

# On the Capacity of the Cognitive Interference Channel with a Relay

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**Abstract**—Interference forwarding has been shown to be beneficial in the interference channel with a relay as it enlarges the strong interference region, allowing the decoding of the interference at the receivers for larger ranges of the channel gains. In this work we demonstrate the benefit of adding a relay to the cognitive interference channel. We pay special attention to the effect of interference forwarding in this configuration. Two setups are presented. In the first, the interference forwarded by the relay is the primary user's signal, and in the second, this is the cognitive user's signal. We characterise the capacity regions of these two models in the case of strong interference. We show that as opposed to the first setup, in the second setup the capacity region is enlarged, compared to the capacity region of the cognitive interference channel, when the relay does not help the intended receiver.

## I. INTRODUCTION

Interference takes the role of noise in multiterminal networks as the first source of impairment to reliable communications. It is unfortunate however that a general approach to dealing with interference is not known. As for the interference channel (IC), its capacity region is only known for the case where both receivers experience strong interference [1]. When the interference level is sufficiently strong at the receivers, these are able to decode the impairment and subtract it from their received signal without incurring in a rate penalty. With that in mind, the interference channel with a relay (IRC) [2] was presented as a model that utilizes a relay to not only increase the intended signal level at one receiver but also to increase the interference level at the other receiver. In that way, the authors of [2] demonstrated that by relaying the interference, process known as interference forwarding (IF), the conditions are created at the interfered receiver for it to be able to decode the impairment without rate penalty. Besides, by considering the scenario where the intended receiver is not benefited by the relayed message and the unintended receiver only receives interference from the relay, it was demonstrated that the rate region is enlarged, leaving no doubt about the benefit of interference forwarding.

In this work we study the benefits of interference forwarding in the cognitive interference channel (CIC) with a relay or cognitive interference relay channel (CIRC). The CIC [3] is a model for unidirectional cooperation at the transmitters where one transmitter (cognitive transmitter) is assumed to have noncausal knowledge of the other transmitter's message

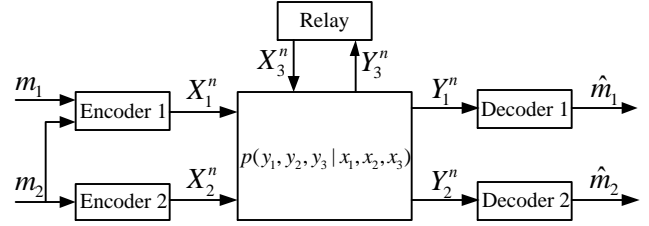


Fig. 1. The cognitive interference relay channel. The cognitive side is denoted by the subscript 1.

(primary transmitter). As opposed to the traditional conception of a cognitive radio (CR), in the CIC both transmitters utilize the channel simultaneously. As the cognitive transmitter has knowledge of both messages, it utilizes its resources to cooperate with the primary user by sending the primary message and also by applying sophisticated encoding techniques to eliminate the effect of the interference at its receiver. For a comprehensive account of the CIC refer to [4]. Fig. 1 depicts the CIRC in the discrete memoryless case where transmitter 1 is cognitive.

Our model is also motivated by practical applications. On the one hand, it is well known that cooperative communications can provide huge benefits, improving the efficiency of the spectrum utilization. On the other hand a secondary system that ideally should not interfere with the primary users can also provide other ways to help the primary users besides the relaying that takes place at the cognitive transmitter. This is achieved in our model by the external relay.

We consider two setups in our analysis of the CIRC. In the first setup, the relay only conveys the primary user's signal, which is interference at the cognitive receiver. In the second setup, the relay only conveys the cognitive user's signal, which turns out as interference at the primary receiver. We characterize the capacity region of both setups in strong interference under certain conditions, namely when there is no rate penalty for decoding both messages at both receivers. The encoding scheme proceeds, as for the CIC in strong interference, by superposition coding, the relay utilizes the decode-forward (DF) [5] encoding scheme and the decoding at the receivers proceeds by backward and simultaneous non-unique decoding. To analyse the benefit due to IF only, we modify both setups

by cutting the link from the relay to the receiver that the relayed message is intended to. In this way the relay will only send interference to the unintended receiver. Through this modification we demonstrate that as opposed to the first setup, in the second setup a real benefit of IF is present as the rate region is enlarged compared to the CIC rate region in strong interference.

The rest of the paper is organized as follows: We present the channel model, definitions and assumptions in Section II. Our main results are in Section III where we present the achievable rates and capacity when certain conditions hold. In Section IV we compute the rate regions in both setups when Gaussian inputs are assumed and compare them with the capacity region of the CIC in strong interference. Numerical results are presented. And we finalize the paper with our conclusions in Section V.

## II. CHANNEL MODEL AND DEFINITIONS

The cognitive interference channel with a relay is a model that extends the interference channel to the case where it is possible to have unidirectional cooperation at the transmitters. Besides a relay that helps in the communication is assumed. This channel configuration is depicted in Fig. 1. The notation of [5] is utilized throughout the paper.

**Definition 1.** The discrete memoryless (DM) CIRC consists of three finite input sets  $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3$ , three finite output sets  $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3$ , and a probability transition function  $p(y_1, y_2, y_3 | x_1, x_2, x_3)$ . The channel is memoryless in the sense that the current received symbols  $(Y_{1i}, Y_{2i}, Y_{3i})$  and the messages and past symbols  $(m_1, m_2, X_1^{i-1}, X_2^{i-1}, X_3^{i-1}, Y_1^{i-1}, Y_2^{i-1}, Y_3^{i-1})$  are conditionally independent given the current transmitted symbols  $(X_{1i}, X_{2i}, X_{3i})$ . A  $(2^{nR_1}, 2^{nR_2}, n)$  code for the DM-CIRC consists of a pair of uniformly distributed messages  $m_1 \in [1 : 2^{nR_1}]$  and  $m_2 \in [1 : 2^{nR_2}]$ , two encoding functions at the transmitters  $X_1^n = f_1(m_1, m_2)$ ,  $X_2^n = f_2(m_2)$ , an encoding function at the relay  $X_{3i} = f_{3i}(Y_3^{i-1})$  and two decoding functions  $\hat{m}_t = g_t(Y_t^n)$ , for  $t = 1, 2$ . The average probability of error is defined as  $P_e^{(n)} = P(\bigcup_t \{\hat{m}_t \neq m_t\})$ . A rate pair  $(R_1, R_2)$  is said to be achievable if there exists a sequence of  $(2^{nR_1}, 2^{nR_2}, n)$  codes such that  $\lim_{n \rightarrow \infty} P_e^{(n)} = 0$ . The capacity region of the DM-CIRC is the closure of the set of all achievable rate regions.

### A. Two setups

We study the benefit of IF in the CIRC. As for the IRC in [2], we study the case where the relay only transmits one of the messages, namely this transmission will be interference for one of the receivers. As opposed to the IRC, the CIRC is asymmetrical, hence we need to consider two separate cases, when the relay transmits the primary user's signal and when it transmits the cognitive user's signal. For this the following assumptions about the relay receiver are made.

**Definition 2.** (Setup 1) The observation  $Y_3$  at the relay is independent of  $X_1$  given  $X_2, X_3$ , which can be stated as

$$p(y_3 | x_1, x_2, x_3) = p(y_3 | x_2, x_3), \quad (1)$$

and implies that  $X_1 \rightarrow (X_2, X_3) \rightarrow Y_3$  form a Markov chain.

**Definition 3.** (Setup 2) The observation  $Y_3$  at the relay is independent of  $X_2$  given  $X_1, X_3$ , which can be stated as

$$p(y_3 | x_1, x_2, x_3) = p(y_3 | x_1, x_3), \quad (2)$$

and implies that  $X_2 \rightarrow (X_1, X_3) \rightarrow Y_3$  form a Markov chain.

These are good assumptions when strong shadowing affects the link from one of the transmitters to the relay in a wireless communication channel, i.e. the relay only "sees" the signal that originates at one transmitter at a time. In the following we define two degradedness conditions that will be utilized for establishing the capacity region in each setup.

**Definition 4.** (Degradedness condition 1) The observation  $Y_2$  at receiver 2 is independent of  $(X_1, X_2)$  given  $(X_3, Y_3)$ ,

$$p(y_2 | x_3, y_3, x_1, x_2) = p(y_2 | x_3, y_3), \quad (3)$$

which means that  $(X_1, X_2) \rightarrow (X_3, Y_3) \rightarrow Y_2$  form a Markov chain.

**Definition 5.** (Degradedness condition 2) The observation  $Y_1$  at receiver 1 is independent of  $(X_1, X_2)$  given  $(X_3, Y_3)$ ,

$$p(y_1 | x_3, y_3, x_1, x_2) = p(y_1 | x_3, y_3), \quad (4)$$

which means that  $(X_1, X_2) \rightarrow (X_3, Y_3) \rightarrow Y_1$  form a Markov chain.

## III. ACHIEVABLE RATES AND CAPACITY

We now present achievable rate regions for both setups. We also introduce two conditions analogous to the strong interference conditions for the CIC [6] under which we establish the capacity region of the DM-CIRC as described by each setup.

**Theorem 1.** An achievable region for the DM-CIRC (setup 1) consists of the set of rate pairs  $(R_1, R_2)$  such that:

$$R_1 \leq I(X_1; Y_1 | X_2, X_3), \quad (5a)$$

$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_1), \quad (5b)$$

$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_2), \quad (5c)$$

$$R_2 \leq I(X_2; Y_3 | X_3), \quad (5d)$$

for some joint input probability distribution that factors as  $p(x_2, x_3)p(x_1 | x_2, x_3)$ .

*Proof.* Encoder 2 cooperates with the relay by employing block Markov encoding. Encoder 1 applies superposition coding. The transmission is done over  $b$  blocks. The details are as follows:

1) *Code generation*: Fix a probability mass function (pmf)  $p(x_2, x_3)p(x_1|x_2, x_3)$  that attains the lower bound. For the transmission in each block  $j$ , generate  $2^{nR_2}$  independent codewords  $x_3^n(m_{2,j-1})$  according to  $\prod_{i=1}^n p(x_{3,i})$ . For each  $m_{2,j-1}$ , generate  $2^{nR_2}$  independent codewords  $x_2^n(m_{2,j}|m_{2,j-1})$  according to  $\prod_{i=1}^n p(x_{2,i}|x_{3,i})$ ;  $m_{2,j-1}, m_{2,j} \in [1 : 2^{nR_2}]$ . For each  $(x_2^n(m_{2,j}|m_{2,j-1}), x_3^n(m_{2,j-1}))$ , generate  $2^{nR_1}$  independent codewords  $x_1^n(m_{1,j}|m_{2,j}, m_{2,j-1})$  according to  $\prod_{i=1}^n p(x_{1,i}|x_{2,i}, x_{3,i})$ ;  $m_{1,j} \in [1 : 2^{nR_1}]$ .

2) *Encoding*: To send  $(m_{1,j}, m_{2,j})$  in block  $j$ , encoder 1 transmits  $x_1^n(m_{1,j}|m_{2,j}, m_{2,j-1})$  and encoder 2 transmits  $x_2^n(m_{2,j}|m_{2,j-1})$ .

3) *Relay encoding*: At the end of block  $j$ , the relay finds the unique message  $\hat{m}_{2,j}$  (message estimate at the relay) such that  $(x_2^n(\hat{m}_{2,j}|\hat{m}_{2,j-1}), x_3^n(\hat{m}_{2,j-1}), y_3^n(j)) \in \mathcal{T}_\epsilon^{(n)}$ . It transmits  $x_3^n(\hat{m}_{2,j})$  in block  $j+1$ .

4) *Backward decoding*: After all blocks are received, decoder 2 proceeds to decode backwards. The unique message  $\hat{m}_{2,j}$  such that  $(x_1^n(\hat{m}_{1,j+1}|\hat{m}_{2,j+1}, \hat{m}_{2,j}), x_2^n(\hat{m}_{2,j+1}|\hat{m}_{2,j}), x_3^n(\hat{m}_{2,j}), y_2^n(j+1)) \in \mathcal{T}_\epsilon^{(n)}$  is found successively with the initial condition  $\hat{m}_{2,b} = 1$ . Decoder 2 decodes  $\hat{m}_{1,j+1}$  non-uniquely. Similarly, decoder 1 finds the unique  $\hat{m}_{1,j+1}$  such that  $(x_1^n(\hat{m}_{1,j+1}|\hat{m}_{2,j+1}, \hat{m}_{2,j}), x_2^n(\hat{m}_{2,j+1}|\hat{m}_{2,j}), x_3^n(\hat{m}_{2,j}), y_1^n(j+1)) \in \mathcal{T}_\epsilon^{(n)}$ . The primary user's message is decoded non-uniquely.

5) *Analysis*: Without loss of generality, the transmission of the message pair (1, 1) in each block is assumed. At the end of block  $j+1$ , decoder 1 finds the unique messages  $m_{1,j+1}$  and  $m_{2,j}$ . The two nontrivial error events are:  $\mathcal{E}_{1,1} = \{m_{1,j+1} \neq 1, m_{2,j} = 1\}$  and  $\mathcal{E}_{1,2} = \{m_{1,j+1} \neq 1, m_{2,j} \neq 1\}$ . By the packing lemma [5], the probability of error is negligible as long as (5a) and (5b) hold. Similarly, at decoder 2 the nontrivial error events are:  $\mathcal{E}_{2,1} = \{m_{1,j+1} = 1, m_{2,j} \neq 1\}$  and  $\mathcal{E}_{2,2} = \{m_{1,j+1} \neq 1, m_{2,j} \neq 1\}$ . The latter error event is dominant and by the packing lemma its probability is negligible as long as (5c) holds. The relay makes an error if the following error event occurs:  $\tilde{\mathcal{E}}_3 = \{\hat{m}_{2,j} \neq 1\}$ . By the packing lemma its probability is negligible as long as (5d) holds.  $\square$

The following region is achievable for the channel under setup 2.

**Theorem 2.** *An achievable region for the DM-CIRC (setup 2) consists of the set of rate pairs  $(R_1, R_2)$  such that:*

$$R_1 \leq I(X_1, X_3; Y_1|X_2), \quad (6a)$$

$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_1), \quad (6b)$$

$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_2), \quad (6c)$$

$$R_1 \leq I(X_1; Y_3|X_2, X_3), \quad (6d)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_3|X_3), \quad (6e)$$

for some joint input probability distribution that factors as  $p(x_2)p(x_1, x_3|x_2)$ .

*Proof.* (Outline). It follows the lines of the proof of Theorem 1. The primary transmitter employs a codebook  $x_2^n$  generated according to  $p(x_2)$ . For each  $x_2^n$ , the cognitive transmitter and the relay employ a codebook  $(x_1^n, x_3^n)$  generated according to  $p(x_1, x_3|x_2)$ . Decoders decode backwards [5].  $\square$

Analogously as for the DM-CIC we define the strong interference conditions for the DM-CIRC as follows:

$$I(X_1, X_3; Y_1|X_2) \leq I(X_1, X_3; Y_2|X_2), \quad (7a)$$

$$I(X_1, X_2, X_3; Y_2) \leq I(X_1, X_2, X_3; Y_1). \quad (7b)$$

Similarly as in the CIC, under these conditions no rate penalty is incurred by decoding both messages at both receivers.

**Theorem 3.** *Under conditions (7) and (3), the rate region of Theorem 1 is the capacity region of the DM-CIRC of setup 1.*

*Proof.* For the converse, (5a) follows from standard techniques by using Fano's inequality [5], or from the cutset bound [7] with the cut  $S = \{X_1, Y_2\}$  and the assumption in (1). Under the condition (7b), (5b) is redundant; (5c) follows from Fano's inequality as  $n(R_1 + R_2)$

$$\begin{aligned} &\leq I(m_2; Y_2^n) + I(m_1; Y_1^n|m_2), \\ &\stackrel{(a)}{\leq} I(X_2^n, X_3^n; Y_2^n) + I(m_1; Y_1^n|m_2, X_2^n, X_3^n), \\ &\leq I(X_2^n, X_3^n; Y_2^n) + I(X_1^n, Y_1^n|X_2^n, X_3^n), \\ &\stackrel{(b)}{\leq} I(X_2^n, X_3^n; Y_2^n) + I(X_1^n; Y_2^n|X_2^n, X_3^n), \\ &\leq \sum_{i=1}^n I(X_{1i}, X_{2i}, X_{3i}; Y_{2i}), \end{aligned} \quad (8)$$

where (a) follows from the independence of the messages and the fact that conditioning reduces entropy, (b) follows by the multiletter characterization of (7a), which can be proved as in [6]. (5d) follows as well by standard techniques from Fano's inequality or by the cutset bound [7]. For the cut  $S = \{X_2, Y_1\}$  we obtain

$$\begin{aligned} R_2 &\leq I(X_2; Y_2, Y_3|X_1, X_3), \\ &= I(X_2; Y_3|X_1, X_3) + I(X_2; Y_2|X_1, X_3, Y_3), \\ &\stackrel{(a)}{=} I(X_2; Y_3|X_1, X_3), \\ &\stackrel{(b)}{=} I(X_2; Y_3|X_3), \end{aligned}$$

where (a) follows from (3), and (b) follows from (1).  $\square$

**Theorem 4.** *Under conditions (7) and (4), the rate region of Theorem 2 is the capacity region of the DM-CIRC of setup 2.*

*Proof.* The proof is similar to the proof of Theorem 3 and the details are omitted.  $\square$

*Remark 1.* In the CIRC of setup 2, condition (2) is not needed as due to the unidirectional cooperation at the cognitive transmitters, the relay is able to decode both messages. Additionally, condition (2) is not necessary in the proof of Theorem 4 as opposed to (1) in the proof of Theorem 3.

For comparison purposes we present next the capacity region of the DM-CIC in strong interference [6]. This region can be obtained from those in Theorem 1 and Theorem 2 by assuming the relay transmission  $X_3$  is known at the receivers and it is independent of the other inputs.

$$R_1 \leq I(X_1; Y_1 | X_2, X_3), \quad (9a)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_2 | X_3). \quad (9b)$$

With the same assumption about  $X_3$ , the strong interference conditions in (7) reduce to the ones for the CIC [6]. We present next a comparison of the region of (9) with the regions of Theorem 1 and Theorem 2.

#### A. Rate region comparison

Here we compare the regions of Theorem 1 and Theorem 2 with the region in (9). We start with the region of setup 1.

1) *CIC vs. CIRC of setup 1*: We attempt to see the benefits of IF in the CIRC of setup 1. In this setup the relay only forwards the primary user's message  $m_2$ . In order to have a fair comparison we assume receiver 2 does not benefit from message forwarding (MF) from the relay (the link from the relay to receiver 2 is off). In strong interference we have (5a) = (9a), (5b) is redundant due to (7b) and (5c) = (9b) as  $X_3 \rightarrow (X_1, X_2) \rightarrow Y_2$  form a Markov chain. Then the two regions coincide as long as (5c)  $\leq$  (5a) + (5d). We can conclude that in this setup, IF offers no direct benefit in terms of enlarging the rate region of the CIC. This can be interpreted as the benefit of IF has already been capitalized by the CIC as its cognitive transmitter does a sort of IF itself. We point out though that the rate region is indeed enlarged when MF to receiver 2 is allowed as we will see in Section IV. IF does indeed change the strong interference conditions with respect to those for the CIC. This can be seen as the CIC may not be in strong interference while the CIRC is, and decoding both messages at the receivers of the CIC is strictly suboptimal, reducing the region of the CIC compared to the region of the CIRC. As for the IRC [8], there may be the case as well when the CIC is in strong interference and the CIRC is not, in which the use of the relay offers no benefit to the cognitive receiver.

2) *CIC vs. CIRC of setup 2*: In the CIRC of setup 2 the relay decodes and forwards the cognitive user's signal, however, due to the superposition structure of the encoding at the cognitive transmitter, it is able to decode the primary user's message as well. Similar as before, we assume the link between the relay and the primary decoder is off. In strong interference (6a) reduces to (9a) as  $X_3 \rightarrow (X_1, X_2) \rightarrow Y_1$  form a Markov chain, (6b) is redundant due to (7b) and (6c) = (9b) +  $I(X_3; Y_2)$ . Then as long as (6a)  $\leq$  (6d) and (6c)  $\leq$  (6e), The rate region of the CIRC is enlarged compared to the rate region of the CIC. It is important to point out though that due to the correlation of the inputs, receiver 2 benefits by both IF and MF. However, in the Gaussian example of the next section the relaying is restricted to the cognitive user's message only. We could make the encoding at the relay to depend only on the cognitive user's codeword for the discrete memoryless setting; however this would introduce an auxiliary

random variable and a direct comparison of the rate regions may not be possible. We will however address this point in a future work.

## IV. GAUSSIAN CHANNEL

The Gaussian CIRC in standard form is described by the following relations:

$$\begin{aligned} Y_1 &= X_1 + h_{12}X_2 + h_{13}X_3 + Z_1, \\ Y_2 &= h_{21}X_1 + X_2 + h_{23}X_3 + Z_2, \\ Y_3 &= h_{31}X_1 + h_{32}X_2 + Z_3, \end{aligned} \quad (10)$$

where  $E[X_t] \leq P_t$ ,  $Z_t \sim \mathcal{N}(0, 1)$  for  $t = 1, 2, 3$  and  $h_{ij}$  are the channel gains. We define the fraction of power that the cognitive transmitter uses to cooperate with the primary user as  $\alpha$  and the fraction of power used for cooperation with the relay as  $\beta$ . We also define  $\bar{\alpha} = 1 - \alpha$ ,  $\bar{\beta} = 1 - \beta$  and  $\alpha, \beta \in [0, 1]$ . We evaluate the rate region of Theorem 1 for:

$$\begin{aligned} X_{20} &\sim \mathcal{N}(0, \bar{\beta}P_2), U_2 \sim \mathcal{N}(0, \beta P_2), X_2 = X_{20} + U_2, \\ X_{10} &\sim \mathcal{N}(0, \bar{\alpha}P_1), X_1 = X_{10} + \sqrt{\alpha P_1/P_2}X_2, \\ X_3 &= \sqrt{P_3/\beta P_2}U_2, \end{aligned}$$

and obtain the following region in the Gaussian case:

$$R_1 \leq \mathcal{C}(\bar{\alpha}P_1), \quad (11a)$$

$$R_1 + R_2 \leq \mathcal{C}(h_{21}^2 P_1 + P_2 + 2h_{21}\sqrt{\alpha P_1 P_2}), \quad (11b)$$

$$R_2 \leq \mathcal{C}(h_{32}^2 \bar{\beta}P_2), \quad (11c)$$

where  $\mathcal{C}(x) = 1/2 \log_2(1 + x)$ . The rate region of Theorem 2 is computed for:

$$\begin{aligned} X_2 &\sim \mathcal{N}(0, P_2), \\ X_{10} &\sim \mathcal{N}(0, \bar{\alpha}\bar{\beta}P_1), U_1 \sim \mathcal{N}(0, \bar{\alpha}\beta P_1), \\ X_1 &= X_{10} + U_1 + \sqrt{\alpha P_1/P_2}X_2, \\ X_3 &= \sqrt{P_3/\bar{\alpha}\beta P_1}U_1, \end{aligned}$$

and the following rate region in the Gaussian case is obtained:

$$R_1 \leq \mathcal{C}(\bar{\alpha}P_1), \quad (12a)$$

$$\begin{aligned} R_1 + R_2 &\leq \mathcal{C}(h_{21}^2 P_1 + P_2 + 2h_{21}\sqrt{\alpha P_1 P_2} \\ &\quad + 2h_{21}h_{23}\sqrt{\bar{\alpha}\beta P_1 P_3} + h_{23}^2 P_3), \end{aligned} \quad (12b)$$

$$R_1 \leq \mathcal{C}(h_{31}^2 \bar{\alpha}\bar{\beta}P_1), \quad (12c)$$

$$R_1 + R_2 \leq \mathcal{C}(h_{31}^2 (1 - \bar{\alpha}\beta)P_1). \quad (12d)$$

In both, (11) and (12), the redundant relations in (5b) and (6b) due to the strong interference conditions are not listed.

#### A. Comparison plots

We first compare the capacity region of the CIC in strong interference with the capacity region of the CIRC of setup 1.

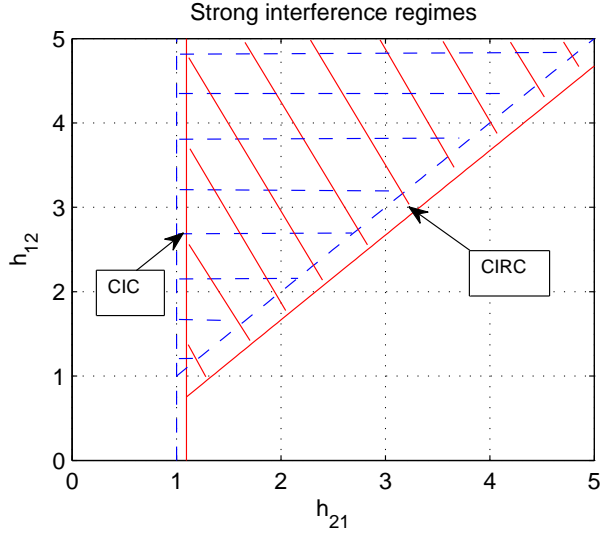


Fig. 2. The strong interference conditions for the CIC and the CIRC for  $P_1 = P_2 = P_3 = 1$ ,  $h_{13}^2 = 0.2$ ,  $h_{32}^2 = 12$ ,  $h_{23} = h_{31} = 0$ .

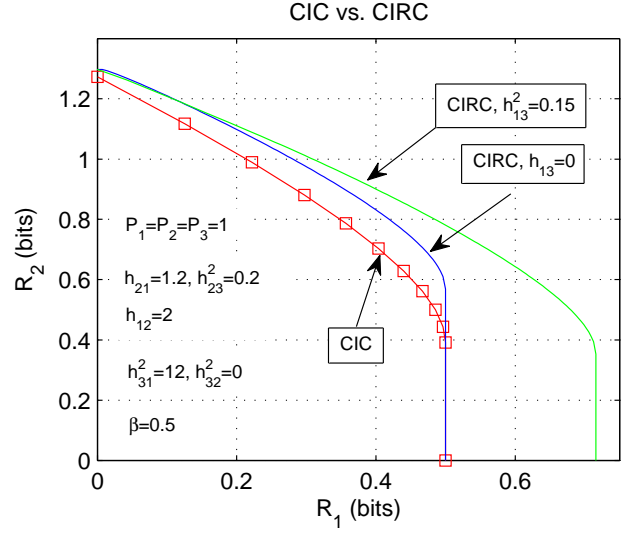


Fig. 4. Capacity regions of the CIC and the CIRC of setup 2 in strong interference.

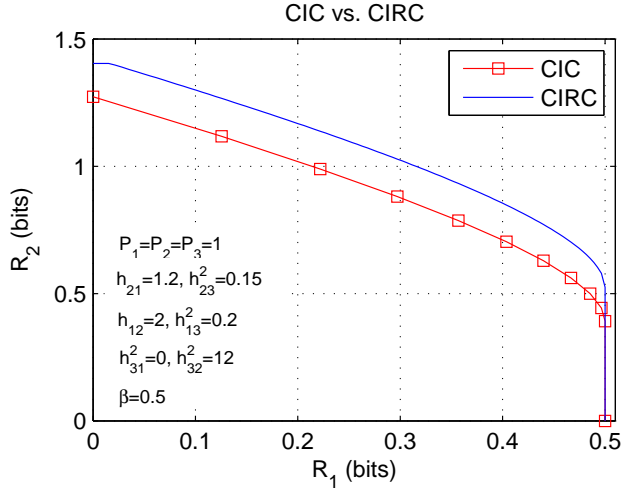


Fig. 3. Capacity regions of the CIC and the CIRC of setup 1 in strong interference.

1) *CIC vs. CIRC of setup 1*: As it was shown in Section III-A1, the capacity region of the CIRC coincides with the capacity region of the CIC in strong interference when  $h_{23} = 0$ . Fig. 2 depicts the strong interference conditions for the CIC and the CIRC. It can be noticed that the condition in (7a) worsens as it pushes  $h_{21}$  away from 1, but condition (7b) improves as it pushes the bar on  $h_{12}$  downwards. When message forwarding is allowed at the relay ( $h_{23} \neq 0$ ), Fig. 3 depicts the capacity regions when both channels are in strong interference. It is noticeable a rate improvement due to cooperation at the relay.

2) *CIC vs. CIRC of setup 2*: For the CIRC of setup 2, Fig. 4 depicts the strong interference capacity regions of the CIC, the CIRC when  $h_{13} = 0$  and the CIRC when both receivers can be reached by the relay ( $h_{13} \neq 0$ ). It can be noticed the rate improvement due to interference and message forwarding.

## V. CONCLUSIONS

In this work we have analysed the CIRC in the discrete and Gaussian cases. The capacity region of such configuration has been characterized in strong interference when the relay only transmits information from one of the transmitters. Due to the asymmetrical nature of the CIRC, two setups were studied, i.e., when the relayed signal is the primary user's and when this is the cognitive user's. We have shown that as opposed to the latter case, in the former case a rate improvement due to IF can be verified. Both setups also benefit by MF when both receivers are reached by the relay. Comparison figures of the capacity regions in this regime have also been provided. As future tasks we will study the case where IF is restricted to the cognitive user's message in the discrete memoryless case and we will also address the case where the relay decodes both signals and transmits both messages simultaneously.

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