = det(\mathbf{I}+\mathbf{BA})$. Furthermore, since we are using the DCT basis for modulation, the input auto-correlation matrix admits the eigen-decomposition $\mathbf{R}_{xx} = \mathbf{C}^t\mathbf{\Sigma}_x\mathbf{C}$.

It can be shown that the *channel block throughput is maximized when $\mathbf{H}_{eqv}$ is diagonalized by the DCT*; i.e. $\mathbf{H}_{eqv} = \mathbf{C}^t\mathbf{D}\mathbf{C}$. In this case, (10) simplifies to

$$\begin{aligned} I(\mathbf{X};\mathbf{Y}) &= \frac{1}{2}\frac{N}{N+2\nu}\log det(\mathbf{I}_N + \frac{1}{\sigma_z^2}\mathbf{D}^2\mathbf{\Sigma}_x) \quad (11) \\ &= \frac{1}{2}\frac{N}{N+2\nu}\sum_{i=1}^{N}\log(1+\frac{d_i^2}{\sigma_z^2}\sigma_{x,i}) \quad (12) \end{aligned}$$

where d_i and $\sigma_{x,i}$ denote the i^{th} elements of the diagonal matrices $\mathbf{D}$ and $\mathbf{\Sigma}_x$, respectively. The optimum channel throughput is achieved by further optimizing the input energy allocation profile $\sigma_{x,i}$ to maximize (12) subject to the average energy constraint

$$\frac{1}{N+2\nu}trace(\mathrm{E}[\mathbf{x}_{k-\nu:k+N+\nu-1}\mathbf{x}_{k-\nu:k+N+\nu-1}^t]) = 1 \qquad (13)$$

which can be expressed as follows

$$\sum_{i=1}^{N}\sigma_{x,i}\gamma_i = N+2\nu\ . \qquad (14)$$

The constants γ_i in (14) are pre-calculated as a function of the DCT matrix $\mathbf{C}$ and the guard matrices $\mathbf{G}_{pre}$ and $\mathbf{G}_{suf}$. The maximization of (12) subject to the constraint in (14) can be performed using standard Lagrange multiplier techniques.

[5]The factors of $\frac{1}{2}$ and $\frac{N}{N+2\nu}$ are due to one-dimensional signalling and guard sequence overhead, respectively. We assume a basis of two for the logarithm, hence, $I(\mathbf{X};\mathbf{Y})$ is measured in bits/block.

B. DCT Optimality Conditions

For $\mathbf{H}_{eqv}$ in (8) to be diagonalizable by the DCT, it must satisfy the conditions in (9). Note that $\mathbf{H}_{info}$ becomes a symmetric Toeplitz matrix if and only if (see (5))

$$h_i = h_{-i} \quad : \quad i = 1, 2, \cdots, \nu \tag{15}$$

which results in a symmetric linear-phase CIR. Then, it remains to choose $\mathbf{G}_{pre}$ and $\mathbf{G}_{suf}$ to make the matrix $\mathbf{H}_{pre}\mathbf{G}_{pre} + \mathbf{H}_{suf}\mathbf{G}_{suf}$ a Hankel matrix that satisfies (9) in addition to Condition (15), i.e.

$$\mathbf{H}_{pre}\mathbf{G}_{pre} + \mathbf{H}_{suf}\mathbf{G}_{suf} = \begin{bmatrix} h_1 & \cdots & h_\nu & 0 & \cdots & \cdots & 0 \\ \vdots & & 0 & \cdots & & & \vdots \\ h_\nu & 0 & \cdots & \ddots & & \ddots & \vdots \\ 0 & \cdots & \ddots & & \ddots & & 0 \\ \vdots & \ddots & & \ddots & & 0 & h_\nu \\ & & \ddots & & 0 & & \vdots \\ 0 & \cdots & \cdots & 0 & h_\nu & \cdots & h_1 \end{bmatrix} \tag{16}$$

It can be easily checked that this condition is satisfied by setting

$$\mathbf{H}_{pre}\mathbf{G}_{pre} = \begin{bmatrix} h_1 & \cdots & h_\nu & \\ \vdots & & 0 & \mathbf{0}_{N\times(N-\nu)} \\ h_\nu & 0 & 0 & \\ & \mathbf{0}_{(N-\nu)\times\nu} & & \end{bmatrix}$$

$$\Rightarrow \quad \mathbf{G}_{pre} = \begin{bmatrix} \mathbf{J}_\nu & \mathbf{0}_{\nu\times(N-\nu)} \end{bmatrix} \tag{17}$$

$$\mathbf{H}_{suf}\mathbf{G}_{suf} = \begin{bmatrix} & \mathbf{0}_{(N-\nu)\times N} & & \\ & 0 & 0 & h_\nu \\ \mathbf{0}_{\nu\times(N-\nu)} & 0 & & \vdots \\ & h_\nu & \cdots & h_1 \end{bmatrix}$$

$$\Rightarrow \quad \mathbf{G}_{suf} = \begin{bmatrix} \mathbf{0}_{\nu\times(N-\nu)} & \mathbf{J}_\nu \end{bmatrix} \tag{18}$$

where we have used the definitions of $\mathbf{H}_{pre}$ and $\mathbf{H}_{suf}$ in Equations (4) and (5), respectively. The expressions for $\mathbf{G}_{pre}$ and $\mathbf{G}_{post}$ in (17) and (18) imply that

$$x_{k-i} = x_{k+i-1} \quad : \quad 1 \le i \le \nu \tag{19}$$

$$x_{k+N+i-1} = x_{k+N-i} \quad : \quad 1 \le i \le \nu \tag{20}$$

This reveals the design strategy for the prefix and suffix sequences : they should be chosen as *symmetric extensions* of the information sequence on both ends with the axes of symmetry at the data-guard boundaries.

C. Receiver Signal Processing

1) Front-End Prefilter: The symmetry condition in (15) can be met in practice by implementing an FIR front-end prefilter, as follows :

- For channels with long memory, denoted by $L \ge (2\nu+1)$, we propose to modify the design criterion for the FIR channel shortening prefilter commonly used in DFT-based multicarrier transceivers (see e.g. [10]) by incorporating the symmetry constraint on the target (shortened) impulse response (TIR), as described briefly next. It was shown in [10] that the channel shortening mean square error (MSE) can be expressed in the following quadratic form

$$\mathrm{MSE} = \mathbf{h}^t\mathbf{R}\mathbf{h} \tag{21}$$

where $\mathbf{h} = \begin{bmatrix} h_{-\nu} & \cdots & h_{-1} & h_0 & h_1 & \cdots & h_\nu \end{bmatrix}^t$ is the shortened CIR and $\mathbf{R}$ is a positive-definite correlation matrix that depends on the original channel and noise characteristics. We can impose the symmetry condition in (15) on $\mathbf{h}$ by defining

$$\mathbf{h} = \tilde{\mathbf{I}}\bar{\mathbf{h}} \tag{22}$$

where $\bar{\mathbf{h}} = \begin{bmatrix} h_0 & \cdots & h_\nu \end{bmatrix}^t$ and the symmetric stacking matrix $\tilde{\mathbf{I}} = \begin{bmatrix} \mathbf{I}_{\nu+1} \\ \mathbf{J}_\nu \;\; \mathbf{0}_{\nu\times 1} \end{bmatrix}$. By combining (21) and (22), the prefilter design problem becomes

$$\text{minimize } \mathrm{MSE} = \bar{\mathbf{h}}^t\tilde{\mathbf{I}}^t\mathbf{R}\tilde{\mathbf{I}}\bar{\mathbf{h}} = \bar{\mathbf{h}}^t\bar{\mathbf{R}}\bar{\mathbf{h}}$$

subject to [6] the unit-norm constraint $\bar{\mathbf{h}}^t\bar{\mathbf{h}} = 1$. The optimum $\bar{\mathbf{h}}$ is well-known to be the eigenvector of $\bar{\mathbf{R}} = \tilde{\mathbf{I}}^t\mathbf{R}\tilde{\mathbf{I}}$ corresponding to its minimum eigenvalue. The optimum symmetric shortened CIR is then calculated using (22).

- For channels with short memory (compared to block length N), we can still use the prefilter design algorithm described above by setting the original CIR length *equal* to the TIR length (i.e. no shortening) and the prefilter only equalizes the CIR to a symmetric one in a MSE sense. Alternatively, to reduce computational complexity, this prefilter can be set to the *time-reversed* (matched) filter to a memory-ν channel resulting in an overall symmetric CIR with memory 2ν. The price paid is a reduction in throughput by a factor of $\frac{N+\nu}{N+2\nu}$ which is ≈ 1 for $N \gg \nu$.

In summary, satisfying the symmetric CIR condition in (15) can be achieved in practice using an FIR prefilter with computational complexity even less [7] than its counterpart in DFT-MCM.

[6] This constraint is introduced to avoid the trivial all-zeros solution.

[7] Because imposing the symmetry condition reduces the dimensionality of the prefilter optimization problem from $2\nu + 1$ to $\nu + 1$.

2) Detection Algorithms: By properly designing the guard sequence and the prefilter, Conditions (15) and (20) are met. Then, starting from (8) we get

$$\begin{aligned} \mathbf{y}_{k:k+N-1} &= \mathbf{H}_{eqv}\mathbf{x}_{k:k+N-1} + \mathbf{z}_{k:k+N-1} && (23)\\ &= \mathbf{C}^t\mathbf{D}\mathbf{C}\mathbf{x}_{k:k+N-1} + \mathbf{z}_{k:k+N-1} && (24)\\ \Rightarrow \mathbf{Y}_{k:k+N-1} &= \mathbf{C}\mathbf{y}_{k:k+N-1} && (25)\\ &= \mathbf{D}\mathbf{X}_{k:k+N-1} + \mathbf{Z}_{k:k+N-1} && (26) \end{aligned}$$

where capital letters denote the DCT-transformed quantities. Since $\mathbf{D}$ is a real diagonal matrix, the components of $\mathbf{X}_{k:k+N-1}$ (the DCT transform of the information vector) are decoupled and can be individually detected by applying a simple zero-forcing (or mean square error) scalar real equalizer followed by a slicer. A block diagram of this DCT-based multi-carrier transceiver architecture is shown in Figure 1.

D. Multi-Input Multi-Output Channels

The DCT-MCM transceiver proposed in this paper can be applied to a system with n_i inputs and n_o outputs by adding the symmetric prefix and suffix guard sequences to the length-N information blocks transmitted from each of the n_i inputs and by imposing the symmetry condition on each of the resulting $n_i n_o$ CIR's when implementing the MIMO FIR prefilter of [11]. As in the single-input single-output (SISO) case described in Section 3.3, we have to modify the design criterion of the prefilter in [11] to incorporate the symmetric TIR constraint by defining (c.f. (22))

$$\mathbf{h} = \begin{bmatrix} \mathbf{I}_{n_o(\nu+1)} \\ \tilde{\mathbf{J}}_{n_o\nu} \quad \mathbf{0}_{n_o\nu \times n_o} \end{bmatrix} \begin{bmatrix} \mathbf{h}_0 \\ \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_\nu \end{bmatrix} \qquad (27)$$

where each of the MIMO TIR coefficients $\mathbf{h}_i$ is an $n_o \times n_i$ matrix and $\tilde{\mathbf{J}}_{n_o\nu}$ is a block reversal matrix (has the identity matrices $\mathbf{I}_{n_o}$ on the main anti-diagonal and zeroes everywhere else).

Remarks :

1) The throughput of the transmission scheme proposed in this paper is double that in [12] which restricts the information sequence to be symmetric as well. In contrast, our scheme only restricts the guard sequence (which is composed of redundant symbols anyway) to be a symmetric extension of the information sequence.
2) Our DCT-MCM transceiver can be extended to the case of complex signalling over passband channels by designing the prefilter so that both the real and imaginary parts of the complex baseband-equivalent overall CIR are symmetric. The complex guard sequence is also set to be a symmetric extension of the complex information sequence in each block.
   An interesting topic for future research is to compare DFT-MCM and DCT-MCM under practical wireless channel impairments such as frequency offset, I/Q imbalance, multipath spread longer than CP leading to IBI and ICI effects, narrow-band interference, channel estimation errors, amplifier non-linearities, and Doppler (mobility) conditions leading to additional IBI/ICI degradation. The enhanced energy concentration property of DCT basis vectors (compared to DFT) could result in more robust performance under these impairments.

IV. Numerical Example

Consider a 2×2 MIMO Typical Urban (TU) wireless channel with 6 symbol-spaced taps that have the power delay profile (in dB) $\begin{bmatrix} -3 & 0 & -2 & -6 & -8 & -10 \end{bmatrix}$. A snap shot (single realization) of the four original CIR's, denoted by $\mathbf{p}_{ij}$ for $1 \le i, j \le 2$ where each is a 6-tap FIR filter, is given in Figure 2. We assume quasi-static fading over the transmission blocks and implement a 16-tap MIMO MMSE prefilter [8] to simultaneously shorten the four FIR channels to symmetric 3-tap ($\nu = 1$) channels. For PAM signalling and an input SNR of 20 dB, the 4 optimum symmetric TIR's are depicted in Figure 3. They can be simultaneously diagonalized by the DCT after adding a symmetric guard sequence (of length $2\nu = 2$) to each information block.

V. Conclusions

The DCT is an optimal modulation/demodulation basis for multi-carrier signalling on FIR frequency-selective channels when the overall CIR is symmetric and both the prefix and suffix guard sequences (of total length equal to the CIR memory) are symmetric extensions of the information sequence in each transmitted block. The first condition can be met by implementing an FIR prefilter whose complexity is less than that of its counterpart in DFT-MCM systems. The second

[8]We modified the *Identity Norm Constraint* (INC) design criterion in [11] by incoporating the symmetry constraint on the TIR given in (27).

condition is met by placing symmetry conditions only on the guard (not the information) sequence, hence, the guard overhead is identical to a DFT-MCM system. Furthermore, all signal processing operations involve only real arithmetic unlike DFT-MCM which involves complex arithmetic even for PAM signals. Finally, we showed how to generalize the DCT-MCM transceiver design to accommodate input energy optimization, complex signalling, and MIMO channels.

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REFERENCES

[1] J. Bingham. Multicarrier modulation for data transmission : An idea whose time has come. *IEEE Communications Magazine*, pages 5–14, May 1990.

[2] S. Kasturia, J. Aslanis, and J.M. Cioffi. Vector Coding for Partial-Response Channels. *IEEE Transactions on Information Theory*, 36(4):741–762, July 1990.

[3] N. Al-Dhahir and S.N.Diggavi. On the Choice of Guard Sequence for Block Transmission over Linear Dispersive Channels. *IEEE Trans. Communications*, pages 938–946, June 2000.

[4] S. Martucci. Symmetric Convolution and the Discrete Sine and Cosine Transforms. *IEEE Transactions on Signal Processing*, pages 1038–1051, May 1994.

[5] Y. Yeh and S. Chen. Efficient Channel Estimation Based on Discrete Cosine Transform. In *ICASSP*, pages 676–679, 2003.

[6] A.H. Sayed. *Fundamentals of Adaptive Filtering*. Wiley, 2003.

[7] V. Sanchez, P. Garcia, A. Peinado, J. Segura, and A. Rubio. Diagonalizing Properties of the Discrete Cosine Transform. *IEEE Transactions on Signal Processing*, pages 2631–2641, November 1995.

[8] N. Ahmed, T. Natarajan, and K. Rao. Discrete Cosine Transform. *IEEE Transactions on Computers*, pages 90–93, January 1974.

[9] T. Cover and J. Thomas. *Elements of Information Theory*. Wiley, 1991.

[10] N. Al-Dhahir and J.M.Cioffi. Efficiently-Computed Reduced-Parameter Input-Aided MMSE Equalizers for ML Detection : A Unified Approach. *IEEE Trans. Information Theory*, pages 903–915, May 1996.

[11] N. Al-Dhahir. FIR Channel-Shortening Equalizers for MIMO ISI Channels. *IEEE Trans. Communications*, 50:213–218, February 2001.

[12] G. Mandyam. On the Discrete Cosine Transform and OFDM Systems. In *ICASSP*, pages 544–547, 2003.

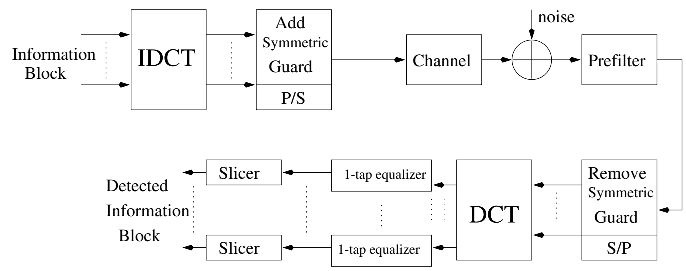

Fig. 1. DCT-MCM Block Diagram

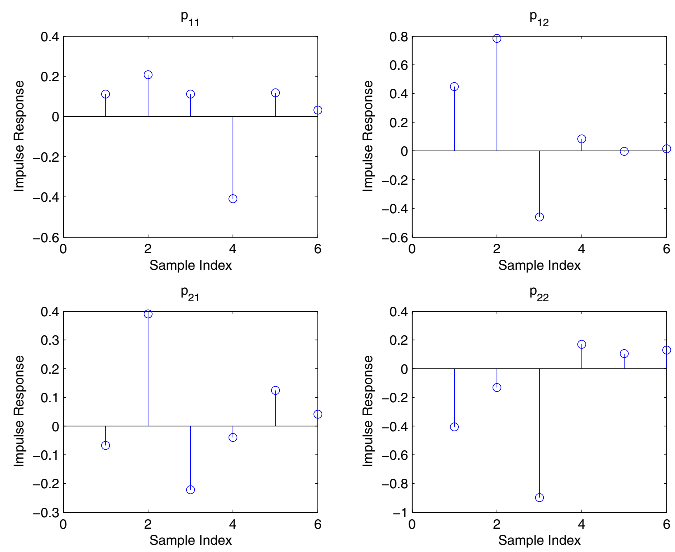

Fig. 2. Typical Urban Wireless MIMO Channel Impulse Response in Example 3

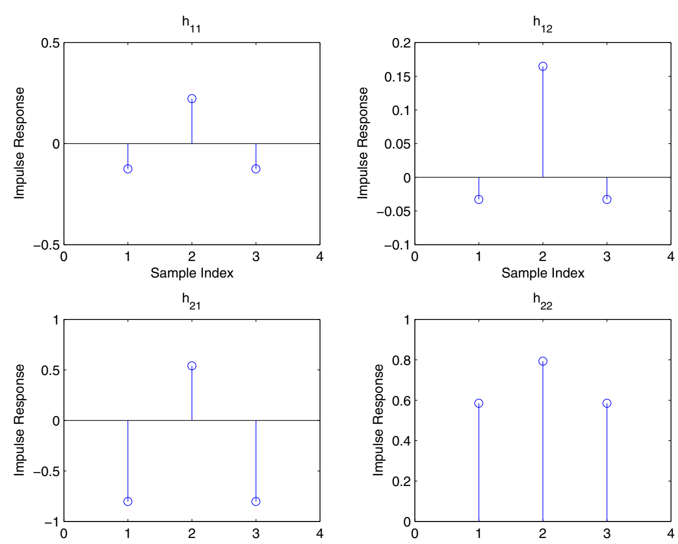

Fig. 3. Overall Symmetric CIR in Example 3 After Prefiltering