Relay Cooperation Schemes for the Multiple Access Relay Channel: Compute-and-Forward and Successive Interference Cancellation

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***Abstract*—In this paper, we propose two relay cooperation schemes for a Multiple Access Relay Channel (MARC). The proposed schemes make use of a lattice-based approach to block Markov encoding. The first scheme applies Compute-And-Forward (CAF) at the relay and Successive Interference Cancellation (SIC) at the destination; whereas the second one applies SIC at both the relay and the destination. A detailed analysis of the rates achieved by these two schemes is also provided. The schemes are further studied under two specific channel settings.**

***Index Terms*—Multiple access relay channel, compute-and-forward, successive interference cancellation, nested lattice codes.**

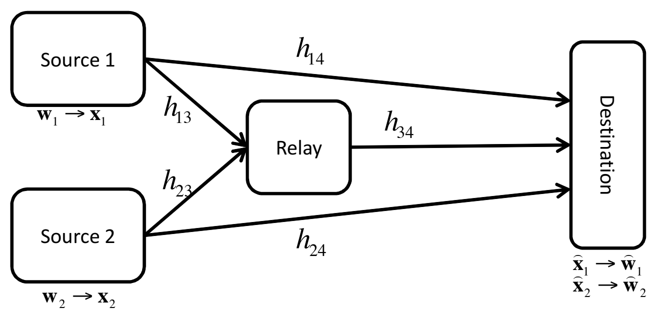

Fig. 1. Two sources multiple access relay channel.

I. Introduction

For a long time, interference has been considered as an obstacle to effective communication in wireless networks. Recently, many schemes have been proposed to exploit interference to achieve higher rates in a wireless network. Interference exploitation schemes sound attractive because they promise higher rates than their counterparts. However, deciding an appropriate interference exploitation scheme with respect to the network topology is not a straight forward task. For example, schemes like Successive Interference Cancellation (SIC) [1], [2], [3], [4] and Compute-And-Forward (CAF) [5], [6], [7], [8] have their own constraints (such as sequential signal-to-interference (SINR) constraint, sequential decoding limit, computation rate constraint) [1], [5]. These constraints not only define the efficacy of these schemes, but also put limits on their use in certain network scenarios.

SIC and CAF have different characteristics when it comes to their effectiveness in a cooperative relaying environment. In fact, the channel conditions plays an important role in determining the use of the schemes. Consider a two-source multiple access channel. If the channel gain from Source 1 to destination is higher than the one from Source 2 to destination, SIC should be used to decode the message of Source 1 first and before decoding that of Source 2. However, if both channel gains (i.e., Source 1 to destination and Source 2 to destination) are equal, the use of SIC may not feasible.

In this paper, we propose two relay cooperation schemes for a MARC. We also derive their achievable sum-rates. The rest of the paper is organized as follows. In Section II, we describes the system model and describe the lattice codes used in our proposed schemes. In Section III, we present our proposed schemes in detail. Section IV shows our results under two different channel settings and Section V concludes the paper.

Following are the descriptions of some symbols and notations used in this paper. The symbols $\mathbf{x}^{S\ell}$ and $\mathbf{t}^{S\ell}$ represent the transmit codeword and unit power lattice point of Source ℓ, respectively. The notation Γ_R represents the computation rate at the relay. The notation $\Gamma_{S_\ell R}$ represents the rate between the Source ℓ and the relay; and similarly the notation $\Gamma_{S_\ell D}$ represents the rate between the Source ℓ and destination. To differentiate the rate of full codeword and its resolution part or meso-lattice point, the notations $\Gamma_{S_\ell D,res}$ and $\Gamma_{S_\ell D,meso}$ are used. Furthermore, $\log^+\{\beta\} = \min\{0, \log \beta\}$.

II. System Model and Lattice Codes Used

A. System Model

Referring to Fig. 1, we consider a MARC with two sources, one relay and one destination. Each of the two message vectors from the two sources, i.e., $\mathbf{w}_1$ and $\mathbf{w}_2$, has a length of k. Moreover, their elements are drawn from a finite field $\mathbb{F}_p$, where p is prime. Each source then encodes the message vector $\mathbf{w}_l$ $(l \in \{1, 2\})$ and maps it $(\mathbb{F}_p^k \rightarrow \mathbb{R}^n)$ to a channel input vector $\mathbf{x}_l$ of length n. The power constraint on the channel input is further given by $||\mathbf{x}_\ell||^2 \leq nP_S$ where P_S is the transmit power of the source.

We denote

- $\mathbf{x}_R$ as the channel input vector of the relay;

- $\mathbf{z}_R$ as the identical and independent distributed (i.i.d.) Gaussian noise vector with zero mean and variance N_R at the relay;
- $\mathbf{y}_R$ as the channel output vector at the relay;
- $\mathbf{z}_D$ as the i.i.d. Gaussian noise vector with zero mean and variance N_D at the destination;
- $\mathbf{y}_D$ as the channel output vector at the destination;
- h_{13} as the channel coefficient between Source 1 and the relay;
- h_{14} as the channel coefficient between Source 1 and the destination;
- h_{23} as the channel coefficient between Source 2 and the relay;
- h_{24} as the channel coefficient between Source 2 and the destination;
- h_{34} as the channel coefficient between the relay and the destination;
- P_R as the transmit power of the relay;
- $||\mathbf{x}_R||^2 \leq nP_R$ as the power constraint for the channel input vector of the relay.

We also assume that the channel coefficients are known at the receiver side. In other words, the relay has full knowledge of h_{13} and h_{23} while the destination has full knowledge of h_{14}, h_{24} and h_{34}.

At each channel use, the channel inputs and outputs are related by

$$\mathbf{y}_R = h_{13}\mathbf{x}_1 + h_{23}\mathbf{x}_2 + \mathbf{z}_R \tag{1}$$

$$\mathbf{y}_D = h_{14}\mathbf{x}_1 + h_{24}\mathbf{x}_2 + h_{34}\mathbf{x}_R + \mathbf{z}_D. \tag{2}$$

The destination observes the noisy sum of all the channel inputs (2) and aims to decode the two transmitted messages individually.

B. Lattice Codes Used

In this section, we review some definitions of lattice codes and describe the nested lattice codes to be used in our proposed schemes.

Definition 1: An n-dimensional lattice Λ is a discrete additive subgroup of the Euclidean space $\mathbb{R}^n$ and is represented as

$$\Lambda = \{\mathbf{t} = \mathbf{G}\mathbf{i} : \mathbf{i} \in \mathbb{Z}^n\} \tag{3}$$

where $\mathbb{Z}$ is the set of integers, $n \in \mathbb{Z}_+$, and $\mathbf{G}$ is an $n \times n$ generator matrix corresponding to the lattice Λ.

Definition 2: The lattice quantizer Q_Λ maps any point $\mathbf{x} \in \mathbb{R}^\mathbf{n}$ to the nearest lattice point $\mathbf{t}$, i.e.,

$$Q_\Lambda(\mathbf{x}) = \arg\min_{\mathbf{t}\in\Lambda} ||\mathbf{x} - \mathbf{t}||. \tag{4}$$

Definition 3: The voronoi region $\mathcal{V}(\mathbf{t})$ of a lattice point $\mathbf{t} \in \Lambda$ contains all the points closest to $\mathbf{t}$, i.e.,

$$\mathcal{V}(\mathbf{t}) = \{\mathbf{x} \in \mathbb{R}^n : Q_\Lambda(\mathbf{x}) = \mathbf{t}\}. \tag{5}$$

Definition 4: The fundamental voronoi region $\mathcal{V}$ is the voronoi region of the origin, i.e.,

$$\mathcal{V}(\mathbf{t}) = \{\mathbf{x} \in \mathbb{R}^n : Q_\Lambda(\mathbf{x}) = 0\}. \tag{6}$$

Definition 5: The modulo operation of $\mathbf{x} \in \mathbb{R}^n$ returns the quantization error with respect to the lattice, i.e.,

$$\mathbf{x} \bmod \Lambda = \mathbf{x} - Q_\Lambda(\mathbf{x}). \tag{7}$$

Definition 6: The second moment per dimension $\sigma^2(\Lambda)$ quantifies the average power of a shaping lattice Λ, i.e.,

$$\sigma^2(\Lambda) = \frac{1}{n\mathrm{Vol}(\mathcal{V})} \int_{\mathcal{V}} ||\mathbf{x}||^2 d\mathbf{x} \tag{8}$$

where Vol(A) is the volume of a set $A \subset \mathbb{R}^n$.

A lattice codebook $\mathcal{L}$ is constructed by intersecting a coding lattice Λ_{coding} with the fundamental voronoi region $\mathcal{V}$ of a shaping lattice, i.e., $\mathcal{L} = \Lambda_{coding} \cap \mathcal{V}$, where the shaping lattice is chosen to enforce the average power constraint.

Our proposed schemes apply lattice-based approach to block Markov encoding. We use doubly nested lattice codes, the structure of which are the same as those used in [9]. The codebooks are defined by the sequence of nested lattices $\Lambda_S \subset \Lambda_m \subset \Lambda_C$, where Λ_S is the shaping lattice which enforces the average power constraint and Λ_C is the coding lattice which contains all codewords. Moreover, the objective of adding the meso-lattice Λ_m in the aforementioned lattice chain is to divide one codeword into two parts — the meso-lattice point and the resolution part.

From the nested lattice chain, we can form the following three codebooks

$$\begin{aligned} \mathcal{C} &= \Lambda_C \cap \mathcal{V}_S \\ \mathcal{C}_0 &= \Lambda_C \cap \mathcal{V}_m \\ \mathcal{C}_1 &= \Lambda_m \cap \mathcal{V}_S \end{aligned} \tag{9}$$

where $\mathcal{V}_S$ and $\mathcal{V}_m$ denote the voronoi regions of the coarse lattice and the meso-lattice, respectively. In (9), the codebook $\mathcal{C}$ contains all the codewords while the codebooks $\mathcal{C}_0$ and $\mathcal{C}_1$, respectively, contain the resolution parts and the meso-lattice points of all codewords. Each meso-lattice point in $\mathcal{C}_1$ is analogous to the centre of a cloud in which codewords may reside whereas each resolution part in $\mathcal{C}_0$ is analogous to the position of a certain codeword within that cloud.

Consequently, referring to Fig. 2, each codeword $\mathbf{t}$ can be distinctively determined by its meso-lattice point $\mathbf{t}_1$ and its resolution information $\mathbf{t}_0$. Furthermore, $\mathbf{t}$, $\mathbf{t}_1$ and $\mathbf{t}_0$ are related by

$$\begin{aligned} \mathbf{t} &= [\mathbf{t}_0 + \mathbf{t}_1] \bmod \Lambda_S \in \mathcal{C} \\ \mathbf{t}_0 &= \mathbf{t} \bmod \Lambda_m \in \mathcal{C}_0 \\ \mathbf{t}_1 &= [\mathbf{t} - \mathbf{t}_0] \bmod \Lambda_S \in \mathcal{C}_1. \end{aligned} \tag{10}$$

For simplicity, we choose to use the unit power codebooks whose codewords can be scaled to satisfy the transmit power constraints. Hence, we select lattices with $\sigma^2(\Lambda_S) = 1$ such that $\mathcal{C}$ and $\mathcal{C}_1$ have unit power. Moreover, we construct a unit power version of $\mathcal{C}_0$, denoted by $\mathcal{C}_0^*$ and defined as $\mathcal{C}_0^* = \Lambda_C^* \cap \mathcal{V}_m^*$. Here and also in subsequent sections, the symbol * represents a scaled version of the original such that the power becomes unity, i.e., $\sigma^2(\Lambda_m^*) = 1$.

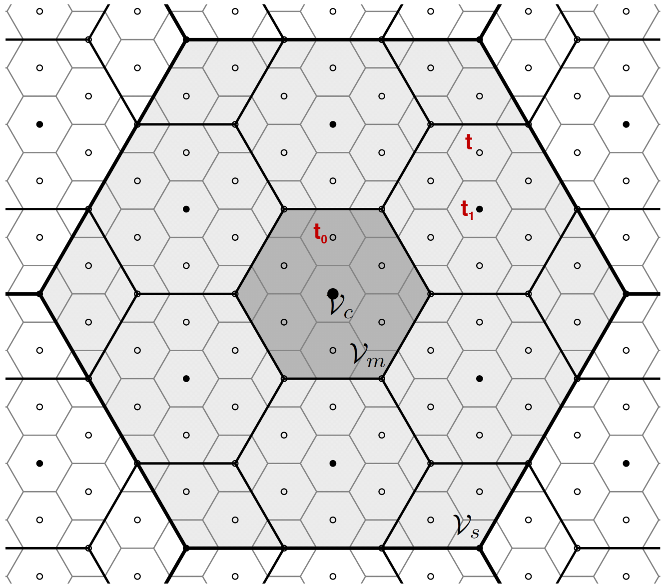

Fig. 2. Doubly-nested lattices for the relay channel. Each lattice point is the sum of a meso-lattice point (medium-sized dots) and a coding-lattice point (white dots) inside the fundamental Voronoi region of the meso-lattice [9].

III. Relay Cooperation Schemes

We assume that each frame consists of $B+1$ time slots. Based on block Markov encoding, we will show how we can transmit B messages within one frame. In the following, we show the transmit and receive signals in the different time slots within a frame. Then, we present our two proposed relay cooperation schemes and derive their achievable sum-rates.

A. Transmit and Receive Signals

1) Time Slot b where $b=1$: Each of the two sources maps its message vector ($\mathbf{w}^{S1}[b]$ and $\mathbf{w}^{S2}[b]$) to the lattice codeword ($\mathbf{t}^{S1}[b]$ and $\mathbf{t}^{S2}[b]$) using

$$\begin{aligned} \mathbf{t}^{S1}[b] &= \phi_1(\mathbf{w}^{S1}[b]) && (11) \\ \mathbf{t}^{S2}[b] &= \phi_2(\mathbf{w}^{S2}[b]) && (12) \end{aligned}$$

where ϕ is a mapping function that maps the message vector $\mathbf{w}$ to the lattice point $\mathbf{t}$. (Note that the subscripts/superscripts are not indicated for simplicity.) The each source apply a dither ($\mathbf{d}^{S1}$ and $\mathbf{d}^{S2}$), which is drawn independently and uniformly over $\mathcal{V}_S$, to form the channel input vector ($\mathbf{x}^{S1}[b]$ and $\mathbf{x}^{S2}[b]$), i.e.,

$$\begin{aligned} \mathbf{x}^{S1}[b] &= \sqrt{P_S}[\mathbf{t}^{S1}[b]-\mathbf{d}^{S1}] \bmod \Lambda_S && (13) \\ \mathbf{x}^{S2}[b] &= \sqrt{P_S}[\mathbf{t}^{S2}[b]-\mathbf{d}^{S2}] \bmod \Lambda_S. && (14) \end{aligned}$$

The relay has not received any information from the sources yet and thus remains idle.

Both the relay and the destination receives the faded and noisy sum of signals of the two sources. The received vectors at the relay $\mathbf{y}_R[b]$ and at the destination $\mathbf{y}_D[b=1]$ can be written as

$$\begin{aligned} \mathbf{y}_R[b] &= h_{13}\mathbf{x}^{S1}[b]+h_{23}\mathbf{x}^{S2}[b]+\mathbf{z}_R && (15) \\ \mathbf{y}_D[b=1] &= h_{14}\mathbf{x}^{S1}[b=1]+h_{24}\mathbf{x}^{S2}[b=1]+\mathbf{z}_D && (16) \end{aligned}$$

2) Time Slot b where $2 \le b \le B$: The encoding process at the sources remain the same and hence (11) to (14) can be applied.

The relay encodes and transmits its own codeword $\mathbf{x}_R[b]$. The encoding process of $\mathbf{x}_R[b]$ is dependent upon the scheme being used.

The relay continues to receive signals from the two sources and hence (15) remain valid.

The received vector at the destination $\mathbf{y}_D[b]$ consists of the faded signals from the sources, the faded signal from the relay, and noise, i.e.,

$$\mathbf{y}_D[b] = h_{14}\mathbf{x}^{S1}[b]+h_{24}\mathbf{x}^{S2}[b]+h_{34}\mathbf{x}_R[b]+\mathbf{z}_D. \quad (17)$$

3) Time Slot $b=B+1$: The sources do not transmit.

The relay transmits its own codeword $\mathbf{x}_R[b=B+1]$.

The relay does not receive any signal from the sources.

The received vector at the destination $\mathbf{y}_D[b=B+1]$ contains the faded signal from the relay and noise, i.e.,

$$\mathbf{y}_D[b=B+1] = h_{34}\mathbf{x}_R[b=B+1]+\mathbf{z}_D. \quad (18)$$

B. First proposed scheme: CAF-SIC scheme

The first proposed relay cooperation scheme uses an amalgamation of CAF and SIC. In this scheme, the relay computes the linear combination of the received channel inputs and forwards part of it to the destination as an individual codeword. The destination then uses SIC to estimate the two messages individually from the noisy sum of all the channel inputs. Details are described as follows.

1) Decoding and Encoding at the Relay: In Time slot b where $1 \le b \le B$, the relay receives the faded noisy sum of the codewords of both sources according to (15). In order for the relay to decode the linear combination of the two received codewords as a separate codeword, we need to fulfill the following rate constraint [5]

$$\Gamma_R < \frac{1}{2}\log^+\left(\frac{1}{||\mathbf{h}_R||^2}+\frac{P_S}{N_R}\right) \quad (19)$$

where $\mathbf{h}_R=[h_{13},h_{23}]$.

After decoding, the relay forms a linear combination of (only) the *resolution* parts of the two received codewords as an individual codeword using another lattice chain $\Lambda_{S,R} \subset \Lambda_{m,R} \subset \Lambda_{C,R}$, where the structure of the chain is the same as those used at the sources. The new codeword is then sent in the next time slot as $\mathbf{x}_R[b+1]$ where

$$\mathbf{x}_R[b+1] = \sqrt{P_R}\left([\mathbf{t}_0^{*R}[b]-\mathbf{d}^{*R}] \bmod \Lambda^*_{m,R}\right). \quad (20)$$

In (20), $\mathbf{t}_0^{*R}[b]$ is a lattice point in $\Lambda^*_{C,R} \cap \mathcal{V}^*_{m,R}$ and $\sigma^2(\Lambda^*_{m,R})=1$.

2) Decoding at the destination:

a) Time Slot b where $b = 1$: The destination receives only the faded noisy sum of channel inputs of both sources, as in (16). Assuming that $h_{14} \geq h_{24}$, $\mathbf{x}^{S1}[b=1]$ can be decoded based on (16) under the following rate constraint.

$$\Gamma_{S_1D} \quad < \quad \frac{1}{2}\log\left(1+\frac{h_{14}^2P_S}{h_{24}^2P_S+N_D}\right). \tag{21}$$

Subsequently, the destination subtracts $h_{14}\mathbf{x}^{S1}[b=1]$ from $\mathbf{y}_D[b=1]$ to obtain $\mathbf{y}'_D[b=1]$, which only contains $h_{24}\mathbf{x}^{S2}[b=1]$ and noise, i.e.,

$$\mathbf{y}'_D[b=1] \quad = \quad h_{24}\mathbf{x}^{S2}[1]+\mathbf{z}_D. \tag{22}$$

The destination performs decoding again on $\mathbf{y}'_D[b=1]$ to obtain only the meso-lattice point (i.e., $\mathbf{t}_1^{S2}[b=1]$) of $\mathbf{x}^{S2}[b=1]$, which gives the following rate constraint.

$$\Gamma_{S_2D,meso} \quad < \quad \frac{1}{2}\log\left(1+\frac{h_{24}^2P_S}{N_D}\right) \tag{23}$$

b) Time Slot b where $2 \leq b \leq B$: The received signal at the destination, given in (17), can be re-written as

$$\begin{aligned}\mathbf{y}_D[b] &= h_{14}\mathbf{x}^{S1}[b]+h_{24}\mathbf{x}^{S2}[b]+h_{34}\mathbf{x}_R[b]+\mathbf{z}_D \\ &= h_{14}\sqrt{P_S}\left([\mathbf{t}^{S1}[b]-\mathbf{d}^{S1}] \bmod \Lambda_S\right) \\ &\quad + h_{24}\sqrt{P_S}\left([\mathbf{t}^{S2}[b]-\mathbf{d}^{S2}] \bmod \Lambda_S\right) \\ &\quad + h_{34}\sqrt{P_R}\left([\mathbf{t}_0^{*R}[b-1]-\mathbf{d}^{*R}] \bmod \Lambda^*_{m,R}\right)+\mathbf{z}_D.\end{aligned} \tag{24}$$

Based on $\mathbf{y}_D[b]$, the destination first decodes the codeword $\mathbf{x}_R[b]$ sent by the relay, which is only possible if

$$\Gamma_{RD} \quad < \quad \frac{1}{2}\log\left(1+\frac{h_{34}^2P_R}{h_{14}^2P_S+h_{24}^2P_S+N_D}\right). \tag{25}$$

Having decoded $\mathbf{x}_R[b]$ (which is a linear combination of the resolution parts of the codewords transmitted by Source 1 and Source 2) and by using the information decoded in the previous time slot (i.e., codeword $\mathbf{x}^{S1}[b-1]$ and meso-lattice point $\mathbf{t}_1^{S2}[b-1]$), the full codeword from Source 2 can be obtained as follows.

- Derive the resolution part (i.e., $\mathbf{t}_0^{S1}[b-1]$) of the codeword from Source 1 based on $\mathbf{x}^{S1}[b-1]$.
- Subtract $\mathbf{t}_0^{S1}[b-1]$ from the $\mathbf{x}_R[b]$ to obtain the resolution part (i.e., $\mathbf{t}_0^{S2}[b-1]$) of the codeword from Source 2.
- Combine the resolution part $\mathbf{t}_0^{S2}[b-1]$ and the meso-lattice point $\mathbf{t}_1^{S2}[b-1]$ to obtain the full codeword from Source 2.

Subsequently, $h_{34}\mathbf{x}_R[b]$ can be subtracted from (24) and a similar method as in the case of $b=1$ can be used to decode the full codeword $\mathbf{x}^{S1}[b]$ and the meso-lattice point $\mathbf{t}_1^{S2}[b]$.

c) Time Slot b where $b = B+1$: The destination receives a noisy signal from the relay, i.e.,

$$\mathbf{y}_D[B+1] = h_{34}\sqrt{P_R}\left([\mathbf{t}_0^{*R}[B]-\mathbf{d}^{*R}] \bmod \Lambda^*_{m,R}\right)+\mathbf{z}_D. \tag{26}$$

At the destination, the decoding process for this block works exactly in the same manner as in the case of Time slot b where $2 \leq b \leq B$, except that there is no interference from the sources.

C. Second proposed scheme: SIC-SIC scheme

In the second proposed scheme, the relay uses SIC to decode the messages from the two sources individually. Then it transmits a weighted sum of (i) the full codeword of one source and (ii) the resolution codeword of the other source. At the destination we again use SIC to estimate the messages individually from the noisy sum of all the channel inputs.

1) Decoding and Encoding at the Relay: In Time slot b where $1 \leq b \leq B$, the relay receives the faded noisy sum of the codewords of both sources according to (15). The relay decodes the strongest signal first. Then it removes the decoded codeword from the noisy signal and estimates the other codeword based on the residual signal. Assuming that $h_{13} \geq h_{23}$, SIC gives following rate constraints

$$\Gamma_{S_1R} \quad < \quad \frac{1}{2}\log\left(1+\frac{h_{13}^2P_S}{h_{23}^2P_S+N_R}\right) \tag{27}$$

$$\Gamma_{S_2R} \quad < \quad \frac{1}{2}\log\left(1+\frac{h_{23}^2P_S}{N_R}\right) \tag{28}$$

Then, the relay forms the weighted sum of (i) the full codeword (i.e., $[\mathbf{t}^{S1}[b]-\mathbf{d}^{S1}] \bmod \Lambda_S$) of the message of Source 1 and (ii) only the resolution codeword (i.e., $[\mathbf{t}_0^{S2}[b]-\mathbf{d}_0^{S2}] \bmod \Lambda_m$) of the message of Source 2, and sends the new codeword in the following time slot, i.e.,

$$\begin{aligned}\mathbf{x}_R[b+1] &= \sqrt{\bar{\beta}P_R}\left([\mathbf{t}^{S1}[b]-\mathbf{d}^{S1}] \bmod \Lambda_S\right) \\ &\quad + \sqrt{\beta P_R}\left([\mathbf{t}_0^{*S2}[b]-\mathbf{d}_0^{S2}] \bmod \Lambda^*_m\right)\end{aligned} \tag{29}$$

where $\beta+\bar{\beta}=1$, and Λ^*_m represents a scaled version of Λ_m that satisfies $\sigma^2(\Lambda^*_m)=1$.

2) Decoding at the destination:

a) Time Slot b where $b = 1$: The destination does not perform any decoding.

b) Time Slot b where $2 \leq b \leq B$: The received signal at the destination, given in (17), can be re-written as

$$\begin{aligned}\mathbf{y}_D[b] &= h_{14}\mathbf{x}^{S1}[b]+h_{24}\mathbf{x}^{S2}[b]+h_{34}\mathbf{x}_R[b]+\mathbf{z}_D \\ &= h_{14}\sqrt{P_S}\left([\mathbf{t}^{S1}[b]-\mathbf{d}^{S1}] \bmod \Lambda_S\right) \\ &\quad + h_{24}\sqrt{P_S}\left([\mathbf{t}^{S2}[b]-\mathbf{d}^{S2}] \bmod \Lambda_S\right) \\ &\quad + h_{34}\sqrt{\bar{\beta}P_R}\left([\mathbf{t}^{S1}[b-1]-\mathbf{d}^{S1}] \bmod \Lambda_S\right) \\ &\quad + h_{34}\sqrt{\beta P_R}\left([\mathbf{t}_0^{*S2}[b-1]-\mathbf{d}_0^{S2}] \bmod \Lambda^*_m\right)+\mathbf{z}_D.\end{aligned} \tag{30}$$

Based on $\mathbf{y}_D[b]$, the destination first decodes the full codeword of Source 1 (i.e., $\mathbf{x}^{S1}[b-1]$) which corresponds to $\mathbf{t}^{S1}[b-1] = [\mathbf{t}_0^{S1}[b-1]+\mathbf{t}_1^{S1}[b-1]] \bmod \Lambda_S$. The decoding is only possible if

$$\Gamma_{S_1D} \quad < \quad \frac{1}{2}\log\left(1+\frac{h_{34}^2P_R\overline{\beta}}{h_{34}^2P_R\beta+h_{14}^2P_S+h_{24}^2P_S+N_D}\right). \tag{31}$$

Then the destination removes $\mathbf{x}^{S1}[b-1]$ from $\mathbf{y}_D[b]$ and uses the residual signal to estimate the resolution part (i.e.,

$\mathbf{t}_0^{S2}[b-1]$) of Source 2. The following rate constraint needs to be satisfied in order for $\mathbf{t}_0^{S2}[b-1]$ to be successfully decoded.

$$\Gamma_{S_2D,res} \quad < \quad \frac{1}{2}\log\left(1+\frac{h_{34}^2 P_R \beta}{h_{14}^2 P_S + h_{24}^2 P_S + N_D}\right) \tag{32}$$

After the successful decoding of $\mathbf{x}^{S1}[b-1]$ and $\mathbf{t}_0^{S2}[b-1]$, the corresponding signals can be further removed from the signal received from the previous time slot, forming $\mathbf{y}'_D[b-1]$ where

$$\mathbf{y}'_D[b-1] = h_{24}\sqrt{P_S}\left([\mathbf{t}_1^{S2}[b-1] - \mathbf{d}^{S2} + \mathbf{z}_D] \bmod \Lambda_S\right). \tag{33}$$

The destination then decodes the meso-lattice point of the codeword of Source 2 (i.e., $\mathbf{t}_1^{S2}[b-1]$) from $\mathbf{y}'_D[b-1]$, which yields the following rate constraint.

$$\Gamma_{S_2D,meso} \quad < \quad \frac{1}{2}\log\left(1+\frac{h_{24}^2 P_S}{N_D}\right) \tag{34}$$

c) Time Slot b where $b = B+1$: The destination receives a noisy signal from the relay, i.e.,

$$\begin{aligned}\mathbf{y}_D[B+1] \\ = \quad & h_{34}\sqrt{\bar{\beta} P_R}\left([\mathbf{t}^{S1}[B] - \mathbf{d}^{S1}] \bmod \Lambda_S\right) \\ & + h_{34}\sqrt{\beta P_R}\left([\mathbf{t}_0^{*S2}[B] - \mathbf{d}_0^{S2}] \bmod \Lambda_m^*\right) + \mathbf{z}_D.\end{aligned} \tag{35}$$

At the destination, the decoding process for this block works exactly in the same manner as in the case of Time slot b where $2 \le b \le B$, except that there is no interference from the sources.

D. Achievable Rates

Based on (19), (21), (23) and (25), the achievable rates of the CAF-SIC relay-cooperation scheme are given by

$$\Gamma_{S_1} \quad < \quad \min\left[\frac{1}{2}\log^+\left(\frac{1}{||\mathbf{h}_R||^2}+\frac{P_S}{N_R}\right), \right. \\ \left. \frac{1}{2}\log\left(1+\frac{h_{14}^2 P_S}{h_{24}^2 P_S + N_D}\right)\right] \tag{36}$$

$$\Gamma_{S_2} \quad < \quad \min\left[\frac{1}{2}\log^+\left(\frac{1}{||\mathbf{h}_R||^2}+\frac{P_S}{N_R}\right), \right. \\ \left(\frac{1}{2}\log\left(1+\frac{h_{34}^2 P_R}{h_{14}^2 P_S + h_{24}^2 P_S + N_D}\right)\right. \\ \left.\left. + \frac{1}{2}\log\left(1+\frac{h_{24}^2 P_S}{N_D}\right)\right)\right]. \tag{37}$$

Similarly, based on (27), (28), (31), (32) and (34), the achievable rates of the SIC-SIC relay-cooperation scheme are given by

$$\Gamma_{S_1} \quad < \quad \min\left[\frac{1}{2}\log\left(1+\frac{h_{13}^2 P_S}{h_{23}^2 P_S + N_R}\right), \right. \\ \left. \frac{1}{2}\log\left(1+\frac{h_{34}^2 P_R \overline{\beta}}{h_{34}^2 P_R \beta + h_{14}^2 P_S + h_{24}^2 P_S + N_D}\right)\right] \tag{38}$$

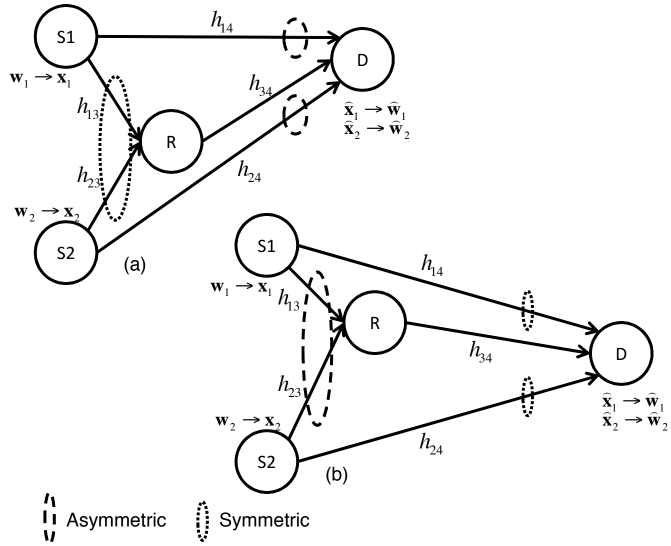

Fig. 3. Multiple access relay channels with two sources. (a) Symmetric source-to-relay channels and asymmetric source-to-destination channels; (b) Asymmetric source-to-relay channels and symmetric source-to-destination channels

$$\Gamma_{S_2} \quad < \quad \min\left[\frac{1}{2}\log\left(1+\frac{h_{23}^2 P_S}{N_R}\right), \right. \\ \left(\frac{1}{2}\log\left(1+\frac{h_{34}^2 P_R \beta}{h_{14}^2 P_S + h_{24}^2 P_S + N_D}\right)\right. \\ \left.\left. + \frac{1}{2}\log\left(1+\frac{h_{24}^2 P_S}{N_D}\right)\right)\right] \tag{39}$$

IV. Simulation Results

A MARC with two sources can be sub-divided into many categories, depending on the channel conditions. Here we investigate two scenarios which are shown in Fig. 3. Figure 3(a) illustrates the case when the source-to-relay channels are symmetric and the source-to-destination channels are asymmetric; whereas Fig. 3(b) illustrates the case when the source-to-relay channels are asymmetric and the source-to-destination channels are symmetric.

Based on the results in Section III-D, we plot the achievable sum-rates of the two proposed schemes, namely the CAF-SIC scheme and the SIC-SIC scheme, against the signal-to-noise ratio (SNR) defined as P_S/N_D. We also plot the achievable sum-rate of the basic CAF scheme in [5] for MARC for comparison. Figure 4 shows the results for the channel settings shown in Fig. 3(a) and Fig. 5 shows the results for the settings in Fig. 3(b).

We observe in Fig. 4 that our proposed CAF-SIC scheme and the basic CAF scheme achieve identical sum-rates whereas our proposed SIC-SIC scheme outperforms the other two schemes when the SNR is low. Based on the results, we suggest using the SIC-SIC scheme at low SNR; and the CAF-SIC scheme or the basic CAF scheme when the SNR is moderate to large for such channel settings.

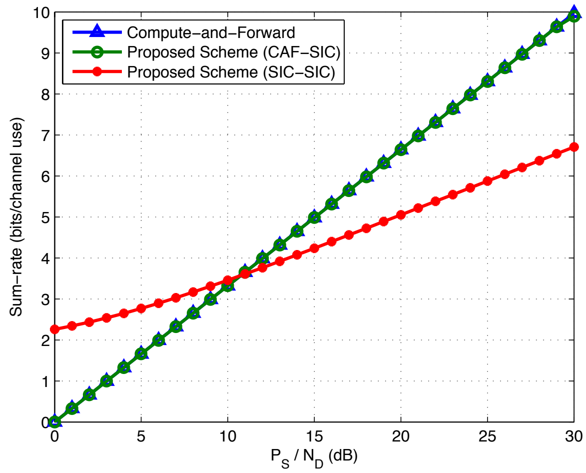

Fig. 4. Achievable sum-rate comparison of proposed schemes (i.e., CAF-SIC and SIC-SIC) and the basic compute-and-forward (when sources to relay channels are symmetric and sources to destination channels are asymmetric). $P_S = P_R$, $N_D = N_R$, $h_{13} = 40, h_{23} = 40, h_{14} = 30, h_{24} = 1$, and $h_{34} = 100$. (It is well known that $||\mathbf{h}|| \leq 1$. The unrealistic values of the channels coefficients are only assumed to emulate radically symmetric and asymmetric channel conditions.)

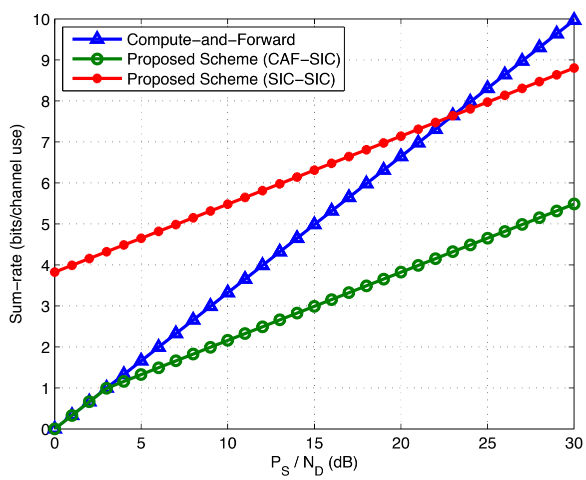

Fig. 5. Achievable sum-rate comparison of proposed schemes (i.e., CAF-SIC and SIC-SIC), and the basic compute-and-forward (when sources to relay channels are asymmetric and sources to destination channels are symmetric). $P_S = P_R$, $N_D = N_R$, $h_{13} = 50, h_{23} = 10, h_{14} = 5, h_{24} = 5$, and $h_{34} = 100$. (It is well known that $||\mathbf{h}|| \leq 1$. The unrealistic values of the channels coefficients are only assumed to emulate radically symmetric and asymmetric channel conditions.)

In Fig. 4, we observe that our proposed SIC-SIC scheme outperforms the basic CAF scheme under low to moderate SNR; whereas the proposed CAF-SIC underperforms over all SNR. Under such channel settings, we should use the SIC-SIC scheme at low to moderate SNR; and the CAF-SIC scheme at high SNR.

V. Conclusion

In this paper, we have proposed two relay cooperation schemes for use in a multiple access relay channel (MARC) with two sources. The foundations of our proposed schemes are successive interference cancellation (SIC) and compute-and-forward (CAF). We have derived the achievable sum-rates of the two proposed schemes, namely the CAF-SIC scheme and the SIC-SIC scheme. The proposed schemes have been studied under two specific channel settings and they can outperform the basic CAF scheme under certain SNR conditions. In the future, we shall continue to study the performance of our proposed schemes under other channel settings. We are also extending our schemes to MARC with multiple sources.

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