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Abstract—In this paper, we investigate downlink resource allocation in two–tier OFDMA heterogeneous networks comprising a macrocell transmitting at a microwave frequency and dual band small cells utilizing both microwave and millimeter wave frequencies. A non - cooperative game theoretic approach is proposed for adaptively switching the small cell transmission frequency based on the location of small cell users and interference to macrocell users. We propose a resource allocation approach which maximizes the sum rate of small cell users while minimizing interference to macrocell users and the total power consumption. The performance of the proposed resource allocation solution is evaluated via rigorous MATLAB simulations.

Index Terms—Heterogeneous network, millimeter wave, 5G, interference coordination, optimization, dual band, resource allocation, OFDMA.

I. INTRODUCTION

Heterogeneous networks (HetNets) utilizes layers of macrocells and high density of deployed low power small cells in order to bring the network closer to end users. This way, radio link quality can be enhanced owing to the reduced distance between the transmitter and the receiver, and the larger number of cells allows for more efficient spectrum reuse and, therefore, larger data rates. However, efficient allocation of radio resources for the large number of small cells is required in order to ensure reliable and high quality of service to both macrocell as well as small cell users.

There has been considerable interest on the development of algorithms for radio resource allocation (RRA) in HetNets within the last few years (see e.g., [1]–[7] and the references therein). There has been several solutions proposed based on the optimization of varying system objectives. However, the research so far have majorly focus on shared spectrum networks utilizing only the macro - wave frequency band. Recently, there has been considerable research interest in cellular systems utilizing the millimeter-wave (mmWave) bands, which offer bandwidths that are orders of magnitude wider than current cellular networks. The available spectrum at these higher frequencies can be up to 200 times greater than all cellular allocations today [8], [9]. Moreover, recent advances in miniaturized electronic circuits design enable multiple mmWave antennas to be placed within a limited space [9]. These multiple antennas can be used to provide very high gain arrays at the base station and/or mobile equipment. [9].

A new design for 5G heterogeneous network operating in both microwave and millimeter wave frequencies was introduced in [10]. This dual band framework offers the potential to explore the differing characteristics of these frequency bands to enhance system throughput and reduce interference. Moreover, user equipments which are served on mmWave and microwave frequency becomes isolated and therefore, do not interfere with each other. In [11], a similar multiband framework is investigated for OFDMA HetNet in 5G applications. The author proposes utilization of mmWave in a multihop relay network having one dual band macrocell BS and single band small cell operating on mmWave frequency. A comparative performance analysis of dual band HetNets is presented in [12]. Motivated by the potentials of these dual band architecture and the spectral benefits of the mmWave frequency band, we propose a dynamic radius architecture and investigate resource allocation for 5G HetNets with Orthogonal Frequency Division Multiple Access (OFDMA) in this paper. This allows small cell BSs to adaptively switch the transmission frequency to a user based on the user location within the small cell coverage area and the interference to other users in the network utilizing the same frequency band. We formulate the RA problem as as the optimization of the total small cell sum rate under a minimum interference to macro users and maximum power constraints.

II. SYSTEM MODEL

A. Dual Band HetNet Network Model

Let us consider the downlink of a two-tier heterogeneous network as shown in Fig. 1. The network consists of a single macro base station (MBS) serving N_m macro user equipment (MUE) and P small cell base stations (SBSs). We assume that each SBS has dual air interface capability such that the small cell user equipment (SUE) can either be served on microwave frequency or millimeter wave frequency depending on the location of the SUE within the small cell (SC). The SC coverage area is divided into two region with radius d_1 and d_2, respectively. We assume that the outer radius d_2 is fixed during the SC deployment and select the inner cell radius, d_1 such that the sum rate of SUEs is maximized while minimizing interference to the MUEs. SUEs that falls withing the inner radius (i.e., user equipments (UEs) with distance from SBS less than d_1) of the SC are served on millimeter wave frequency while those outside the inner radius operates on microwave frequency. We assume that the outer radius d_2 is fixed during the SC deployment and select the inner cell radius, d_1 such that the sum rate of SUEs is maximized while minimizing interference to the MUEs. This design isolates
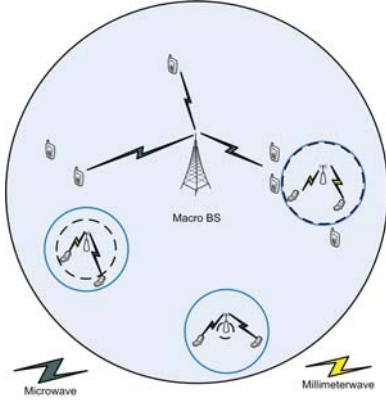
Fig. 1. Schematic illustration of a adaptive radii two-tier dual–band heterogeneous network operating at both microwave and millimeter wave frequencies.

UEs operating on different frequencies thereby reducing both the intra-tier and cross–tier interference to MUEs. We denote the number of users in the pth SC as N_p and the total number of users in the network as N such that $N = N_m + \sum_{p=1}^{P} N_p$.

We consider Orthogonal Frequency Division Multiple Access (OFDMA) transmission with a binary resource block (RB) allocation structure such that the available bandwidth on each frequency band is divided into M resource blocks. We assume that each UE in a cell can only be assigned one RB. The MBS can allocate all micro – wave RBs to its users. All the millimeter – wave RBs can also be allocated to small cell UEs in the inner coverage area of the SC. The small cell uses spectrum sharing to serve the users in the outer cell on the micro – wave RBs based on a specified resource allocation policy. We assume that the MBS has a central control node which collect relevant information to enhance allocation decisions and supply the selection information to the SCs.

B. Resource Allocation Policy

The SCs has to make a number of decisions regarding the transmission to associated SUEs. These include:

- The transmission frequency on which each user will be served considering the location of the UE
- The RB to be used for transmission to SUEs operating on micrometer wave frequency giving the state of the resource blocks in the macrocell and interference to MUEs.
- The transmit power to each user.

Let the transmission frequency set be denoted as $f = \{f_{mc} \quad f_{mm}\}^T$, we denote the frequency selection variables at small cell p for downlink transmission to UE k as $\varphi_{[p,k]} \in \{[0\,1], [1\,0]\}$. The transmission frequency is the obtained as $f_{[p,k]} = \varphi_{[p,k]}f$. The set of all frequency is denoted as $\varphi = [\varphi_{[1,1]}, \cdots, \varphi_{[1,K]}, \cdots, \varphi_{[P,K]}]$. We also denote RB allocation binary indicator variables, $\vartheta_{[p,k,m]} \in \{0, 1\}$, such that $\vartheta_{[p,k,m]} = 1$ when the k UE is served by the pth SBS on RB n and zero otherwise. The allocation variables for all SCs and the associated UEs can be combined into

a vector $\boldsymbol{\vartheta} = [\vartheta_{[1,1,1]}, \cdots, \vartheta_{[P,K,M]}]$ which describes the RB allocation policy for all SCs. We assume that the SBSs select the transmit power for each user from a finite set of power levels, $\mathcal{U} = [1, 2, \cdots, U]$. Each SCs select a suitable combination of transmit frequency, RB and power such that the interference cause to MUEs is below a specified threshold. We denote the transmit power selected by the pth SC for transmission on the nth RB as $g_{[p,n]}$. A vector, $\mathbf{g} = [g_{[1,1]}, \cdots, g_{[P,N]}]$ comprizing of all the selected power levels describe the power allocation policy of the SCs.

C. SINR Modelling

The received signal at the nth user on the mth RB from BS p can be written as

$$y_{[n,p,m]} = h_{[n,p,m]}s_{[n,p,m]} + \sum_{\substack{q=1 \\ q \neq p}}^{N_p} h_{[n,p,m]}s_{[n,q,m]}$$
$$+ \sum_{\substack{r=0 \\ r \neq n}}^{P} h_{[r,p,m]} \sum_{u=1}^{N_r} s_{[r,u,m]} + \rho_{[n]} \quad (1)$$

where $h_{[i,j,k]}$ and $s_{[i,j,k]}$ denote the complex channel impulse response of the link and the transmitted signal from the jth BS to the ith UE on the kth resource block, respectively. The transmitted signal are normalized such that $\mathbb{E}[|s_{[i,j,k]}|^2] = 1$. $\rho_{[n]}$ denotes the additive white Gaussian noise with variance $\sigma_{[n]}^2$ at the nth UE. Without loss of generality, $p = 0$ in (1) corresponds to UEs in the MBS. We assume that the channels on each RB are iid complex Gaussian random variables, i.e., $\mathbf{h}_{[i,j,k]} \sim \mathcal{CN}(0, PL_{[i,j,k]})$. We use the close-in free space reference distance model, such that the path loss (in dB) between the ith BS and the j UE on RB k is defined as [13], [14]

$$PL_{[}i,j,k] = PL(d_0) + 10\Gamma \log\left(\frac{d_{[i,j,k]}}{d_0}\right) + \mathcal{X}_\sigma, \quad \text{for} \quad d \geq d_0 \quad (2)$$

Here, $PL(d_0)$ denotes the free space path loss in dB. $PL(d_0)$ can be expressed in terms of the carrier frequency, f and speed of light, c as

$$PL(d_0) = 10 \log\left(\frac{4\pi f d_o}{c}\right)^2, \quad (3)$$

$d_{[0]}$ and $d_{[i,j,k]}$ are the reference distance and distance between the ith BS and jth UE, respectively. Γ is the path loss exponent and X_σ denotes the shadow fading component which is modelled as a zero mean Gaussian random variable with variance σ. Using (1), the signal to interference and noise ratio (SINR) for the $[n, p, m]$th SUE is obtained as

$$\gamma_{[n,p,m]} = \frac{g_{[p,m]}|\mathbf{h}_{[n,p,m]}|^2}{\sum_{\substack{q=1 \\ q \neq p}}^{N_p} |\mathbf{h}_{[n,p,m]}|^2 + \sum_{\substack{r=1 \\ r \neq n}}^{P} |\mathbf{h}_{[r,p,m]}|^2 + \sigma_{[p]}^2} \quad (4)$$

Here $\sigma_{[p]}^2 = B_{RB}N_0$, where N_0 and RB denote the thermal noise and the resource block bandwidth, respectively. For the macro cell UEs the SINR expression in (4) reduces to

$$\gamma_{[n,0,m]} = \frac{g_{[0,m]}|\mathbf{h}_{[n,0,m]}|^2}{\sum_{\substack{r=1 \\ r \neq n}}^{P} g_{[r,m]}|\mathbf{h}_{[n,r,m]}|^2 + \sigma_{[p]}^2} \quad (5)$$

Using either (4) and (5) and the Shannon's capacity formula, the per user rate for the nth UE in cell p can be expressed as

$$R_{[n,p,m]} = R_{\text{RB}} \log_2 \left[1 + \gamma_{[n,p,m]} \right] \quad (6)$$

III. PROBLEM FORMULATION

Let $\nu_{[u,m]} \in \{0,1\}$ denote the binary indicator for power level assignment to resource block such that $\nu_{[u,m]} = 1$ when power level u is assigned to the m RB and zero otherwise. The rate equation in (6) can now be expressed in terms of the resource allocation decision variables as

$$R_{[n,p]} = \sum_{m=1}^{M} \sum_{u=1}^{U} \nu_{[u,m]} \vartheta_{[p,k,m]} R_{\text{RB}} \log_2 \left[1 + \gamma_{[n,p,m]} \right] \quad (7)$$

It should be noted that the SINR expression in (4), and hence, the rate in (7) is dependent on the choice of transmission frequency. By summing (7) over the SUEs, we obtain

$$R_{[n]} = \sum_{p=1}^{P} \sum_{m=1}^{M} \sum_{u=1}^{U} \nu_{[u,m]} \vartheta_{[p,k,m]} R_{\text{RB}} \log_2 \left[1 + \gamma_{[n,p,m]} \right] \quad (8)$$

The DBDR – RA problem can therefore be formulated as

$$\max_{\boldsymbol{\vartheta}, \boldsymbol{\nu}, \boldsymbol{\varphi}} \sum_{n=1}^{N} \sum_{p=1}^{P} \sum_{m=1}^{M} \sum_{u=1}^{U} \nu_{[u,m]} \vartheta_{[p,k,m]} R_{\text{RB}} \log_2 \left[1 + \gamma_{[n,p,m]} \right] \quad (9)$$

subject to

$$\nu_{[u,m]} \in \{0,1\}, \forall m = 1, 2, \cdots, M \quad (10)$$

$$\vartheta_{[p,k,m]} \in \{0,1\}, \forall k \in \mathcal{K}_p; \forall p \in \mathcal{P} \quad (11)$$

$$\sum_{p=1}^{P} \left(\sum_{k=1}^{N_p} \vartheta_{[p,k,m]} \right) g_{[p,m]} |\mathbf{h}_{[n,0,m]}|2 \leq I_m^{\max}; \forall m \quad (12)$$

$$\sum_{p=1}^{P} \left(\sum_{k=1}^{N_p} \vartheta_{[p,k,m]} \right) g_{[p,m]} \leq G_{\max}; \forall p \quad (13)$$

$$d_{\min} \leq d_1 \leq d_2 \quad (14)$$

$$g_{[p,m]} \geq 0, \forall p, m \quad (15)$$

$$\varphi_{[p,k]} = \begin{cases} [0\,1]; & 0 \leq d_{[p,k]} \leq d_1 \\ [1\,0]; & d_1 < d_{[p,k]} \leq d_2 \end{cases} \quad (16)$$

The objective in (9) maximizes the system sum rate under the constraints in (10) - (16). Constraints (10) and (11) are the RB selection and small cell frequency switching binary indicators, respectively. Constraint (13) ensures that the total network power consumption does not exceed the maximum allowable threshold, $G_{\max}$. (15) indicates that the transmit power for each cell on a selected RB is non-zero. Constraint (14) specifies the minimum and maximum threshold for the adaptive inner radius of each SC. The constraint in (16) performs the frequency switching between microwave and millimeter wave based on the optimized value of the distance variable in (13). Finally, constraint (12) denotes the overall maximum interference that a MUE can tolerate from all shared-spectrum SCs on the nth RB in order to meet the MUE's rate requirement. Since a minimum rate requirement

at a MUE is directly related to the minimum required SINR, $\gamma_{[n,0,m]}^{\min}$, the total interference that the nth macro cell UE can tolerate on the mth RB can be obtained from

$$I_m^{\max} = \frac{g_{[0,m]} |\mathbf{h}_{[n,0,m]}|2}{\gamma_{[n,0,m]}^{\min}} - B_{\text{RB}} N_0 \quad (17)$$

On the other hand, the resource allocation problem for the DBFR architecture involves power and RB allocation for the SUEs operating on the micro-wave frequency. The frequency selection is performed once for each users prior to resource allocation. The DBFR - RA problem can be obtained from (9) - (16) be removing the distance/radius and frequency selection constraints. In the next section, we present the proposed allocation method for DBDR. Similar procedure can applied to the DBFR optimization problem.

IV. ADAPTIVE RADIUS RESOURCE ALLOCATION

The proposed scheme is a two stage optimization involving location aware frequency selection and resource (RB and power level) allocation.

A. Adaptive Frequency Switching

Given the distances between SUEs and the corresponding small cell base stations, the goal of the SC frequency selection stage is to adaptively switch the transmit frequency for each SUE from microwave to millimeter wave and vice-versa. We assume that the SC has knowledge of the channel status information and required distances. This can typically be obtained from channel estimation and/or feedback. The SC frequency switching problem is formulated in our research as a minimization of the total interference to MUEs, thus

$$\min_{\mathbf{f}} \sum_{p=1}^{P} \left(\sum_{k=1}^{N_p} \vartheta_{[p,k,m]} \right) g_{[p,m]} |\mathbf{h}_{[n,0,m]}|2 \quad (18)$$

subject to

$$\vartheta_{[p,k,m]} \geq 0, \forall k \in \mathcal{K}_p; \forall p \in \mathcal{P}$$

$$f \in [f_{\text{mc}} \quad f_{\text{mm}}] \quad (19)$$

Since the optimization variable in (18) can only take any two values at a time, we propose a simple P - player non-cooperative strategic game to obtain the optimal combination of transmission frequencies for each SC in the network. The SBs act as players in the proposed game. The pth player in the game chooses a combination of N_p frequencies for transmission to its users in order to minimize interference to neighboring MUEs. For simplicity reasons, we assume that each SB has knowledge of the location of all MUEs. We define the non-cooperative switching game in a strategic form as the triplet:

$$\mathcal{G} = (\mathcal{P}, \{\mathcal{S}_{[p]}\}_{p \in \mathcal{P}}, \{\mathfrak{u}_{[p]}\}_{p \in \mathcal{P}})$$

where $\mathcal{P} = \{1, \cdots, P\}$ denotes the set of players, $\mathcal{S}_{[p]}$ is the set of strategies for the pth player and $\mathfrak{u}_{[p]}(\mathbf{s})$ is the *utility function*[1] associated to player for a combination of strategies $\mathbf{s} = [s_{[1]}, \cdots, s_{[P]}] = [s_{[p]}, \mathbf{s}_{[/p]}]$. Here,

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[1]Utility function is also referred as reward/payoff function. These terms will be used interchangeably throughout this paper.

$\mathbf{s}_{[/p]} = [s_{[1]}, \cdots, s_{[p-1]}, s_{[p+1]}, \cdots, s_{[P]}]$ represents the set of actions taken by all other players other than the pth player. Since the switching game involve the small cells (i.e., players) choosing the transmission frequency for their associated users between specific microwave and millimeter wave frequencies, the set of feasible strategies, $\mathcal{S}_{[p]}$ for the pth player is a set containing the two frequency values, i.e., $\mathcal{S}_{[p]} = \{f_{\mathrm{mc}} \quad f_{\mathrm{mm}}\}$. In order to guarantee the existence and improve the efficiency of the game's Nash Equilibrium (NE), we utilized a pricing mechanism [15] in the formulation of the reward function. The priced utility function for the switching game is defined as

$$\hat{\mathfrak{u}}_{[p]}(\mathbf{s}) = \mathfrak{u}_{[p]}(\mathbf{s}) - \beta \mathcal{S}_{[p]} \tag{20}$$

where β denote the pricing factor. The pricing factor is chosen such that the players are appropriately penalized for the actions taken. Since the goal of our switching game is to minimize the interference to MUEs as given in (18), the priced utility function in (20) can be expressed as

$$\hat{\mathfrak{u}}_{[p]}(\mathbf{s}) = \sum_{k=1}^{N_p} \vartheta_{[p,k,m]} g_{[p,m]} |\mathbf{h}_{[n,0,m]}|^2 - \beta \mathcal{S}_{[p]} \tag{21}$$

A summary of the game - theoretic SC frequency switching stage of the DRDB - RA scheme is presented in Algorithm 1.

Algorithm 1 Algorithm for Non - Cooperative Game Based SC Frequency Switching
Input: Channel gains, frequency set, $f = \{f_{\mathrm{mc}} \quad f_{\mathrm{mm}}\}$
Output: $f_{[k,p]}; \quad \forall k, p;$
  Initialisation : $\{f_{[k,p]}\}_{k=1}^{N_p}$
1: **while** no convergence **do**
2:   something
3:   **for** $p = 1$ to P **do**
4:     $d_{[n,p]} \leftarrow$ estimate user distances and interference
5:     substitute measured values into (21)
6:     $\mathbf{f}_{[p]} \leftarrow \max \hat{\mathfrak{u}}_{[p]}(\mathbf{s})$;select frequencies for the pth SC
7:   **end for**
8: **end while**
9: update $f_{[k,p]}$

We now present a method for allocating the transmission resources using the selected frequency values in the SC switching stage.

B. Joint Resource Allocation

The binary decision variables, $\nu_{[u,m]}$ and $\vartheta_{[p,k,m]}$ make the resource allocation formulation in (9) a Mixed - Inter Problem Optimization Problem (MIOP). Solving an MIOP require the application of integer programming, which has been proven to have very high computational complexity. In this research, we utilize the classical method called Branch and Bound (BB) for solving the DBDR - RA problem. This method involves finding the optimal solution to the relaxation of the problem without the integer constraints via standard optimization methods. Relaxing the binary constraints in (9)

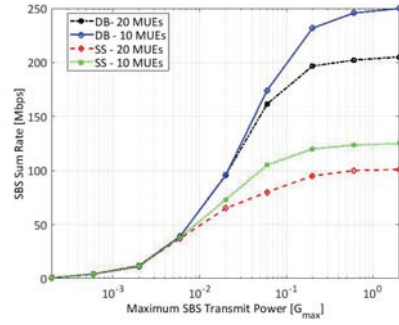| Parameter	Macro-cell	Small-cell
Number of BSs	1	(10 - 20)
Carrier Frequency	2.1 GHz	(2.1,38) GHz
Cell radius	900 m	80 m
SC Generation	Uniform Distribution	
Bandwidth	10 MHz	
Pathloss Exponent	4	(4,3.8)
Shadow fading SD	8 dB	

Fig. 2. Effect of small cell maximum power constraint on SBS sum rate.

such that their values can either be equal to or greater than zero, we obtain

$$\max_{\boldsymbol{\vartheta}, \boldsymbol{\nu}, \boldsymbol{\varphi}} \sum_{n,m,p,u} \nu_{[u,m]} \vartheta_{[p,k,m]} R_{\mathrm{RB}} \log_2 \left[1 + \frac{\gamma_{[n,p,m]}}{\nu_{[u,m]} \vartheta_{[p,k,m]}} \right] \tag{22}$$

subject to $\quad (12), (13), (14), (15), (16),$

$$\nu_{[u,m]} \geq 0, \forall m = 1, 2, \cdots, M \tag{23}$$

$$\vartheta_{[p,k,m]} \geq 0, \forall k \in \mathcal{K}_p; \forall p \in \mathcal{P} \tag{24}$$

The relaxed problem in (22) is a convex optimization problem, which can be solved using any of the standard approaches for solving convex problems. There is however, no guarantee, that the relaxed parameters will be integers in the solution. In this paper, we utilize the Lagrangian method in MATLAB optimization toolbox for solving the problem.
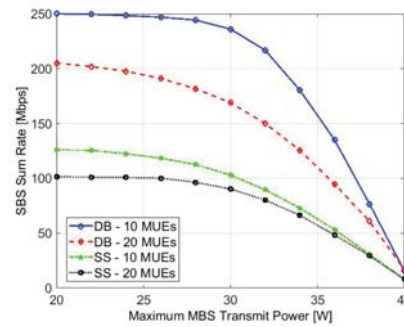
V. SIMULATION AND RESULTS

Fig. 3. Effect of macro cell maximum power constraint on the sum rate performance of SBSs. MBS maximum power between 20 W and 40 W and SBS maximum power of 2 W.

In this section, we evaluate the performance of the proposed adaptive radius resource allocation via numerical simulations in MATLAB and comparison with classical shared spectrum heterogeneous network utilizing only microwave frequency band. We consider the downlink of a two-tier heterogeneous network with the parameters in Table I. We study the effect of the maximum transmit power of the cells on sum rate performance in Fig. 2. We plot the sum rate of the small cells as a function of maximum SBS transmit power with different number of macro-cell users in the network. We observe that the sum rate increases with increasing transmit power. This is expected since increasing the transmit power increases the signal to noise and interference ratio (SINR) and hence, the information rate. However, the rate of increase of the sum rate decreases as the transmit power increases. For instant, while the rate for the dual band architecture with 10 MUEs increases by about 44 Mbps when the power is increased from 0.1 W to 0.5 W, the increase is only about 5 Mbps for a similar rise in transmit power from 0.5 W to 1 W. This is expected since increasing the transmit power of the small cells increases the co-tier interference. However, with transmit power below 1 W, the increase in co-tier interference is not significant enough to affect the SINR and hence, the capacity. Fig. 2 also shows the effect of increasing macro user density of the rate performance of the small cells. We observe that for both the dual band (DB) and shared spectrum (SS) architectures, the rate diminishes when the number of MUEs is increased from 10 to 20. This is reasonable since increasing the number of users in the network increases the number of shared sub-carriers and hence, the cross-tier interference. As expected, Fig. 2 shows that the proposed dual band network offers significant improvement in rate performance over the entire power region considered when compared the classical shared spectrum (SS) network. The rate improvement however, increases with increasing transmit power. A plausible explanation for the is that the adaptive switching of transmission frequency by the SBSs reduces the cross-tier interference from the macro cell to the small cells.

In Fig. 3, we illustrate the effect of varying the MBS maximum power constraints on the sum rate of the SBSs. The MBS power is varied between 20 W and 40 W while the maximum power for the SBSs is fixed at 2 W. Contrary to the trend in Fig. 2, the sum rate decreases as the MBS transmit power increases. This is expected since the macro to small cell cross-tier interference power and hence, the total interference, increases with increasing MBS transmit power. This leads to a reduction in the SINR and hence, the transmission rate.

VI. Conclusion

We have investigated the downlink resource allocation in two–tier OFDMA heterogeneous networks comprising a macrocell transmitting at a microwave frequency and dual band small cells utilizing both microwave and millimeter wave frequencies in this. The problem is formulated such that the frequency at which a small user is served and the radius of the inner region is adaptively optimized. The performance of the dual band network architecture and the proposed resource allocation solution is evaluated via simulations. Results show improved performance compared to the classical single band shared spectrum architecture. Our current research is deriving analytical solutions for the joint optimization problem.

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