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Achievable DoF for 2-user MIMO Relay Interference Channels with Outdated Channel State Information

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***Abstract*—In this paper, we investigate the achievable degree of freedom (DoF) of a fully connected 2-user multiple-input multiple-output (MIMO) relay interference channel with M antennas at each transmitter and N antennas at each receiver when channel state information (CSI) is not available at transmitters and a delayed version of the CSI of direct links from the transmitters to the receivers is known at the relay. We propose a two-phase transmission scheme and obtain different sum-DoF gains (inner bound) under various configurations. We found the achievable sum-DoF values for different antenna configurations at the relay node for the MIMO relay interference channels. The results show that if the relay node has more than $2M$ antennas, our proposed scheme can achieve the outer bound when the instantaneously CSI is known at the relay and at the receivers. This result implies that with the help of a relay with enough antennas, the requirement for instantaneous CSI can be relaxed to the delayed CSI. Moreover, when compared to the MIMO interference channel with no relay, the presence of relay with delayed global CSI can boost the DoF performance even when there is no CSI at the transmitters.**

I. INTRODUCTION

The proliferation of wireless devices makes interference among them increasingly the major limitation for further improvement in both capacity and performance. Adding relays appropriately in the system is one way to reduce the interference and to boost the quality of service. The information theoretic approach studies the fundamental limits of the systems and, in particular, degree of freedom (DoF) is an important metric used to measure and characterise the asymptotic performance of a communication system as the signal-to-noise ratio approaching infinity. In these situations, interference becomes the limiting factor and interference alignment (IA) is the one of the well-known techniques in alleviating effect of interference.

The K-user interference channel has $\frac{K}{2}$ sum-DoF, indicating that each user shares half the channel, and can be achieved by interference alignment [1]. Then [2], [3] extended the work to MIMO interference channel with M antennas at each transmitter and N antennas at each receiver, and the DoF outer bound $min(M,N)K$ if $K \leq R$ and $\frac{max(M,N)}{R+1}K$ if $K > R$, where $R = \left\lfloor \frac{max(M,N)}{min(M,N)} \right\rfloor$ are obtained in [2]. [4] investigated the impact of the presence of relay in the system and found that relay cannot increase the DoF if all channels are fully connected with all nodes having instantaneous global channel state information (CSI). While CSI at the receivers are more readily to be obtained, CSI at the transmitters or CSIT are much more difficult to achieve. A more relaxed and probably practical assumption is the delayed, instead of instantaneous, CSIT where transmitters know a delayed version of the CSI usually obtained through feedback channels from the receivers. With this relaxed assumption, [5] introduced the concept of retrospective interference alignment and a tight outer bound was derived in [6] for the two-user MIMO interference channel. Specifically, authors in [7] explored the value of CSIT and showed how delayed CSIT increases the DoF. The sum-DoF of K-user MISO broadcast channel with delayed CSIT is given as $\frac{K}{1+\frac{1}{2}+\ldots+\frac{1}{K}}$, which is larger than the case with no CSIT [8]. An interesting result shown in [9] is that with the presence of a MIMO relay, the $\frac{KM}{2}$ sum-DoF can be achieved in a K-user $M \times M$ MIMO interference channel with no CSIT. Thus, relay can eliminate the need of CSIT provided that relay has instantaneous global channel state information. Moreover, in K-user MISO interference channels without CSIT, [10] showed that having relay, even with only delayed CSI feedback, can improve the sum-DoF. Intuitively, if the transmitters and the relay are fully cooperated, we can treat them as a huge node and DoF of this MIMO relay interference channel is bounded by broadcast channel under the same assumption.

In our work, we are interested to exploit the DoF result in a 2-user MIMO relay interference channel without CSIT and outdated CSI at the relay and the receivers. Based on the idea of retrospective interference alignment, one interesting result is that even the relay and the receiver only know delayed version of channel state information from the transmitters to the receivers, we can achieve the optimal sum-DoF in [9] which requires instantaneous global CSI at the relay and local CSI at the receivers. Meanwhile, we also find an inner bound for the case when the relay has fewer antennas.

$Notations$: $\mathbf{A}$, $\mathbf{A}^T$, $\mathbf{A}^H$ stand for matrix, transpose, Hermitian transpose, respectively. $\mathbf{A}_{i,j}$ is the entry on ith row and jth column of $\mathbf{A}$. Tr($\mathbf{A}$) , rank($\mathbf{A}$), $null(\mathbf{A})$, dim($\mathbf{A}$) stand for the trace , rank, null space, dimension of $\mathbf{A}$, respectively.

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$\mathbb{E}[\cdot]$ denotes expectation. $\lfloor x \rfloor$ denotes the largest integer no greater than x. $\mathbf{I}$ is identity matrix. $\mathbf{O}(p \times q)$ represents the null matrix with p rows and q columns.

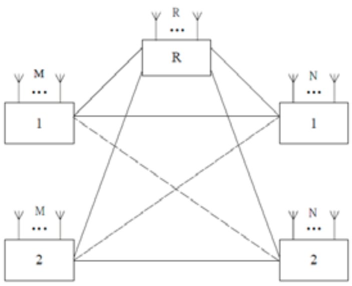

Fig. 1. 2-user $M \times N$ MIMO relay interference channel.

II. System Model

We consider a 2-user $M \times N$ MIMO interference channel with a MIMO relay as depicted in Figure-1, which consists of 2 transmission pairs with M antennas at each transmitter, N antennas at each receiver and R antennas at the relay. Each transmitter sends an independent message, W_i for the i^{th} transmitter, to its corresponding receiver through the help of the relay. In this work, we assume the relay knows the instantaneous CSI of its incoming and outcoming links,and that every receiver knows the instantaneous CSI of its incoming links. The relay also has the CSIs of all direct links from transmitters to receivers with a unit time slot delay, every receiver knows the direct links information from all transmitters to the other receivers with a unit time slot delay, which is the outdated CSI, but the transmitters have no CSI. The relay node is assumed to operate at half duplex mode, i.e. can only receive or transmit at any given time. The signals transmitted from user i and from the relay in the t th time slot, denoted by $\mathbf{X}_i(t) \in \mathbb{C}^M$, and $\mathbf{X}_R(t) \in \mathbb{C}^R$, respectively, are subject to the power constraints $\mathbb{E}[tr(\mathbf{X}_i(t)\mathbf{X}_i(t)^H)] \leqslant P$ and, $\mathbb{E}[tr(\mathbf{X}_R(t)\mathbf{X}_R(t)^H)] \leqslant P$, respectively. The received signals at the relay and receiver i at time t, denoted by $\mathbf{Y}_R(t) \in \mathbb{C}^R$ and, $\mathbf{Y}_i(t) \in \mathbb{C}^N$, respectively, can be expressed as:

$$\mathbf{Y}_R(t) = \sum_{j=1}^{2} \mathbf{H}^{[R,j]}(t)\mathbf{X}_j(t) + \mathbf{Z}_R(t), \tag{1}$$

and

$$\mathbf{Y}_i(t) = \sum_{j=1}^{2} \mathbf{H}^{[i,j]}(t)\mathbf{X}_j(t) + \mathbf{Z}_i(t), \tag{2}$$

where $\mathbf{H}^{[i,j]}(t) \in \mathbb{C}^{N \times M}$ represents the channel gain matrix from transmitter node j to receiver node i and, $\mathbf{H}^{[R,j]}(t) \in \mathbb{C}^{R \times M}$ represents the channel gain matrix from transmitter node j to the relay node. Since the relay operates in half-duplex mode, the received signal at the receiver i when the relay is transmitting can be expressed by

$$\mathbf{Y}_{i,R}(t) = \mathbf{H}^{[i,R]}(t)\mathbf{V}\mathbf{X}_R(t) + \mathbf{Z}_i(t). \tag{3}$$

where $\mathbf{V} \in \mathbb{C}^{R \times R}$ is the transmit beamforming vector at the relay node and $\mathbf{H}^{[i,R]}(t) \in \mathbb{C}^{N \times R}$ is the channel gain matrix from the relay node to receiver node i. All the channel coefficients are drawn from an independent and identically distributed (i.i.d.) continuous distribution. $\mathbf{Z}_i(t)$ and $\mathbf{Z}_R(t)$ denote the noise at receiver i and the relay node which are assumed to be i.i.d. complex Gaussian random variables follow $\mathcal{CN}(0, 1)$.

A rate tuples $\{R_i(SNR)\}$ for $i \in \{1, ..., K\}$ is achievable if the error probability $P_e^n = Pr(\bigcup_i \hat{W}_i \neq W_i)$ can be made arbitrarily small when n is sufficiently large. We define the degrees of freedom (DoF), which captures the number of independent data streams from the transmitters to the receivers, as

$$DoF = \lim_{SNR \to \infty} \frac{C_{sum}(SNR)}{log(SNR)}. \tag{4}$$

where $C_{sum}(SNR) = sup \sum_{i=1}^{K} R_i(SNR)$.

III. Two-Phase Transmission Scheme

In this section, we propose a new two-phase transmission scheme that can achieve the Degrees of Freedom with Retrospective Interference Alignment method under different relay antenna configurations. The proposed transmission scheme consists two phases. In phase 1, transmitters send their messages to corresponding receivers. Then in phase 2, transmitters kept silent and the relay broadcasts the encoding messages.

A. Achievable Scheme for $R = M > N \geq \frac{M}{2}$

When the number of antennas at the relay equals to the number of antennas at each transmitter $(R = M)$, we will show that our scheme can achieve a sum-DoF of $\frac{2M}{3}$.

$Phase 1$: There are two time slots in phase 1. The two users take turn to transmit $\mathbf{X_1}(1) = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_M^1 \end{bmatrix}^T$ and $\mathbf{X_2}(2) = \begin{bmatrix} x_1^2 & x_2^2 & \cdots & x_M^2 \end{bmatrix}^T$. The received signals at receiver node i at the two time slots are then

$$\mathbf{Y}_i(1) = \mathbf{H}^{[i,1]}(1)\mathbf{X}_1(1) + \mathbf{Z}_i(1), \tag{5}$$

$$\mathbf{Y}_i(2) = \mathbf{H}^{[i,2]}(2)\mathbf{X}_2(2) + \mathbf{Z}_i(2). \tag{6}$$

$Phase 2$: Phase 2 has only 1 time slot. Since the relay has the same number of antennas as the transmitters, the relay can decode all the transmitted signals in Phase 1. At Phase 2, since both the relay and the receivers know the CSI of the direct links at the previous time slots, the relay can send a linear combination of the symbols to do interference alignment using the delayed direct links channel state information.

We notice that in equation (5) and (6), there are M signal symbols but receiver i only has N equations. Thus the aim is for the relay to use the overheard signals to create additional useful equations in Phase 2 to ensure decodability at the receiver nodes. In this situation, there is no need to even do interference alignment. We simply pick $2(M-N)$ antennas in the relay to transmit and pick $(M-N)$ antennas in the receiver

$$\mathbf{X_R}(3)=\begin{bmatrix}\mathbf{H}_{s1}(1)\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\mathbf{X_1}(1)+\begin{bmatrix}\mathbf{0}\\ \mathbf{H}_{s2}(2)\\ \mathbf{0}\end{bmatrix}\mathbf{X_2}(2)=\begin{bmatrix}\mathbf{x}_{1,R}(3)\\ \mathbf{x}_{2,R}(3)\\ \mathbf{0}\end{bmatrix} \tag{7}$$

to receive. In order to do this, $R=M\geq 2(M-N), N\geq (M-N)$ need to be satisfied, which is $N\geq \frac{M}{2}$. So the effective channel information matrix is simply $\mathbf{H}'^{[i,R]}(3)\in\mathbb{C}^{(M-N)\times 2(M-N)}$. Thus, we can design a linear beamforming matrix $\mathbf{H}_{si}(t)\in\mathbb{C}^{(M-N)\times M}$ at the relay to combine the transmitted signals in the first phase based on the overheard equations, such that $\mathbf{H}_{si}(t)\prec\mathbf{H}^{[j,i]}(t),\forall i,j\in\{1,2\},j\neq i$. Where $\mathbf{P}\prec\mathbf{Q}$ means that $span(\mathbf{P})\subset span(\mathbf{Q})$, the set of column vectors of $\mathbf{P}$ is a subset of the set of column vectors of $\mathbf{Q}$. For example, the relay can transmit a linear combination of the symbols as shown in (7) on the top of this page.

It's worth mentioning that the dimension of the null space of $\mathbf{H}'^{[i,R]}(3)$ is $M-N$. Hence, the beamforming vector $\mathbf{V}=[\mathbf{V}_1\quad\mathbf{V}_2]$ can be constructed such that $\mathbf{V}_1\in null(\mathbf{H}'^{[2,R]}(3))$, $\mathbf{V}_2\in null(\mathbf{H}'^{[1,R]}(3))$. Then, the received signal in Phase 2 at the receiver i is,

$$\mathbf{Y}_{i,R}(3)=\mathbf{H}'^{[i,R]}(3)\mathbf{V_i}\mathbf{X}_{i,R}(3)+\mathbf{Z}_i(3). \tag{8}$$

Consequently, we now have M linear equations at receiver i to decode M desired signals because $\begin{bmatrix}\mathbf{H}^{[i,i]}(i)\\ \mathbf{H}'^{[i,R]}(3)\mathbf{V_i}\mathbf{H}_{si}(i)\end{bmatrix}$ is full rank *almost surely* for any continuous distribution. As the receivers now know previous channel coefficients, the symbols can be decoded if SNR is high enough. Thus, $2M$ transmitted symbols can be completely decoded in a 3-time-slots transmission scheme, and achieve a sum-DoF of $2M/3$.

B. Achievable Scheme for $R\geq KM=2M, \frac{M}{2}\leq N<M$

When the number of antennas at the relay is equal to or larger than the total number of antennas at all the transmitters, i.e. $R\geq KM$ or in this case $R\geq 2M$, we will show in this section that a modified version of our scheme can achieve a sum-DoF of $\frac{2MN}{M+N}$. This result matches the upper bound in [9] when the direct links channel state information is known at both relay and receivers instantaneously.

Since the relay has enough antennas to decode all the transmitted signals, we can simply reduce the first phase in the aforementioned section to only one time slot. That means both transmitters send $\mathbf{X_1}(1)=\begin{bmatrix}x_1^1 & x_2^1 & \cdots & x_M^1\end{bmatrix}^T$ and $\mathbf{X_2}(1)=\begin{bmatrix}x_1^2 & x_2^2 & \cdots & x_M^2\end{bmatrix}^T$ simultaneously. Clearly the interference at the receivers has more terms and the received signal at receiver i becomes

$$\mathbf{Y}_i(1)=\mathbf{H}^{[i,1]}(1)\mathbf{X}_1(1)+\mathbf{H}^{[i,2]}(1)\mathbf{X}_2(1)+\mathbf{Z}_i(1), \tag{9}$$

The received signal in (9) provides N linear equations for user i to decode M signal symbols in the presence of interference. Intuitively, in phase 2, the broadcasting phase, the relay can spend one time slot to transmit related interference terms to do interference purification and then use an additional time slot to transmit another signals to provide needed linear equations to help user i to decode the signal symbols. Using this approach, we can achieve a sum-DoF of $\frac{2M}{3}$ which is exactly the same as previous section.

However, we can do better. In the first phase, we can use μ time slots to transmit $2M\mu$ symbols in total by doing multiple access for μ times, where μ is the symbol extension coefficient defined by $\mu=\left\lfloor\frac{N}{M-N}\right\rfloor\geq 1$. i.e. $\mathbf{X_1}(\mu)=\begin{bmatrix}x^1_{(\mu-1)M+1} & x^1_{(\mu-1)M+2} & \cdots & x^1_{\mu M}\end{bmatrix}^T$ and $\mathbf{X_2}(\mu)=\begin{bmatrix}x^2_{(\mu-1)M+1} & x^2_{(\mu-1)M+2} & \cdots & x^2_{\mu M}\end{bmatrix}^T$. The re-

$$\mathbf{X_R}(t)=\begin{bmatrix}\mathbf{0}\\ \mathbf{H}^{[2,1]}(t-\mu)\\ \mathbf{0}\end{bmatrix}\mathbf{X_1}(t-\mu)+\begin{bmatrix}\mathbf{H}^{[1,2]}(t-\mu)\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\mathbf{X_2}(t-\mu)=\begin{bmatrix}\mathbf{X}_{1,R}(t)\\ \mathbf{X}_{2,R}(t)\\ 0\end{bmatrix},\forall t\in\{\mu+1,\mu+2,\cdots,2\mu\} \tag{11}$$

$$\begin{aligned}\mathbf{X_R}(2\mu+1) &= \begin{bmatrix}\mathbf{H}_{s1}(1)\\ \mathbf{O}((3N-M)\times M)\end{bmatrix}\mathbf{X_1}(1)+\cdots+\begin{bmatrix}\mathbf{O}((2N-M)\times M)\\ \mathbf{H}_{s1}(\mu)\\ \mathbf{O}(N\times M)\end{bmatrix}\mathbf{X_1}(\mu)\\ &+ \begin{bmatrix}\mathbf{O}(N\times M)\\ \mathbf{H}_{s2}(1)\\ \mathbf{O}((2N-M)\times M)\end{bmatrix}\mathbf{X_2}(1)+\cdots+\begin{bmatrix}\mathbf{O}((3N-M)\times M)\\ \mathbf{H}_{s2}(\mu)\end{bmatrix}\mathbf{X_2}(\mu)=\begin{bmatrix}\mathbf{X}^1_{1,R}(2\mu+1)\\ \vdots\\ \mathbf{X}^N_{1,R}(2\mu+1)\\ \mathbf{X}^1_{2,R}(2\mu+1)\\ \vdots\\ \mathbf{X}^N_{2,R}(2\mu+1)\end{bmatrix}.\end{aligned} \tag{12}$$

$$
\begin{aligned}
\mathbf{Y_i} \quad = & \begin{bmatrix} \mathbf{H}^{[i,i]}(1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^{[i,i]}(2) & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{H}^{[i,i]}(\mu) \\ \multicolumn{4}{c}{\mathbf{O}(\mu N \times \mu M)} \\ \mathbf{G}^{[i,R]}(2\mu+1)\begin{bmatrix} \mathbf{H}_{si}(1) \\ \mathbf{O}((2N-M)\times M) \end{bmatrix} & \mathbf{G}^{[i,R]}(2\mu+1)\begin{bmatrix} \mathbf{O}((M-N)\times M) \\ \mathbf{H}_{si}(2) \\ \mathbf{O}((3N-2M)\times M) \end{bmatrix} & \cdots & \mathbf{G}^{[i,R]}(2\mu+1)\begin{bmatrix} \mathbf{O}((2N-M)\times M) \\ \mathbf{H}_{si}(\mu) \end{bmatrix} \end{bmatrix} \begin{bmatrix} \mathbf{X_i}(1) \\ \mathbf{X_i}(2) \\ \vdots \\ \mathbf{X_i}(\mu) \end{bmatrix} \\
+ & \begin{bmatrix} \mathbf{H}^{[i,j]}(1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^{[i,j]}(2) & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{H}^{[i,j]}(\mu) \\ \mathbf{G}^{[i,R]}(\mu+1)\mathbf{H}^{[i,j]}(1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}^{[i,R]}(\mu+2)\mathbf{H}^{[i,j]}(2) & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{G}^{[i,R]}(2\mu)\mathbf{H}^{[i,j]}(\mu) \\ \multicolumn{4}{c}{\mathbf{O}(N \times \mu M)} \end{bmatrix} \begin{bmatrix} \mathbf{X_j}(1) \\ \mathbf{X_j}(2) \\ \vdots \\ \mathbf{X_j}(\mu) \end{bmatrix} + \mathbf{Z_i} = \begin{bmatrix} \mathbf{Y_i}(1) \\ \vdots \\ \mathbf{Y_i}(2\mu) \\ \mathbf{Y_i}(2\mu+1)^1 \\ \vdots \\ \mathbf{Y_i}(2\mu+1)^N \end{bmatrix}
\end{aligned} \tag{13}
$$

ceived signal at receiver i at the μth time slot is

$$\mathbf{Y}_i(\mu) = \mathbf{H}^{[i,1]}(\mu)\mathbf{X}_1(\mu) + \mathbf{H}^{[i,2]}(\mu)\mathbf{X}_2(\mu) + \mathbf{Z}_i(\mu). \tag{10}$$

Phase 2 consists of $\mu + 1$ time slots. In the first μ time slots, the relay sends interference terms so that all the interference can be eliminated at the receivers using retrospective interference alignment. The last time slot is used to transmit additional signals to create additional linear equations to make sure there are enough equations to solve for the signal symbols. If the relay uses $2N$ antennas to do the transmission, we can construct the transmit signal at the relay as shown in (11) and (12). It is the same as the previous section, we construct a matrix $\mathbf{H}_{si}(t) \in \mathbb{C}^{(M-N)\times M}$ at the relay based on the overheard equations in the transmission phase, such that $\mathbf{H}_{si}(t) \prec \mathbf{H}^{[j,i]}(t), \forall i,j \in \{1,2\}, j \neq i$. In order to do beamforming at the relay node, the relay using the information of the channel matrix $\tilde{\mathbf{H}}^{[i,R]}(t) \in \mathbb{C}^{N\times 2N}$,$\forall i \in \{1,2\}, t \in \{\mu+1, \mu+2, \cdots, 2\mu+1\}$. Since the dimension of the null space of $\tilde{\mathbf{H}}^{[i,R]}(t)$ is N, $\forall i \in \{1,2\}, t \in \{\mu+1, \mu+2, \cdots, 2\mu+1\}$. So beamforming vector $\mathbf{V}(t) = [\mathbf{V}_1(t) \quad \mathbf{V}_2(t)]$ can be constructed such that $\mathbf{V}_1(t) \in null(\tilde{\mathbf{H}}^{[2,R]}(t))$, $\mathbf{V}_2(t) \in null(\tilde{\mathbf{H}}^{[1,R]}(t)), \forall t \in \{\mu{+}1, \mu{+}2, \cdots, 2\mu{+}1\}$. Consequently, we have finished all transmission tasks over $2\mu+1$ time slots. The received signals structure during phase 1 and 2 at receiver i, is shown as equation (13) on the top of this page, where $\mathbf{G}^{[i,R]}(t) = \tilde{\mathbf{H}}^{[i,R]}(t)\mathbf{V_i}(t)$ and $j \neq i$.

There are total $(2\mu + 1)$ time slots and each time slot contributes N linear equations. So from the expression (13), there are a total of $(2\mu{+}1)N$ linear equations with μN of them used for aligning interference terms. With proper manipulation, we can eliminate the interference signals. Thus we could solve μM desired symbols through μM linear equations. In summary, we use μ time slots to transmit $2M\mu$ symbols successfully, which achieve a sum-DoF of $\frac{2MN}{M+N}$ when μ is an integer with minimum value 1.

$Remark$: In [9], a tight upper bound $d_{sum} \leq \frac{MN}{M+N} \times K$ when there is no CSIT and global CSI is known instantaneously at the relay and the receiver is derived when $N \in (0, KM)$. Our result shows that if the direct links channel state information is a delayed version instead of instantaneous, as long as the relay node has enough antennas, the upper bound can be achieved and is the actual DoF.

C. Achievable Scheme for $R = M+1 \geq 2\mu, \frac{M+1}{2} \leq N < M$

So far, we have considered the cases when relay has the same number of antennas as each transmitter (i.e. $R = M$) and when relay has sufficient antennas (i.e. $R \geq KM = 2M$) to decode all symbols in one time slot. In this section, we consider the case when the number of antennas at the relay is between these two numbers (i.e. $M < R < 2M$). Let us consider the case where $R = M+1$ to illustrate the idea of our scheme. Obviously, we can do at least as good as the case in $Section - III - A$ and achieve a sum-DoF of $\frac{2M}{3}$. However, the following scheme shows that the additional antenna at the relay can be exploited to increase the DoF. This transmission strategy consists of 2 phases, with 2μ time slots in phase 1 and $\mu + 1$ in phase 2.

$Phase 1$: In the first time slot, user 1 sends $\mathbf{X_1}(1) =$

$$\mathbf{X_R}(t) = \begin{bmatrix} \mathbf{X}_{1,R}(t) \\ \mathbf{X}_{2,R}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{s1}(t')\mathbf{X_1}(t') \\ \mathbf{X_2}(t') \\ \mathbf{X_1}(t'+1) \\ \mathbf{H}_{s2}(t'+1)\mathbf{X_2}(t'+1) \end{bmatrix}. \tag{18}$$

$$
\mathbf{Y_1} = \begin{bmatrix}
\mathbf{H}^{[1,1]}(1) & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \dot{\mathbf{H}}^{[1,1]}(2) & \cdots & \mathbf{0} & \mathbf{0} \\
\vdots & \vdots & \ddots & \cdots & \cdots \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{H}^{[1,1]}(2\mu-1) & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \dot{\mathbf{H}}^{[1,1]}(2\mu) \\
\mathbf{\Gamma_1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\
\vdots & \vdots & \ddots & \cdots & \cdots \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Gamma}_\mu & \mathbf{0} \\
\mathbf{0} & \mathbf{\Upsilon_1} & \cdots & \mathbf{0} & \mathbf{\Upsilon}_\mu
\end{bmatrix}
\begin{bmatrix}
\mathbf{X_1}(1) \\ \mathbf{X_1}(2) \\ \vdots \\ \mathbf{X_1}(2\mu-1) \\ \mathbf{X_1}(2\mu)
\end{bmatrix}
+ \begin{bmatrix}
\dot{\mathbf{H}}^{[1,2]}(1) & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{H}^{[1,2]}(2) & \cdots & \mathbf{0} & \mathbf{0} \\
\vdots & \vdots & \ddots & \cdots & \cdots \\
\mathbf{0} & \mathbf{0} & \cdots & \dot{\mathbf{H}}^{[1,2]}(2\mu-1) & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H}^{[1,2]}(2\mu) \\
\mathbf{\Lambda_1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\
\vdots & \vdots & \ddots & \cdots & \cdots \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Lambda}_\mu & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}
\end{bmatrix}
\begin{bmatrix}
\mathbf{X_2}(1) \\ \mathbf{X_2}(2) \\ \vdots \\ \mathbf{X_2}(2\mu-1) \\ \mathbf{X_2}(2\mu)
\end{bmatrix}
+ \mathbf{Z_1} = \begin{bmatrix}
\mathbf{Y_1}(1) \\ \mathbf{Y_1}(2) \\ \vdots \\ \mathbf{Y_1}(3\mu) \\ \mathbf{Y_1}(3\mu+1)
\end{bmatrix} \tag{19}
$$

$\begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_M^1 \end{bmatrix}^T$ and user 2 uses only one antenna to send one symbol $\mathbf{X_2}(1) = \begin{bmatrix} x_1^2 \end{bmatrix}$. Then in the second time slot, user 1 only picks one antenna to send $\mathbf{X_1}(2) = \begin{bmatrix} x_{M+1}^1 \end{bmatrix}$ and user 2 sends $\mathbf{X_2}(2) = \begin{bmatrix} x_2^2 & x_3^2 & \cdots & x_{M+1}^2 \end{bmatrix}^T$. In the remaining $2(\mu-1)$ time slots, user 1 and 2 just repeat the same step in the first two time slots to generate another $2(\mu-1)R$ symbols, for example, the signals in the third and fourth time slots are $\mathbf{X_1}(3) = \begin{bmatrix} x_1^1(3) & x_2^1(3) & \cdots & x_M^1(3) \end{bmatrix}^T$, $\mathbf{X_2}(3) = \begin{bmatrix} x_1^2(3) \end{bmatrix}$, $\mathbf{X_1}(4) = \begin{bmatrix} x_{M+1}^1(4) \end{bmatrix}$ and $\mathbf{X_2}(4) = \begin{bmatrix} x_2^2(4) & x_3^2(4) & \cdots & x_{M+1}^2(4) \end{bmatrix}^T$. As illustrated before, there is one transmitter sending only one symbol in a specific time slot. Thus in the receiver side, we have

$$\mathbf{Y}_1(t) = \mathbf{H}^{[1,1]}(t)\mathbf{X}_1(t) + \dot{\mathbf{H}}^{[1,2]}(t)\mathbf{X_2}(t) + \mathbf{Z}_1(t), \tag{14}$$

$$\mathbf{Y}_1(t') = \dot{\mathbf{H}}^{[1,1]}(t')\mathbf{X_1}(t') + \mathbf{H}^{[1,2]}(t')\mathbf{X_2}(t') + \mathbf{Z}_1(t'). \tag{15}$$

at receiver 1 and

$$\mathbf{Y}_2(t) = \mathbf{H}^{[2,1]}(t)\mathbf{X_1}(t) + \dot{\mathbf{H}}^{[2,2]}(t)\mathbf{X_2}(t) + \mathbf{Z}_2(t), \tag{16}$$

$$\mathbf{Y}_2(t') = \dot{\mathbf{H}}^{[2,1]}(t')\mathbf{X_1}(t') + \mathbf{H}^{[2,2]}(t')\mathbf{X}_2(t') + \mathbf{Z}_2(t'). \tag{17}$$

at receiver 2 for $t \in \{1, 3, \cdots, 2\mu-1\}, t' \in \{2, 4, \cdots, 2\mu\}$, where $\dot{\mathbf{H}}^{[i,j]}(t) \in \mathbb{C}^{N\times 1}$,$\forall i, j \in \{1, 2\}$.

$Phase2$: During phase 1, there are R symbols arriving at the relay node at every time slot and the relay can completely decode the incoming symbols. Thus we design the linear combination symbols transmitted by the relay in the $2\mu+1$th, $2\mu+2$th,$\cdots$, $3\mu-1$th and the 3μth time slot, by applying the same construction method to create a matrix $\mathbf{H}_{si}(t) \in \mathbb{C}^{(M-N)\times M}$, such that $\mathbf{H}_{si}(t) \prec \mathbf{H}^{[j,i]}(t), \forall i, j \in \{1, 2\}, j \neq i$, which is expressed in (18) at the bottom of pervious page, where $(t, t') = \{(2\mu+1, 1), (2\mu+2, 3), \cdots, (3\mu-1, 2\mu-3), (3\mu, 2\mu-1)\}$. So we require $R = M+1 \geq 2(M-N)+2$, in other words $N \geq \frac{M+1}{2}$ so that it can send the symbols successfully.

In the $(3\mu+1)$th time slot, transmitted signal at the relay is simple as $\mathbf{X_R}(3\mu+1) = \begin{bmatrix} \mathbf{X_1}(2) \; \mathbf{X_1}(4) \cdots \mathbf{X_1}(2\mu) \; \mathbf{X_2}(1) \; \mathbf{X_2}(3) \cdots \mathbf{X_2}(2\mu-1) \end{bmatrix}^T$. To ensure the successful transmission, we need $R = M+1 \geq 2\mu$ antennas at the relay node. Then the relay transmits signal by multiplying a beamforming vector which is similar as before so that the desired signals could arrive to the corresponding receiver. The dimension of the null space of $\mathbf{H}^{[i,R]}(t)$ is $R-N$, $\forall i \in \{1, 2\}, t \in \{2\mu+1, \cdots, 3\mu+1\}$. So there exists beamforming matrix $\mathbf{V}(t) = \begin{bmatrix} \mathbf{V}_1(t) & \mathbf{V}_2(t) \end{bmatrix}$ such that $\mathbf{V}_1(t) \in null(\mathbf{H}^{[2,R]}(t))$, $\mathbf{V}_2(t) \in null(\mathbf{H}^{[1,R]}(t)$, $\forall t \in \{2\mu+1, \cdots, 3\mu+1\}$.

Up to now we have done all the steps of this transmission scheme and the whole receiving structures at receiver side are constructed. Using receiver 1 as an example. The whole receiving part during the two phases is shown in expression (19) where

$$\mathbf{\Gamma_1} = \mathbf{H}^{[1,R]}(2\mu+1)\mathbf{V_1}(2\mu+1)\begin{bmatrix} \mathbf{H}_{s1}(1) \\ \mathbf{O}(1\times M) \end{bmatrix},$$
$$\mathbf{\Gamma}_\mu = \mathbf{H}^{[1,R]}(3\mu)\mathbf{V_1}(3\mu)\begin{bmatrix} \mathbf{H}_{s1}(2\mu-1) \\ \mathbf{O}(1\times M) \end{bmatrix},$$
$$\mathbf{\Lambda_1} = \mathbf{H}^{[1,R]}(2\mu+1)\mathbf{V_1}(2\mu+1)\begin{bmatrix} \mathbf{O}^T(M-N)\times 1 & 1 \end{bmatrix}^T,$$
$$\mathbf{\Lambda}_\mu = \mathbf{H}^{[1,R]}(3\mu)\mathbf{V_1}(3\mu)\begin{bmatrix} \mathbf{O}^T(M-N)\times 1 & 1 \end{bmatrix}^T,$$
$$\mathbf{\Upsilon_1} = \mathbf{H}^{[1,R]}(3\mu+1)\mathbf{V_1}(3\mu+1)\begin{bmatrix} 1 & \mathbf{O}^T(\mu-1)\times 1 \end{bmatrix}^T,$$
$$\mathbf{\Upsilon}_\mu = \mathbf{H}^{[1,R]}(3\mu+1)\mathbf{V_1}(3\mu+1)\begin{bmatrix} \mathbf{O}^T(\mu-1)\times 1 & 1 \end{bmatrix}^T.$$

We want to recover μR independent symbols from $(2\mu+1)N+\mu$ linear equations. From expression (19), we can see that the interference terms fall within a μN dimensional space. Thus we have enough dimension in the signal space to decode all the μR desired signals using this transmission strategy. Similar argument can be made for receiver 2. Ultimately combining the results for the two receivers, we can achieve a sum-DoF of $\frac{2\mu R}{3\mu+1}$, which lies between $\frac{2M}{3}$ and $\frac{2MN}{M+N}$.

$Remark$: If we add more antennas at the relay node and keep everything else the same, which means a 2-user system with $R = M + \zeta$, where $\zeta \in \{2, 3, ..., N\}$, by applying the same mechanism, we can achieve a sum-DoF of $\frac{2\mu R}{4\mu+1}$. This result is inferior to the one when there are $(M + 1)$ antennas at the relay. Hence, our scheme when applied to this 2-user $M \times N$ model, with relay having $(M + 1)$ antennas, cannot increase DoF gain even if we add an additional antenna to the relay. But if the relay has $(M + \zeta)$ antennas, we can ignore the redundant antennas and do at least as good as the case introduced in this section.

IV. Discussion and Comparison

In this section, we compare our DoF inner bound with different types of channel models.

Numerical results of the proposed scheme by setting $(M, N) = (3, 2)$ are provided in Fig.2, which match our theoretical DoF values. When the relay has 6 antennas, the curve has the largest slope. While the sum rate performance has the slowest increasing speed when $R = 3$.

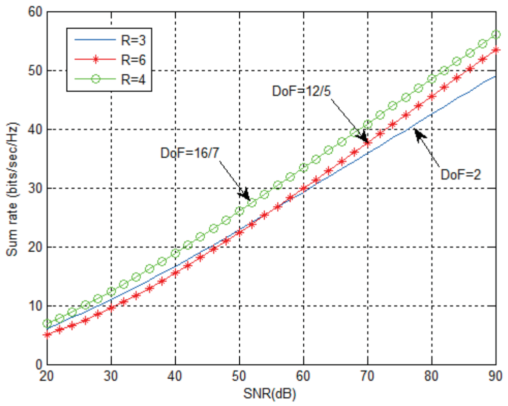

Fig. 2. Achievable sum rates versus SNR for 2-user 3×2 interference channel.

In [8], the optimal sum-DoF for a K-user $M \times N$ MIMO interference channel with no CSIT is $\frac{N}{K} \times K = N$ for $N \in (0, KM)$. Comparing this result with our result in $Section-III-A$, when $N \leq \frac{2M}{3}$, we can see that the relay is helpful to increasing the DoF even though the receiver knows only the delayed global CSI. Intuitively, MIMO system must have more multiplexing gain than MISO system. An upper bound has already derived in [9] when $M \neq N$, for example, the optimal DoF gain of 2 user $M \times N$ interference channel with no CSIT is $\frac{2MN}{M+N}$. But it can be achieved when relay has more than M antennas with instantaneous global CSI rather than at least $2M$ antennas in this paper. That's the penalty due to the lack of the instantaneous global CSI. However, when the relay has more than $2M$ antennas, the CSI requirement can be relaxed and still achieve the same sum-DoF. An interesting result is that as mentioned in the remark in $Section-III-C$, if we add one more antenna at relay from $M + 1$ to $M + 2$, the DoF characterization drops under the same mechanism which is counter-intuition. That is because if we add one more antenna, one more independent symbol appears in phase 1. That increases the difficulty to cancel interference. But we can achieve the same DoF using another proposed scheme. Since we only investigate the 2-user case, it's not that obvious to illustrate the merit of retrospective interference alignment because there's only one interference user. However, examples shown in this paper lower the complexity of the system with delayed CSI.

V. Conclusion

In this paper, we focus on a scenario where a 2-user MIMO relay interference channel has outdated channel state information. We introduced a two-phase transmission scheme based on retrospective interference alignment to achieve different DoF inner bounds under different antenna configurations at the relay node. We explain the mechanism when the number of antennas at the relay is $R = M, M < R < 2M, R \geq KM = 2M$ and show that when $R \geq 2M$, it can achieve the optimal sum-DoF. Hence, we do not need to require instantaneous global CSI at the relay. The proposed scheme can be extended to the more general case with arbitrary number of K, M, R, and N. Furthermore, an outer bound of this channel with the same assumption will need to be derived.

Reference

[1] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *Information Theory, IEEE Transactions on*, vol. 54, no. 8, pp. 3425–3441, 2008.

[2] T. Gou and S. A. Jafar, "Degrees of freedom of the K-user MN MIMO interference channel," *Information Theory, IEEE Transactions on*, vol. 56, no. 12, pp. 6040–6057, 2010.

[3] A. Ghasemi, A. S. Motahari, and A. K. Khandani, "Interference alignment for the K user MIMO interference channel," in *Information Theory Proceedings (ISIT), 2010 IEEE International Symposium on*. IEEE, 2010, pp. 360–364.

[4] V. R. Cadambe and S. A. Jafar, "Degrees of freedom of wireless networks with relays, feedback, cooperation, and full duplex operation," *Information Theory, IEEE Transactions on*, vol. 55, no. 5, pp. 2334–2344, 2009.

[5] H. Maleki, S. A. Jafar, and S. Shamai, "Retrospective interference alignment over interference networks," *Selected Topics in Signal Processing, IEEE Journal of*, vol. 6, no. 3, pp. 228–240, 2012.

[6] C. S. Vaze and M. K. Varanasi, "The degrees of freedom region and interference alignment for the MIMO interference channel with delayed CSIT," *Information Theory, IEEE Transactions on*, vol. 58, no. 7, pp. 4396–4417, 2012.

[7] M. A. Maddah-Ali and D. Tse, "Completely stale transmitter channel state information is still very useful," *Information Theory, IEEE Transactions on*, vol. 58, no. 7, pp. 4418–4431, 2012.

[8] C. S. Vaze and M. K. Varanasi, "The degree-of-freedom regions of MIMO broadcast, interference, and cognitive radio channels with no CSIT," *Information Theory, IEEE Transactions on*, vol. 58, no. 8, pp. 5354–5374, 2012.

[9] S. Chen and R. S. Cheng, "On the achievable degrees of freedom of a K-user MIMO interference channel with a MIMO relay," *Wireless Communications, IEEE Transactions on*, vol. 12, no. 8, pp. 4118–4128, 2013.

[10] W. Shin, B. Lee, B. Shim, and J. Lee, "A MIMO Relay with Delayed Feedback Can Improve DoF in K-user MISO Interference Channel with No CSIT," *to appear in IEEE Transactions on Vehicular Technology*, 2016.