Optimal Scheduling and Power Allocation for Wireless Powered Two-Way Relaying Systems

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***Abstract*—We consider a two-way wireless powered cooperative system where the relay not only helps to forward the information for the user nodes, but also acts as an energy beacon. Assuming that due to the hardware limitation, harvesting energy and information transmission cannot be performed simultaneously, we propose a novel three-phase energy harvesting and transmission protocol. In the first phase, the relay broadcasts radio frequency (RF) energy signals which is harvested by both user nodes. In the second and third phase, the user nodes communicate with each other via the relay node. Thus, in order to maximize the network throughput, it is critical to investigate the tradeoff between the durations of the wireless energy harvesting phase and the information transfer phases. In particular, we maximize the throughput for the proposed wireless powered two-way relaying systems by jointly optimizing the durations of wireless energy harvesting and information transmission phases, and the power allocation for transmissions. The optimal solution is obtained, and through simulations, we show the effectiveness of the proposed scheme as compared to the benchmark schemes.**

I. Introduction

Traditional battery-powered wireless communication systems usually suffer from limited lifetime and high cost of maintenance [1]. In some application scenarios, it may be dangerous or even impossible to replace the batteries, e.g., in a high radioactive environment or for medical implanted sensors. Thus it is critical to seek alternative solutions for such kind of communication nodes. Compared to other renewable energy sources such as solar and wind, harvesting energy from radio-frequency signals enjoys the stability and the advantage that information can be transmitted simultaneously with energy. This provides a promising way to prolong the lifetime of communication networks [2].

Simultaneous wireless information and power transfer (SWIPT) has emerged as a popular research topic. In [3], the fundamental tradeoff between the rates of wireless energy transfer and wireless information transfer was characterized. The tradeoff lies in the fact that entropy rate in an RF signal determines the quantity of information, while the average squared value of RF signals account for its power [4]. Several channel models have been studied afterwards, including the point-to-point channel, the multi-user channel, and the relay channel [5].

A related research topic taking a different approach in energy harvesting, is called wireless powered communications networks (WPCN). Unlike the simultaneous information and power transfer in SWIPT, an access point (AP) in WPCN system first transfers energy to the users, and then the users transmit information using the harvested energy. Many different models have been considered in the literature. A point-to-point system has been investigated in [6]. In [6], a hybrid access point (HAP) which transfers energy in the first phase and receives information in the second phase was considered. The optimal energy transferring time that maximizes the system throughput was obtained for the high signal-to-noise ratio (SNR) case. A multi-user network was discussed in [7], where a hybrid access point transfers energy to a set of users in the downlink and receives information from users in the uplink. The optimal time allocation schemes were derived for sum throughput maximization and common throughput maximization.

Recently, wireless powered two-way relaying system is attracting more interest. The authors in [8] investigated the impact of relaying strategies to throughput maximization for two-way relay channels with all wireless powered nodes. In [9], the authors analyzed the outage probability and ergodic capacity in two-way amplify-and-forward wireless powered relay channel, where each block is simply divided into downlink phase and uplink phase. Cognitive relay networks were also investigated in [10], where the outage performance of cooperative cognitive relay (CR) network with an energy harvesting relay was studied.

In this paper, we consider a new wireless powered two-way relaying system where the relay not only acts as an energy beacon but also helps forward the information. We propose a three-phase protocol and formulate the sum throughput maximization problem under the energy causality, time duration, and relay's total energy constraint. The optimal time and power allocation scheme is obtained numerically by the proposed algorithms. Simulation results demonstrate the effectiveness of the optimal solution.

The rest of the paper is organized as follows. We present the system model in Section II. In Section III, we formulate the sum throughput maximization problem and develop the optimal solution. Simulation results are presented in Section IV. Finally, Section V concludes the paper.

II. System Model

As shown in Fig. 1 and Fig. 2, we consider a two-way relaying system, where the relay is denoted by R, and the two users are denoted by U_1 and U_2, respectively. It is assumed

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that the relay and all user nodes are equipped with single antenna for simplicity. The length of the frame is denoted as T. In each frame, the relay first broadcasts energy to user nodes in phase 1 using $\tau_0 T$ amount of time, where $0 < \tau_0 < 1$ is the fraction of frame length allocated for energy transferring. Meanwhile, the user nodes harvest energy from the received signals. Owing to the hardware limitation, we assume the relay is operating under half-duplex mode, and thus, the remaining time period is equally divided into two phases, i.e., phase 2 and phase 3, each is with length $\tau_1 T$. We assume a normalized unit frame time $T = 1$ in the sequel without loss of generality. Thus,

$$\tau_0 + 2\tau_1 \leq 1 \tag{1}$$

Specifically, decode-and-forward relaying protocol is adopted as the relaying strategy. In phase 2, the users transmit its own information to the relay node with the harvested energy, and the relay will decode the received information (if possible). In the phase 3, the relay re-encodes the received information and helps forward it to the destination. It is assumed the relay has maximum transmit power P_{max}.

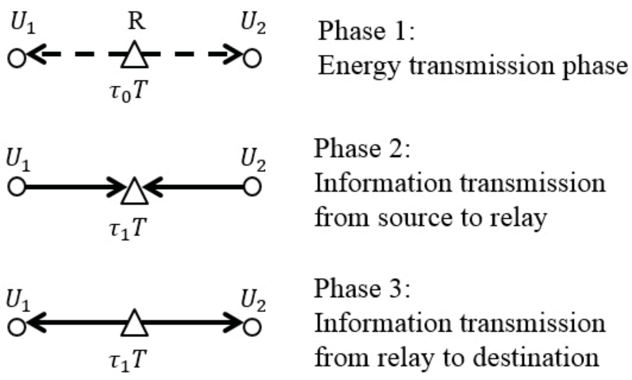

Fig. 1: The 3-phase protocol.

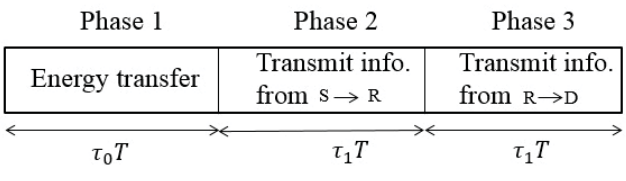

Fig. 2: Frame structure.

The channel gains from U_i to the relay is denoted by $\tilde{h}_i, \quad i = 1, 2$. It is assumed that both channels are quasi-static flat-fading, i.e., $\tilde{h}_1$ and $\tilde{h}_2$ remain constant during each block, but change independently from one block to another. For simplicity, we further assume channel reciprocity and no direct link exists between two user nodes. Also, we assume the relay knows both $\tilde{h}_1$ and $\tilde{h}_2$ perfectly at the beginning of each block, and makes all the decisions.

In phase 1, the transmitted signal of the relay is denoted by x_0, which is assumed to be an arbitrary complex random signal satisfying $E[|x_0|^2] = P_0$. The received signal at U_i in phase 1 is expressed as

$$y_i = \tilde{h}_i x_0 + z_i, \quad i = 1, 2 \tag{2}$$

where y_i and z_i denote the received signal and noise at U_i, respectively. It is assumed that P_0 is large that energy harvested from the noise can be neglected. Thus the energy harvested by U_i in phase 1 is

$$E_i = \gamma_i |\tilde{h}_i|^2 P_0 \tau_0, \quad i = 1, 2 \tag{3}$$

where $0 < \gamma_i < 1, \quad i = 1, 2$, is the energy harvesting efficiency at each user. It is assumed $\gamma_1 = \gamma_2 = \gamma$ for convenience. The power that U_i can use is

$$P_i = E_i / \tau_1, \quad i = 1, 2 \tag{4}$$

The complex baseband signal transmitted by U_i is denoted by $x_i, \quad i = 1, 2$, where x_i is a circularly symmetric complex Gaussian (CSCG) random variable with zero mean and variance P_i, i.e., $x_i \sim \mathcal{CN}(0, P_i), \quad i = 1, 2$. With the assumption of separate channels, the signal received at the relay from U_i is expressed as

$$y_{r,i} = \tilde{h}_i x_i + n_{r,i}, \quad i = 1, 2, \tag{5}$$

where $n_{r,i}$ is the receiving noise at the relay corresponding to U_i during phase 2. It is assumed that $n_{r,i} \sim \mathcal{CN}(0, \sigma^2), \forall i$. The relay then decodes both user nodes' information and forward them in the next phase. Let $h_i^2 = \frac{|\tilde{h}_i|^2}{\sigma^2}$, $\alpha = \gamma\sigma^2$. Therefore, the achievable rate from U_i to the relay can be expressed as

$$R_{i,r} = \tau_1 \log_2 \left(1 + \frac{\alpha h_i^4 E_{ui}}{\tau_1}\right), \quad i = 1, 2 \tag{6}$$

where $E_{ui} \leq E_u$ is the energy that U_i used for transmission and $E_u = P_0 \tau_0$ is the relay's transmitting energy in phase 1. Next we denote the relay's transmitting signals to U_1 and U_2 in phase 3 as $x_{r,1}, x_{r,2}$, respectively, where $E[|x_{r,1}|^2] = P_{R,1}$ and $E[|x_{r,2}|^2] = P_{R,2}$ are relay's transmit power for U_1 and U_2. Thus the received signal at U_i in phase 3 is expressed as

$$y_{3,i} = h_i x_{r,i} + n_{3,i}, \quad i = 1, 2 \tag{7}$$

It is assumed that the noise at the receiver side $n_{3,i} \sim \mathcal{CN}(0, \sigma^2), \ i = 1, 2$. The achievable rate in bits/second/Hz (bps/Hz) for U_i in phase 3 can be expressed as

$$R_{r,i} = \tau_1 \log_2 \left(1 + h_j^2 \frac{E_{pi}}{\tau_1}\right), \quad i, j = 1, 2, \ j \neq i. \tag{8}$$

$E_{pi} = P_{R,i}\tau_1, \ i = 1, 2$, is the energy relay used to forward U_i's information.

The achievable throughput for U_i is expressed as

$$R_i(\tau_1, \boldsymbol{E}) = \min(R_{i,r}, R_{r,i}), \quad i = 1, 2, \tag{9}$$

where $\boldsymbol{E} = [E_{u1}, E_{u2}, E_{p1}, E_{p2}]$. Then the sum rate for the two-way relay system can be expressed as

$$\begin{aligned} R_{sum} &= R_1 + R_2 \\ &= \sum_{i=1}^{2} \min(R_{i,r}, R_{r,i}) \\ &= \min\left(\tau_1 \log_2(1 + \frac{\alpha h_1^4 E_{u1}}{\tau_1}), \tau_1 \log_2(1 + h_2^2 \frac{E_{p1}}{\tau_1})\right) \\ &+ \min\left(\tau_1 \log_2(1 + \frac{\alpha h_2^4 E_{u2}}{\tau_1}), \tau_1 \log_2(1 + h_1^2 \frac{E_{p2}}{\tau_1})\right) \end{aligned} \tag{10}$$

III. Optimal Time and Power Allocation in Two-Way Relaying

In this section, we focus on the sum throughput maximization problem. If more time is allocated to the energy transfer phase, the user nodes will harvest more power, which may lead to a higher throughput. On the other hand, there remains less time for the information transmission phase, which may result in a lower throughput. Therefore, it is critical to investigate the tradeoff between different phases. The problem is then formulated as follows.

$$\begin{aligned} (P1): \quad & \max_{\boldsymbol{E},\tau_1} \quad R_{sum} \\ & \text{s.t.} \quad 0 < \tau_1 < \frac{1}{2} \\ & \qquad E_{ui} \leq P_{max}(1 - 2\tau_1) && (11) \\ & \qquad E_{p1} + E_{p2} \leq P_{max}\tau_1 && (12) \\ & \qquad E_{p1} \geq 0, E_{p2} \geq 0 && (13) \\ & \qquad E_{ui} \geq 0, \ i = 1, 2. && (14) \end{aligned}$$

First, we have the following lemma.

Lemma 3.1: The optimal solutions satisfy

$$\alpha h_1^4 E_{u1} = h_2^2 E_{p1} \tag{15}$$
$$\alpha h_2^4 E_{u2} = h_1^2 E_{p2} \tag{16}$$

Proof: E_{ui} denotes the actual energy that U_i used for transmission. $E_{ui} \leq E_u$. Note that this is equivalent to prove $R_{i,r} = R_{r,i}, \ i = 1, 2$ for the optimal solutions. This is easy to verify as for any given solution, if $R_{i,r} < R_{r,i}, \ i = 1, 2$, we can always reduce E_{pi} to achieve the same sum throughput with less energy consumption. Therefore, the optimal solutions always satisfy (15), (16). ∎

By observing $(P1)$, it is noted that the problem can be simplified for given τ_1. Thus we solve the problem in two steps. First, we find the optimal power allocation scheme $\boldsymbol{E}$ for a given τ_1. Then, based on $\boldsymbol{E}$, we find the sum throughput-maximizing τ_1 in its feasible region. However, in order to find the optimal power allocation scheme for arbitrary τ_1, we need to consider the objective function precisely, which is not straightforward at first sight. To tackle this problem, we try to find some insights of it. When τ_1 is small, it is very likely that $R_{i,r} > R_{r,i}$. In this case the objective function can be expressed as $R_{r,1} + R_{r,2}$, which is easy to tackle. When τ_1 is large, the reverse is true. It is very likely that $R_{i,r} < R_{r,i}$ and the objective function can be expressed as $R_{1,r} + R_{2,r}$. More specifically, the following lemma gives the critical points of τ_1 that divides the feasible region into four subregions. We assume $h_1 > h_2$ without loss of generality. For $h_2 > h_1$, it can be analyzed similarly. Let $\beta = h_1^2/h_2^2$ in the rest of the paper.

Lemma 3.2: There exist $\tau_{11} = \alpha h_2^2/(\beta + 2\alpha h_2^2)$, $\tau_{12} = \alpha\beta h_1^2/(1+2\alpha\beta h_1^2)$ and $\tau_{13} = \alpha(\beta h_1^2 + h_2^2/\beta)/[1+2\alpha(\beta h_1^2 + h_2^2/\beta)]$ such that the feasible region of τ_1 is divided into $(0, \tau_{11}], (\tau_{11}, \tau_{12}], (\tau_{12}, \tau_{13}], (\tau_{13}, \frac{1}{2}]$.

Proof: The point is that we need to obtain the exact expression of the objective function, at least in each range of τ_1. It is observed that when τ_1 is very small, most of the time is allocated to the energy transfer phase. Due to the maximum power constraint at the relay, there is limited energy left at the relay for phase 3. In this case, it is unnecessary for the user nodes to consume all the energy in information transmission phase because the relay even does not have enough energy to forward it. This corresponds to region A in Fig. 3. It is shown that in region A, the corner point $(P_{max}(1 - 2\tau_1), P_{max}\tau_1)$ is always below line l_2. As τ_1 increases, the corner point moves close to l_2 and when τ_1 reaches some value, the corner point will be on l_2. That is to say, there exists a critical point τ_{11} which satisfies $P_{max}\tau_{11} = \alpha h_2^2 P_{max}(1 - 2\tau_{11})/\beta$. i.e., there exists $\tau_{11} = \alpha h_2^2/(\beta + 2\alpha h_2^2)$ such that for $0 < \tau_1 \leq \tau_{11}$, the bottleneck is at the relay's side and the objective function can be expressed as $R_{r,1} + R_{r,2}$. As τ_1 increases, the time allocated to the energy transfer phase decreases and on the other side, the available energy at the relay increases. It becomes possible for the relay to forward U_2's information when U_2 consumes all its harvested energy. However, this may not be true for U_1, as U_1 can harvest more energy. In fact, there exists another critical point τ_{12} which satisfies $P_{max}\tau_{12} = \alpha\beta h_1^2 P_{max}(1 - 2\tau_{12})$, such that beyond this point the relay has enough energy to forward U_1's information when U_1 consumes all its harvested energy. When τ_1 becomes very large, very little time is allocated to the energy transfer phase. It becomes optimal for the user nodes to use up the harvested energy. Specifically, there exists a critical point τ_{13} which satisfies $E_{ui} = P_{max}(1 - 2\tau_{13}), \ i = 1, 2$, and $E_{p1} + E_{p2} = P_{max}\tau_{13}$, such that for $\tau_1 \in (\tau_{13}, \frac{1}{2})$, $R_{i,r} \leq R_{r,i}, \ i = 1, 2$. Consequently, the feasible region of τ_1 is divided into four subregions according to the above critical points. ∎

As a result, we conclude the optimal scheduling and power allocation schemes for different subregions in the following lemmas.

Lemma 3.3: For $0 < \tau_1 \leq \tau_{11}$, the optimal scheduling is $\tau_1 = \tau_{11}$ and the corresponding power allocation scheme is

$$\boldsymbol{E} = \begin{cases} [0, \beta P_{max}\tau_1/(\alpha h_2^2), \\ \qquad 0, P_{max}\tau_1] & \text{if } \overline{E}_{p1} < 0, \\ [E_{p1}/(\alpha\beta h_1^2), \beta E_{p2}/(\alpha h_2^2), \\ \qquad \overline{E}_{p1}, P_{max}\tau_1 - E_{p1}] & \text{if } \overline{E}_{p1} > 0. \end{cases} \tag{17}$$

where $\overline{E}_{p1} = (1 + h_1^2 P_{max} - \beta)\tau_1/(2h_1^2)$.

Proof: When τ_1 is small, it corresponds to region A in Fig. 3. As mentioned before, the sum throughput is bounded

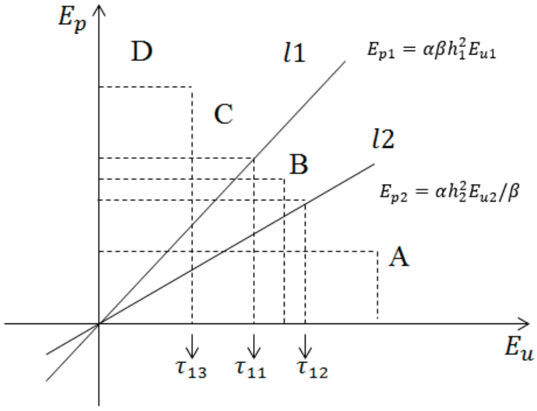

Fig. 3: The relationships between the optimal E_u and E_p under different τ_1

by E_{p1}, E_{p2}. The objective function can be expressed as

$$\begin{aligned} & f_1(\tau_1, E_{p1}, E_{p2}) \\ &= \tau_1 \log_2\left(1 + h_2^2 \frac{E_{p1}}{\tau_1}\right) + \tau_1 \log_2\left(1 + h_1^2 \frac{E_{p2}}{\tau_1}\right) \end{aligned} \quad (18)$$

Note that for given $\tau_1 \in (0, \tau_{11})$, $E_{p1} + E_{p2} = P_{max}\tau_1$ is always true for the optimal solution. As when it is not met, we can always increase E_{p1} or E_{p2} to achieve a larger sum throughput. Thus the objective function can be further expressed as

$$\begin{aligned} & f_1(\tau_1, E_{p1}) \\ &= \tau_1 \log_2\left(1 + h_2^2 \frac{E_{p1}}{\tau_1}\right) + \tau_1 \log_2\left(1 + h_1^2 \frac{P_{max}\tau_1 - E_{p1}}{\tau_1}\right) \\ &= \tau_1 \log_2(g(E_{p1})) \end{aligned} \quad (19)$$

where $g(E_{p1}) = -\frac{h_1^2 h_2^2}{\tau_1^2} E_{p1}^2 + \frac{h_2^2 + h_1^2 h_2^2 P_{max} - h_1^2}{\tau_1} E_{p1} + h_1^2 P_{max} + 1$ is a quadratic function of E_{p1}. After checking the second order condition of $g(E_{p1})$, we find that there exists an unique $\overline{E}_{p1}$ such that $g^{''}(\overline{E}_{p1}) = 0$. As $0 < E_{p1} \leq P_{max}\tau_1$, If $\overline{E}_{p1} < 0$, then the objective function $f_1(\tau_1) = \tau_1 \log_2(1 + h_1^2 P_{max})$, which is an increasing function of τ_1. If $0 < \overline{E}_{p1} < P_{max}\tau_1$, $g(E_{p1}^*) = g(\overline{E}_{p1})$ and again it is irrelative to τ_1. Thus the objective function $f_1(\tau_1) = \tau_1 \log_2(g(\overline{E}_{p1}))$ is also an increasing function of τ_1. Therefore, for $\tau_1 \in (0, \tau_{11})$, the optimal $\tau_1 = \tau_{11}$. The corresponding power allocation scheme is then determined by lemma 3.1. ■

Lemma 3.4: For $\tau_{11} < \tau_1 \leq \tau_{12}$, the optimal scheduling and the corresponding power allocation scheme are

$$\tau_1 = \begin{cases} \tau_{11} & \text{if } f_2^{'}(\tau_{11}) < 0, f_2^{'}(\tau_{12}) < 0, \\ \tau_{12} & \text{if } f_2^{'}(\tau_{11}) > 0, f_2^{'}(\tau_{12}) > 0, \\ \overline{\tau}_1 & \text{if } f_2^{'}(\tau_{11}) > 0, f_2^{'}(\tau_{12}) < 0. \end{cases} \quad (20)$$

$$\boldsymbol{E} = \begin{bmatrix} E_{p1}/(\alpha\beta h_1^2), P_{max}(1 - 2\tau_1), \\ \quad P_{max}\tau_1 - E_{p2}, \alpha h_2^2 P_{max}(1 - 2\tau_1)/\beta \end{bmatrix}. \quad (21)$$

where f_2 represents the simplified objective function and $\overline{\tau}_1$ satisfies $f_2^{'}(\overline{\tau}_1) = 0$.

Proof: This corresponds to region B in Fig. 3. First note that for given $\tau_1 \in (\tau_{11}, \tau_{12}]$, $E_{u2} = P_{max}(1 - 2\tau_1), E_{u1} < E_{u2}$. $E_{p1} + E_{p2} = P_{max}\tau_1$ hold for the optimal solution. If $E_{u2} < P_{max}(1 - 2\tau_1)$, then the same power allocation can be achieved with a larger τ_1 that satisfies $E_{u2} = P_{max}(1 - 2\tau_1)$, which can lead to a higher sum throughput. If $E_{p1} + E_{p2} < P_{max}\tau_1$, then we can increase E_{p1} until $E_{p1} + E_{p2} = P_{max}\tau_1$ to achieve a higher sum throughput. Therefore, from the above observations and lemma 3.1, the objective function can be expressed as

$$\begin{aligned} f_2(\tau_1) &= \tau_1 \log_2\left(1 + h_2^2 \frac{E_{p1}}{\tau_1}\right) + \tau_1 \log_2\left(1 + h_1^2 \frac{E_{p2}}{\tau_1}\right) \\ &= \tau_1 \log_2\left(1 + h_2^2 P_{max} + \frac{2\alpha h_2^6}{h_1^2} P_{max} - \frac{\frac{\alpha h_2^6}{h_1^2} P_{max}}{\tau_1}\right) \\ &\quad + \tau_1 \log_2\left(1 - 2\alpha h_2^4 P_{max} + \frac{\alpha h_2^4 P_{max}}{\tau_1}\right) \\ &= \tau_1 \log_2\left(M - \frac{N}{\tau_1}\right) + \tau_1 \log_2\left(P + \frac{Q}{\tau_1}\right) \end{aligned} \quad (22)$$

where $M = 1 + h_2^2 P_{max} + 2\alpha h_2^4 P_{max}/\beta$, $N = \alpha h_2^4 P_{max}/\beta$, $P = 1 - 2\alpha h_2^4 P_{max}$, $Q = \alpha h_2^4 P_{max}$. The second derivative $f_2^{''} = -N^2/[\tau_1(\tau_1 M - N)^2] - Q^2/[\tau_1(\tau_1 P + Q)^2] < 0$. Thus, $f_2^{'} < f_2^{'}(\tau_{12})$, $f_2^{'} > f_2^{'}(\tau_{11})$. If $f_2^{'}(\tau_{12}) > 0, f_2^{'}(\tau_{11}) < 0$, then the unique optimal τ_1 is achieved at $f_2^{'}(\tau_1) = 0$ for $\tau_1 \in (\tau_{12}, \tau_{11})$. If $f_2^{'}(\tau_{12}) > 0$, $f_2^{'}(\tau_{11}) > 0$, then the optimal $\tau_1 = \tau_{11}$. If $f_2^{'}(\tau_{12}) < 0$, $f_2^{'}(\tau_{11}) < 0$, then the optimal $\tau_1 = \tau_{12}$. The corresponding power allocation scheme is then determined by lemma 3.1. ■

Lemma 3.5: For $\tau_{12} < \tau_1 \leq \tau_{13}$, the optimal scheduling and power allocation scheme correspond to the one with higher sum throughput in the following two cases.
case 1:

$$\tau_1 = \begin{cases} \tau_{12} & \text{if } f_3^{'}(\tau_{12}) < 0, f_3^{'}(\tau_{13}) < 0, \\ \tau_{13} & \text{if } f_3^{'}(\tau_{12}) > 0, f_3^{'}(\tau_{13}) > 0, \\ \overline{\tau}_1 & \text{if } f_3^{'}(\tau_{12}) > 0, f_3^{'}(\tau_{13}) < 0. \end{cases} \quad (23)$$

$$\boldsymbol{E} = \begin{bmatrix} E_{p1}/(\alpha\beta h_1^2), P_{max}(1 - 2\tau_1), \\ \quad P_{max}\tau_1 - E_{p2}, (\alpha h_2^2 P_{max}(1 - 2\tau_1))/\beta \end{bmatrix}. \quad (24)$$

case 2:

$$\tau_1 = \begin{cases} \tau_{12} & \text{if } f_4^{'}(\tau_{12}) < 0, f_4^{'}(\tau_{13}) < 0, \\ \tau_{13} & \text{if } f_4^{'}(\tau_{12}) > 0, f_4^{'}(\tau_{13}) > 0, \\ \overline{\tau}_1 & \text{if } f_4^{'}(\tau_{12}) > 0, f_4^{'}(\tau_{13}) < 0. \end{cases} \quad (25)$$

$$\boldsymbol{E} = \begin{bmatrix} P_{max}(1 - 2\tau_1), \beta E_{p2}/(\alpha h_2^2), \\ \quad \alpha\beta h_1^2 P_{max}(1 - 2\tau_1), P_{max}\tau_1 - E_{p1} \end{bmatrix}. \quad (26)$$

where f_3, f_4 are the simplified objective functions.

Proof: This case corresponds to region C in Fig. 3. First note that E_{u1} or E_{u2} must be $P_{max}(1 - 2\tau_1)$ for the optimal solution. Since if for the optimal τ_1, both E_{u1} and E_{u2} are less than $P_{max}(1 - 2\tau_1)$, then we can increase τ_1 until $\max(E_{u1}, E_{u2}) = P_{max}(1 - 2\tau_1)$, which can result in a higher throughput. Moreover, E_{u1}, E_{u2} can not be equal to $P_{max}(1 - 2\tau_1)$ at the same time since $\tau_1 < \tau_{13}$.

$E_{p1} + E_{p2} = P_{max}\tau_1$ also holds for the optimal solution in this case. Since if $E_{p1} + E_{p2} < P_{max}\tau_1$ for the optimal τ_1, then we can increase E_{p1} or E_{p2} until $E_{p1} + E_{p2} = P_{max}\tau_1$, which will lead to a higher sum throughput. However, unlike the case in lemma 3.4 which $E_{u1} < E_{u2}$ for sure, the complicated part here is that E_{u2} may be less than E_{u1}. When $E_{u1} < E_{u2}$, $E_{u2} = P_{max}(1 - 2\tau_1)$, $E_{p2} = \alpha h_2^2 E_{u2}/\beta$, $E_{p1} = P_{max}\tau_1 - E_{p2}$. The objective function $f_3(\tau_1)$ is the same as that in lemma 3.4. Thus this part follows lemma 3.4. When $E_{u1} > E_{u2}$, $E_{u1} = P_{max}(1 - 2\tau_1)$, $E_{p1} = \alpha\beta h_1^2 E_{u1}$, $E_{p2} = P_{max}\tau_1 - E_{p1}$. The objective function can be expressed as

$$\begin{aligned} & f_4(\tau_1) \\ & = \tau_1 \log_2\left(1 - 2\alpha h_1^4 P_{max} + \alpha h_1^4 P_{max}/\tau_1\right) \\ & + \tau_1 \log_2\left[1 + h_1^2 P_{max}(1 + 2\alpha\beta h_1^2 - \alpha\beta h_1^2/\tau_1)\right] \end{aligned} \tag{27}$$

Note that the structure of f_4 is the same as f_3. Thus there also exists an unique optimal τ_1 for $\tau_1 \in (\tau_{12}, \tau_{13}]$ and the details for this part are omitted. ∎

Lemma 3.6: For $\tau_{13} < \tau_1 < \frac{1}{2}$, the optimal scheduling and the corresponding power allocation scheme are

$$\tau_1 = \begin{cases} \tau_{13} & \text{if } f_5'(\tau_{13}) < 0, \\ \overline{\tau}_1 & \text{if } f_5'(\tau_{13}) > 0, \end{cases} \tag{28}$$

$$\boldsymbol{E} = \left[\begin{array}{l} P_{max}(1 - 2\tau_1), P_{max}(1 - 2\tau_1), \\ \quad \alpha\beta h_1^2 P_{max}(1 - 2\tau_1), \alpha h_2^2 P_{max}(1 - 2\tau_1)/\beta \end{array} \right]. \tag{29}$$

where f_5 represents the simplified objective function and $\overline{\tau}_1$ satisfies $f_5'(\overline{\tau}_1) = 0$.

Proof: This case corresponds to region D in Fig. 3. When $\tau_1 > \tau_{13}$, little time is allocated to the energy transfer phase, thus user nodes have limited energy. The bottleneck of the throughput lies in the links from user nodes to the relay. Here $E_{u1} = E_{u2} = P_{max}(1 - 2\tau_1)$, $E_{p1} + E_{p2} < P_{max}\tau_1$. The objective function can be expressed as

$$\begin{aligned} f_5(\tau_1) &= \tau_1 \log_2\left(1 + \frac{\alpha h_1^4 E_{u1}}{\tau_1}\right) + \tau_1 \log_2\left(1 + \frac{\alpha h_2^4 E_{u2}}{\tau_1}\right) \\ &= \tau_1 \log_2\left(1 - 2\alpha h_1^4 P_{max} + \frac{\alpha h_1^4 P_{max}}{\tau_1}\right) \\ & \quad + \tau_1 \log_2\left(1 - 2\alpha h_2^4 P_{max} + \frac{\alpha h_2^4 P_{max}}{\tau_1}\right) \end{aligned} \tag{30}$$

Let $g(\tau_1) = \tau_1 \log(A + \frac{B}{\tau_1})$, it can be derived that

$$g'' = -B^2\tau_1(\tau_1 A + B)^2 < 0 \tag{31}$$

Thus $f_5'' < 0$. $f_5' > f_5'(\frac{1}{2})$, $f_5' < f_5'(\tau_{13})$.

$$\begin{aligned} & f_5'\left(\frac{1}{2}\right) = -\alpha h_1^4 P_{max} - \alpha h_2^4 P_{max} < 0 \\ & f_5'(\tau_{13}) = \log\left(1 + \frac{h_1^6 h_2^2 P_{max}}{h_1^6 + h_2^6}\right) \\ & - \frac{h_1^4 P_{max}(h_1^2 h_2^2 + 2\alpha(h_1^6 + h_2^6))}{h_1^6 + h_2^6 + h_1^6 h_2^2 P_{max}} + \log\left(1 + \frac{h_1^2 h_2^6 P_{max}}{h_1^6 + h_2^6}\right) \\ & - \frac{h_2^4 P_{max}(h_1^2 h_2^2 + 2\alpha(h_1^6 + h_2^6))}{h_1^6 + h_2^6 + h_1^2 h_2^6 P_{max}} \end{aligned} \tag{32}$$

If $f_5'(\tau_{13}) < 0$, the optimal $\tau_1 = \tau_{13}$ for $\tau_1 \in (\tau_{13}, \frac{1}{2})$. If $f_5'(\tau_{13}) > 0$, then the optimal τ_1 satisfies $f_5'(\tau_1) = 0$ for $\tau_1 \in (\tau_{13}, \frac{1}{2})$. The solution is unique since $f_5'' < 0$. The corresponding power allocation scheme is then determined by lemma 3.1. ∎

Naturally, we have the following theorem which gives the optimal scheduling and power allocation scheme for the original problem.

Theorem 3.1: The optimal scheduling and power allocation scheme correspond to the case with the largest sum throughput among the four subregions.

Proof: The whole feasible region of τ_1 is divided into four subregions and it is proved in each subregion there exists an unique optimal τ_1. It is straightforward that the theorem holds and the optimal scheduling and power allocation scheme are determined according to lemma (3.3-3.6). ∎

IV. Simulation Results

In this section, we investigate the performance of the proposed algorithms in wireless powered two-way relaying system through simulations. In the simulations, we let the bandwidth be 1MHz, the channel power gains $|\tilde{h}_i|^2 = 10^{-3}\lambda d_i^{-\theta}$, where λ is an exponentially distributed random variable with mean 1, d_i is the distance between U_i and R, and θ is the path-loss exponent. The AWGN noise power at the receivers is assumed to be -60 dBm.

Fig. 4 shows the comparison of the sum throughput under the proposed scheduling and two benchmark schemes versus different relay's maximum transmission power P_{max} and path-loss exponents. One benchmark scheme is equipartition, i.e., $\tau_0 = \tau_1 = \frac{1}{3}$. The other benchmark scheme corresponds to $\tau_0 = \frac{1}{2}, \tau_1 = \frac{1}{4}$. We set $d_1 = d_2 = 10m$. As shown in Fig. 4, when $\theta = 2, P_{max} = 4W$, the sum throughput under our proposed scheduling is about 27 % and 64% higher than the two benchmark schemes.

Fig. 5 illustrates the sum throughput improvement of the proposed scheduling compared to the two benchmark schemes under different relay's maximum transmission power. It is shown that for the free space scenario which corresponds to $\theta = 2$, the improvement is about 27 % compared to the equal partition scheme and more than 60 % compared to the other scheme. Even for the worst case, there is still about 13 % increase of the sum throughput. Essentially, the performance of the two benchmark schemes depends on the channel condition. This is because that the channel condition determines the optimal τ_1, and to some extent, the difference between the optimal τ_1 and the benchmark τ_1, i.e., $\frac{1}{3}, \frac{1}{2}$, affects the benchmark schemes' performance.

Fig. 6 reports the value of optimal τ_1 versus the relay's maximum transmit power. From Fig. 6, it is shown that the optimal τ_1 increases with P_{max}. Less time is allocated to the energy transfer phase when the relay's maximum transmit power is larger. This makes sense as when the transmit power is larger, the energy can be transmitted in a shorter time such that more time is allocated to the information transfer phase. In addition, the relay should always transmit with the maximum power in the energy transfer phase in order to shorten τ_0.

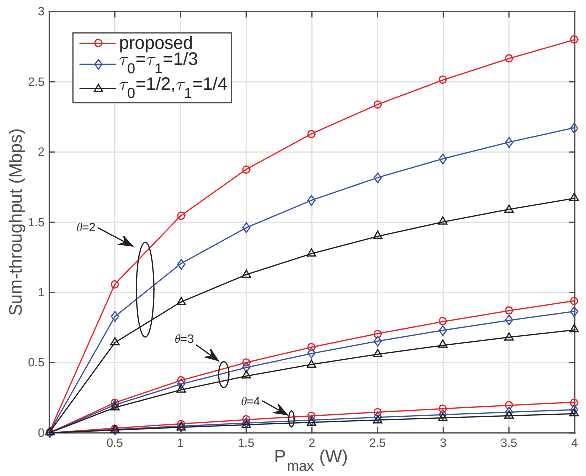

Fig. 4: Sum throughput versus relay's maximum transmission power under different time allocation schemes and different path-loss exponents.

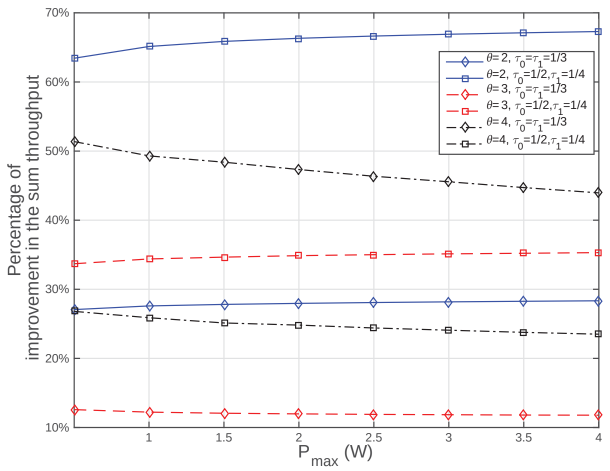

Fig. 5: The sum throughput improvement of the proposed scheduling compared to the other benchmark schemes under different relay's maximum transmission power.

However, it may not be true for the relay in the information transfer phase, i.e., phase 3. Since it may be unnecessary for the relay to transmit at the maximum power. In fact, there are two tradeoffs in this problem. One is the tradeoff between the length of different phases, i.e., τ_0, τ_1. The other one is the tradeoff between the energy allocation, i.e., E_{p1}, E_{p2}. Although these two tradeoffs are somehow related, their effects to the sum throughput are different, which makes the problem difficult.

V. Conclusion

This paper has studied the wireless powered two-way relaying system where the relay not only helps forward information but also acts as energy beacon. The sum throughput maximization problem has been investigated by jointly considering time scheduling and the power allocation scheme. We have obtained the maximum sum throughput by optimizing the time and power allocation. Simulation results has demonstrated the effectiveness of the optimal solution. It is interesting to extend our work to more general networks, such as multiple relays or multiple S-D pairs scenarios.

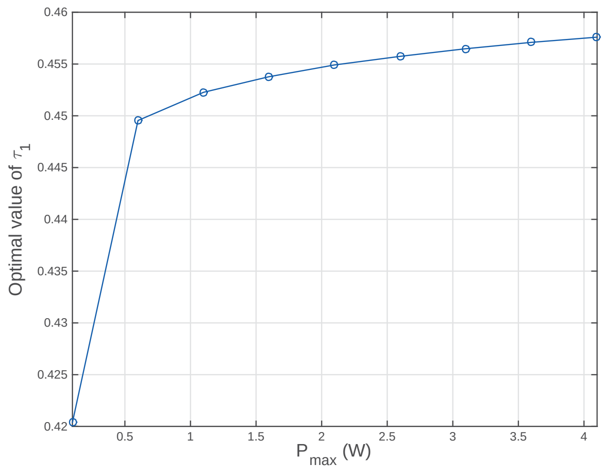

Fig. 6: The optimal τ_1 versus relay's maximum transmission power.

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