Network Coding For Data Dissemination: It Is Not What You Know, But What Your Neighbors Don't Know

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***Abstract*— We propose a linear network coding scheme to disseminate a finite number of data packets in arbitrary networks. The setup assumes a packet erasure channel, slotted time, and that nodes cannot transmit and receive information simultaneously. The dissemination process is completed when all terminals can decode the original data packets. We also assume a perfect knowledge of the information at each of the nodes, but not necessarily a perfect knowledge of the channel. A centralized controller decides which nodes should transmit, to what set of receiver nodes, and what information should be broadcasted. We show that the problem can be thought of as a scheduling problem, which is hard to solve. Thus, we consider the use of a greedy algorithm that only takes into account the current state of the system to make a decision. The proposed algorithm tries to maximize the impact on the network at each slot, i.e. maximize the number of nodes that will benefit from the coded packet sent by each active transmitter. We show that our scheme is considerably better, in terms of the number of slots to complete transmission, than schemes that choose the node with more information as the transmitter at every time slot.**

I. Introduction

Network coding was introduced by Ahlswede *et al* [1]. Network coding considers the nodes to have a set of functions that operate upon received or generated data packets. Today's networks constitute a subset of the coded packet networks, in which each node performs two main functions: forwarding and replicating a packet. A classical network's task is to transport packets provided by the source nodes unmodified. In contrast, network coding considers information as an algebraic entity, on which one can operate. Reference [1] showed that network coding achieves multicast capacity. Work in [2] and [3] showed that linear codes are sufficient to implement any feasible multicast connection. Also, [3] provides an algebraic framework for studying this subset of coded networks.

The problem of data dissemination has been widely studied for routing scenarios, focusing on theoretical analysis, e.g. [6], and protocol design, e.g. [5]. More recently, Reference [7] studied the effect of using network coding showing significant improvement over routing in terms of completion time. Reference [8] provides a wireless medium access control combined with network coding for multi-hop content distribution. The authors focus on a protocol that uses a content-directed

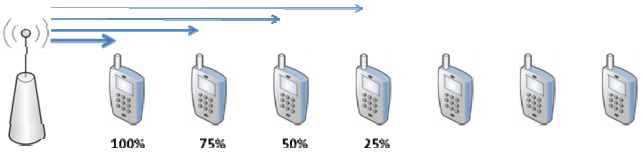

Fig. 1. Network Example: J mobile devices require M data packets, but not all of them can obtain them directly from the Base Station. Each node can transmit to I nodes.

medium access control (MAC), through which transmission priority is given to those nodes based on the rank of the coefficient matrix associated with the coded content the node holds, i.e. nodes with more information are given higher priority.

This paper advocates for the combination of network coding and medium access strategies, similar to the idea in Reference [8]. However, we illustrate that giving priority to the nodes with the most information in the network is not necessarily going to promote a faster dissemination of the data. The main objective of this paper is to determine key ideas to help in the development of ad-hoc protocols that combine network coding and MAC considerations.

In particular, we focus on the problem of minimizing the completion time to disseminate information assuming a time slotted system. This problem can be stated as a scheduling problem which is in hard to solve in general. We propose a heuristic to solving the problem, in which the nodes with the greatest impact on the network at each time slot should transmit, instead of choosing the node with the most information. Starting with a toy example for a linear meshed network, i.e. nodes deployed in a line, different medium access strategies are compared with each other in terms of the mean completion time. We show that our scheme can obtain considerable gains with respect to choosing transmitters in terms of their knowledge. Even in small networks and moderate number of data packets to transmit, roughly a twofold improvement can be obtained. Although the examples and simulation results focus on linear meshed networks, we emphasize that the description and analysis of our algorithm is valid for any network and any

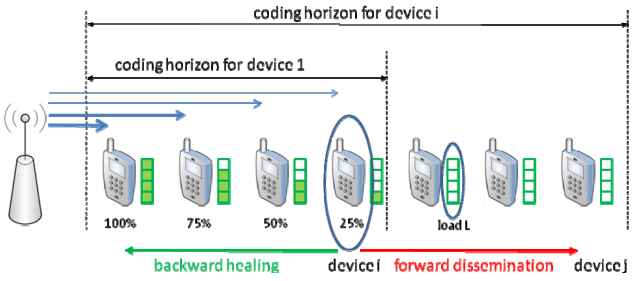

Fig. 2. Network Example: J mobile devices require M data packets, but not all of them can obtain them directly from the Base Station. Each node can transmit to I nodes.

starting distribution of (coded) packets of the nodes. In fact, our analysis considers routing as a particular case. Also, the problem of linear meshed networks is interesting in itself for some applications, e.g. underwater acoustic networks [9] [10].

In order to understand the combination of network coding and medium access strategies, we assume the following scenario: A set of J mobile devices wants to receive the same number of M packets from the base station. The mobile devices are lined up with different distances to the base station as shown in Figure 1. The coverage of the base station is sufficient to reach $I < J$ mobile devices. Furthermore, we assume that the devices with longer distances to the base station will receive less information. For illustration we assume that the packet receiving probability, after the base station is broadcasting a number of coded packets, varies between 25% and 100% in our example with $J = 8$ and $I = 4$. The question at this point is how to continue disseminating the information, as different strategies will have an impact on the overall number of transmitted packets to satisfy all mobile devices. Let us discuss some simple possibilities.

- Base Station Centered: The base station continues the transmission of coded packets until all stations in its coverage range have understood the full information (all 100%). Note that nodes closer to the base station will need less time to gather all information, but the base station needs to continue to satisfy all devices in its coverage range. Once all devices in the coverage of the base station are satisfied, optimally the device with full knowledge that is farther downstream (farther away from the base station) will start to relay the information to the rest. Network coding helps in this example to compensate for packet erasures, as the base station does not need to know about which packets have been received so far. It only has to focus on delivering enough linear combinations to the nodes. Thus, the base station transmits random linear combinations until all devices have a sufficiently high number to decode all packets.
- Progressive Base Station: The first mobile device that receives the full information will start to transmit and the base station will stop automatically. The mobile deice with full information will act as base station until another device farther downstream has all information. Obviously this approach has the advantage that more devices can be reached and that those missing information are now closer to the source, which in turn will lead to lower packet loss probability. This scheme will perform as well as or better than the Base Station Centered scheme.
- Greater Impact at each time slot (Greedy Algorithm): For the last approach, it is not the node closest to the base that goes first, in general. We look at the received packets so far by each mobile device to determine which nodes should transmit. It is important to note that the received packets in the example are uncorrelated , i.e. the packets marked as 25 % are not necessarily contained in those marked as 50%. This assumption opens the door for a new strategy. For the following discussion we introduce the term coding horizon, which is roughly the number of devices one transmitting device can reach by broadcasting information. The very first mobile device in the line has the coding horizon of 4, while mobile device i (the last one that received information from the base station) has a coding horizon of 7 (Figure 2). In this simple example, there is clearly a drift of information from left to right. Note that in this example, devices with a small coding horizon have collected more packets so far, but devices with fewer packets have a larger coding horizon and will therefore reach more neighbors. We observe that node i has more impact on the network in this time slot because it can benefit more nodes with the transmission of a single coded data packet. Therefore, each relaying action is divided into a backward healing and a forward dissemination part. In case mobile device i is sending a packet, obviously all devices to the right of it are seeing that information for the first time (forward dissemination). Simultaneously, there might be devices to the left that are also interested in these packets (backwards healing). This is the main component of our heuristic scheme.

Note that the Progressive Base Station scheme will perform as well as or better than the Base Station Centered scheme. For this reason, we will focus on comparing Progressive Base Station scheme to the Greater Impact scheme.

The paper is organized as follows. In Section II, we discuss motivating examples that illustrate benefits of using nodes with the greatest impact to the network, instead of nodes with the most knowledge. In Section III, we outline the set up of the problem and present our scheme as a greedy algorithm to solve a more general but hard problem. Section IV provides numerical results for different scenarios. Conclusions are summarized in Section V.

II. Motivating Examples

Before starting with a formal analysis of the problem, let us consider two examples that illustrate the advantage of 1) choosing the transmitter node in order to provide the greatest impact to the system at each time slot versus choosing the node that has the most information, and 2) the advantage of breaking ties between sets of transmitters with the same impact

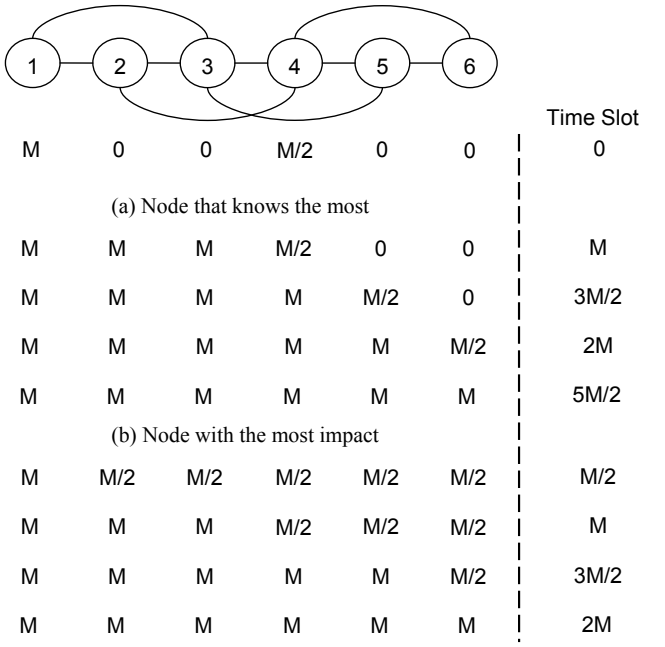

Fig. 3. Motivating Example 1: Data dissemination when choosing (a) node with the most knowledge, and (b) node with the most impact

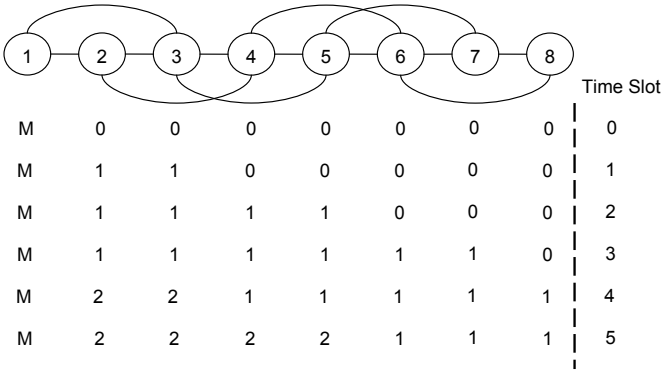

Fig. 4. Motivating Example 2: Data dissemination exploiting parallel transmission

on the network by choosing the one that includes nodes with the least information.

- Example 1: Let us consider a network with no packet erasures where each node wants to receive M data packets. Each node can contact 2 neighbors to the left and 2 to the right, as in Figure 3. The leftmost node has all M packets while a middle node has $M/2$ linear combinations. Figure 3 (a) shows the data dissemination process in time if we choose the node with the most information and that is further downstream (Progressive Base Station scheme in the introduction). This scheme takes $\frac{5M}{2}$ time slots to complete the dissemination. Figure 3 (b) shows the same procedure when the node with the highest impact is chosen at the beginning. We break ties without considering parallel transmissions and trying to keep as close a behavior to the first approach. This is a simplified version of the Greater Impact scheme. Note that this very naive scheme requires only $2M$ time slots to disseminate all information to the nodes, just by choosing node 4 as the first transmitter. Thus, choosing the node with the most information requires 25% more time to complete transmission in this simple example, even with a vanilla version of our scheme. If we performed the dissemination with a scheme similar to 1 of the introduction, $3M$ time slots would be required to complete the transmission, i.e. 50% more time than choosing the middle node, which has greater impact to the network, at the beginning.
- Example 2: We consider a similar setup as the previous example. However, we study the case of a network with K nodes in which only one node has all the information. Assuming no packet erasures, choosing the node with the most knowledge at every time slot will transmit all of its information to its N nodes further downstream. At this point, the node further downstream starts transmitting to its N neighbors until those neighbors have all information. This process is repeated until all nodes have all data. The time to complete transmission $T_c^{(1)}$ under this scheme is

$$T_c^{(1)} = M \left\lceil \frac{K-1}{N} \right\rceil. \tag{1}$$

However, if we use a scheme that chooses the node with the greatest impact to the network but that breaks ties in favor of sets of transmitters that will benefit nodes with the least information, there will be a considerable reduction in the completion time. This happens because the system will be able to take advantage of parallel non-interfering transmissions. Figure 4 illustrates the effect of such a scheme when $N = 2$ and $K = 8$. We observe that every 3 time slots the same sequence of transmitting events occurs. This is valid for larger K. Using this insight, the time to complete transmission $T_c^{(2)}$ under this scheme is

$$T_c^{(2)} = 3(M-1) + \left\lceil \frac{K-1}{N} \right\rceil. \tag{2}$$

It is simple to show that $T_c^{(2)} \leq T_c^{(1)}$ for every value of K, N and M of importance. However, $T_c^{(2)}$ is strictly less than $T_c^{(1)}$ for $K > 4N$. Let us define the gain G in this case as

$$G = \frac{T_c^{(1)}}{T_c^{(2)}} = \frac{M \left\lceil \frac{K-1}{N} \right\rceil}{3(M-1) + \left\lceil \frac{K-1}{N} \right\rceil} \tag{3}$$

which represents how much more time it takes to complete the dissemination when we use a scheme that chooses the node with greater knowledge instead of trying to take advantage of the spatial diversity. We can show that G can be made arbitrarily large. For example,

$$\lim_{M \to \infty} G = \frac{1}{3} \left\lceil \frac{K-1}{N} \right\rceil \tag{4}$$

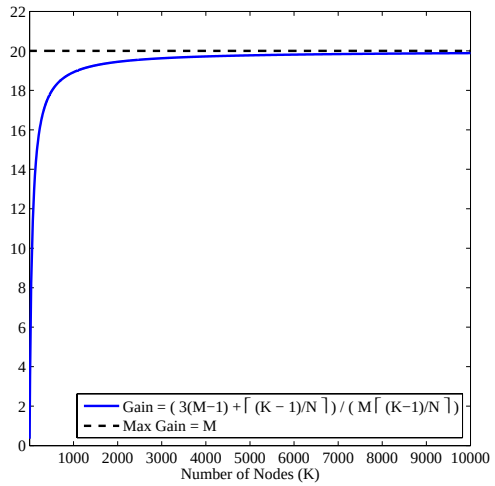

Fig. 5. Gain G when M is fixed and the size of the network is increased. Parameters used are $M = 20$ and $N = 1$

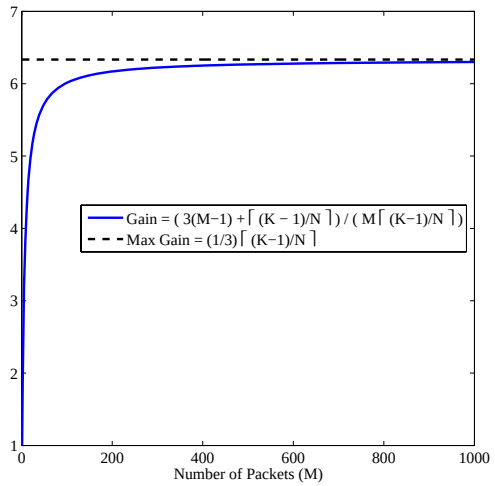

Fig. 6. Gain G when K is fixed and we change the number of transmitted packets M. Parameters used are $K = 20$ and $N = 1$

for a fixed value of K. This is related to the case of a fixed network and a large number of packets that need to be disseminated. On the other hand, if we have a fixed number of packets but our network is large with respect to the number of packets

$$\lim_{K\to\infty} G = M. \tag{5}$$

Figure 5 illustrates the gain for a fixed value of M when we increase the size of the network. For this example, $M = 20$ and $N = 1$. Figure 6 illustrates the gain for a fixed network size K when we vary the number of packets to be transmitted.

III. Problem Setup

Let us formalize our problem. In particular, we focus in networks where each of the nodes has some information but it wants all M data packets present in the network. In our previous examples, we generally considered a single node, e.g. node 1 in Figure 7, that wanted to transmit M data packets to

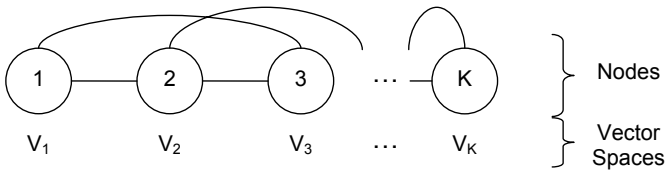

Fig. 7. Network of interest. Each node i has at the beginning a vector space V_i. If we used no coding, each vector space would be spanned by a subset of individual packets $P_a, \forall a = 1, ..., M$ where M is the total number of packets to disseminate. If we use coding, each vector space is spanned by a set of linear combinations of $P_a, \forall a = 1, ..., M$.

all other $K-1$ nodes in the network. Let us assume that time is slotted. A data packet or coded packet is transmitted in a single slot. Each node i has a vector space $V_i(t)$ at time t. If we used no coding, each vector space would be spanned by a subset of individual packets $P_a, \forall a = 1, ..., M$ where M is the total number of packets to disseminate. If we use coding, each vector space is spanned by a set of linear combinations of $P_a, \forall a = 1, ..., M$.

We consider the network to be modeled as a hypergraph $G = (N, A)$, where N is the set of nodes and A is the set of hyperarcs. This is an extension of the graph, in which we are capturing the broadcast nature of the channel. A hyperarc (i, J) represents a connection between a node i (the transmitter) and a set of nodes J (the receivers).

We also assume a perfect knowledge of the information of the nodes and that we can operate under different channel conditions, and have different knowledge about the transmission channel. We assume a centralized controlled that decides which hyperarcs should be active at each slot, i.e. which nodes should transmit and to what set of receiver nodes, and what information should be transmitted through each hyperarc.

In general, the decision made in a time slot will affect decisions in the following slots. Let us consider the case of no erasures first. We can think about this problem as a scheduling problem, where we have a set of schedules S from which we can choose $s(t)$at time t. Each schedule $s \in S$ is a set of hyperarcs that are active in that schedule, e.g. $s = \left\{(i_1, J^{(1)}), (i_2, J^{(2)}), ..., (i_m, J^{(m)})\right\}$. In general, each schedule should only include hyperarcs with different transmitters, i.e. $i_a \neq i_b, \forall a \neq b$. However, we impose no conditions on the set of receivers J^a of each node a. This allows us to cause collisions in one node during a time slot if this means a higher benefit for the system overall.

Let us define $B^s_{iJ^{(i)}j}(t)$ as the benefit that node j gets from node i when i uses hyperarc $(i, J^{(i)})$ at time t. This benefit $B^s_{iJ^{(i)}j}(t)$ will depend on what coded packet node i transmits at time t, say $q_{i,J^{(i)}}(t) \in V_i(t)$. Let us define $W_{s(t),\bar{q}(t)}$ at time t as the weight for a schedule $s(t)$ given that $\bar{q}(t) = [q_{i,J^{(i)}}(t)]$ coded packets are being transmitted by the active transmitters of that schedule. This weight represents the impact of that schedule over the system in that time slot, i.e. how many nodes are seeing an increase in dimension of their vector space if that

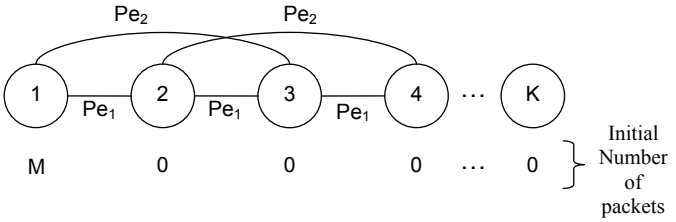

Fig. 8. Simulation Setup: linear meshed network, with each node with at most $N = 2$ neighbors upstream (closer to node 1) and $N = 2$ downstream. Node 1 has all M data packets at the beginning. The objective is to disseminate those packets to every receiver in the least amount of time.

schedule is chosen. We can define $W_{s(t),\bar{q}(t)}$ as

$$W_{s(t),\bar{q}(t)} = \sum_{i=1}^{m(s(t))} \left[\sum_{j \in J^{(i)}} B^s_{iJ^{(i)}j} \right] \tag{6}$$

where $m(s(t))$ is the number of hyperarcs in schedule $s(t)$. The objective is to find the sequence $U(1), U(2), ...,$ that minimizes the time to disseminate all data packets to all nodes of the network, where $U(t) = (s(t), \bar{q}(t))$.

If we assume that the network started with $W_{ini} = \sum_{k=1}^{K} \dim(V_k)$ and that at the end of the process the system should have a total of MK packets, then the problem can be formulated as

$$\begin{aligned} &\min_{U(1),U(2),...} n \\ &\text{subject to} \\ &U(t) = (s(t), \bar{q}(t)), \forall t \\ &s(t) \in S, \forall t \\ &q_{i,J^{(i)}}(t) \in V_i(t), \forall t, i, J^{(i)} \\ &MK - W_{ini} = \sum_{t=1}^{D} W_{s(t),\bar{q}(t)}, \forall D \geq n \end{aligned}$$

Solving this scheduling problem is hard even in the absence of packet erasures or perfect knowledge of the channel. Let us focus on a greedy algorithm that tries to maximize the impact on the network at each time slot.

Let us use greedy algorithms that only takes into account the current state of the system to make a decision, i.e. we try to find the set of hyperarcs s that will have the node with the most information or the set s that has greater impact in the network in the current slot for the Progressive Base Station and Greater Impact schemes, respectively. From its perspective, at any time slot the network can be modelled as a set of nodes N with a vector space V_i associated to each node i (Figure 7).

Let us define v_{iJ} as the vector selected for transmission in hyperarc (i, J). This choice is made to maximize the impact of the transmission from i to each of his receivers in J. One way to state this problem is to choose v_{iJ} so that we increase as much as possible the dimensions of the vector spaces of each of the receivers. This is,

$$v_{iJ} = \arg \max_{q_{iJ} \in V_i} \sum_{j \in J} \dim \left(\{V_j, q_{iJ}\} \right). \tag{7}$$

Note that this is valid for network coding. The simplest way of generating v_{iJ} is to create a random linear coded packet over a large enough field size. This coded packet is generated from the packets or linear combinations that span the vector space of node V_i. If no coding is allowed, we have to impose the additional constrain on q_{iJ} to be a single packet, i.e. the vector q_{iJ} will have a very specific structure. At any time slot we choose a schedule depending on the scheme we use.

The Progressive Base Station scheme will choose the schedule such that

$$s^*_{PBS} = \arg \max_{s \in S} \left(\max_{(i,J) \in s} |V_i| \right) \tag{8}$$

and we break ties in favor of nodes that have more neighbors with incomplete information. This approach is similar in its essence to the work in [8], because it gives nodes with the most information a priority to access the channel.

The Greater Impact greedy algorithm will choose as schedule

$$s^*_{GI} = \arg \max_{s \in S} W_s \tag{9}$$

where W_s is the weight for each schedule given the choice of q_{iJ}.

Similar to the full problem, W_s represents the impact of a schedule s over the system in that slot. Thus,

$$W_s = \sum_{i=1}^{m(s)} \left[\sum_{j \in J^{(i)}} B^s_{iJ^{(i)}j} \right] \tag{10}$$

where $m(s)$ is the number of hyperarcs in schedule s, $B^s_{iJ^{(i)}j}$ is the benefit that node j gets from node i when i uses hyperarc $(i, J^{(i)})$.

In the case of no erasures,

$$B^s_{iJ^{(i)}j} = \begin{cases} 1 & \text{if } Z_{iJ^{(i)}j} \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where $Z_{iJ^{(i)}j} \equiv \dim(V_j) < \dim\left(\{V_j, v_{iJ^{(i)}}\} \right)$, $j \notin J^{(k)} \forall k \neq i$, $j \neq i_a, \forall a$

If we had perfect Channel State Information (CSI),

$$B^s_{iJ^{(i)}j} = \begin{cases} C_{iJ^{(i)}j} & \text{if } Z_{iJ^{(i)}j} \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

where $C_{iJ^{(i)}j}$ is the channel state of the channel from i to j when i transmits through hyperarc $iJ^{(i)}$. $C_{iJ^{(i)}j} = 1$ if the channel will cause no erasure, and $C_{iJ^{(i)}j} = 0$ otherwise.

If we have no CSI but we have knowledge (or estimates) of the packet erasure probability, the Progressive Base Station

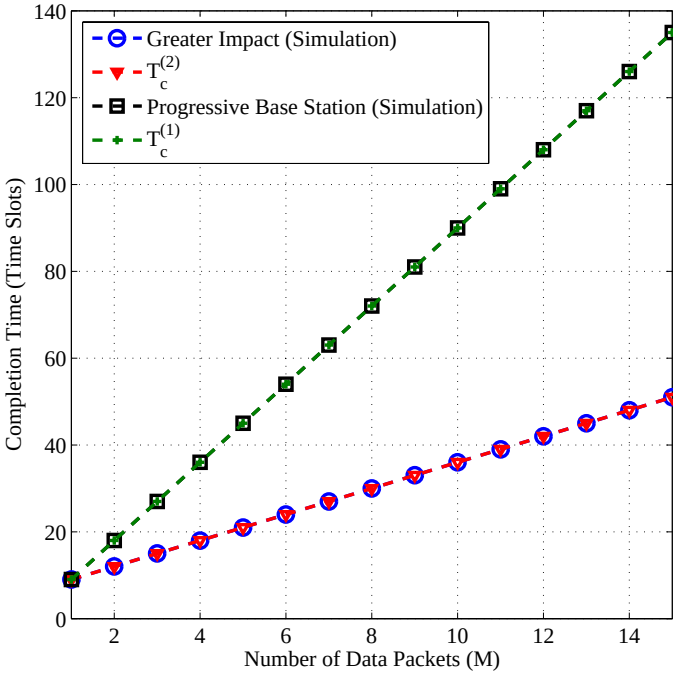

Fig. 9. Completion time of schemes on a linear meshed network of 10 nodes with different number of data packets to be disseminated. Each node can only contact one neighbor upstream and one downstream but with no packets being erased, i.e. $Pe_1 = 0, Pe_2 = 1$.

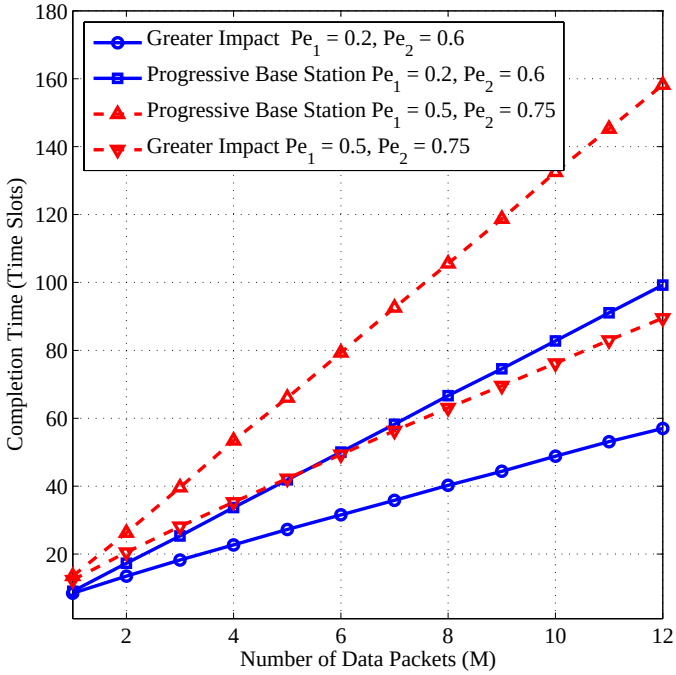

Fig. 10. Completion time of schemes on a linear meshed network of 10 nodes with different number of data packets to be disseminated. Each node can contact $N = 2$ neighbors upstream and 2 downstream.

scheme remains the same. For the Greater Impact scheme, we can define

$$B^s_{iJ^{(i)}j} = \begin{cases} 1 - P_{iJ^{(i)}j} & \text{if } Z_{iJ^{(i)}j} \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

where $P_{iJ^{(i)}j}$ is the packet erasure probability of the channel from i to j when i transmits through hyperarc $iJ^{(i)}$.

Note that this last approach is equivalent to choosing the schedule s^* that maximizes the average weight of the schedule, i.e. $s^* = \arg\max_{s \in S} E[W_s]$. Note that

$$E[W_s] = \sum_{i=1}^{m(s)} \left[\sum_{j \in J^{(i)}} E\left[B^s_{iJ^{(i)}j} \right] \right] \tag{14}$$

when

$$B^s_{iJ^{(i)}j} = \begin{cases} C_{iJ^{(i)}j} & \text{if } Z_{iJ^{(i)}j} \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

and $E\left[C_{iJ^{(i)}j} \right] = 1 - P_{iJ^{(i)}j}$.

IV. Numerical Results

This section provides numerical examples that compare the performance of two different network coding schemes we have discussed so far. Namely, our proposed scheme which chooses at each time slot the schedule that will provide the greatest impact on the network ('Greater Impact'), and the scheme that chooses a schedule based on the node that has the most knowledge in the network ('Progressive Base Station'). We assume that the available schedules are the same for both schemes. Also, we simplify the problem by only allowing schedules that do not generate interference in some nodes, which could be beneficial in some cases. Note that this is not a restriction of the analysis but a means to simplify the simulation. The comparison between the schemes is carried out in terms of average time to complete transmission of M data packets under different packet erasure probability scenarios.

Our results focus on linear meshed networks with one or two neighbors upstream and downstream. The packet erasure probability to the closest two neighbors is Pe_1, while Pe_2 corresponds to the packet erasure probability for the two neighbors farther away. Finally, we assume that one node at the edge of the network has all information at the beginning and that all other nodes have no information (See Figure 8).

Figure 9 shows that the results obtained in Section II for Motivating Example 2 match the simulation results. In particular, we observe that our 'Greater Impact' scheme has the same completion time to that in $T_c^{(2)}$ for $N = 1$ neighbor upstream and 1 neighbor downstream, $K = 10$ nodes in the network, and a range of data packets M. The 'Progressive Base Station' approach shows similar performance to $T_c^{(1)}$. Figure 9 shows that a considerable reduction in completion time can be found by choosing the schedule with the greatest impact to the network and by breaking ties in favor of schedules that benefit nodes with the least information. This is related to the opportunistic overhearing idea presented in [11] for routing, i.e. if we allow overhearing of the data packets the dissemination time can be considerably reduced, because

each transmission will be useful for more than one node. The figure shows the degradation in performance we would get if we performed routing of the coded packets from one node to its neighbor without exploiting the broadcast nature of wireless channels.

Figure 10 shows the performance of the two schemes when coded packets can suffer erasures. Each node has $N = 2$ neighbors upstream and downstream, except nodes at the edges of the linear meshed network. This figure shows that for different scenarios our 'Greater Impact' scheme shows much better performance. Note that both schemes are allowed the same schedules. The main difference is the way each scheme chooses amongst the different schedules.

Figure 11 illustrates the gain in completion time for different pairs (Pe_1, Pe_2) of packet erasure probabilities. This figure illustrates the gains for a network of $K = 10$ nodes. We expect larger gains if we increase M or the number of nodes K present in the network. However, it is clear that there is a considerable advantage of our scheme even for moderate network size and M.

Figure 11 shows that the gain from using the Greater Impact scheme instead of the Progressive Base Station scheme increases with packet erasures for our example with $N = 2$. We observe this by comparing the result for $(Pe_1 = 0, Pe_2 = 0)$ to those with non-zero packet erasures, except the case $(Pe_1 = 0, Pe_2 = 1)$ which corresponds to $N = 1$. The gain for the case of $(Pe_1 = 0, Pe_2 = 0)$ is a propagation gain, i.e. the Greater Impact scheme propagates coded packets faster to other nodes, allowing parallel transmissions. The same applies for $(Pe_1 = 0, Pe_2 = 1)$. For the cases of random erasures, the gain is in part due to the propagation gain and in part because the Greater Impact scheme chooses schedules based on their impact to the network without focusing the decision on a single node. Figure 11 shows that for $K = 10$ nodes and $M = 12$ packets with different packet erasure pairs (Pe_1, Pe_2) we observe a gain of about 1.8. This means that Progressive Base Station takes 80% more time to complete the data dissemination process than what it would take if we used the Greater Impact scheme.

Figure 11 shows that the case of $(Pe_1 = 0, Pe_2 = 1)$ (in essence $N = 1$) has a larger gain than $(Pe_1 = 0, Pe_2 = 0)$ $(N = 2)$, although the former requires more time complete the transmission for both schemes. This illustrates that allowing overhearing of the coded packets, which is represented by the ability of the nodes to transmit to more than one neighbor, reduces the gain of using Greater Impact with respect to Progressive Base Station, for the same number of nodes in the network K. However, this gain will increase as we increase the network size or as we increase the number of packets to transmit. Even for a small network of $K = 10$ nodes we get a gain of 1.6 when we transmit $M = 12$ packets with no erasures, which is to say that Progressive Base Station takes 60% more time to complete the data dissemination process than what it would take if we used the Greater Impact scheme.

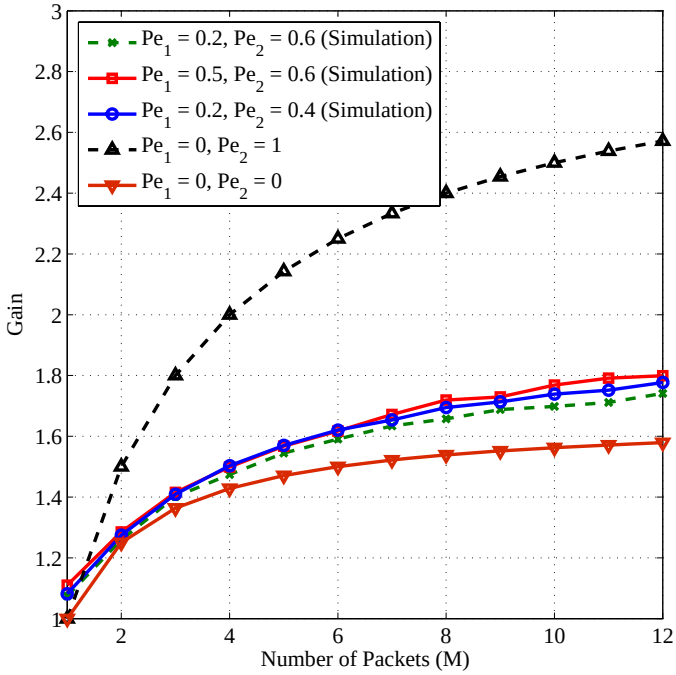

Fig. 11. Gain in completion time on a linear meshed network of 10 nodes with different number of data packets to be disseminated. Each node can contact $N = 2$ neighbors upstream and 2 downstream.

V. Conclusions

We present an analysis and numerical results that show that choosing a schedule based on its impact on the network a each time slot provides considerably better performance than scheme that choose schedules giving priority to nodes that know the most information. In fact, even for small networks and moderate number of packets to transmit we can expect large gains in terms of completion time.

Future research will consider an extension of the principles proposed in this work to the case of a random arrivals of new packets to the system (e.g. from a base station) that have to be disseminated to all nodes in the network. One metric of interest is the throughput performance of our system. More importantly, future work will focus on using the principles proposed in this paper to provide distributed MAC protocols that improve performance of practical systems. In particular, we will consider distributed protocols that provide a higher priority to a node based on the number of neighbors that require some of the information of that node. One of the main challenges will be to develop a protocol that can allow nodes to quickly and efficiently gather information of the degrees of freedom (number of linear combinations of packets) that their neighbors have or require, without incurring into much overhead.

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REFERENCES

[1] Ahlswede, R., Cai, N., Li, S. Y. R., Yeung, R. W., "Network Information Flow", IEEE Trans. Inf. Theory, vol. 46, no. 4, pp. 1204-1216, Jul. 2000

[2] Li, S.Y.R., Yeung, R. W., Cai, N., "Linear Network Coding", IEEE Trans. Inf. Theory, vol. 49, pp. 371, Feb. 2003

[3] Koetter, R., Médard, M., "An algebraic Approach to network coding", IEEE/ACM Trans. Netw., vol. 11, no. 5, pp. 782-795, Oct. 2003

[4] Ho, T., Médard, M., Koetter, R., Karger, D.R., Effros, M., Shi, J., Leong, B.,"A Random Linear Network Coding Approach to Multicast", Trans. Info. Theory, vol. 52, no. 10, pp.4413-4430, Oct. 2006

[5] Kulik, J., Heinzelman, W., Balakrishnan, H.,"Negotiation-Based Protocols for Disseminating Information in Wireless Sensor Networks," Wireless Networks, 8, pp. 169185, 2002

[6] Boyd, S., Ghosh, A., Prabhakar,B., Shah, D.," Gossip algorithms: Design, analysis, and applications," In Proc. IEEE Infocom'05, Miami, Mar. 2005.

[7] Deb, S., Médard, M., Choute, C.,"Algebraic Gossip: A Network Coding Approach to Optimal Multiple Rumor Mongering," Trans. Info. Theory, vol. 52, no. 6, pp.2486-2507, Jun. 2006

[8] Cheng, C. M., Kung, H. T., Lin, C.K., Su, C.Y., Vlah, D.,"Rainbow: A Wireless Medium Access Control Using Network Coding for Multi-hop Content Distribution," In Proc. MILCOM'08, San Diego, CA, Nov. 2008

[9] Stojanovic, M.,"Capacity of a Relay Acoustic Link," in Proc. IEEE Oceans'07 Conference, Vancouver, Canada, Oct. 2007

[10] Zhang, W., Stojanovic, M., Mitra, U., "Analysis of A Simple Multihop Underwater Acoustic Network," in Proc. Third ACM International Workshop on Underwater Networks (WUWNeT'08) / MobiCom 2006, San Francisco, CA, Sept. 2008

[11] Biswas, S., Morris, R.,"ExOR: Opportunistic MultiHop Routing for Wireless Networks", In the Proc. SIGCOMM 2005, Philadelphia, PA, Aug. 2005