Linear Beamforming Assisted Receiver for Binary Phase Shift Keying Modulation Systems

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ABSTRACT

The paper considers adaptive beamforming assisted receiver for multiple antenna aided multiuser systems that employ binary phase shift keying (BPSK) modulation. The standard minimum mean square error (MMSE) design is based on the principle of minimising the mean square error (MSE) between the beamformer's desired output and complex-valued beamformer output. Since the desired output for BPSK systems is real-valued, minimising the MSE between the beamformer's desired output and real-part of the beamformer output can significantly improve the bit error rate (BER) performance, and we refer to this alternative MMSE design as the real-valued MMSE (RV-MMSE) to contrast to the standard complex-valued MMSE (CV-MMSE). The minimum BER (MBER) design however still outperforms the RV-MMSE solution, particularly for overloaded systems where degree of freedom of the antenna array is smaller than the number of BPSK users. Adaptive implementation of this RV-MMSE design is realised using a least mean square (LMS) type adaptive algorithm, which we refer to as the RV-LMS, in comparison to the standard CV-LMS algorithm. The RV-LMS adaptive beamformer has the same computational complexity as the adaptive least bit error (LBER) algorithm, imposing half of the computational requirements of the CV-LMS algorithm.

I. INTRODUCTION

The ever-increasing demand for mobile communication capacity has motivated the development of adaptive antenna array assisted spatial processing techniques [1]–[10] in order to further improve the achievable spectral efficiency. A technique that has shown real promise in achieving substantial capacity enhancements is the use of adaptive beamforming with antenna arrays. Through appropriately combining the signals received by the different elements of an antenna array, adaptive beamforming is capable of separating signals transmitted on the same carrier frequency, and thus provides a practical means of supporting multiusers in a space division multiple access scenario. Classically, the beamforming process is carried out by minimising the mean square error (MSE) between the desired output and the actual array output. For a communication system, however, it is the bit error rate (BER) that really matters. Adaptive beamforming based on directly minimising the system's BER has been proposed for binary phase shift keying (BPSK) and quadrature phase shift keying modulation schemes [11],[12].

This paper specifically considers adaptive beamforming for BPSK systems. The standard minimum MSE (MMSE) design [13] seeks the complex-valued (CV) beamformer's weight vector that minimises the MSE between the beamformer's desired output and the CV beamformer output. We will refer to this MMSE solution as the CV-MMSE. A practical rule is that, the number of antennas should not be smaller than the number of users supported, and the CV-MMSE beamforming has the capacity of supporting up to the same number of users as the number of antenna elements as this will ensure a sufficient degree of freedom to cancel the interfering signal sources. For BPSK systems, however, the beamformer's desired output, namely the desired user's transmitted symbol, is real-valued (RV). We show that by minimising the MSE between the beamformer's desired output and the real part of the beamformer output, the achievable system's BER performance can significantly be enhanced. We will refer to this alternative MMSE design as the RV-MMSE, in contrast with the standard CV-MMSE. Moreover, using the RV-MMSE design, the system should be capable of supporting up to twice the number of users as the number of antenna elements, since the signal of each antenna array element is two-dimensional or CV. A drawback of this RV-MMSE design is that, unlike the case of the CV-MMSE solution, there exists no closed-form solution and numerical optimisation based on gradient algorithm has to be applied to arrive at a numerical solution.

The minimum BER (MBER) beamforming design [11] is the true optimal solution and it generally outperforms the RV-MMSE solution, particularly for overloaded systems where degree of freedom of the antenna array is smaller than the number of BPSK users. The CV-MMSE solution can adaptively be implemented using the least mean square (LMS) algorithm [13], and we will refer to this standard LMS algorithm as the CV-LMS. We derive an adaptive implementation of the RV-MMSE design based on a LMS-type adaptive algorithm, which we refer to as the RV-LMS. The computational complexity of this RV-LMS algorithm is similar to that of the adaptive MBER algorithm known as the least bit error rate (LBER) [11],[14], imposing only half of the computational requirements of the CV-LMS algorithm.

II. SYSTEM MODEL

The system consists of M users, and each user transmits a BPSK signal on the same carrier frequency $\omega = 2\pi f$. The receiver is equipped with a linear antenna array consisting of L uniformly spaced elements. Assume that the channel is narrow-band which does not induce intersymbol interference. Then the symbol-rate received signal samples can be expressed as

$$x_l(k) = \sum_{i=1}^{M} A_i b_i(k) e^{j\omega t_l(\theta_i)} + n_l(k) = \bar{x}_l(k) + n_l(k), \quad (1)$$

for $1 \le l \le L$, where $t_l(\theta_i)$ is the relative time delay at element l for source i with θ_i being the direction of arrival for source i, $n_l(k)$ is a complex-valued Gaussian white noise with $E[|n_l(k)|^2] = 2\sigma_n^2$,

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TABLE I

LOCATIONS OF USERS IN TERMS OF ANGLE OF ARRIVAL FOR THE SIMULATION.

user i	1	2	3	4	5	6	7	8	9	10
AOA θ	0°	10°	-20°	30°	-45°	50°	60°	-55°	-35°	-60°

A_i is the channel coefficient for user i, and $b_i(k)$ is the kth symbol of user i which takes the value from the BPSK symbol set $\{\pm 1\}$. Source 1 is the desired user and the rest of the sources are interfering users. The desired-user signal to noise ratio is SNR$= |A_1|^2\sigma_b^2/2\sigma_n^2$ and the desired signal to interferer i ratio is SIR$_i = A_1^2/A_i^2$, for $2 \leq i \leq M$, where $\sigma_b^2 = 1$ is the symbol energy. The received signal vector $\mathbf{x}(k) = [x_1(k)\, x_2(k) \cdots x_L(k)]^T$ is given by

$$\mathbf{x}(k) = \mathbf{P}\mathbf{b}(k) + \mathbf{n}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k), \tag{2}$$

where $\mathbf{n}(k) = [n_1(k)\, n_2(k) \cdots n_L(k)]^T$, the system matrix $\mathbf{P} = [\mathbf{p}_1\, \mathbf{p}_2 \cdots \mathbf{p}_M] = [A_1\mathbf{s}_1\, A_2\mathbf{s}_2 \cdots A_M\mathbf{s}_M]$ with the steering vector for source i $\mathbf{s}_i = [e^{j\omega t_1(\theta_i)}\, e^{j\omega t_2(\theta_i)} \ldots e^{j\omega t_L(\theta_i)}]^T$, and the transmitted user symbol vector $\mathbf{b}(k) = [b_1(k)\, b_2(k) \cdots b_M(k)]^T$.

A linear beamformer is employed, whose soft output is given by

$$y(k) = \mathbf{w}^H\mathbf{x}(k) = \mathbf{w}^H(\bar{\mathbf{x}}(k) + \mathbf{n}(k)) = \bar{y}(k) + e(k) \tag{3}$$

where $\mathbf{w} = [w_1\; w_2 \cdots w_L]^T$ is the beamformer weight vector and $e(k)$ is Gaussian distributed with zero mean and $E[|e(k)|^2] = 2\sigma_n^2\mathbf{w}^H\mathbf{w}$. The beamformer's hard decision is given by

$$\hat{b}_1(k) = \mathrm{sgn}(y_R(k)), \tag{4}$$

where $\hat{b}_1(k)$ is the estimate of $b_1(k)$ and $y_R(k) = \Re[y(k)]$ denotes the real part of $y(k)$.

III. BEAMFORMER DESIGNS

The task of designing the beamformer (3) is to choose the beamformer's weight vector $\mathbf{w}$ according to some design criterion.

A. Complex-Valued Minimum Mean Square Error Design

Classically, the beamformer's weight vector $\mathbf{w}$ is determined by minimising the MSE metric of

$$J_{\mathrm{MSE}}(\mathbf{w}) = E[|b_1(k) - y(k)|^2]. \tag{5}$$

Setting the gradient of $J_{\mathrm{MSE}}(\mathbf{w})$

$$\nabla J_{\mathrm{MSE}}(\mathbf{w}) = -2\mathbf{p}_1 + 2\left(\mathbf{P}\mathbf{P}^H + 2\sigma_n^2\mathbf{I}_L\right)\mathbf{w} \tag{6}$$

to zero leads to the following closed-form CV-MMSE solution [13]

$$\mathbf{w}_{\mathrm{CMMSE}} = \left(\mathbf{P}\mathbf{P}^H + 2\sigma_n^2\mathbf{I}_L\right)^{-1}\mathbf{p}_1, \tag{7}$$

where $\mathbf{I}_L$ denotes the $L \times L$ identity matrix. An adaptive implementation of the CV-MMSE solution can readily be realised using the CV-LMS algorithm [13]

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\left(b_1(k) - y(k)\right)^*\mathbf{x}(k), \tag{8}$$

where μ is the step size.

B. Real-Valued Minimum Mean Square Error Design

For BPSK systems, the beamformer's desired output $b_1(k)$ is RV. The CV-MMSE solution minimises the MSE (5), which can be decomposed into

$$\begin{aligned} J_{\mathrm{MSE}}(\mathbf{w}) &= E[(b_1(k) - y_R(k))^2] + E[y_I^2(k)] \\ &= J_{\mathrm{rpMSE}}(\mathbf{w}) + J_{\mathrm{ipMSE}}(\mathbf{w}), \end{aligned} \tag{9}$$

where $y_I(k) = \Im[y(k)]$. It is clearly that the CV-MMSE solution attempts to simultaneously minimise the MSE between the desired signal and the real part of the beamformer's output as well as the energy of the imaginary part of the beamformer's output. However, the beamformer's decision depends only on $y_R(k)$. Minimising $J_{\mathrm{ipMSE}}(\mathbf{w})$ does not contribute to improving the beamformer's performance. Rather it imposes a unnecessary constraint on the solution and wastes the antenna array resource.

It is also clear that a more intelligent way of designing the beamformer is to minimise the MSE between the desired output and the real part of the beamformer's output

$$J_{\mathrm{rpMSE}}(\mathbf{w}) = E[(b_1(k) - y_R(k))^2]. \tag{10}$$

The RV-MMSE solution is defined by

$$\mathbf{w}_{\mathrm{RMMSE}} = \arg\min_{\mathbf{w}} J_{\mathrm{rpMSE}}(\mathbf{w}). \tag{11}$$

The gradient of $J_{\mathrm{rpMSE}}(\mathbf{w})$ is

$$\begin{aligned} \nabla J_{\mathrm{rpMSE}}(\mathbf{w}) &= E[-(b_1(k) - y_R(k))\mathbf{x}(k)] \\ &= -\mathbf{p}_1 + \left(\mathbf{P}\mathbf{P}_R^T + \sigma_n^2\mathbf{I}_L\right)\mathbf{w}_R \\ &\quad + \left(\mathbf{P}\mathbf{P}_I^T + \sigma_n^2\mathbf{I}_L\right)\mathbf{w}_I, \end{aligned} \tag{12}$$

where $\mathbf{P} = \mathbf{P}_R + j\mathbf{P}_I$ and $\mathbf{w} = \mathbf{w}_R + j\mathbf{w}_I$. It is seen from (12) that there exists no closed-form solution for this RV-MMSE design. Numerical optimisation has to be applied to obtain a $\mathbf{w}_{\mathrm{RMMSE}}$ based on for example the steepest descent method or the simplified conjugate gradient algorithm [11],[14],[15]. To derive a sample-by-sample adaptive implementation of this RV-MMSE solution, the stochastic gradient, namely $-(b_1(k) - y_R(k))\mathbf{x}(k)$, can be used, leading to the following RV-LMS algorithm

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\left(b_1(k) - y_R(k)\right)\mathbf{x}(k). \tag{13}$$

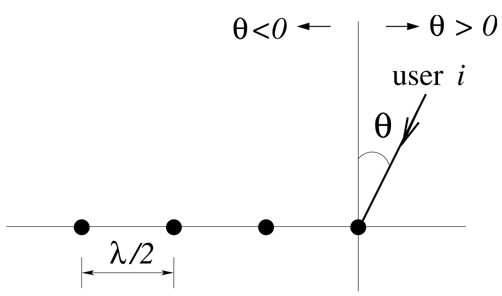


Fig. 1. Geometric structure of the four-element linear array having $\lambda/2$ spacing used in the simulation, where λ is the wavelength.

C. Minimum Bit Error Rate Design

As recognized by [11], the best strategy is to choose $\mathbf{w}$ by directly minimising the system's BER. Following the notations used in [11],[14], let us denote the $N_b = 2^M$ number of possible transmitted symbol sequences of $\mathbf{b}(k)$ as $\mathbf{b}^{(q)}$, $1 \leq q \leq N_b$. Denote furthermore the first element of $\mathbf{b}^{(q)}$, corresponding to the desired symbol $b_1(k)$, as $b_1^{(q)}$. The noise-free part of the beamformer's output $\bar{y}(k)$ assumes values from the signal state set

$$\mathcal{Y} = \{\bar{y}^{(q)} = \mathbf{w}^H \bar{\mathbf{x}}^{(q)} = \mathbf{w}^H \mathbf{P} \mathbf{b}^{(q)},\ 1 \leq q \leq N_b\}, \tag{14}$$

and $\mathcal{Y}$ can be partitioned into the two subsets conditioned on the value of $b_1(k)$

$$\mathcal{Y}^{(\pm)} = \{\bar{y}^{(q,\pm)} \in \mathcal{Y} : b_1(k) = \pm 1\}. \tag{15}$$

Thus $\bar{y}_R(k)$ can only take the values from the set

$$\mathcal{Y}_R = \{\bar{y}_R^{(q)} = \Re[\bar{y}^{(q)}],\ 1 \leq q \leq N_b\}, \tag{16}$$

and $\mathcal{Y}_R$ can be divided into the two subsets conditioned on $b_1(k)$

$$\mathcal{Y}_R^{(\pm)} = \{\bar{y}_R^{(q,\pm)} \in \mathcal{Y}_R : b_1(k) = \pm 1\}. \tag{17}$$

The conditional probability density function (PDF) of $y(k)$ given $b_1(k) = +1$ is a Gaussian mixture defined by

$$p(y|+1) = \frac{1}{N_{sb}} \sum_{q=1}^{N_{sb}} \frac{1}{2\pi\sigma_n^2 \mathbf{w}^H \mathbf{w}} e^{-\frac{|y - \bar{y}^{(q,+)}|^2}{2\sigma_n^2 \mathbf{w}^H \mathbf{w}}}, \tag{18}$$

where $\bar{y}^{(q,+)} \in \mathcal{Y}^{(+)}$ and $N_{sb} = N_b/2$ is the size of $\mathcal{Y}^{(+)}$. Thus the marginal conditional PDF of $y_R(k)$ is

$$p(y_R|+1) = \frac{1}{N_{sb}} \sum_{q=1}^{N_{sb}} \frac{1}{\sqrt{2\pi\sigma_n^2 \mathbf{w}^H \mathbf{w}}} e^{-\frac{\left(y_R - \bar{y}_R^{(q,+)}\right)^2}{2\sigma_n^2 \mathbf{w}^H \mathbf{w}}}, \tag{19}$$

where $\bar{y}_R^{(q,+)} \in \mathcal{Y}_R^{(+)}$. The BER of the beamformer with the weight vector $\mathbf{w}$ can be shown to be [11],[14]

$$P_E(\mathbf{w}) = \frac{1}{N_{sb}} \sum_{q=1}^{N_{sb}} Q\left(g^{(q,+)}(\mathbf{w})\right), \tag{20}$$

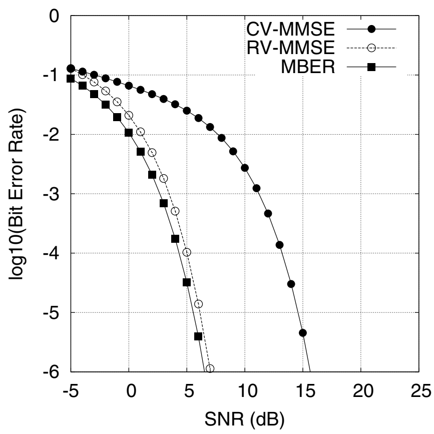
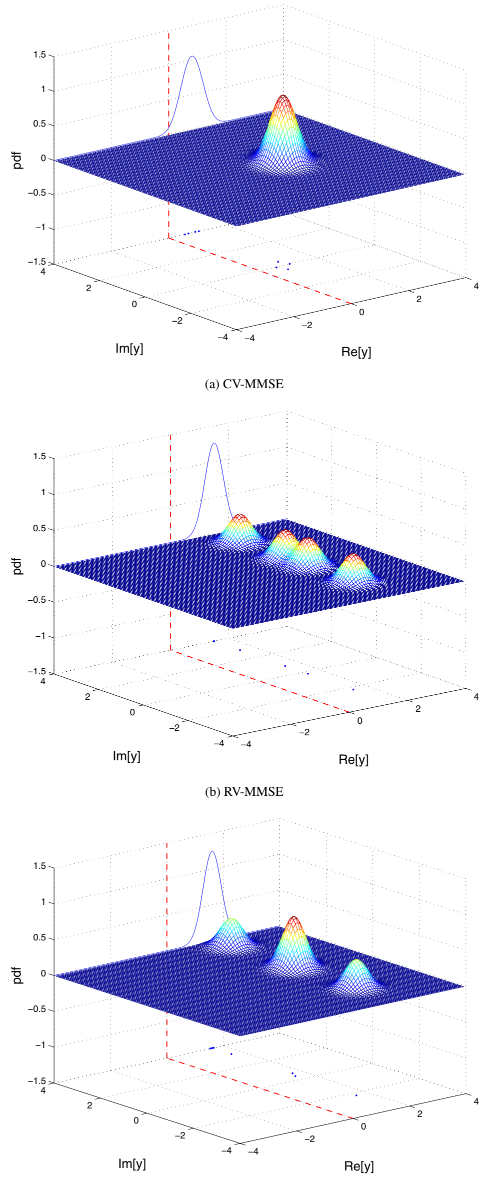


Fig. 2. BER comparison of three beamforming designs for the four-element array system supporting 3 users.



(a) CV-MMSE

(b) RV-MMSE

(c) MBER

Fig. 3. Conditional probability density functions $p(y|+1)$ (surfaces), marginal conditional probability density functions $p(y_R|+1)$ (curves), signal state subsets $\mathcal{Y}^{(+)}$ and $\mathcal{Y}_R^{(+)}$ (points) for the four-element array system supporting 3 users with SNR= 7 dB. The beamformer weight vector is normalised to a unit length.

where

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-\frac{v^2}{2}}\, dv \tag{21}$$

and

$$g^{(q,+)}(\mathbf{w}) = \frac{\mathrm{sgn}(b_1^{(q)}) \bar{y}_R^{(q,+)}}{\sigma_n \sqrt{\mathbf{w}^H \mathbf{w}}}. \tag{22}$$

The BER can alternatively be computed based on the other subset $\mathcal{Y}_R^{(-)}$. Note that the BER is invariant to a positive scaling of $\mathbf{w}$.

The MBER solution for the beamformer is then defined as the weight vector that minimises the error probability (20)

$$\mathbf{w}_{\text{MBER}} = \arg\min_{\mathbf{w}} P_E(\mathbf{w}). \tag{23}$$

The gradient of $P_E(\mathbf{w})$ with respect to $\mathbf{w}$ is given by

$$\nabla P_E(\mathbf{w}) = \frac{1}{2N_{sb}\sqrt{2\pi}\sigma_n\sqrt{\mathbf{w}^H\mathbf{w}}} \sum_{q=1}^{N_{sb}} e^{-\frac{\left(\bar{y}_R^{(q,+)}\right)^2}{2\sigma_n^2\mathbf{w}^H\mathbf{w}}}$$

$$\times \text{sgn}\left(b_1^{(q)}\right)\left(\frac{\bar{y}_R^{(q,+)}\mathbf{w}}{\mathbf{w}^H\mathbf{w}} - \bar{\mathbf{x}}^{(q,+)}\right), \tag{24}$$

where $\bar{y}_R^{(q,+)} = \Re[\mathbf{w}^H\bar{\mathbf{x}}^{(q,+)}] \in \mathcal{Y}_R^{(+)}$. Given the gradient (24), the optimisation problem (23) can be solved using a gradient-based algorithm [11],[14],[15]. Following the derivations presented in [11],[14], an adaptive implementation of the MBER solution can be realised using the LBER algorithm which takes the form of

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\frac{\text{sgn}(b_1(k))}{2\sqrt{2\pi}\rho_n} e^{-\frac{y_R^2(k)}{2\rho_n^2}}\mathbf{x}(k), \tag{25}$$

where ρ_n is the kernel width.

D. *Comparison of Three Designs*

The CV-MMSE solution minimises the MSE between $b_1(k)$ and $y(k)$. Therefore, the associated conditional signal subset $\mathcal{Y}^{(+)}$ must have a symmetric distribution with respect to $\Re[y]$ and $\Im[y]$ axes. This imposes an unnecessary constraint and limits the achievable BER performance, since only the distribution of $\mathcal{Y}_R^{(+)}$ influences the BER performance. By removing the unnecessary constraint on $y_I(k)$, the RV-MMSE solution has more freedom in designing a more favourable distribution of $\mathcal{Y}_R^{(+)}$, leading to an improved BER. The minimum distance between the decision threshold $y_R = 0$ and the subset $\mathcal{Y}_R^{(+)}$ ultimately determines the BER. Minimising $J_{\text{rpMSE}}(\mathbf{w})$ does not guarantees maximising this minimum distance. The MBER solution ensures that this minimum distance is maximised and, therefore, the MBER design generally provides a smaller BER than the RV-MMSE design. In terms of the computational requirements per weight updating, it can be shown that the RV-LMS and LBER algorithms have a similar complexity, which is about half of the complexity required by the CV-LMS algorithm.

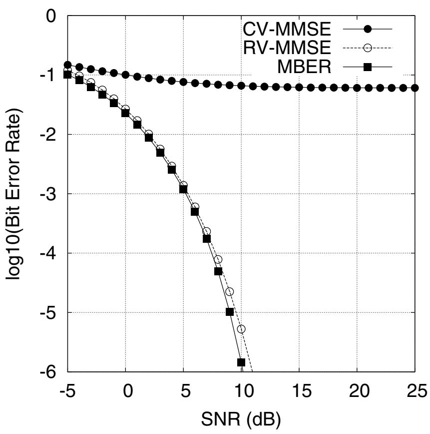
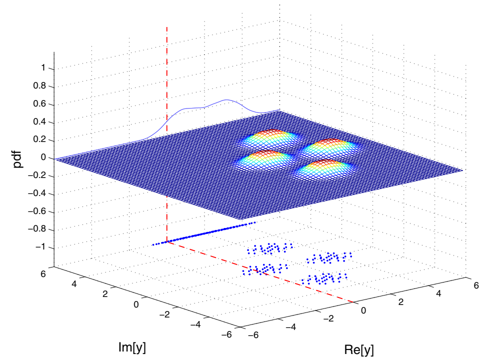
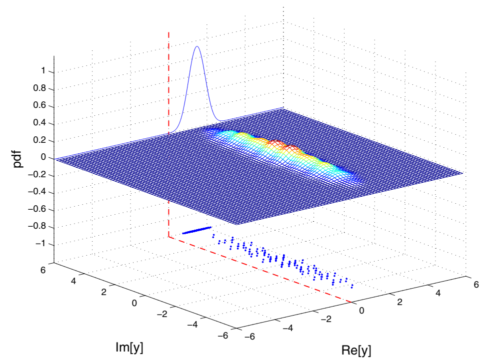


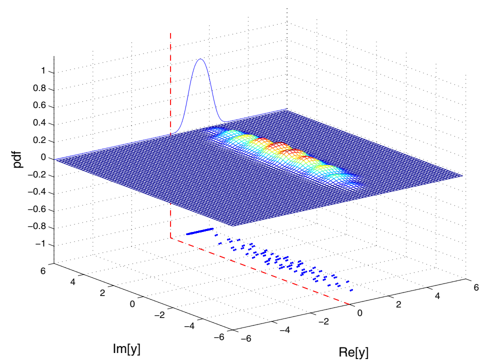
Fig. 4. BER comparison of three beamforming designs for the four-element array system supporting 8 users.



(a) CV-MMSE



(b) RV-MMSE



(c) MBER

Fig. 5. Conditional probability density functions $p(y|+1)$ (surfaces), marginal conditional probability density functions $p(y_R|+1)$ (curves), signal subsets $\mathcal{Y}^{(+)}$ and $\mathcal{Y}_R^{(+)}$ (points) for the four-element array system supporting 8 users with SNR= 8 dB. The beamformer weight vector is normalised to a unit length.

In order for the CV-MMSE solution to perform adequately, sufficient antenna array resource is required so that the interfering signals can be cancelled. Thus, in order to ensure a correct separation of $\mathcal{Y}_R^{(+)}$ and $\mathcal{Y}_R^{(-)}$ by the decision threshold $y_R = 0$, it is generally required that the number of users is no more than the number of array elements. For the CV-MMSE beamformer, therefore, a system is overloaded if $M > L$. By intelligently concentrating on the real part of the beamformer's output, the RV-MMSE design effectively doubles the degree of freedom in beamforming, since each input

$x_l(k)$ is CV or two-dimensional. Thus, the RV-MMSE design is capable of supporting users up to twice the number of array elements. Therefore, for the RV-MMSE design, a system is overloaded only if $M > 2L$. The MBER design is no restricted by this limit and is capable of supporting more users.

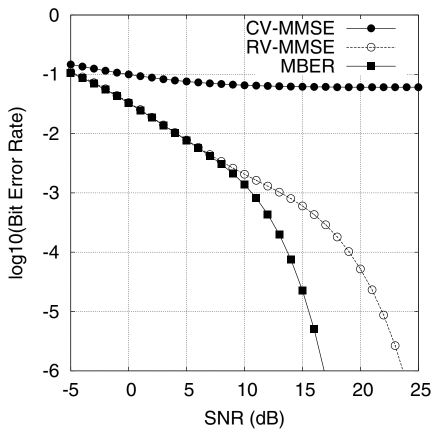
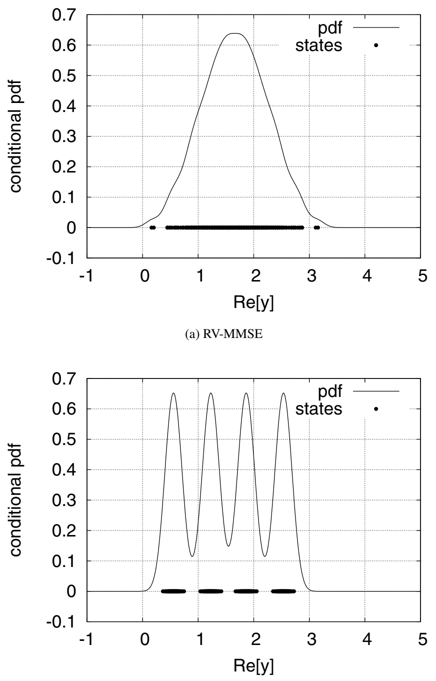


Fig. 6. BER comparison of three beamforming designs for the four-element array system supporting 9 users.



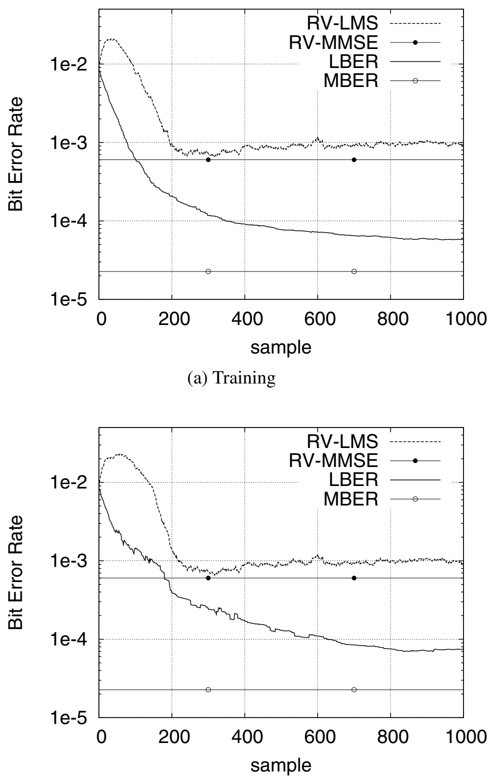
(a) RV-MMSE

(b) MBER

Fig. 7. Marginal conditional probability density functions $p(y_R|+1)$ (curves) and signal subsets $\mathcal{Y}_R^{(+)}$ (points) for the four-element array system supporting 9 users with SNR$= 15$ dB. The beamformer weight vector is normalised to a unit length.

IV. Simulation Study

The simulated system consisted of a four-element linear antenna array and supported up to $M = 10$ users. Fig. 1 shows the array geometric structure and Table I lists the locations of users with respect to the antenna array. The simulated channel conditions were $A_i = 1.0 + j0.0$ for all users and, therefore, SIR$_i = 0$ dB for all i. Fig. 2 compares the BER performance of the three beamformer designs when only the first 3 users were active. Given SNR= 7 dB, Fig. 3 depicts the conditional PDFs $p(y|+1)$, marginal conditional PDFs $p(y_R|+1)$, signal subsets $\mathcal{Y}^{(+)}$ and $\mathcal{Y}_R^{(+)}$ for the three designs, where the beamformer weight vector $\mathbf{w}$ was normalised to a unit length. It can be seen from Fig. 3 (a) that the distribution $p(y|+1)$ is symmetric for the CV-MMSE solution. By contrast, the RV-MMSE and MBER designs are not restricted by this symmetric constraint and spread $p(y|+1)$ more widely along the $\Im[y]$ axis, resulting in a more favourable distribution of $p(y_R|+1)$. It can also be seen from Fig. 3 (a) that the CV-MMSE solution is able to correctly separate $\mathcal{Y}_R^{(-)}$ and $\mathcal{Y}_R^{(+)}$ and thus provide an adequate BER performance as seen in Fig. 2.



(a) Training

(b) Decision directed adaptation after 40-symbol training

Fig. 8. Learning curves of the adaptive RV-LMS and LBER algorithms averaged over 100 runs for the four-element array system supporting 9 users with SNR$= 15$ dB. The step size $\mu = 0.005$ for the RV-LMS, the step size $\mu = 0.01$ and kernel variance $\rho_n^2 = 2\sigma_n^2$ for the LBER.

When the number of users was increased to $M > 4$, the CV-MMSE solution was no longer able to provide this desired separation, resulting in a high BER floor. Fig. 4 compares the BER performance of the three beamformer designs when the first 8 users were active, while Fig. 5 shows the conditional PDFs $p(y|+1)$, marginal conditional PDFs $p(y_R|+1)$, signal subsets $\mathcal{Y}^{(+)}$ and $\mathcal{Y}_R^{(+)}$ for the three designs, given SNR= 8 dB. It can be seen from Fig. 5 (a) that some points of $\mathcal{Y}_R^{(+)}$ are in the wrong side of $y_R = 0$ for the CV-MMSE solution, resulting in the high BER floor as shown in Fig. 4. By contrast, the RV-MMSE design is still capable of obtaining a distribution that is similar to the MBER design.

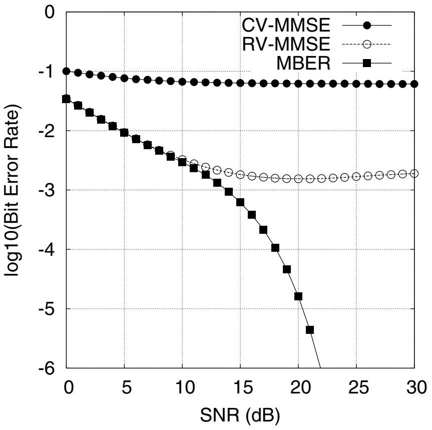


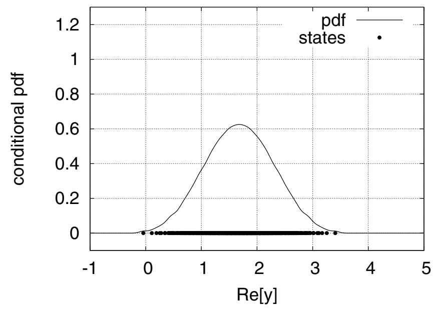
Fig. 9. BER comparison of three beamforming designs for the four-element array system supporting 10 users.

Fig. 6 compares the BER performance of the three beamformer designs when the first 9 users were active, while Fig. 7 shows the marginal conditional PDFs $p(y_R|+1)$ and signal subsets $\mathcal{Y}_R^{(+)}$ for the RV-MMSE and MBER designs, given SNR$=15$ dB. The RV-LMS and LBER algorithms were investigated, and Fig. 8 shows the convergence performance of the two adaptive algorithms averaged over 100 runs, given SNR$=15$ dB. In Fig. 8 (a), training was carried out over the whole length, while in Fig. 8 (b), after 40-symbol training, the decision directed (DD) adaptation was invoked by substituting $\hat{b}_1(k)$ for $b_1(k)$.

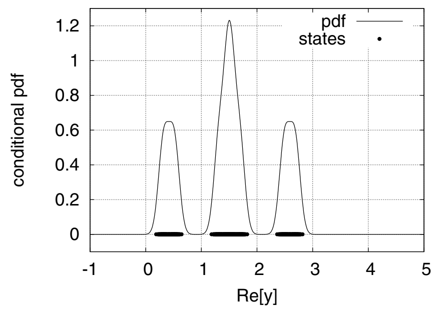
Finally, Fig. 9 compares the BER performance of the three beamformers when all the 10 users were active, while Fig. 10 shows the marginal conditional PDFs $p(y_R|+1)$ and signal subsets $\mathcal{Y}_R^{(+)}$ for the RV-MMSE and MBER designs, given SNR$=20$ dB. Note that in Fig. 10 (a) a point of $\mathcal{Y}_R^{(+)}$ is in the wrong side of the decision threshold $y_R=0$. It is seen that the RV-MMSE was no longer capable of separating $\mathcal{Y}_R^{(-)}$ and $\mathcal{Y}_R^{(+)}$ correctly and exhibited a BER floor, since the system was overloaded. By contrast, the MBER design was still able to separate $\mathcal{Y}_R^{(-)}$ and $\mathcal{Y}_R^{(+)}$ correctly and provided a much better BER performance than the RV-MMSE design.

V. Conclusions

An alternative MMSE design has been considered for beamforming assisted BPSK receiver, which minimises the MSE between the real-valued desired output and the real part of the complex-valued beamformer output. This RV-MMSE design offers significant performance enhancement over the standard CV-MMSE design. A drawback of this RV-MMSE design, as compared with the CV-MMSE design, is that there exists no closed-form solution and numerical optimisation based on a gradient algorithm has to be used. It has been demonstrated that the RV-MMSE beamforming solution is capable of obtaining a BER performance that is close to the optimal MBER solution for supporting BPSK users up to twice of the number of antenna array elements. The MBER design is capable of supporting more users than the RV-MMSE design. Adaptive algorithms for implementing these three beamforming designs have also been compared.



(a) RV-MMSE



(b) MBER

Fig. 10. Marginal conditional probability density functions $p(y_R|+1)$ (curves) and signal subsets $\mathcal{Y}_R^{(+)}$ (points) for the four-element array system supporting 10 users with SNR$=20$ dB. The beamformer weight vector is normalised.

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