Outage Performance of AF-based Time Division Broadcasting Protocol in the Presence of Co-channel Interference

Xiaochen Xia, Youyun Xu, Kui Xu, Dongmei Zhang, and Ning Li
Institute of Communication Engineering, PLA University of Science and Technology
Nanjing 210007, P. R. China
Email: Xia1382084@gmail.com

***Abstract*—In this paper, we investigate the outage performance of time division broadcasting (TDBC) protocol in independent but non-identical Rayleigh flat-fading channels, where all nodes are interfered by a finite number of co-channel interferers. We assume that the relay operates in the amplified-and-forward mode. A tight lower bound as well as the asymptotic expression of the outage probability is obtained in closed-form. Through both theoretic analyses and simulation results, we show that the achievable diversity of TDBC protocol is zero in the interference-limited scenario. Moreover, we study the impacts of interference power, number of interferers and relay placement on the outage probability. Finally, the correctness of our analytic results is validated via computer simulations.**

I. Introduction

Recently, two-way relaying (TWR) or bi-directional relaying has emerged as a powerful technique to improve the spectral efficiency of wireless network [1]. A number of relaying protocols have been proposed, such as amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF). For AF relaying, two popular TWR protocols are analog network coding (ANC) [2], which requires two time slots to complete the information exchange between two terminal nodes, and TDBC [3], which needs three time slots. However, TDBC protocol can use the direct link between two terminals even under a half-duplex constraint [3][4], thus can provide higher diversity gain.

Several previous works have investigated the TWR network using TDBC for Rayleigh fading channels, in which relay and terminals are only perturbed by additive white Gaussian noise (AWGN) [5]-[7]. The outage performance of AF-based TDBC protocol in Rayleigh fading channels was analyzed in [5][6] and the diversity-multiplexing tradeoff (DMT) was also obtained. In [7], the authors considered relay selection scheme for TDBC protocol and analyzed the outage performance with optimal relay selection. However, signals of terminals (or relay) are often corrupted by co-channel interference (CCI) from other sources that share the same frequency resources in wireless networks [8]. Moreover, for the wireless scenarios with dense frequency reuse, co-channel interference may dominate the AWGN. Therefore, it is necessary to take the effect of CCI into serious consideration in the analysis and design of the practical TDBC protocol. In [9], the performance

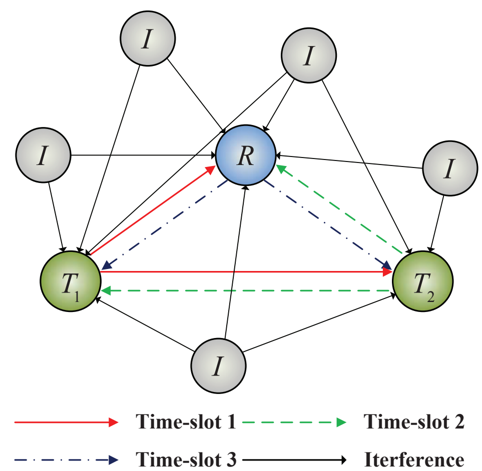

Fig. 1: The TDBC with a finite number of co-channel interferers, where I denotes the co-channel interferer.

of ANC protocol corrupted by equal-power interferers was studied, where closed-form expressions of the average bit error rate and outage probability were presented. Outage probability of the cooperative relaying using DF protocol with CCI has been analyzed in [10]. However, for AF-based TDBC protocol, the effect of CCI is still unknown.

In this work, we study the AF-based TDBC protocol where all the nodes (terminals and relay) are interfered by a finite number of co-channel interferers in independent but non-identical Rayleigh flat-fading channels. The system model is described in the next section. In section III, the CDF of the upper bounded SINR is analyzed. Based on the results, a lower bound as well as asymptotic expression of outage probability is obtained. In section IV, the effects of interference power, number of interferers and relay placement on the outage probability are studied.

II. System model

We study the TWR network which consists of two terminals and a relay node, as shown in Fig. 1, in which terminal T_1 and terminal T_2 exchange statistically independent messages with the help of a relay R. Each node is equipped with a single antenna and operates in the half-duplex mode, that is, a node cannot transmit and receive simultaneously.

The TDBC protocol can be achieved within three time slots, that is, terminal T_1 transmits during the first time slot, while T_2 and R listen. In time slot 2, T_2 transmits while T_1 and R listen. It is assumed that both terminals and the relay are interfered by a finite number of co-channel interferers. Denoting L_R, L_1 and L_2 as the total numbers of interferers that affect node R, T_1 and T_2, respectively, the received signals at the relay and T_i during the first two time slots are expressed as

$$\begin{aligned} y_{iR} &= \sqrt{E_i} h_i S_i + \sqrt{E_I} \sum_{k=1}^{L_R} d_{R,k} I_{R,k}^{i} + n_{iR}, \\ y_{ji} &= \sqrt{E_j} h_0 S_j + \sqrt{E_I} \sum_{k=1}^{L_i} d_{T_i,k} I_{T_i,k}^{j} + n_{ji}, \end{aligned} \tag{1}$$

respectively, where $i, j \in \{1, 2\}$ and $i \neq j$. E_i and E_I denote the transmit powers of T_i and interferers, respectively. h_1, h_2 and h_0 represent the channel coefficients belonging to the links $T_1 \to R$, $T_2 \to R$ and $T_1 \to T_2$, respectively. The channel reciprocity is assumed. Moreover, $d_{N,k}$ indicates the channel coefficient of link between node N and the kth interferer that affects N, where $N \in \{T_1, T_2, R\}$. All links are assumed to be independent but non-identical Rayleigh flat-fading. S_i denotes the unit-power symbol transmitted by T_i. $I_{N,k}^{m}$ indicates the unit-power interference signal of kth interferer that affects node N during the mth time slot, where $N \in \{T_1, T_2, R\}$ and $m \in \{1, 2, 3\}$. Finally, n_{iR}/n_{ji} denote the AWGN and $n_{iR}/n_{ji} \sim \mathcal{CN}(0, 1)$.

In time slot 3, R transmits the combined information to terminals T_1 and T_2. The combined signal to be transmitted by R can be written as $S_R = \mathcal{A}_1 y_{1R} + \mathcal{A}_2 y_{2R}$. $\mathcal{A}_1$ and $\mathcal{A}_2$ denote combining coefficients which can be determined as follow[1] [11]:

$$\mathcal{A}_i = \sqrt{\frac{\omega_i}{\omega_1 E_1 |h_1|^2 + \omega_2 E_2 |h_2|^2 + E_I \sum_{k=1}^{L_R} |d_{R,k}|^2 + 1}}, \tag{2}$$

where $i \in \{1, 2\}$. $\omega_i \in [0, 1]$ is the power allocation number and $\omega_1 + \omega_2 = 1$. Then the received signal at terminal T_i during the third time slot can be written as

$$\begin{aligned} y_{Ri} &= \sqrt{E_r} h_i S_R + \sqrt{E_I} \sum_{k=1}^{L_i} d_{T_i,k} I_{T_i,k}^{3} + n_{Ri} \\ &= \sqrt{E_r E_1} \mathcal{A}_1 h_i h_1 S_1 + \sqrt{E_r E_2} \mathcal{A}_2 h_i h_2 S_2 + \sqrt{E_I} \sum_{k=1}^{L_i} d_{T_i,k} I_{T_i,k}^{3} \\ &+ \sqrt{E_r E_I} h_i \mathcal{A}_1 \sum_{k=1}^{L_R} d_{R,k} I_{R,k}^{1} + \sqrt{E_r E_I} h_i \mathcal{A}_2 \sum_{k=1}^{L_R} d_{R,k} I_{R,k}^{2} \\ &+ \sqrt{E_r} \mathcal{A}_1 h_i n_{1R} + \sqrt{E_r} \mathcal{A}_2 h_i n_{2R} + n_{Ri}, \end{aligned} \tag{3}$$

where $n_{Ri} \sim \mathcal{CN}(0, 1)$ is the AWGN and E_r is the transmit power of R. In the following, we assume equal power allocation[2] between T_1, T_2 and R, i.e., $E_1 = E_2 = E_r = E$. Since T_i knows its own transmitted symbols, it can cancel the self-interference component in y_{Ri}. Therefore, after performing maximal-ratio combining, the instantaneous SINR at terminal T_i can be expressed as in (4), where $i, j \in \{1, 2\}$ and $i \neq j$. By substituting the expressions of $\mathcal{A}_1$ and $\mathcal{A}_2$ into (4) and performing some manipulations, γ_{T_i} can be rewritten as [9]

$$\gamma_{T_i} \approx \gamma_{T_i,D} + \frac{\gamma_{T_i,1} \gamma_{T_i,2}}{\gamma_{T_i,1} + \gamma_{T_i,2}}, \tag{5}$$

where $\gamma_{T_i,D} = \frac{E|h_0|^2}{E_I \sum_{k=1}^{L_i} |d_{T_i,k}|^2 + 1}$ is the received SINR of link $T_j \to T_i$. Moreover, $\gamma_{T_i,1}$ and $\gamma_{T_i,2}$ are given by

$$\begin{aligned} \gamma_{T_i,1} &= \frac{E|h_i|^2}{E_I \sum_{k=1}^{L_i} |d_{T_i,k}|^2 + 1} \\ \gamma_{T_i,2} &= \frac{\omega_j E |h_j|^2}{E_I \sum_{k=1}^{L_R} |d_{R,k}|^2 + \omega_i E_I \sum_{k=1}^{L_i} |d_{T_i,k}|^2 + \omega_i + 1}. \end{aligned} \tag{6}$$

III. Outage Probability Analysis

A. Lower Bound of the Exact Outage Probability

In this section, the outage probability of the AF-based TDBC protocol in the presence of CCI is studied. For brevity of analysis and without loss of generality, we focus on the outage probability at terminal T_1 in the rest of this work. By definition, the outage event occurs when the mutual information at T_1 falls below the target rate R_t, or equivalently, the output SINR at T_1 is below the target SINR φ. Therefore, the outage probability at terminal T_1 can be written as

$$P_{T_1}^{OUT}(R_t) = \Pr(I_{T_1} < R_t) = F_{\gamma_{T_1}}(\varphi), \tag{7}$$

where $I_{T_1} = \frac{1}{3} \log(1 + \gamma_{T_1})$ indicates the mutual information[3] at terminal T_1, and $\varphi = 2^{3R_t} - 1$. $F_\Psi(\eta)$ represents the CDF of random variable (RV) Ψ. However, it is very difficult to obtain the exact expression of $F_{\gamma_{T_1}}(\varphi)$ in closed-form. To circumvent this obstacle, we introduce a tight upper bound on the received SINR at T_1 by employing a widely used inequality, i.e., $\nu_1 \nu_2 / (\nu_1 + \nu_2) \le \min\{\nu_1, \nu_2\}$, where ν_1 and ν_2 are positive numbers, then we shall have

$$\gamma_{T_1} \le \gamma_{T_1}^{UB} = \gamma_{T_1,D} + \min\{\gamma_{T_1,1}, \gamma_{T_1,2}\}. \tag{8}$$

Next, we will determine the CDF of the upper bounded SINR. For convenience of analysis, letting $X \triangleq E|h_1|^2$, $Y \triangleq E|h_2|^2$, $Z \triangleq E|h_0|^2$, $S \triangleq E_I \sum_{k=1}^{L_R} |d_{R,k}|^2$ and $T \triangleq E_I \sum_{k=1}^{L_1} |d_{T_1,k}|^2$. Note that X , Y and Z are exponential RVs with means $E\Omega_1$, $E\Omega_2$ and $E\Omega_0$, respectively, where Ω_i indicates the variance of h_i, $i \in \{0, 1, 2\}$. Letting $\gamma_m = \min\{\gamma_{T_1,1}, \gamma_{T_1,2}\}$, then with the help of total probability theorem, the CDF of $\gamma_{T_1}^{UB}$ conditioned on S and T can be written as in (9), where $F_{\gamma_{m|\{S,T\}}}(x)$ is the CDF of γ_m

[1] As in [9], here it is assumed that R knows the channel gains of links $T_1 \to R$ and $T_2 \to R$, and the total interference power (instantaneous) at R. Moreover, it is assumed that T_i knows the channel gains of links $T_1 \to R$, $T_2 \to R$, $T_1 \to T_2$ as well as the total interference powers (instantaneous) at R and T_i.

[2] Similar as in [6] and [10], the assumption of equal power allocation dose not make the analysis in this work lose generality because the variances of the channel coefficients between T_1, T_2 and R may be different.

[3] Herein, three time slots required for the TDBC protocol account for the pre-log factor of 1/3.

$$\gamma_{T_i} = \frac{E|h_0|^2}{E_I \sum_{k=1}^{L_i} |d_{T_i,k}|^2 + 1} + \frac{\mathcal{A}_j^2 E^2 |h_1|^2 |h_2|^2}{\left(\mathcal{A}_1^2 + \mathcal{A}_2^2\right) E|h_i|^2 \left(E_I \sum_{k=1}^{L_R} |d_{R,k}|^2 + 1\right) + E_I \sum_{k=1}^{L_i} |d_{T_i,k}|^2 + 1}, \tag{4}$$

$$\begin{aligned} F_{\gamma_{T_1}^{UB}|\{S,T\}}(\varphi) &= 1 - \Pr\left(\gamma_{T_1,D} > \varphi \middle| T\right) - \Pr\left(\gamma_{T_1,D} < \varphi, \gamma_m > \varphi - \gamma_{T_1,D} \middle| S,T\right) \\ &= 1 - \int_{\varphi}^{\infty} f_{\gamma_{T_1,D}|T}(r)\, dr - \int_0^{\varphi} \left(1 - F_{\gamma_{m|\{S,T\}}}(\varphi - r)\right) f_{\gamma_{T_1,D}|T}(r)\, dr \end{aligned} \tag{9}$$

conditioned on S and T which can be written as

$$\begin{aligned} F_{\gamma_{m|\{S,T\}}}(x) &= 1 - \prod_{i=1}^{2} \left(1 - F_{\gamma_{T_1,i}|\{S,T\}}(x)\right) \\ &= 1 - \exp\left(-\frac{T+1}{E\Omega_1} x\right) \exp\left(-\frac{S + \omega_1 T + 1 + \omega_1}{\omega_2 E \Omega_2} x\right) \end{aligned} \tag{10}$$

and $f_{\gamma_{T_1,D}|T}(x)$ is the PDF of $\gamma_{T_1,D}$ conditioned on T which can be expressed as

$$f_{\gamma_{T_1,D}|T}(x) = \frac{T+1}{E\Omega_0} \exp\left(-\frac{T+1}{E\Omega_0} x\right). \tag{11}$$

Then the CDF of $\gamma_{T_1}^{UB}$ can be obtained by averaging the conditioned CDF with respect to the PDFs of S and T, i.e.,

$$\begin{aligned} F_{\gamma_{T_1}^{UB}}(\varphi) &= \int_0^{\infty} \int_0^{\infty} f_S(s) f_T(t)\, F_{\gamma_{T_1}^{UB}|\{S,T\}}(\varphi)\, ds dt \\ &= 1 - \underbrace{\int_0^{\infty} \int_{\varphi}^{\infty} f_T(t)\, f_{\gamma_{T_1,D}|T}(r) dr dt}_{P_1(\varphi)} \\ &\quad - \int_0^{\infty} \int_0^{\infty} \int_0^{\varphi} f_S(s)\, f_T(t)\, f_{\gamma_{T_1,D}|T}(r) \\ &\quad \times \left(1 - F_{\gamma_{m|\{S,T\}}}(\varphi - r)\right) dr ds dt, \end{aligned} \tag{12}$$

where $f_S(s)$ and $f_T(t)$ are the PDFs of RVs S and T, respectively. Note that T is the sum of a finite number of exponential RVs with different means. Hence with the help of [12], the PDF of T can be written as $f_T(t) = \sum_{k=1}^{L_1} \frac{p_k}{E_I} \exp\left(-\frac{t}{E_I \rho_{1,k}}\right)$, where $\rho_{1,k}$ is the variance of $d_{T_1,k}$ and $p_k = \prod_{j=1, j\neq k}^{L_1} \frac{1}{\rho_{1,k} - \rho_{1,j}}$ for $L_1 \geq 2$ and $p_k = \frac{1}{\rho_{1,k}}$ for $L_1 = 1$. Substituting the expressions of $f_T(t)$ and $f_{\gamma_{T_1,D}|T}(r)$ into (12), we can obtain

$$P_1(\varphi) = \exp\left(-\frac{\varphi}{E\Omega_0}\right) \sum_j \frac{p_j E\Omega_0 / E_I}{\varphi + E\Omega_0 / E_I \rho_{1,j}}. \tag{13}$$

Moreover, denoting $\rho_{R,k}$ as the variance of $d_{R,k}$, the PDF of S can be given by $f_S(s) = \sum_{k=1}^{L_R} \frac{q_k}{E_I} \exp\left(-\frac{s}{E_I \rho_{R,k}}\right)$, where $q_k = \prod_{j=1, j\neq k}^{L_R} \frac{1}{\rho_{R,k} - \rho_{R,j}}$ for $L_R \geq 2$ and $q_k = \frac{1}{\rho_{R,k}}$ for $L_R = 1$. By substituting the PDFs of S and T into (12) and using the results of Appendix, the third term in the right-hand side of (12) (denoted by $P_2(\varphi)$) can be evaluated as

$$\begin{aligned} P_2(\varphi) &= \frac{\omega_2}{E_I^2} \exp\left(-\frac{\varphi}{E\lambda_1}\right) \sum_j \sum_k \frac{p_j q_k}{\varphi + \beta_{j,k}} \\ &\times \left(\frac{E^2 \Omega_2 \lambda_2}{\varphi + E\lambda_2 / E_I \rho_{1,j}} - \frac{E^2 \Omega_0 \Omega_2}{\varphi + E\Omega_0 / E_I \rho_{1,j}} \exp\left(-\frac{\Phi_b}{E}\varphi\right)\right. \\ &\left. + \left(1 + \frac{E\Omega_0}{\varphi + \beta_{j,k}}\right) \Theta_k(\varphi) + \left(\frac{1}{\omega_2} + \frac{E\Omega_0 \Omega_2 \Phi_a}{\varphi + \beta_{j,k}}\right) \Xi_j(\varphi)\right), \end{aligned} \tag{14}$$

where $\Phi_a = \frac{1}{\Omega_0} - \frac{1}{\Omega_1} - \frac{\omega_1}{\omega_2}\frac{1}{\Omega_2} \neq 0$, $\Phi_b = \frac{1}{\Omega_0} - \frac{1}{\Omega_1} - \frac{1+\omega_1}{\omega_2 \Omega_2}$, $\beta_{j,k} = \frac{E\Omega_0}{E_I}\left(\frac{1}{\rho_{1,j}} + \frac{\omega_2 \Phi_a \Omega_2}{\rho_{R,k}}\right)$, $\lambda_1 = \left(\frac{1}{\Omega_1} + \frac{\omega_1 + 1}{\omega_2}\frac{1}{\Omega_2}\right)^{-1}$ and $\lambda_2 = \left(\frac{1}{\Omega_1} + \frac{\omega_1}{\omega_2}\frac{1}{\Omega_2}\right)^{-1}$. $\Theta_k(\varphi)$ and $\Xi_j(\varphi)$ can be expressed[4] as in (15) and (16) at the top of the next page, where $\vartheta_j = \frac{\varphi}{\Phi_a}\left(\frac{1}{\Omega_1} + \frac{\omega_1}{\omega_2}\frac{1}{\Omega_2}\right) + \frac{E}{\Phi_a E_I \rho_{1,j}}$. $\mathrm{Ei}(\cdot)$ and $\phi(\cdot)$ are the exponential integral [13, 3.351.6] and lower incomplete gamma function [13, 8.350.1], respectively. Besides, for the case of $\Phi_a = 0$, $P_2(\varphi)$ can be written as [Appendix]

$$\begin{aligned} P_2(\varphi) &= \frac{\omega_2}{E_I^2} \exp\left(-\frac{\varphi}{E\lambda_1}\right) \sum_j \sum_k p_j q_k \\ &\times \frac{1}{\varphi + E\Omega_0 / E_I \rho_{1,j}} \left(1 + \frac{E\Omega_0}{\varphi + E\Omega_0 / E_I \rho_{1,j}}\right) \Theta_k(\varphi). \end{aligned} \tag{17}$$

Then we will test the convergence of the infinite series involved in the expressions of $\Theta_k(\varphi)$ and $\Xi_j(\varphi)$. Defining

$$\Delta_l(\eta_1, \eta_2, \eta_3, \eta_4) = \frac{\eta_1^l \eta_4^{-(l+1)}}{\left(\eta_2 \varphi + \eta_3\right)^{l+1}} \phi\left(l+1, \varphi \eta_4\right), \tag{18}$$

where $0 < \eta_1 \leq \eta_2$ and $\eta_3 > 0$. Then it can be shown that $\Theta_k(\varphi) = \Delta_l\left(\frac{1}{E\Omega_2}, \frac{1}{E\Omega_2}, \frac{\omega_2}{E_I \rho_{R,k}}, \frac{\Phi_b}{E}\right)$ $(\Phi_b > 0)$ and $\Xi_j(\varphi) = \exp\left(-\frac{\Phi_b}{E}\varphi\right) \Delta_l\left(\frac{\Phi_a}{E}, \frac{1}{E\Omega_0}, \frac{1}{E_I \rho_{1,j}}, -\frac{\Phi_b}{E}\right)$ $(\Phi_b < 0 < \Phi_a)$, thus it is sufficient to prove that the infinite series $\sum_{l=1}^{\infty} \Delta_l(\eta_1, \eta_2, \eta_3, \eta_4)$ is convergent. Using [13, 3.381.1], it can be shown that

$$\begin{aligned} &\lim_{l\to\infty} \sqrt[l]{\Delta_l(\eta_1, \eta_2, \eta_3, \eta_4)} \\ &= \lim_{l\to\infty} \sqrt[l]{\frac{\eta_1^l}{(\eta_2\varphi + \eta_3)^{l+1}} \int_0^{\varphi} r^l \exp(-\eta_4 r) dr} \\ &\leq \lim_{l\to\infty} \sqrt[l]{\frac{\eta_1^l \varphi^l}{(\eta_2\varphi + \eta_3)^{l+1}} \int_0^{\varphi} \exp(-\eta_4 r) dr} \\ &= \frac{\eta_1 \varphi}{\eta_2 \varphi + \eta_3} < 1 \end{aligned} \tag{19}$$

[4] Note that the series expression of $\Theta_k(\varphi)$ (for $\Phi_b > 0$ in (15)) is also valid for the case of $\Phi_b < 0$. However, we present another closed-form expression without infinite series to facilitate the computation of $\Theta_k(\varphi)$ when $\Phi_b < 0$. Similarly, it can be seen the series expression of $\Xi_j(\varphi)$ (for $\Phi_b < 0 < \Phi_a$ in (16)) is always valid except for the case ($\Phi_a = 0$ or $\Phi_b = 0$).

$$\Theta_k(\varphi)=\begin{cases}\sum\limits_{l=0}^{\infty}\frac{E\Omega_2}{(\varphi+\omega_2E\Omega_2/E_I\rho_{R,k})^{l+1}}\left(\frac{\Phi_b}{E}\right)^{-(l+1)}\phi\left(l+1,\frac{\varphi\Phi_b}{E}\right),\Phi_b>0\\ E\Omega_2\exp\left(-\frac{\Phi_b}{E}\left[\varphi+\frac{\omega_2E\Omega_2}{E_I\rho_{R,k}}\right]\right)\left(\mathrm{Ei}\left(\frac{\Phi_b}{E}\left[\varphi+\frac{\omega_2E\Omega_2}{E_I\rho_{R,k}}\right]\right)-\mathrm{Ei}\left(\frac{\omega_2\Phi_b\Omega_2}{E_I\rho_{R,k}}\right)\right),\Phi_b<0\\ E\Omega_2\ln\left(\frac{E_I\rho_{R,k}}{\omega_2E\Omega_2}\varphi+1\right),\Phi_b=0\end{cases}\tag{15}$$

$$\Xi_j(\varphi)=\begin{cases}\frac{E}{\Phi_a}\exp\left(\frac{\Phi_b\vartheta_j}{E}\right)\left(\mathrm{Ei}\left(-\frac{\Phi_b(\varphi+\vartheta_j)}{E}\right)-\mathrm{Ei}\left(-\frac{\Phi_b\vartheta_j}{E}\right)\right),\Phi_a>\Phi_b>0\\ \exp\left(-\frac{\Phi_b\varphi}{E}\right)\sum\limits_{l=0}^{\infty}\frac{E\Phi_a^l\Omega_0^{l+1}}{(\varphi+E\Omega_0/E_I\rho_{1,j})^{l+1}}\left(-\frac{\Phi_b}{E}\right)^{-(l+1)}\phi\left(l+1,-\frac{\varphi\Phi_b}{E}\right),\Phi_b<0<\Phi_a\\ \frac{E}{\Phi_a}\exp\left(-\frac{\Phi_b}{E}\left[\varphi-\frac{\varphi+\frac{E\Omega_0}{E_I\rho_{1,j}}}{\Omega_0\Phi_a}\right]\right)\left(\mathrm{Ei}\left(-\frac{\Phi_b}{E}\frac{\varphi+\frac{E\Omega_0}{E_I\rho_{1,j}}}{\Omega_0\Phi_a}\right)-\mathrm{Ei}\left(\frac{\Phi_b}{E}\left[\varphi-\frac{\varphi+\frac{E\Omega_0}{E_I\rho_{1,j}}}{\Omega_0\Phi_a}\right]\right)\right),\Phi_b<\Phi_a<0\\ \frac{E}{\Phi_a}\ln\left(\frac{1+\varphi E_I\rho_{1,j}/E\Omega_0}{1+\varphi E_I\rho_{1,j}/\lambda E}\right),\Phi_b=0\end{cases}\tag{16}$$

By the root test [14], it can be seen that the infinite series in (15) and (16) are always convergent when $\varphi<\infty$.

Finally, the lower bound of the outage probability for the AF-based TDBC protocol in the presence of CCI can be derived by substituting (13) and (14) (or (17)) in to (12), i.e.,

$$P_{T_1}^{OUT-LB}(R_t)=F_{\gamma_{T_1}^{UB}}(\varphi)=1-P_1(\varphi)-P_2(\varphi).\tag{20}$$

B. Asymptotic Analysis

To offer an intuitive observation into the effect of CCI on the outage performance, we develop asymptotic analysis on the outage probability based on the analysis of subsection A. According to [15][16], the asymptotic expression can be derived by performing McLaurin series expansion to $F_{\gamma_{T_1}^{UB}}(\varphi)$ and taking only the first two order terms. Wherein the McLaurin series expansion of $\Theta_k(\varphi)$ can be given by

$$\Theta_k(\varphi)=\Theta_k(0)+\Theta_k^{(1)}(0)\varphi+\frac{\Theta_k^{(2)}(0)}{2}\varphi^2+\mathcal{O}(\varphi^2),\tag{21}$$

where $\mathcal{O}(\delta)$ indicates the higher order term of δ and $\Theta_k^{(n)}(0)$ ($n=1,2$) can be determined by[5] [13, 0.410]:

$$\begin{aligned}\Theta_k^{(1)}(0)&=\left[g(\varphi,r)|_{r=\varphi}\right]_{\varphi=0}\\ \Theta_k^{(2)}(0)&=\left[\left.\frac{dg(\varphi,r)}{d\varphi}\right|_{r=\varphi}+\frac{d}{d\varphi}\left(g(\varphi,r)|_{r=\varphi}\right)\right]_{\varphi=0},\end{aligned}\tag{22}$$

where $g(\varphi,r)=\left(\frac{\varphi-r}{E\Omega_2}+\frac{\omega_2}{E_I\rho_{R,k}}\right)^{-1}\exp\left(-\frac{\Phi_b}{E}r\right)$. Moreover, the McLaurin series expansion of $\Xi_j(\varphi)$ can be calculated using the similar method as in the above. Finally, the asymptotic expression of $F_{\gamma_{T_1}^{UB}}(\varphi)$ can be written as[6]

$$\begin{aligned}F_{\gamma_{T_1}^{UB}}(\varphi)\approx\left(\frac{E_I}{E}\right)^2\frac{\varphi^2}{2\Omega_0}\Bigg[&\sum_j\sum_k\frac{p_jq_k\rho_{R,k}^2}{\omega_2\Omega_2}\left(\frac{\rho_{1,j}}{E_I}+\rho_{1,j}^2\right)\\ &\sum_j p_j\left(\frac{\rho_{1,j}}{E_I^2\lambda_1}+\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)\frac{\rho_{1,j}^2}{E_I}+\frac{2\rho_{1,j}^3}{\lambda_2}\right)\Bigg].\end{aligned}\tag{23}$$

Through the asymptotic expression, it can be seen that when the ratio of useful power to interference power is constant, the AF-based TDBC protocol dose not achieve any diversity.

IV. Simulation Results

In this section, we provide the simulation results to verify our theoretical analyses on the outage probability. It is assumed that T_1, T_2 and R are located in a straight line and R is between T_1 and T_2. The distance between two terminals is normalized to 1 and the path loss exponent is set to 4 [17], thus the variances of h_0, h_1 and h_2 can be computed as $\Omega_0=1$, $\Omega_1=D_1^{-4}$ and $\Omega_2=(1-D_1)^{-4}$, respectively, where D_1 indicates the normalized distance between T_1 and R. The normalized distances between node N and the interferers that interfere N are assumed to be evenly distributed on the interval $(\alpha_1,\alpha_2)=(1,1.5)$, where $N\in\{T_1,T_2,R\}$. Hence, $\rho_{R,k}$ and $\rho_{i,k}$ can be determined by $\rho_{R,k}=(\alpha_1+(k-1)(\alpha_2-\alpha_1)/(L_R-1))^{-4}$ and $\rho_{i,k}=(\alpha_1+(k-1)(\alpha_2-\alpha_1)/(L_i-1))^{-4}$.

In Fig. 2, the outage performance at T_1 is presented as a function of the transmit power E, where E/E_I is fixed

[5]Note the $\Theta_k^{(n)}(\varphi)$ here is obtained by taking the derivative of its integral expression which is given by (26).

[6]The asymptotic expression here is obtained based on the expression of $F_{\gamma_{T_1}^{UB}}(\varphi)$ in the case of $\Phi_a\neq0$, however, for the case of $\Phi_a=0$, it can be verified that this expression is also valid.

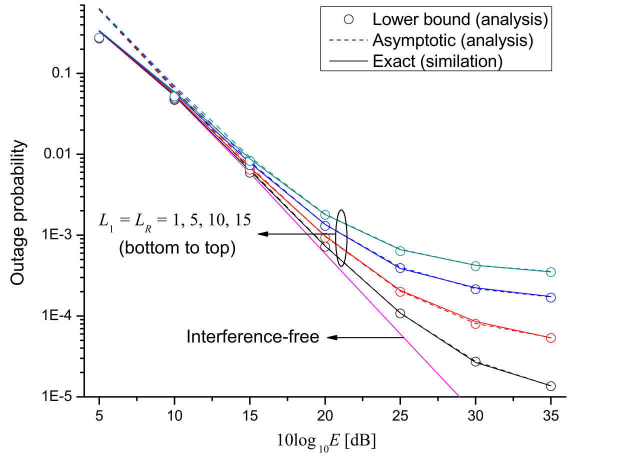

Fig. 2: Outage performance at T_1 with fixed E/E_I, where D_1 = 0.5, $\omega_1 = 0.5$ and $R_t = 1$ bit/s/Hz.

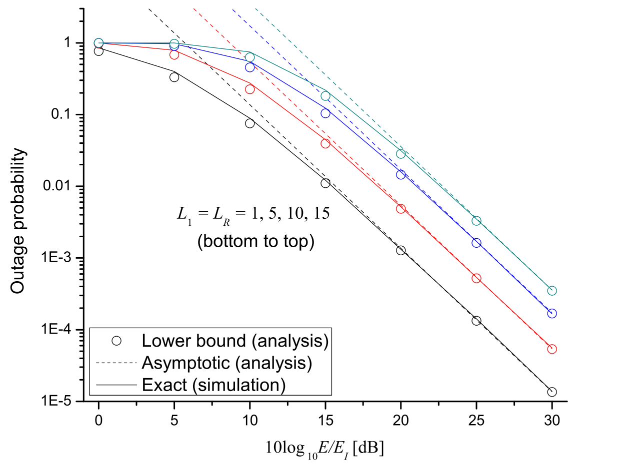

Fig. 3: Outage performance at T_1 versus E/E_I, where E_I = 5dB, D_1 = 0.5, $\omega_1 = 0.5$ and $R_t = 1$ bit/s/Hz.

at 30dB. The outage performance of the scenario without interference is also presented as a benchmark. It is seen that the outage performance degrades as the numbers of interferers increase. Furthermore, the slope of outage probability curves are steep in the low SNR region ($E < 15$dB). This is because the power of AWGN dominates the interference power. While a performance floor can be observed in the high SNR region due to the dominant role of interference power. This phenomenon also indicates that the achievable diversity order of the TDBC protocol in the interference-limited scenario is zero. Fig. 3 studies the outage performance at T_1 against E/E_I. From Fig. 3, it can be seen that the outage probability increases as the numbers of interferers as well as E_I/E increase as expected. Finally, a fine agreement between the analytic results and simulations can be observed from the figures.

Fig. 4 investigates the effect of relay placement on the outage performance, where numbers of interferers that affect T_1 and R may different. Without loss of generality, we set $L_1 = 1$ and let L_R increase from 1, 5, 10 to 15. Then we examine the outage performance at T_1 as a function of D_1. It is seen from the figure that the optimal relay placement moves toward T_2 as the number of interferers (and total interference power) that affects the relay increases. This is because the AF operation adopted by the relay. From equation (4), we can see that, for given (α_1, α_2), when L_R increases, to decrease the amplified interference (i.e., the term $\left(\mathcal{A}_1^2 + \mathcal{A}_2^2\right) E|h_1|^2 \left(E_I \sum_{k=1}^{L_R} |d_{R,k}|^2 + 1\right)$), the relay should move toward T_2 to decrease $|h_1|^2$.

V. Conclusions

In this paper, we study the effect of CCI on the AF-based TDBC protocol. Lower bound of the outage probability is derived and is shown to provide a good match with the simulation results. Meanwhile, a simpler asymptotic expression of outage probability is also provided. We show through both analytic and simulation results that the achievable diversity of the TDBC protocol in the interference limited scenario is zero. Moreover, we investigate the effect of relay placement on the outage probability and show that when only consider the outage performance at one terminal (e.g. T_1), as the number of interferers that interferes the relay increases, the optimal relay placement needs to move toward T_2 in order to obtain the optimal outage probability at T_1.

Appendix

Substituting the PDFs of S and T into (12) and interchanging the integration order, we can obtain

$$\begin{aligned} P_2(\varphi) &= \frac{1}{E_I^2} \exp\left(-\left(\frac{1}{\Omega_1} + \frac{\omega_1+1}{\omega_2\Omega_2}\right)\frac{\varphi}{E}\right) \sum_j \sum_k \frac{p_j q_k}{E\Omega_0} \\ &\int_0^{\varphi} \exp\left(-\frac{\Phi_b}{E} r\right) \int_0^{\infty} \exp\left(-\left[\frac{\varphi - r}{\omega_2 E\Omega_2} + \frac{1}{E_I \rho_{R,k}}\right] s\right) ds \\ &\int_0^{\infty} \exp\left(-\left[\frac{\Phi_a r}{E} + \left(\frac{1}{\Omega_1} + \frac{\omega_1}{\omega_2}\frac{1}{\Omega_2}\right)\frac{\varphi}{E} + \frac{1}{E_I \rho_{1,j}}\right] t\right) \\ &\times (t+1)\, dt dr. \end{aligned} \tag{24}$$

Case1 ($\Phi_a \neq 0$): In this case, solving the integrals with respect to s and t, we can yield

$$\begin{aligned} &P_2(\varphi) = \tfrac{\omega_2}{E_I^2} \exp\left(-\left(\tfrac{1}{\Omega_1} + \tfrac{\omega_1+1}{\omega_2\Omega_2}\right)\tfrac{\varphi}{E}\right) \\ &\times \sum_j \sum_k \frac{p_j q_k}{\varphi + \frac{E\Omega_0}{E_I}\left(\frac{1}{\rho_{1,j}} + \frac{\omega_2 \Phi_a \Omega_2}{\rho_{R,k}}\right)} \\ &\times \left\{ \left[\frac{E\Omega_2}{\frac{\varphi}{E}\left(\frac{1}{\Omega_1} + \frac{\omega_1}{\omega_2}\frac{1}{\Omega_2}\right) + \frac{1}{E_I \rho_{1,j}}} - \frac{E\Omega_2}{\frac{\varphi}{E\Omega_0} + \frac{1}{E_I \rho_{1,j}}} \exp\left(-\tfrac{\Phi_b}{E}\varphi\right)\right] \right. \\ &+ \left(1 + \frac{E\Omega_0}{\varphi + \frac{E\Omega_0}{E_I}\left(\frac{1}{\rho_{1,j}} + \frac{\omega_2\Phi_a\Omega_2}{\rho_{R,k}}\right)}\right) \Theta_k(\varphi) \\ &\left. + \left(\frac{1}{\omega_2} + \frac{E\Omega_0\Omega_2\Phi_a}{\varphi + \frac{E\Omega_0}{E_I}\left(\frac{1}{\rho_{1,j}} + \frac{\omega_2\Phi_a\Omega_2}{\rho_{R,k}}\right)}\right) \Xi_j(\varphi) \right\}, \end{aligned} \tag{25}$$

where $\Theta_k(\varphi)$ and $\Xi_j(\varphi)$ are expressed as

$$\begin{aligned} \Theta_k(\varphi) &= \int_0^{\varphi} \frac{1}{\frac{\varphi - r}{E\Omega_2} + \frac{\omega_2}{E_I \rho_{R,k}}} \exp\left(-\tfrac{\Phi_b}{E} r\right) dr \\ \Xi_j(\varphi) &= \int_0^{\varphi} \frac{1}{\frac{\Phi_a}{E} r + \left(\frac{1}{\Omega_1} + \frac{\omega_1}{\omega_2}\frac{1}{\Omega_2}\right)\frac{\varphi}{E} + \frac{1}{E_I \rho_{1,j}}} \exp\left(-\tfrac{\Phi_b}{E} r\right) dr \end{aligned} \tag{26}$$

When $\Phi_b > 0$, to solve the integral $\Theta_k(\varphi)$, we apply Taylor series expansion $\left(\frac{\varphi - r}{E\Omega_2} + \frac{\omega_2}{E_I \rho_{R,k}}\right)^{-1} = E\Omega_2 \sum_{l=0}^{\infty} r^l \Big/ \left(\varphi + \omega_2 E\Omega_2 / E_I \rho_{R,k}\right)^{l+1}$. Then based on [13, 3.381.1], the integral can be solved into

$$\Theta_k(\varphi) = \sum_{l=0}^{\infty} \frac{E\Omega_2}{\left(\varphi + \frac{\omega_2 E\Omega_2}{E_I \rho_{R,k}}\right)^{l+1}} \left(\frac{\Phi_b}{E}\right)^{-(l+1)} \phi\left(l+1, \frac{\varphi \Phi_b}{E}\right). \tag{27}$$

When $\Phi_b < 0$, $\Theta_k(\varphi)$ can be solved by replacing $(\varphi - r)$ with t, and then using the integral result reported in [13, 3.352.1],

$$\begin{aligned} \Theta_k(\varphi) &= \exp\left(-\frac{\Phi_b}{E}\varphi\right) \int_0^{\varphi} \frac{1}{\frac{t}{E\Omega_2} + \frac{\omega_2}{E_I \rho_{R,k}}} \exp\left(\frac{\Phi_b}{E} t\right) dt \\ &= E\Omega_2 \exp\left(-\frac{\Phi_b}{E}\left[\varphi + \frac{\omega_2 E\Omega_2}{E_I \rho_{R,k}}\right]\right) \\ &\times \left(\mathrm{Ei}\left(\frac{\Phi_b}{E}\left[\varphi + \frac{\omega_2 E\Omega_2}{E_I \rho_{R,k}}\right]\right) - \mathrm{Ei}\left(\frac{\omega_2 \Phi_b \Omega_2}{E_I \rho_{R,k}}\right)\right). \end{aligned} \tag{28}$$

On the other hand, using [13, 3.352.1] on $\Xi_j(\varphi)$ when $\Phi_a > \Phi_b > 0$, we can yield $\Xi_j(\varphi) = \frac{E}{\Phi_a} \exp\left(\frac{\Phi_b \vartheta_j}{E}\right)\left(\mathrm{Ei}\left(-\frac{\Phi_b(\varphi + \vartheta_j)}{E}\right) - \mathrm{Ei}\left(-\frac{\Phi_b \vartheta_j}{E}\right)\right)$. To solve the integral $\Xi_j(\varphi)$ when $\Phi_b < 0 < \Phi_a$, similar approach as in the case of $\Phi_b > 0$ for $\Theta_k(\varphi)$ can be used, then we obtain

$$\begin{aligned} \Xi_j(\varphi) &\overset{\varphi - r = t}{=} \exp\left(-\frac{\Phi_b \varphi}{E}\right) \int_0^{\varphi} \frac{1}{\frac{\varphi}{E\Omega_0} + \frac{1}{E_I \rho_{1,j}} - \frac{\Phi_a}{E} t} \exp\left(\frac{\Phi_b}{E} t\right) dt \\ &= \exp\left(-\frac{\Phi_b \varphi}{E}\right) \sum_{l=0}^{\infty} \frac{E \Phi_a^l \Omega_0^{l+1}}{\left(\varphi + E\Omega_0 / E_I \rho_{1,j}\right)^{l+1}} \\ &\times \left(-\frac{\Phi_b}{E}\right)^{-(l+1)} \phi\left(l+1, -\frac{\varphi \Phi_b}{E}\right). \end{aligned} \tag{29}$$

Using the similar method as in the case of $\Phi_b < 0$ for $\Theta_k(\varphi)$, it can be shown that, when $\Phi_b < \Phi_a < 0$,

$$\begin{aligned} \Xi_j(\varphi) &= -\frac{E}{\Phi_a} \exp\left(-\frac{\Phi_b}{E}\left[\varphi - \frac{\varphi + \frac{E\Omega_0}{E_I \rho_{1,j}}}{\Omega_0 \Phi_a}\right]\right) \\ &\times \left(\mathrm{Ei}\left(\frac{\Phi_b}{E}\left[\varphi - \frac{\varphi + \frac{E\Omega_0}{E_I \rho_{1,j}}}{\Omega_0 \Phi_a}\right]\right) - \mathrm{Ei}\left(-\frac{\Phi_b}{E} \frac{\varphi + \frac{E\Omega_0}{E_I \rho_{1,j}}}{\Omega_0 \Phi_a}\right)\right). \end{aligned} \tag{30}$$

Moreover, it is very easy to verify that $\Theta_k(\varphi)$ and $\Xi_j(\varphi)$ can be expressed as in (15) and (16) when $\Phi_b = 0$, thus the derivations details for this case are omitted.

Case2 $(\Phi_a = 0)$: In this case, it is easy to verify by using the equation $\Phi_a = 0$ that the sum of the first term and third term in bracket $\{\cdot\}$ of (25) equals to zero, thus $P_2(\varphi)$ can be simplified to (17).

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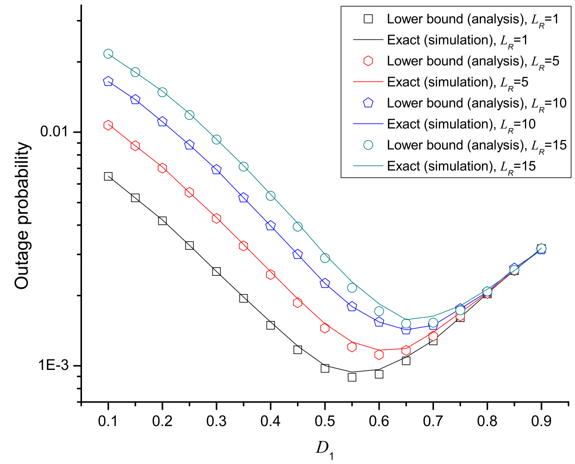

Fig. 4: Outage performance at T_1 versus relay placement, $E=$ 30dB, $E_I=10$dB, $\omega_1=0.5$, $R_t=1$ bit/s/Hz and $L_1=1$.

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