# Full-Rate Full-Diversity Achieving MIMO Precoding with Partial CSIT

Biswajit Dutta, Somsubhra Barik and A. Chockalingam Department of ECE, Indian Institute of Science, Bangalore 560012, India

#### Abstract

In this paper, we consider a  $n_t \times n_r$  multiple-input multiple-output (MIMO) channel subjected to block fading. Reliability (in terms of achieved diversity order) and rate (in number of symbols transmitted per channel use) are of interest in such channels. We propose a new precoding scheme which achieves both full diversity  $(n_t n_r th order$ diversity) as well as full rate ( $n_t$  symbols per channel use) using partial channel state information at the transmitter (CSIT), applicable in MIMO systems including  $n_r < n_t$ asymmetric MIMO. The proposed scheme achieves full diversity and improved coding gain through an optimization over the choice of constellation sets. The optimization maximizes  $d_{min}^2$  for our precoding scheme subject to an energy constraint. The scheme requires feedback of  $n_t - 1$  angle parameter values, compared to  $2n_t n_r$  real coefficients in case of full CSIT. Error rate performance results for  $3 \times 1$ ,  $3 \times 2$ ,  $4 \times 1$ ,  $8 \times 1$ precoded MIMO systems (with  $n_t = 3, 3, 4, 8$  symbols per channel use, respectively) show that the proposed precoding achieves 3rd, 6th, 4th and 8th order diversities, respectively. These performances are shown to be better than other precoding schemes in the literature; the better performance is due to the choice of the signal sets and the feedback angles in the proposed scheme.

Keywords – MIMO precoding, asymmetric MIMO, full rate, full diversity, partial CSIT, optimum con-

stellation sets.

### 1 Introduction

Multiple-input multiple-output (MIMO) techniques can achieve high data rates and spatial diversity in wireless communications over fading channels [1],[2]. Spatial multiplexing (V-BLAST) with  $n_t$  antennas at the transmitter achieves the full rate of  $n_t$  symbols per channel use, but does not achieve transmit diversity. Space constraints in user terminals like mobile/portable receivers make asymmetric MIMO configuration with  $n_r < n_t$  antennas at the receiver to be a preferred choice. Precoding techniques can improve performance through the use of channel state information at the transmitter (CSIT). Several precoding methods use CSIT and achieve diversity benefits, but compromise on the achieved rate; e.g., only one symbol per channel use is achieved in transmit beamforming [3]. It is desirable that precoding methods achieve high rates (preferably the full rate of  $n_t$  symbols per channel use as in V-BLAST) and high diversity orders (preferably the full cliversity order of  $n_t n_r$ )<sup>1</sup> with partial CSIT. Our work reported in this paper addresses this problem.

A vast body of research in the literature has addressed the problem of MIMO precoding. In particular, precoding using partial CSIT/limited feedback has been of interest because providing full CSIT (which refers to the full knowledge of all the channel gains between transmit and receive antennas) through feedback can be too expensive [4]-[7].

Precoding in spatial multiplexing (V-BLAST) systems has been considered for achieving high rates and transmit diversity [9]-[14]. Precoding schemes in [10]-[12] incur some loss in rate. For example, the Grassmannian subspace packing based precoding in [10] does not allow simultaneous transmission of more than  $n_t-1$  streams (i.e., achievable rate is  $\leq n_t-1$  symbols per channel use). The precoding scheme in [13] achieves full rate asymptotically<sup>2</sup> using D-BLAST architecture with partial CSIT, but achieves full diversity only for min $(n_t, n_r) = 2$ and less diversity orders for other cases. Other recent works have proposed to improve the diversity gain of singular value decomposition (SVD) precoding<sup>3</sup> by pairing good and bad subchannels, and jointly coding information across each pair of subchannels [15],[16]. In the schemes in [15],[16], though the achieved diversity orders are high, namely  $(n_t - \frac{n_s}{2} + 1)(n_r - \frac{n_s}{2} + 1)$  where  $n_s \leq n_r$  is even number of streams, full diversity is not guaranteed. Also,

<sup>&</sup>lt;sup>1</sup>In this paper, 'full rate' is defined as  $n_t$  symbols per channel use regardless of the number of receive antennas, and 'full diversity' is defined as  $n_t n_r$ th diversity.

<sup>&</sup>lt;sup>2</sup>The scheme in [13] achieves a rate of 2T - 1 symbols per T channel uses in  $2 \times 2$  system, which achieves close to full rate (of  $n_t = 2$ ) for large number of channel uses.

<sup>&</sup>lt;sup>3</sup>SVD precoding achieves the full rate of  $n_t$  symbols per channel use but its diversity order is only 1.

Precoding	Channel	Rate	Diversity		
	Knowledge				
Grassmannian	Partial CSIT	Not full	Full		
packing scheme [3]		1  symbol/chl. use	$n_t n_r$		
Grassmannian	Partial CSIT	Not full	Not Full		
packing scheme [10]		$< n_t$ symbols/chl. use			
SVD precoding	Full CSIT	Not full	Not full		
		$min(n_t, n_r)$ symbols/chl. use	1		
$E-d_{min}$ Precoder in [15]*	Full CSIT	Not full rate for $n_r < n_t$	Not full		
		$n_s$ symbols/chl. use	$(n_t - \frac{n_s}{2} + 1)(n_r - \frac{n_s}{2} + 1)$		
		$n_s \leq n_r,  n_t \geq n_r,  n_s$ even	$n_s = \min(n_t, n_r), n_s$ even		
X-, Y-precoders in $[16]^*$	Full CSIT	same as in $[15]$	same as in $[15]$		
Precoder in $[13]^{**}$	Partial CSIT	Asymptotically full	Not full for		
		2T-1 symbols in T chl. uses	$\min(n_t, n_r) \neq 2$		
		for $n_t = n_r = 2$			
Precoder in [14]	Partial CSIT	Full	Not full		
	one angle	$n_t$ symbols/chl. use			
Proposed	Partial CSIT	Full	Full		
	$n_t - 1$ angles	$n_t$ symbols/chl. use	$n_t n_r$		

Table	1:	Comparison	of required	channel	knowledge,	achieved	rates	and	$\operatorname{diversity}$	orders	in	different	pre-
coding	sche	emes.											

\*  $E-d_{min}$  and X-, Y-Codes in [15] and [16] differ mainly in complexity and applicability to higher-order QAM. \*\* Precoding scheme in [13] uses D-BLAST architecture which needs multiple channel uses to asymptotically achieve full rate.

these schemes use full CSIT. In addition, the number of symbols transmitted per channel use  $n_s \leq n_r$ , and so for  $n_r < n_t$  full rate is not achieved. The precoding scheme in [14] uses partial CSIT in the form of a single angle parameter feedback. Though this scheme achieves full rate and high diversity, it failed to achieve full diversity. A summary of the required channel knowledge, achieved rates and diversity orders in various precoding schemes are given in Table-1.

In the above context, the significance of our contribution in this paper is that we propose a novel precoding scheme that achieves both *full rate of*  $n_t$  symbols per channel use as well as *full diversity of*  $n_t n_r$  using partial CSIT. The scheme requires the feedback of only  $n_t - 1$ angle parameter values, compared to  $2n_t n_r$  real coefficients in case of full CSIT. The proposed scheme achieves full diversity and coding gain through an optimization over the choice of constellation sets. The optimization maximizes  $d_{min}^2$  for our precoding scheme subject to an energy constraint. We present codeword error performance results for  $n_t = 3, 4, 8$  and  $n_r = 1, 2$ . We analytically prove the full diversity ( $n_t n_r$ th diversity) of the proposed scheme, and simulation results are shown to validate the result. It is further shown that the proposed scheme performs better than other existing schemes. This better performance is attributed to the choice of signal sets and feedback angles in the proposed scheme.

Notation: Vectors are denoted by lower case boldface letters, and matrices are denoted by upper case boldface letters.  $(.)^*$ ,  $(.)^H$ ,  $(.)^T$  and tr(.) denote conjugation, hermitian, transpose and trace operators, respectively.  $\mathbf{A}_{xy}$  will be used to denote the (x, y)th entry of matrix  $\mathbf{A}$ .

The rest of the paper is organized as follows. The system model is presented in Section 2. The proposed full rate, full diversity precoding scheme, and choice of constellation sets are presented in Section 3. Performance results and discussions are presented in Section 4. Conclusions are given in Section 5.

## 2 System Model

Consider a precoded V-BLAST system with  $n_t$  antennas at the transmitter and  $n_r$  antennas at the receiver. Let  $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$  denote the channel gain matrix, whose entries  $h_{pq}$  are distributed i.i.d  $\mathcal{CN}(0,1), \forall p = 1, \dots, n_r, \forall q = 1, \dots, n_t$ . Let  $\mathbf{F}$  denote the precoder matrix of size  $n_t \times n_t$ , which is known to both the transmitter and receiver. For a given channel matrix  $\mathbf{H}$ , the receiver computes  $n_t - 1$  number of real feedback parameters and sends them to the transmitter. Given the feedback parameters, the transmitter forms the precoding matrix  $\mathbf{F}$ . Let  $\mathbf{x}$  denote the  $n_t \times 1$  complex data symbol vector of the form

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_{n_t} \end{bmatrix}^T, \tag{1}$$

where  $x_i$  is transmitted by the *i*th antenna.  $x_i$ 's will take values from a suitable complex constellation, the choice of which will be discussed later. The received signal vector,  $\mathbf{y} \in \mathbb{C}^{n_r \times 1}$ , at the receiver is given by

$$\mathbf{y} = \mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{n}, \tag{2}$$

where  $\mathbf{n} \in \mathbb{C}^{n_r \times 1}$  is the noise vector with its entries distributed as i.i.d.  $\mathcal{CN}(0, \sigma^2)$ .

### 3 Proposed Precoding Scheme

Consider the precoded V-BLAST system described in Section 2. Let

$$\mathbf{F} = \begin{bmatrix} \mathbf{a} \ \mathbf{a} \ \cdots \ \mathbf{a} \end{bmatrix}, \tag{3}$$

where the column vector  $\mathbf{a} = [a_1 \cdots a_{n_t}]^T \in \mathbb{C}^{n_t \times 1}$  and  $|a_i| = 1, \forall i = 1, \cdots, n_t$ . At high SNRs, symbol error probability  $P_e$  decays as SNR<sup> $-n_tn_r$ </sup>, provided the received distance squared between each pair of codewords  $\mathbf{x}_k, \mathbf{x}_l$   $(k \neq l)$ , denoted by  $d_{k,l}^2$ , is a Chi-square  $\chi^2_{2n_tn_r}$ distributed variable with  $2n_tn_r$  degrees of freedom [14]. In general,

$$d_{k,l}^2 = \|\mathbf{HF}\Delta\mathbf{x}\|^2, \tag{4}$$

where  $\Delta \mathbf{x} = \mathbf{x}_k - \mathbf{x}_l, \ k \neq l$ . Now, (4) can be written as

$$d_{k,l}^{2} = \operatorname{tr}(\mathbf{H}\mathbf{F}\Delta\mathbf{x}\Delta\mathbf{x}^{H}\mathbf{F}^{H}\mathbf{H}^{H})$$
$$= \operatorname{tr}(\mathbf{F}^{H}\mathbf{H}^{H}\mathbf{H}\mathbf{F}\Delta\mathbf{x}\Delta\mathbf{x}^{H}).$$
(5)

It can be shown that the (p,q)th element of  $\mathbf{H}^{H}\mathbf{HF}$  is given by

$$(\mathbf{H}^{H}\mathbf{H}\mathbf{F})_{pq} = \sum_{m=1}^{n_{t}} a_{m} \Big(\sum_{o=1}^{n_{r}} h_{op}^{*} h_{om}\Big).$$
(6)

It may be noted that all the elements of the *p*th row of the matrix  $\mathbf{H}^{H}\mathbf{HF}$  are identical.

$$\Rightarrow (\mathbf{F}^{H}\mathbf{H}^{H}\mathbf{H}\mathbf{F})_{pq} = \sum_{n=1}^{n_{t}} a_{n}^{*} \Big(\sum_{m=1}^{n_{t}} a_{m}\Big(\sum_{o=1}^{n_{r}} h_{on}^{*}h_{om}\Big)\Big)$$
$$= \sum_{n=m=1}^{n_{t}} |a_{n}|^{2} \Big(\sum_{o=1}^{n_{r}} |h_{on}|^{2}\Big) + \sum_{n\neq m}^{n_{t}} a_{n}^{*}a_{m}\Big(\sum_{o=1}^{n_{r}} h_{om}^{*}h_{om}\Big).$$
(7)

Moreover, it is clear that all the entries of the matrix  $\mathbf{F}^H \mathbf{H}^H \mathbf{H} \mathbf{F}$  are identical. So it amounts to

$$\mathbf{F}^{H}\mathbf{H}^{H}\mathbf{H}\mathbf{F} = \left(\sum_{n=m=1}^{n_{t}} |a_{n}|^{2} \left(\sum_{o=1}^{n_{r}} |h_{on}|^{2}\right) + 2\mathcal{R}\left(\sum_{m,n,n>m}^{n_{t}} a_{n}^{*}a_{m}\sum_{o=1}^{n_{r}} h_{on}^{*}h_{om}\right)\right) \mathbf{B}, \quad (8)$$

where  $\mathbf{B}_{pq} = 1$ . In order to ensure that  $d_{k,l}^2$  is distributed as a Chi-square  $\chi^2_{2n_tn_r}$  variable, we require that in (8),

$$\mathcal{R}\Big(\sum_{m,n,n>m}^{n_t} a_n^* a_m\Big(\sum_{o=1}^{n_r} h_{on}^* h_{om}\Big)\Big) = 0.$$
(9)

Let  $a_i = |a_i|e^{j\theta_i}$ ,  $\sum_{o=1}^{n_r} h_{on}^* h_{om} = |\sum_{o=1}^{n_r} h_{on}^* h_{om}|e^{j\alpha_{nm}}$ , where  $j = \sqrt{-1}$ . Then, with  $|a_i| = 1$ ,  $\forall i = 1, \dots, n_t$ , (9) becomes

$$\sum_{m,n,n>m}^{n_t} \left| \sum_{o=1}^{n_r} h_{on}^* h_{om} \right| \cos(\theta_m - \theta_n + \alpha_{nm}) = 0.$$
 (10)

For a given realization of **H**, the receiver needs to compute  $\theta_m$ 's to satisfy (10). Again, (10) can be rewritten as

$$\sum_{n=2}^{n_t} \sum_{m=1}^{n-1} \left| \sum_{o=1}^{n_r} h_{on}^* h_{om} \right| \cos(\theta_m - \theta_n + \alpha_{nm}) = 0.$$
(11)

Let the inner summation of (11) with index m be equal to 0 for each n. For n = 2,

$$\cos(\theta_1 - \theta_2 + \alpha_{21}) = 0. \tag{12}$$

Let  $\theta_1 = 0$ . Hence,  $\theta_2 = \alpha_{21} - \pi/2$ . Next, for any n > 2,

$$\sum_{m=1}^{n-1} \left| \sum_{o=1}^{n_r} h_{on}^* h_{om} \right| \cos(\theta_m - \theta_n + \alpha_{nm}) = 0 \quad (13)$$

$$\Rightarrow \quad \sum_{m=1}^{n-1} \left| \sum_{o=1}^{n_r} h_{on}^* h_{om} \right| (\cos(\theta_m + \alpha_{nm}) \cos \theta_n + \sin(\theta_m + \alpha_{nm}) \sin \theta_n) = 0.$$

$$\theta_n = \tan^{-1} \left( \frac{-\sum_{m=1}^{n-1} |\sum_{o=1}^{n_r} h_{on}^* h_{om}| (\cos(\theta_m + \alpha_{nm}))}{\sum_{m=1}^{n-1} |\sum_{o=1}^{n_r} h_{on}^* h_{om}| \sin(\theta_m + \alpha_{nm})} \right).$$
(14)

Using (14),  $\theta_n$  can be recursively calculated, given the values of  $\theta_1$  to  $\theta_{n-1}$ . From (5),(8),(9) and (14), after simplification, we have

$$d_{k,l}^2 = \left(\sum_{n=1}^{n_t} \sum_{o=1}^{n_r} |h_{on}|^2\right) \left|\sum_{i=1}^{n_t} \Delta x_i\right|^2,\tag{15}$$

where

$$\Delta \mathbf{x} = \begin{bmatrix} \Delta x_1 \ \Delta x_2 \ \cdots \ \Delta x_{n_t} \end{bmatrix}^T.$$
(16)

### 3.1 Choice of Constellation Sets

From (15), it is clear that full diversity is guaranteed if, for each codeword pair  $\mathbf{x}_k, \mathbf{x}_l, k \neq l$ in the codebook,

$$\sum_{i=1}^{n_t} \Delta x_i \neq 0. \tag{17}$$

Let  $x_i, \forall i = 1, \dots, n_t$ , take values from some set  $C_i \subseteq a_i \mathbb{Z}[j] = \{a_i z : z \in \mathbb{Z}[j]\}$ , where  $\mathbb{Z}[j] = u + jv$ , u and v are integers, i.e., regular QAM constellations scaled by  $a_i$ . Clearly

then,  $\Delta x_i$  belongs to some other set  $D_i \subseteq a_i \mathbb{Z}[j]$  for each *i*. It may be noted that  $-\Delta x_i \in D_i$ . Now, let  $a_1 = 1$  and choose  $a_2$  such that

$$D_1 \cap D_2 = \{0\}. \tag{18}$$

Again, define  $D_1 + D_2 \stackrel{\triangle}{=} \{\Delta x_1 + \Delta x_2 \text{ s.t } \forall \Delta x_1 \in D_1, \Delta x_2 \in D_2\}$ . Next, choose  $a_3$  such that

$$D_3 \cap (D_1 + D_2) = \{0\}.$$
(19)

Proceeding in this way,  $a_i$  is chosen such that

$$D_i \cap (D_1 + \dots + D_{i-1}) = \{0\}.$$
(20)

This implies that for such choice of  $a_i$ 's and  $\Delta x_i \in D_i$ , (17) is satisfied.

A trivial choice of  $a_i$ 's could be  $\sqrt{p_i}$ , where  $p_i$ 's are distinct prime numbers in  $\mathbb{Z}$ . This shows that the solution set of  $a_i$ 's satisfying (20) is non-empty. For these values of  $a_i$ 's, (17) holds true for all codeword pairs in the codebook ensuring full diversity. Since the number of elements of  $C_i$  is finite, the cardinality of  $D_i$  is also finite. This enables us to exhaustively enumerate all the elements of  $D_i$  as functions of  $a_i$ . Next, we have to choose  $a_i$ 's  $\in \mathbb{C}$  such that (20) is satisfied  $\forall i = 1, \dots, n_t$ . Thus, there exist scaled QAM constellations for which the proposed precoding scheme guarantees full diversity. A parallel argument can be made for PSK constellation to ensure full diversity.

Now, the  $d_{min}^2$  parameter of a codebook determines its coding gain as given by the error probability expression at high SNR. Furthermore,  $d_{min}^2$  is maximized if  $min(\sum_{i=1}^{n_t} \Delta x_i)$  is maximized in (15). Hence, after scaling the constellation sets  $C_i$ 's by appropriate  $a_i$ 's, we propose to rotate/scale each  $C_i$  by angles ( $\phi_i$ 's) and scaling real numbers ( $b_i$ 's), ensuring that the average constellation power never exceeds the total transmit power constraint and also the condition (20) holds true  $\forall i = 1, \dots, n_t$ . The corresponding  $\min(\sum_{i=1}^{n_t} \Delta x_i)$  is computed over all  $C_i$ 's. The optimum rotation angle ( $\phi_{i,opt}$ ) and scaling real number ( $b_{i,opt}$ ) for each  $C_i$ are chosen by computer search for which  $\min(\sum_{i=1}^{n_t} \Delta x_i)$  is maximum. This fixes the choice of constellation sets  $C_i$ 's that guarantee full transmit diversity as well maximizes  $d_{min}^2$  subject to transmit power constraint.

#### 3.2 A Simplified Approach to Choose Constellation Sets

The treatment described in the previous subsection (Section 3.1) is feasible for constellations of small cardinality. The computational complexity involved in choosing large-sized constellation sets makes the choice cumbersome. In this subsection, we illustrate a suboptimal mechanism to choose large constellations. Our choice of  $\mathbf{F}$  in (3) ensures that the effective transmitted vector  $\mathbf{Fx}$  becomes

$$\left(\sum_{i=1}^{n_t} x_i\right) \left[\begin{array}{ccc} a_1 & a_2 & \cdots & a_{n_t}\end{array}\right]^T.$$

$$(21)$$

It then implies that each of the  $n_t$  antennas effectively transmits the symbol  $\sum_{i=1}^{n_t} x_i$ . Now, for a given energy constraint, it is known that square QAM constellations (i.e.,  $4^f$  sized constellation sets, where f is a positive integer) achieve better  $d_{min}^2$  over PAM and PSK constellations. Hence, we propose to choose constellation sets for each  $x_i$  in such a way that  $\sum_{i=1}^{n_t} x_i$  takes values from QAM constellation. Such a choice guarantees full diversity since for each codeword **x** transmitted the corresponding effective symbol  $\sum_{i=1}^{n_t} x_i$  maps to a unique point in some QAM constellation satisfying condition (17).

#### **3.3** Constellation Sets Obtained for $n_t = 3, 4, 8, 16$

Based on the treatment described in the above subsections (Sections 3.1, 3.2), we carried out a computer search to obtain full diversity achieving constellation sets for various  $n_t$ . Let  $aQ_M = \{aq : q \in Q_M, a \in \mathbb{R}\}$ , where  $Q_M$  denotes *M*-QAM constellation. The obtained constellation sets for  $n_t = 3, 4, 8, 16$  are given in Table-2. Figures 1(a) and 1(b) show the plots of the obtained constellation sets for  $n_t = 4$  with of 1 bit/symbol and 2 bits/symbol, respectively.

### 4 Results and Discussions

We evaluated the codeword error rate (CER) performance of the proposed precoding scheme in asymmetric MIMO systems with  $n_r < n_t$  through simulations. In particular, we illustrate the CER performance of the proposed scheme for  $3 \times 1$ ,  $3 \times 2$ ,  $4 \times 1$  and  $8 \times 1$  full-rate MIMO as a function of the average received SNR per receive antenna. ML decoding is used.

Plots of two performance measures, namely, i) CER and ii) pdf of the normalized  $d_{min}^2$ , are

No. of Tx antennas	Modulation	Modulation	Modulation		
	(1  bit/symbol)	(2 bits/symbol)	(4 bits/symbol)		
$n_t = 3$	$x_1 \in \{\pm 1\}$	$x_1 \in \mathcal{Q}_4$	$x_1 \in \mathcal{Q}_{16}$		
	$x_2 \in \{\pm j\}$	$x_2 \in \frac{1}{2}\mathcal{Q}_4$	$x_2 \in \frac{1}{14}\mathcal{Q}_{16}$		
	$x_3 \in \{\pm 0.675 e^{j\frac{\pi}{4}}\}$	$x_3 \in \frac{1}{4}\mathcal{Q}_4$	$x_3 \in \frac{1}{28}\mathcal{Q}_{16}$		
$n_t = 4$	$x_1 \in \{\pm 1\}$	$x_1 \in \mathcal{Q}_4$	$x_1 \in \mathcal{Q}_{16}$		
	$x_2 \in \{\pm j\}$	$x_2 \in \frac{1}{2}\mathcal{Q}_4$	$x_2 \in \frac{1}{14}\mathcal{Q}_{16}$		
	$x_3 \in \{\pm \frac{1}{2}\}$	$x_3 \in \frac{1}{4}\mathcal{Q}_4$	$x_3 \in \frac{1}{28}\mathcal{Q}_{16}$		
	$x_4 \in \{\pm \frac{1}{2}j\}$	$x_4 \in \frac{1}{8}\mathcal{Q}_4$	$x_4 \in \frac{1}{56}\mathcal{Q}_{16}$		
$n_t = 8$	$x_1 \in \{\pm 1\}$	$x_1 \in \mathcal{Q}_4$	$x_1 \in \mathcal{Q}_{16}$		
	$x_2 \in \{\pm j\}$	$x_2 \in \frac{1}{2}\mathcal{Q}_4$	$x_2 \in \frac{1}{14}\mathcal{Q}_{16}$		
	$x_3 \in \{\pm \frac{1}{2}\}$	$x_3 \in \frac{1}{4}\mathcal{Q}_4$	$x_3 \in \frac{1}{28}\mathcal{Q}_{16}$		
	$x_4 \in \{\pm \frac{1}{2}j\}$	$x_4 \in \frac{1}{8}\mathcal{Q}_4$	$x_4 \in \frac{1}{56}\mathcal{Q}_{16}$		
	$x_5 \in \{\pm \frac{1}{4}\}$	$x_5 \in \frac{1}{16}\mathcal{Q}_4$	$x_5 \in \frac{1}{1+2}\mathcal{Q}_{16}$		
	$x_6 \in \{\pm \frac{1}{4}j\}$	$x_6 \in \frac{1}{32}Q_4$	$x_6 \in \frac{1}{224}Q_{16}$		
	$x_7 \in \{\pm \frac{1}{8}\}$	$x_7 \in \frac{1}{64}\mathcal{Q}_4$	$x_7 \in \frac{1}{448}Q_{16}$		
	$x_8 \in \{\pm \frac{1}{8}j\}$	$x_8 \in \frac{1}{128}\mathcal{Q}_4$	$x_8 \in \frac{1}{896}Q_{16}$		
$n_t = 16$	$x_1 \in \{\pm 1\}$	$x_1 \in \mathcal{Q}_4$	$x_1 \in \mathcal{Q}_{16}$		
	$x_2 \in \{\pm j\}$	$x_2 \in \frac{1}{2}\mathcal{Q}_4$	$x_2 \in \frac{1}{14}\mathcal{Q}_{16}$		
	$x_3 \in \{\pm \frac{1}{2}\}$	$x_3 \in \frac{1}{4}Q_4$	$x_3 \in \frac{1}{28}Q_{16}$		
	$x_4 \in \{\pm \frac{1}{2}j\}$	$x_4 \in \frac{1}{8}Q_4$	$x_4 \in \frac{1}{56}Q_{16}$		
	$x_5 \in \{\pm \frac{1}{4}\}$	$x_5 \in \frac{1}{16}\mathcal{Q}_4$	$x_5 \in \frac{1}{1+2}Q_{16}$		
	$x_6 \in \{\pm \frac{1}{4}j\}$	$x_6 \in \frac{1}{32}Q_4$	$x_6 \in \frac{1}{224} \mathcal{Q}_{16}$		
	$x_7 \in \{\pm \frac{1}{8}\}$	$x_7 \in \frac{1}{64}\mathcal{Q}_4$	$x_7 \in \frac{1}{448} \mathcal{Q}_{16}$		
	$x_8 \in \{\pm \frac{1}{8}j\}$	$x_8 \in \frac{1}{128}\mathcal{Q}_4$	$x_8 \in \frac{1}{896} \mathcal{Q}_{16}$		
	$x_9 \in \{\pm \frac{1}{16}\}$	$x_9 \in \frac{1}{256}Q_4$	$x_9 \in \frac{1}{1792} \mathcal{Q}_{16}$		
	$x_{10} \in \{\pm \frac{1}{16}j\}$	$x_{10} \in \frac{1}{512}\mathcal{Q}_4$	$x_{10} \in \frac{1}{3584} \mathcal{Q}_{16}$		
	$x_{11} \in \{\pm \frac{1}{32}\}$	$x_{11} \in \frac{1}{1024}\mathcal{Q}_4$	$x_{11} \in \frac{1}{7168} \mathcal{Q}_{16}$		
	$x_{12} \in \{\pm \frac{1}{32}j\}$	$x_{12} \in \frac{1}{2048} \mathcal{Q}_4$	$x_{12} \in \frac{1}{14336} \mathcal{Q}_{16}$		
	$x_{13} \in \{\pm \frac{1}{64}\}$	$x_{13} \in \overline{4096} \mathcal{Q}_4$	$x_{13} \in \overline{_{28672}} \mathcal{Q}_{16}$		
	$x_{14} \in \{\pm \frac{1}{64} j\}$	$x_{14} \in \overline{x_{192}} \mathcal{Q}_4$	$x_{14} \in \frac{1}{57344} \mathcal{Q}_{16}$		
	$x_{15} \in \{\pm \frac{1}{128}\}$	$x_{15} \in \frac{1}{16384} \mathcal{Q}_4$	$x_{15} \in \frac{1}{114688} \mathcal{Q}_{16}$		
	$x_{16} \in \{\pm \frac{1}{128} j\}$	$x_{16} \in \frac{1}{32768}Q_4$	$x_{16} \in \frac{1}{229376} Q_{16}$		

Table 2: Full diversity achieving constellation sets for  $n_t = 3, 4, 8, 16$  with 1 bit/symbol, 2 bits/symbol and 3 bits/symbol obtained by computer search.



Figure 1: Plots of constellation sets for  $n_t = 4$  with 1 bit/symbol and 2 bits/symbol.

shown in two separate sub-figures in each of Figs. 2, 3, 4 for  $3 \times 1$ ,  $4 \times 1$  and  $8 \times 1$  MIMO, respectively. For comparison purposes, we have plotted the CER performance and  $d_{min}^2$  pdf of the full-rate precoding scheme in [14]. We show the comparison only with [14] because both the proposed scheme and the scheme in [14] achieve the full rate of  $n_t$  symbols per channel use in  $n_r < n_t$  asymmetric MIMO. Whereas, the other precoding schemes in Table-1 lose rate and do not achieve full rate of  $n_t$  symbols per channel use in  $n_r < n_t$  asymmetric MIMO. Mimol.

In Fig. 2, CER plots for the rates of 3 and 6 bits/channel use are shown. For 3 bits/cu, the constellation points used in the simulation for the proposed scheme are as given in the entries of  $n_t = 3$  and 1 bit/symbol modulation in Table-2. Likewise, for 6 bits/cu, the constellation points used are from the  $n_t = 3$  and 2 bits/symbol modulation in Table-2. Appropriate constellations from Table-2 are used for the plots of different rates shown in Figs. 3 and 4. Since constellation optimization is not done in the precoding scheme in [14], we have used regular modulation alphabets (e.g., BPSK, 4-QAM) and matched the bits/cu in both schemes for fair comparison. For example, 6 bits/cu plot in Fig. 2 for the scheme in [14] is simulated using 4-QAM.

From the CER plots of in Figs. 2 to 4, we can see that the proposed precoding achieves better diversity orders compared to the precoding scheme in [14]. Indeed, as predicted by the analysis, the proposed scheme achieves the full diversity of  $n_t n_r = 3, 4, 8$  in Figs. 2, 3 and 4, respectively. This can be verified by observing that pdfs of the  $d_{min}^2$  in the proposed scheme match with the theoretical  $\chi^2_{2n_tn_r}$  pdf (Chi square distribution with  $2n_tn_r$  degrees of freedom). For example, in Fig. 3, the proposed scheme's pdf matches with that of the  $\chi^2_8$  pdf, verifying the achievability of  $n_tn_r = 4$ th order diversity. Similar matches with Chi square distribution are observed in other figures as well. Thus, the analytical claim of full diversity of  $n_tn_r$  in the proposed precoding scheme is validated through simulations as well. It is further noted that the pdfs in the scheme in [14] do not match with those of the theoretical  $\chi^2_{2n_tn_r}$  pdfs, indicating that the scheme in [14] does not achieve full diversity. Finally, Fig. 5 shows the performance of the proposed scheme in a  $3 \times 2$  MIMO system with 3 and 6 bits/cu, where the full  $n_tn_r = 6$ th order diversity is found to be achieved.

### 5 Conclusion

We presented a partial CSIT based precoding scheme, which achieved both full diversity  $(n_t n_r \text{th diversity})$  as well as full rate  $(n_t \text{ symbols per channel use})$  with a feedback requirement of only  $n_t - 1$  real angular parameters, applicable in MIMO systems including asymmetric MIMO with  $n_r < n_t$ . The full diversity was achieved by choosing optimized constellation sets. Through analysis and simulations we established the full diversity achievability in the proposed scheme.

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Figure 2: Comparison of a) CER performance and b) pdf of the normalized  $d_{min}^2$  of the proposed precoder with that in [14] for  $3 \times 1$  MIMO system.



Figure 3: Comparison of a) CER performance and b) pdf of the normalized  $d_{min}^2$  of the proposed precoder with that in [14] for  $4 \times 1$  MIMO system.



Figure 4: Comparison of a) CER performance and b) pdf of the normalized  $d_{min}^2$  of the proposed precoder with that in [14] for  $8 \times 1$  MIMO system.



Figure 5: CER performance and pdf of the normalized  $d_{min}^2$  of the proposed precoder in a  $3 \times 2$  MIMO system.